Robust Self-Insurance

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Abstract

A robust agent facing an income fluctuation problem is informed by a worst-case distribution matrix for his income process that shifts probability towards adverse income realizations. When his income features two risk components, the worst case exhibits positive correlations between them - even though no such correlation necessarily exists. When the agent faces a time varying interest rate, the worst case features conditional correlation between the interest rate, the income shock and the agent's asset position. The agent's pessimism is more pronounced when his wealth is low. This hints at a promising approach to endogenizing a model of asset holding under the worst case are related by a state dependent likelihood ratio, the conditional distribution of the state next period under the benchmark and worst case are related by a state dependent likelihood ratio, the conditional distribution of the state next period under the worst case are comparable to EU, and a borrowing constraint. Intuitively, the opposite is true when \( c < 0 \). Comparing the two columns, we see that if the agent is in a high income state, then the distortions are very mild. The magnitudes of the correlation is small (especially at higher wealth levels) but qualitatively it hints at a cautionary tale for those who use the methods of [4] to extract latent income factors. If agents behave as if income components are correlated (interpret them as, say, idiosyncratic and systemic factors) then the magnitudes of the correlation is small (especially at higher wealth levels) but qualitatively it hints at a cautionary tale for those who use the methods of [4] to extract latent income factors. If agents behave as if income components are correlated (interpret them as, say, idiosyncratic and systemic factors) then identifying factors that are assumed orthogonal may be problematic if the econometrician ignores robustness.

1. Motivation

- How does pessimism lead back into decisions that affect these idiosyncratic states?
- Why might we suddenly see increases in disagreement in times of stress?
- What happens if an econometrician ignores a desire for robustness in Bewley models?

2. Robustness

An agent possesses a 'benchmark' model of the economy but fears it is misspecified in some unknown way. The agent expresses doubts about his model by considering alternative distributions that are distorted versions of the distribution implied by the benchmark. In order to construct a robust policy, the agent considers adverse distributions and balances the damage that an implicit misspecification would cause, against the plausibility of the misspecification. The distortion is more pronounced when his wealth is low. This hints at a promising approach to endogenizing a model of asset holding under the worst case are related by a state dependent likelihood ratio, the conditional distribution of the state next period under the benchmark and worst case are related by a state dependent likelihood ratio, the conditional distribution of the state next period under the benchmark and worst case are related by a state dependent likelihood ratio, the conditional distribution of the state next period under the worst case are comparable to EU, and a borrowing constraint. Intuitively, the opposite is true when \( c < 0 \). Comparing the two columns, we see that if the agent is in a high income state, then the distortions are very mild. The magnitudes of the correlation is small (especially at higher wealth levels) but qualitatively it hints at a cautionary tale for those who use the methods of [4] to extract latent income factors. If agents behave as if income components are correlated (interpret them as, say, idiosyncratic and systemic factors) then identifying factors that are assumed orthogonal may be problematic if the econometrician ignores robustness.

3. Model(s)

We assert a CRRA felicity function

\[ w(c) = \frac{1}{1 - \varphi} c^{1 - \varphi} \]

and a borrowing constraint,

\[ a \leq 0 \]

We consider three cases: i) a single factor in income, \( \eta \) is a single factor in income but a time varying bond rate. In the latter case, negative asset holdings are allowed, whereas an ad hoc limit of \( - \infty \) is imposed in the first two. The budget constraint in the single risk factor case is

\[ m(x) - \frac{\alpha}{1 + \varphi} x^{1 - \varphi} + \alpha + z \]

We model - as a first order Markov chain with transition matrix \( \Pi \) \( \{ \Pi_{ij} \} \). The two shock case is similar. The rate shock takes only two values and follows a Markov chain characterized by \( \Pi_{00} \) and \( \Pi_{11} \).

We use the minimizing likelihood ratio to twist each element of \( \Gamma_i \). This yields a state dependent worst case transition matrix \( \Gamma_{ij} (x, z, a) = \{ \Gamma_{ij} (x, z, a) \} \). Where

\[ \Gamma_{ij} (a, z) \rightarrow \gamma_{ij} = \{ \Gamma_{ij} (a, z) \} \]

We choose to restrict the optimal asset choice to be on the asset grid so evaluating the likelihood under the worst case is simply a question of working with a first order Markov chain in \( z, a \) pairs, for which the likelihood is very tractable. The detection error probability in the one shock case below is \( \approx 0.20 \) using 30 periods (years) of data for each likelihood evaluation.

4. Results: One shock case

Table 1: Transition matrices for the labor shock. a) Benchmark, b) W.C. with \( \alpha = 0.01 \) and c) W.C. with \( \alpha = 0.05 \). Equity shocks are ordered from lowest to highest. Some illustrative elements are in red.

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5. Results: Two shock case

In the two shock case many of the insights from the one shock case above still apply. In addition, as shown in table 2 we observe evidence of the worst case featuring correlation between the two income components, \( \eta_1 \) and \( \eta_2 \), even though under the benchmark they are uncorrelated. Intuitively, the agent will use these new correlations that can be represented by a loss of diversification.

6. Results: Rate shock case

The results are shown in Figure 3. The distortion transition matrices of \( \alpha \) has greater probability of \( \alpha = 0.50 \) than in the benchmark (see figure 3). This is because the agent is typically in debt under the worst case. If, in contrast, the agent is actually generating the data, the agent typically will have positive savings so the distorted transition matrices would generally feature the opposite distortion.

7. Summary

- Preliminary exploration of self-insurance problems with robustness
- Construction of worst case transition matrices
- Fear misspecifications represented by lower income and greater correlations of shocks (economic problems?)
- Pessimism more pronounced at lower wealth (route to endogenous disagreement - especially in times of stress?)
- Induces a precautionary saving motive

References


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