Conventional and Unconventional Monetary Policy and FTPL

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The views expressed are mine and do not necessarily reflect official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or its Board of Governors.
Explore fiscal/monetary interaction in two simple monetary models:
- Simple cash-in-advance endowment economy with Fisherian properties.
- Model with production and secured credit.

Consider some unconventional monetary policies:
- Balance sheet expansion.
- Helicopter drops.

Just examples – no general results here.
Fisherian Cash-in-Advance Economy

- Deterministic world, continuum of identical households with unit mass:
  \[ \sum_{t=0}^{\infty} \beta^t u(x_t) \]
- Fixed endowment \( y \) forever. Cannot consume own output – purchase goods from other households. Consumption goods must be purchased with currency.
Minimal Government

- Government grants monopoly note issue powers to a central bank. Assume costless enforcement of the monopoly.
- Central bank constrained to issuing currency and reserves backed by loans to the private sector.
- Reserves are interest-bearing liabilities of the central bank, which are not acceptable in retail exchange, but are convertible one-for-one into currency and vice-versa.
- Government issues shares in the central bank to the private sector – share dividends are the central bank’s profits.
- reserves + currency = loans at the beginning of each period – no central bank capital.
- Looks like role envisioned for the Federal Reserve System in 1913 (or current ECB, roughly), absent commodity standard and collateral for central bank loans.
Central Bank’s Constraints

\[ m_0 = l_0 \]

\[ m_t - m_{t-1} \frac{R_t}{\pi_t} + c_{t-1} \left( \frac{R_t - 1}{\pi_t} \right) - l_t + \frac{R_t}{\pi_t} l_{t-1} = \tau_t^b, \ t = 1, 2, ... \]

\[ c_t \leq m_t \]

\[ m_t = l_t \]

- \( \pi_t = \frac{P_t}{P_{t-1}} \) is the gross inflation rate, and \( P_t \) is the price level.
- \( m_t, c_t, l_t \) are total central bank liabilities, currency, and loans outstanding, in real terms.
- \( R_t = \) gross nominal interest rate.
- \( \tau_t^b = \) central bank dividend.
Optimization by households:

\[ u'(x_t) = \beta \frac{R_{t+1}}{\pi_{t+1}} u'(x_{t+1}) \]

Markets in currency and reserves clear.
Cash-in-advance constraint binds.
Stationary Equilibrium

- Central bank sets $R$, $M_t = M_0 \mu^t$, and nominal dividend payments $T^b_t = T_0 \mu^t$.
- Implies equilibrium inflation rate is constant
  \[ \pi = \beta R \]
- And central bank must choose $\mu$ consistent with its interest rate policy
  \[ \mu = \beta R \]
- So, the central bank’s dividend each period is
  \[ \tau^b = \frac{y}{\beta R} (R - 1), \]
- Then, $T_0$ determines the initial price level
  \[ P_0 = \frac{T_0 \beta R}{y (R - 1)}. \]
The size of the central bank’s balance sheet in real terms is \( \frac{M_0}{P_0} \geq y \), and the quantity of reserves is then

\[
z = y \left[ \frac{M_0(R - 1) - T_0\beta R}{T_0\beta R} \right]
\]

Finally, the price of shares in the central bank is a constant

\[
q = \frac{y (R - 1)}{R (1 - \beta)}
\]
Economy is purely Fisherian – nominal interest rate set by the central bank determines the inflation rate.

With reserves outstanding, an expansion in the central bank’s balance sheet, holding constant $R$ and nominal dividend payments by the central bank, is irrelevant – $M_0$ does not matter for quantities or prices.

Central bank determines price level and inflation without any fiscal support, other than that it was granted a monopoly, and must distribute its profits to the public.
Consolidated government budget constraints

\[ m_0 + b_0 = \tau_0^f \]

\[ m_t - m_{t-1} \frac{R_t}{\pi_t} + c_{t-1} \left( \frac{R_t - 1}{\pi_t} \right) + b_t - \frac{R_t}{\pi_t} b_{t-1} = \tau_t^f + \tau_t^b, \ t = 1, 2, \ldots \]

\[ \tau_t^b = c_{t-1} \left( \frac{R_t - 1}{\pi_t} \right) \]

\[ \tau_t^f = m_t + b_t - \frac{R_t}{\pi_t} (m_{t-1} + b_{t-1}) \]

Here, central bank holds government debt as assets, and remits profits period-by-period to the fiscal authority.

Assume the fiscal authority never issues so little government debt that it constrains the central bank – though if it did, central bank could revert to making loans to support its currency issues.
Equilibrium

- Looks the same as in the world with minimal government, except we have to say what the fiscal authority does.
- Suppose fiscal authority chooses $\tau_0^f$ and $\tau_t^f = \tau^f$, for $t = 1, 2, \ldots$.
  Then,
  \[ \tau_0^f = -\frac{\beta \tau^f}{1 - \beta} \]

- In a stationary equilibrium, $\tau_0^f$ determines the real stock of consolidated government liabilities outstanding forever.
- As in other case, increasing $M_0$ is irrelevant – no effect on prices or quantities, only expands the central bank’s balance sheet, and reduces government debt outstanding.
- Suppose a helicopter drop:
  - constant $R$ and nominal remittances by the central bank to fiscal authority held constant.
  - increase $\tau_0^f$, and “monetize” it – equal increase in $\frac{M_0}{P_0}$.
  - has no effect on prices or quantities, just like the increase in reserve balances.
• Household now has preferences

\[
\sum_{t=0}^{\infty} \beta^t \left[ \theta u(x_t^m) + (1 - \theta) u(x_t^b) - n_t \right],
\]

• \(x_t^m\) denotes goods purchased with currency, \(x_t^b\) goods purchased with secured credit, \(n_t\) is labor input.

• Household’s constraints

\[
\begin{align*}
b_{t+1} + z_{t+1} + \theta x_t^m & \leq \frac{(b_t + z_t) R_t + c_t}{\pi_t} + \tau^f_t + \tau^b_t \\
(1 - \theta)x_t^b & \leq R_{t+1} (b_{t+1} + z_{t+1}) \\
b_{t+1} + z_{t+1} + c_{t+1} + \theta x_t^m + (1 - \theta)x_t^b & = n_t + \frac{(b_t + z_t) R_t + c_t}{\pi_t} + \tau^f_t + \tau^b_t
\end{align*}
\]
Marginal intratemporal rate of substitution = gross nominal interest rate

\[ \frac{u'(x^m_t)}{u'(x^b_t)} = R_{t+1} \]

Asset pricing: currency

\[ 1 = \frac{u'(x^m_t) - 1}{u'(x^m_t)} + \beta \frac{u'(x^m_{t+1})}{\pi_{t+1} u'(x^m_t)} \]

助攻 liquidity premium fundamental

Asset pricing: government debt and reserves

\[ \frac{1}{R_{t+1}} = \frac{u'(x^b_t) - 1}{u'(x^m_t)} + \beta \frac{u'(x^m_{t+1})}{\pi_{t+1} u'(x^m_t)} \]

助攻 liquidity premium fundamental
\[ \nu = m + b \text{ [value of consolidated government debt constant]} \]

- Solve for consumption \((x^m, x^b)\):

\[ \theta x^m u'(x^m) + (1 - \theta)x^b u'(x^b) = \nu u'(x^m) \]

\[ u'(x^m) = Ru'(x^b) \]

- Results:
  - Higher \(\nu\): \(x^m, x^b\) increase, welfare goes up, real interest rate rises, liquidity premia fall, inflation falls.
  - Higher \(R\): \(x^m\) falls, \(x^b\) rises, real interest rate rises, liquidity premium falls, inflation rises.
  - \(R = 1\) is suboptimal, given \(\nu\).
More on Stationary Equilibrium with Binding Collateral Constraint

- Transfers in each period $t = 1, 2, \ldots$,

$$\tau^b = \frac{\theta x^m}{\beta} \left( \frac{1}{u'(x^b)} - \frac{1}{u'(x^m)} \right)$$

$$\tau^f = \nu \left(1 - \frac{1}{\beta u'(x^b)} \right)$$

- Real interest rate:

$$\frac{R}{\pi} = \frac{1}{\beta u'(x^b)}$$

present value of fiscal transfers = $\frac{\nu}{(1 - \beta)} \left[1 - \frac{1}{u'(x^b)} \right]$ 

present value of central bank’s remittances = $\frac{\theta x^m}{(1 - \beta)} \left( \frac{1}{u'(x^b)} - \frac{1}{u'(x^m)} \right)$
Increase in the balance sheet, everything else held constant, is still neutral – no effects on quantities or prices.

- Helicopter drop – increase in $v$ – not neutral. Has the effects above.
  - monetization of the debt increase irrelevant – will just be held as reserves.
  - welfare goes up, real interest rate rises, inflation rate falls.
Suppose Reserves Inferior Collateral

- Rewrite collateral constraint:

  \[(1 - \theta)x_t^b \leq R_{t+1} (b_{t+1} + \gamma z_{t+1}), \quad 0 < \gamma < 1\]

- Why? For example, include banks and “balance sheet costs,” i.e. what causes the fed funds rate to be lower than the interest rate on reserves.

- Implies that an increase in central bank balance sheet size is not neutral – reduces real interest rate and increases inflation.

- Why? Tightens collateral constraint and reduces welfare. QE is bad.
Key Conclusions

- In a world with no inefficiencies, can conveniently separate central bank determination of inflation and the price level from fiscal policy.
- With all assets playing some role as liquid assets, things are not so simple.
- In a model with binding collateral constraints, where government debt serves as collateral:
  - Central bank balance sheet expansion is at best neutral, at worst welfare-reducing.
  - Increasing government debt increases welfare, but irrelevant (or reduces welfare) to monetize the debt increase (helicopter drop).