The Bidder Exclusion Effect

Dominic Coey   Brad Larsen   Kane Sweeney

eBay Research Labs   Stanford and NBER   eBay Research Labs

December 2015

Motivation

Empirical work in auctions often requires complex estimation steps and strong assumptions on data generating process

Example question: How would revenue improve if seller were to use optimal reserve price?

Typical assumptions:

- Bidders are symmetric
- Bidders have private values
- Bidders' values are independent
- Number of participants independent of bidders' values
- Number of participants correctly observed
- Data comes from auctions with non-binding reserve prices

An alternative: The bidder exclusion effect

Bidder exclusion effect: The fall in expected revenue when one bidder is randomly removed from an auction

Easy to calculate (or find bounds on)

- *without* complex estimation or requiring instruments
- with correlated private or common values, asymmetries, unobserved heterogeneity, unknown number of bidders

The bidder exclusion effect gives insight into some key issues

- How should I model entry? (i.e. are valuations independent of the number of bidders who enter)
- Do reserve prices matter? (Bulow and Klemperer 1996)
- What are seller's losses from mergers/collusion of bidders?

Data requirements: highest and second-highest bids

Related literature

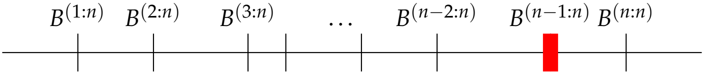
- Robust, sufficient statistics for welfare analysis
  Feldstein (1999); Chetty (2009); Einav, Finkelstein, and Cullen (2010); Jaffe and Weyl (2013)
- Auction models and tests of entry/independence of valuations and N
  Li and Zheng (2009); Aradillas-Lopez, Gandhi, and Quint (2013b); Marmer, Shneyerov, and Xu (2013); Roberts and Sweeting (2013); Gentry and Li (2013)
- How much do reserve prices matter?
  Bulow and Klemperer (1996) (theoretical)
  Li, Perrigne, and Vuong (2003); Paarsch (1997); Krasnokutskaya (2011)
- Effects of collusion/mergers
  Baldwin, Marshall, and Richard (1997); Li and Zhang (2015)
- Obtaining auction results with bounds
  Haile and Tamer (2003); Aradillas-Lopez, Gandhi, and Quint (2013a)

The bidder exclusion effect is easy to estimate

Consider a private values, sealed second-price auction with n bidders

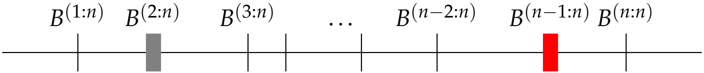
Let $B^{(1:n)}, ..., B^{(n-1:n)}, B^{(n:n)}$ represent the bidders' bids (and valuations) ordered from smallest to largest

Revenue is $B^{(n-1:n)}$

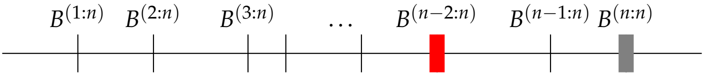


The bidder exclusion effect is easy to estimate

When we drop a bid at random, if any of lowest $n-2$ bids are dropped, revenue remains $B^{(n-1:n)}$



If either of top two bids are dropped, revenue falls to $B^{(n-2:n)}$



The bidder exclusion effect is easy to estimate

If a bidder is randomly excluded,

- with probability $\frac{n-2}{n}$ revenue is unchanged
- with probability $\frac{2}{n}$ revenue falls from $B^{(n-1:n)}$ to $B^{(n-2:n)}$

The bidder exclusion effect here is $\frac{2}{n}E\left[B^{(n-1:n)} - B^{(n-2:n)}\right]$

This holds even if values are correlated and asymmetric

Bounds on bidder exclusion available with symmetric common values, "low" bidding (Haile and Tamer 2003), unknown n, first-price auctions, or binding reserve prices

Naïve approach to estimation

Want to estimate causal effect on revenue from excluding a random bidder

Naïve approach: regress auction price on number of bidders. Only correct if

- Number of bidders correctly observed
- Number of bidders varies exogenously

IV approaches also limited; only give LATE for “complier” auctions

Our approach:

- no instruments
- no exogenous variation in n
- yields bounds when n unobserved, but bounded

Model overview and strategy

- Let N be a random variable representing the number of bidders, with realizations n
- For $k \leq m \leq n$, let $B^{k:m,n}$ be the k^{th} smallest bid in m bidder auctions, where the m bidders are selected uniformly at random from the n bidders in auctions which had exactly n bidders enter
- Define bidder exclusion effect as

$$\Delta(n) \equiv E(B^{n-1:n}) - E(B^{n-2:n-1,n}) \tag{1}$$

- $\Delta(n)$ is a *counterfactual* object: in the $n-1$-bidder auction, all bidders know 1 bidder has been dropped prior to the bidding, and can adjust their bids accordingly
- Distributions of $B^{k:m,m}$ and $B^{k:m,n}$ for $m < n$ may be different

Model overview and strategy

- The following two objects are *not* counterfactual; can always be estimated with data on top two bids and number of entrants
- Effect of removing a *bid* at random (rather than a *bidder*), with all other bids remaining unchanged:

$$\Delta^{bid}(n) \equiv \frac{2}{n} E(B^{n-1:n} - B^{n-2:n}) \tag{2}$$

- Difference in *observed* revenue between auctions in which n bidders entered vs. $n-1$-bidders entered:

$$\Delta^{obs}(n) \equiv E(B^{n-1:n}) - E(B^{n-2:n-1}) \tag{3}$$

Model overview and strategy

- These three objects are key to paper:

$$\Delta(n) \equiv E(B^{n-1:n}) - E(B^{n-2:n-1,n})$$
$$\Delta^{bid}(n) \equiv \frac{2}{n} E(B^{n-1:n} - B^{n-2:n})$$
$$\Delta^{obs}(n) \equiv E(B^{n-1:n}) - E(B^{n-2:n-1})$$

- If valuations are not independent of N (entry is selective), $\Delta(n) \neq \Delta^{obs}(n)$

  $\Rightarrow$ test for selective entry
- We demonstrate conditions where $\Delta(n) \leq \Delta^{bid}(n)$

  $\Rightarrow$ bound revenue benefit of reserve prices using Bulow and Klemperer (1996)

  and bound revenue losses from merger/collusion

Identifying and estimating bidder exclusion effect

Point-identified case:

Proposition

In ascending auctions with private values and no reserve price where bidders bid their values, for all $n > 2$ the bidder exclusion effect $\Delta(n) = \Delta^{bid}(n) \equiv \frac{2}{n}E(B^{n-1:n} - B^{n-2:n})$.

Estimation: Estimate sample analog of $\Delta^{bid}(n)$

With covariate vector X, estimate sample analog of

$$\Delta^{bid}(n|X) \equiv \frac{2}{n}E(B^{n-1:n} - B^{n-2:n}|X),$$

Does not rely on symmetry

Low bidding

Low bidding (similar to Haile and Tamer 2003 setting): Second-highest bidder bids his value, and lower bidders' bids are less than or equal to their values ($B^{n-2:n} \leq V^{n-2:n}$)

Proposition

In ascending auctions with private values and no reserve price, then in the low bidding case, for all $n > 2$ the bidder exclusion effect $\Delta(n) \leq \Delta^{bid}(n) \equiv \frac{2}{n} E(B^{n-1:n} - B^{n-2:n})$.

Above proposition can also be extended to precisely the Haile and Tamer (2003) setting. Requires additional notation (Appendix)

Common values

Proposition

In ascending button auctions with symmetric common values and no reserve price, for all $n > 2$ *the bidder exclusion effect* $\Delta(n) < \Delta^{bid}(n) \equiv \frac{2}{n} E(B^{n-1:n} - B^{n-2:n})$.

Intuition: Removing a random bid and assuming all other bids remain unchanged *overstates* decline in revenue due to reduced winner's curse

Results in Bikhchandani, Haile, and Riley (2002) can be used to show above will hold for *any* symmetric, separating equilibrium

Above proposition is the first positive identification result for common values ascending auctions

Unobserved number of bidders

If number of bidders in each auction (N) unknown, but $\underline{n} \leq N$ with $\underline{n}$ known:

Corollary

In ascending auctions with no reserve price, if $2 < \underline{n} \leq N$ *then* $E(\Delta(N)) \leq \frac{2}{\underline{n}} E(B^{N-1:N} - B^{N-2:N})$.

Also holds under low bidding or common values

Other approaches for unknown N require IPV assumption (Song 2004). First positive identification result for non-IPV ascending auctions with unknown N

Putting the bidder exclusion effect to work

Using bidder exclusion to choose between entry models

A common assumption in auctions lit, with different names:

- "Valuations independent of N" (Aradillas-López, Gandhi, Quint 2013)
- "Exogenous participation" (Athey and Haile 2002, 2007)
- "Non-selective entry" (Roberts and Sweeting 2013)

Consider the following entry framework

- There is a set of potential entrants for each auction
- If they pay an entry cost, they can bid

If bidders observe a signal about values before entering, valuations may not be independent of N

With an estimate of the bidder exclusion effect, this is testable!

Using bidder exclusion to choose between entry models

Consider private values auctions, where bids=values

Compare the bidder exclusion effect, $\Delta(n)$ ($= \Delta^{bid}(n)$ with PV), to the average revenue difference between n and $n-1$ bidder auctions, $\Delta^{obs}(n)$

Simple t-test of $\Delta^{obs}(n) = \Delta^{bid}(n)$

$$E(B^{n-1:n}) - E(B^{n-2:n-1}) = \frac{2}{n} E(B^{n-1:n} - B^{n-2:n})$$

- $\Delta^{obs}(n) = \Delta^{bid}(n)$, if $n-1$ bidder auctions are just like n bidder auctions with one bidder randomly removed
- $\Delta^{obs}(n) > \Delta^{bid}(n)$, if bidders' values lower in $n-1$ than n bidder auctions

Using bidder exclusion to choose between entry models

Can also incorporate covariates, i.e. test whether $\Delta^{obs}(n) = \Delta^{bid}(n)$ holds, conditional on some variables X

Define

$$T(n \mid X) \equiv E(B^{n-1:n} \mid X) - E(B^{n-2:n-1} \mid X) - \frac{2}{n}E(B^{n-1:n} - B^{n-2:n} \mid X)$$

Test hypothesis that $T(n \mid X) = 0$ for all realizations of X

Test can be performed parametrically or nonparametrically (on nonparametrics see Chetverikov 2011; Andrews and Shi (2013); Chernozhukov, Lee, and Rosen 2013)

Using bidder exclusion to choose between entry models

What about the cases (low bidding, common values) where $\Delta^{bid}(n) \geq \Delta(n)$?

As before, if valuations and signals independent of N, then $\Delta(n) = \Delta^{obs}(n)$. Testable implication: $\Delta^{bid}(n) \geq \Delta^{obs}(n)$

- If $\Delta^{bid}(n) < \Delta^{obs}(n)$, evidence of valuations *increasing* with N

Unlike private values case, can't detect presence of selective entry when $\Delta^{obs}(n) \leq \Delta^{bid}(n)$

Appendix extends test to allow for bidder asymmetries, first price auctions, and binding reserve prices

How powerful is bidder exclusion test? Evidence from Monte Carlo simulations

Consider alternative test: Compare mean values of *all* bids in n and $n+1$-bidder auctions (infeasible in ascending auctions)

Compare this alternative "mean comparison test" to bidder exclusion test

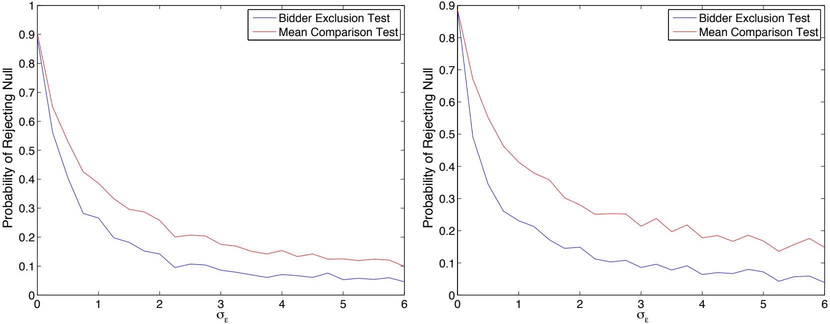
Monte Carlo setup

- 10 potential bidders, with iid private values $\sim \log N(\theta, 1)$
- θ is random variable, unknown to bidders
- Bidders get common signal $\delta = \theta + \varepsilon$

$$\begin{pmatrix} \delta \\ \theta \\ \epsilon \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 + \sigma_\epsilon^2, & 1, & \sigma_\epsilon^2 \\ 1, & 1, & 0 \\ \sigma_\epsilon^2, & 0, & \sigma_\epsilon^2 \end{pmatrix} \right)$$

- As σ_ϵ increases, ratio of noise to signal increases, δ becomes less informative
- Potential bidders pay entry cost 0.5 to learn values
- Play mixed entry strategies, entering with a probability p which depends on δ
- As $\sigma_\epsilon \to \infty$, signal δ is uninformative about θ and p no longer varies with δ (corresponds to Levin and Smith 1994)

How powerful is bidder exclusion test? Evidence from Monte Carlo simulations



(a) 3 and 4 Bidder Auctions

(b) 4 and 5 Bidder Auctions

Null hypothesis: Valuations are independent of N (i.e. no selective entry)

Bidder exclusion test less powerful, still performs well

Using bidder exclusion to bound effects of reserve prices

Large portion of theoretical and empirical auctions literature focuses on optimal reserve prices

How important are they in practice?

Bulow and Klemperer (1996): the seller gains more from one extra bidder and no reserve than from setting an optimal reserve price:

$$\begin{aligned}\Delta(n) \equiv & Rev(n, r = 0) - Rev(n-1, r = 0) \\ & > Rev(n-1, r = r^{OPT}) - Rev(n-1, r = 0)\end{aligned}$$

Bidder exclusion effect bounds the impact of optimal reserve price, without complex estimation approach

Note: Bulow and Klemperer (1996) results apply to *common values* as well

Using bidder exclusion to bound effects of reserve prices

Proposition

In ascending auctions with no reserve price, if

i) *The setting is one of private values with low bidding or a symmetric common values button auction*
ii) *Change in revenue from adding a random bidder is smaller than from removing a random bidder*
iii) *Bidders are symmetric and have increasing marginal revenue*

Then for all $n > 2$ the increase in expected revenue from using the optimal reserve price is less than $\Delta^{bid}(n) \equiv \frac{2}{n} E(B^{n-1:n} - B^{n-2:n})$

"Marginal revenue" defined as in Bulow and Klemperer (1996), Bulow and Roberts (1989). We prove (i) + (iii) $\Rightarrow$ (ii) in certain settings

Using Bidder Exclusion to Bound Impact of Mergers/Collusion

Assume asymmetric bidders with correlated private values

If i and j merge or collude, let M_k be willingness-to-pay of bidder $k \neq i, j$, and $M_{i,j}$ denote the willingness-to-pay of the joint entity

Assumption

Willingness-to-pay does not decrease under mergers/collusion: When any two bidders i and j merge or collude, $M_k \geq V_k$ for all $k \neq i, j$, and $M_{i,j} \geq \max\{V_i, V_j\}$.

In mergers, assumption allows for production efficiencies

In collusion, assumption is satisfied by models of collusive bidding (Graham, Marshall, and Richard 1990; Asker 2009)

Using Bidder Exclusion to Bound Impact of Mergers/Collusion

Proposition

Under above assumptions, the decrease in expected revenue from two bidders merging or colluding is bounded above

(i) *by* $\frac{1}{n-1}\Delta(n)$ *when the two bidders are randomly selected*

(ii) *by* $E(B^{n-1:n} - B^{n-2:n})$ *when the two bidders are not randomly selected*

If N unobserved, can obtain average loss

Easily extends to more than two bidders colluding/merging

Random matching may be good upper bound; using Asker (2009) data, we show cartel wins less often than it would under random matching (incentives for *low-value* bidders to join ring due to side payments)

Testing for selective entry at timber auctions

Setting: US Timber Auctions. Major source of auction studies (of entry, reserve prices, collusion/mergers)

Many studies assume valuations independent of N (i.e. no selective entry)

Data from ascending auctions in California between 1982 and 1989 with ≥ 3 entrants

1,086 auctions

Contains all bids, and other auction-level information: appraisal variables, measures of local industry activity, and other sales characteristics

See Athey, Coey, and Levin (2013)

Estimating the bidder exclusion effect at timber auctions

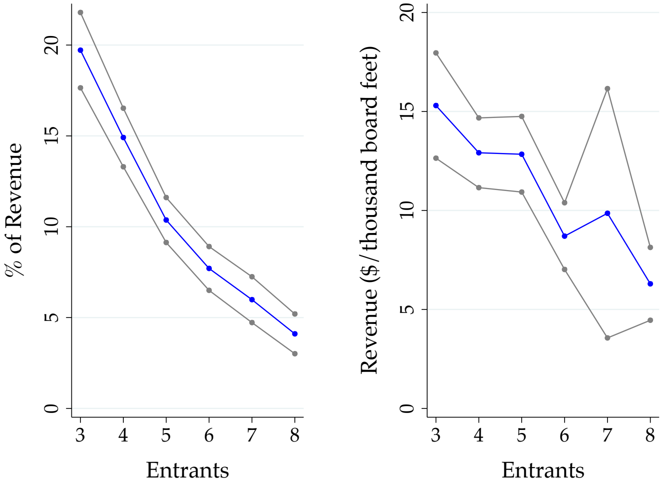


Figure: Bidder exclusion effect conditional on # bidders

Testing for selective entry at timber auctions

Without conditioning on auction-level heterogeneity

Entrants (n)	3	4	5	6	7	8
$T(n) = \Delta^{obs}(n) - \Delta^{bid}(n)$	28.55***	13.97	30.66***	5.98	-1.86	16.78
	(6.87)	(7.92)	(5.95)	(12.15)	(13.00)	(15.43)
Sample Size	497	496	456	350	243	164

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Actual difference in revenue between $n-1$ and n bidder auctions, $\widehat{\Delta^{obs}}(n)$, is higher than bidder exclusion effect (or upper bound on it), $\widehat{\Delta^{bid}}(n)$

$\Rightarrow$ Suggests valuations may be increasing with N (i.e. positive selection in entry)

Testing for selective entry at timber auctions

Conditional on auction-level covariates

Full sample, conditional on covariates:

Entrants (n)	3	4	5	6	7	8
$T(n) = \Delta^{obs}(n) - \Delta^{bid}(n)$	20.32***	17.02***	13.81***	-5.91	12.66*	-5.42
	(5.33)	(4.21)	(3.63)	(4.75)	(5.78)	(17.94)
Sample Size	497	496	456	350	243	164

Loggers only, conditional on covariates:

Entrants	3	4	5	6
$T(n) = \Delta^{obs}(n) - \Delta^{bid}(n)$	14.00	39.54*	17.51	-19.04
	(16.65)	(19.29)	(9.92)	(25.66)
Sample Size	149	138	109	76

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Covariates: appraisal variables, timber species, intensity of local competition

Estimating an upper bound on the bidder exclusion effect at auto auctions

Upper bound on bidder exclusion effect can be estimated conditioning nonparametrically on observables X:

$$\Delta^{bid}(n|X) = \frac{2}{n} E\left[B^{n-1:n} - B^{n-2:n} | X\right]$$

At auto auctions, realizations of N not observed

Assuming lower bound on N known, $\underline{n} \leq N$, yields another upper bound on bidder exclusion effect:

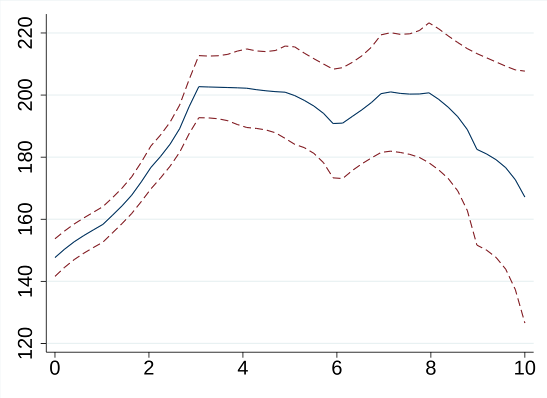
$$E\left[\Delta^{bid}(n|X) \middle| X\right] \leq \frac{2}{\underline{n}} E\left[B^{N-1:N} - B^{N-2:N} | X\right]$$

In auto auction data, we assume $\underline{n} = 3$.

Auto auction data

- 6,003 sales of used-cars
- 2nd and 3rd order statistics
- Car and auction-level characteristics:
  - make, model, age
  - age $\rightarrow$ 2 years old on average
  - mileage $\rightarrow$ 33,000 on average
  - # online bidders $\rightarrow$ 35 on average

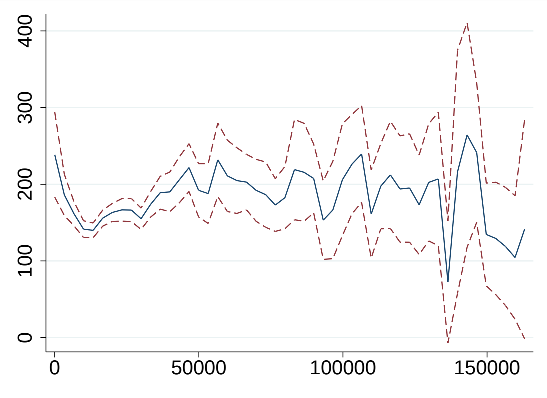
Bidder exclusion effect conditional on car age



(a) Age

Figure: Estimated upper bound on bidder exclusion effect, conditional on car age (in years)

Bidder exclusion effect conditional on car mileage

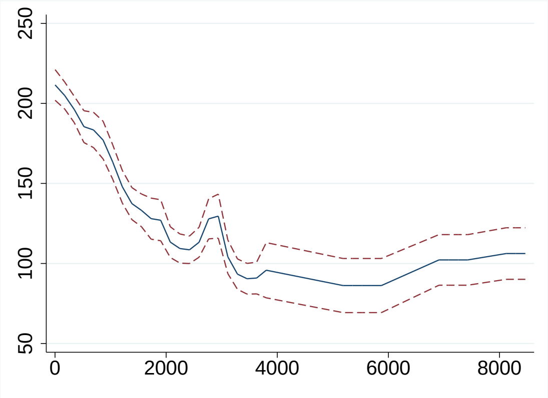


(a) Mileage

Figure: Estimated upper bound on bidder exclusion effect, conditional on car mileage

Bidder exclusion effect conditional on market thickness

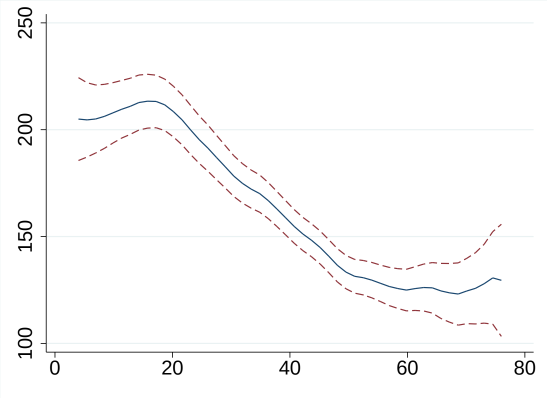
Market thickness = total # at make-model-age level



(a) Market thickness

Figure: Estimated upper bound on bidder exclusion effect, conditional on market thickness

Bidder exclusion effect conditional on # online bidders



(a) # Online bidders

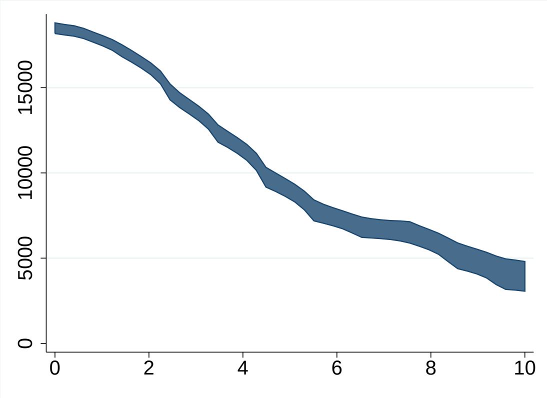
Figure: Estimated upper bound on bidder exclusion effect, conditional on # online bidders

How important are optimal reserve prices at auto auctions?

Bidder exclusion effect leads to bounds on revenue, ranging from no-reserve auction to optimal reserve price auction

$$\underline{\pi}_j(X_j) = b_j^{n_j-1:n_j}(X_j)$$
$$\overline{\pi}_j(X_j) = b_j^{n_j-1:n_j}(X_j) + \frac{2}{\underline{n}}\left(b_j^{n_j-1:n_j}(X_j) - b_j^{n_j-2:n_j}(X_j)\right)$$

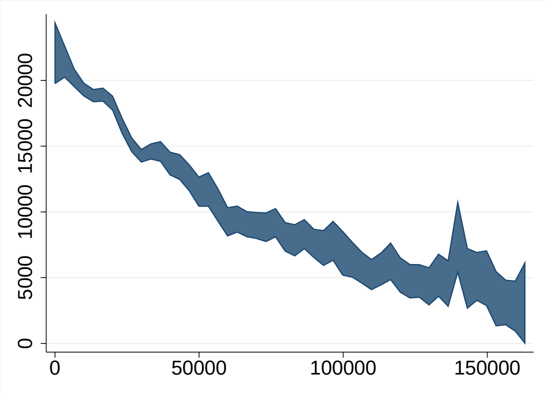
Revenue bounds conditional on car age (in years)



(a) Age

Figure: Revenue bounds (from no reserve price to adding an additional bidder), conditional on car age (in years)

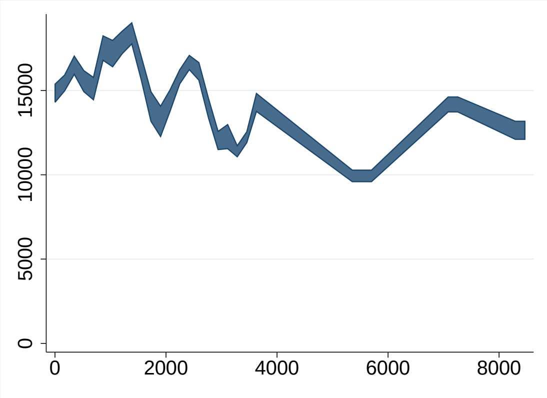
Revenue bounds conditional on car mileage



(a) Mileage

Figure: Revenue bounds (from no reserve price to adding an additional bidder), conditional on car mileage

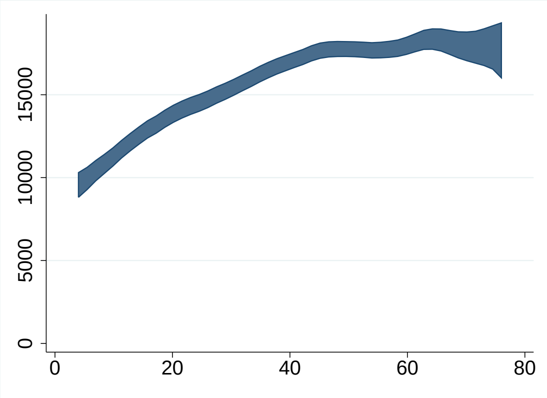
Revenue bounds conditional on market thickness



(a) Market thickness

Figure: Revenue bounds (from no reserve price to adding an additional bidder), conditional on market thickness

Revenue bounds conditional on # online bidders



(a) # Online bidders

Figure: Revenue bounds (from no reserve price to adding an additional bidder), conditional on # online bidders

Bidder exclusion effect as a benchmark for other studies

Four-year-old car $\Rightarrow$ bidder exclusion effect bounded above by \$333

Comparison to other studies:

- Tadelis and Zettelmeyer (2011): Effect of information disclosure through condition reports = + \$643
- Lacetera, Larsen, Pope, and Sydnor (2014): one-standard-deviation improvement in auctioneer performance = + \$348
- Hortaçsu, Matvos, Syverson, and Venkataraman (2013): 1,000 point increase in credit default swap spread of car manufacturer = - \$68

Extensions

Other auction settings:

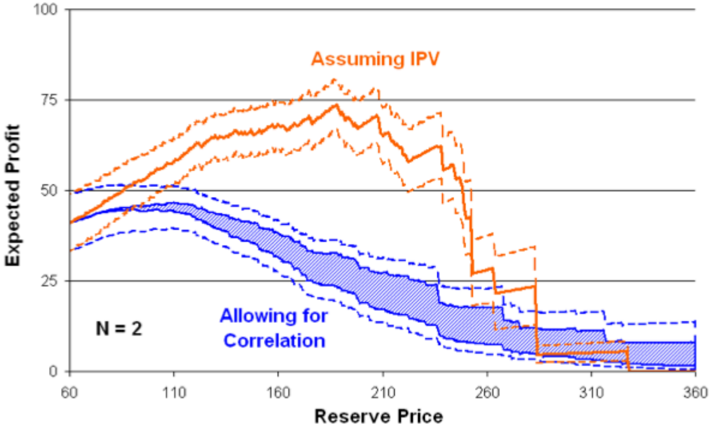
- Haile and Tamer (2003) setting
- First price auctions
- Binding reserve prices
- Asymmetric bidders

May also serve as a specification check. Example: assume IPV, simulate optimal reserve benefit, compare to bidder exclusion effect

Approach may yield other similar tools for analysis of auctions which, like the bidder exclusion effect, is more robust to a wider range of environments and requires less complex estimation than typical empirical auctions approaches

Specification test using bidder exclusion effect/Bulow-Klemperer insight

Example: Aradillas-Lopez, Gandhi, and Quint (2013)

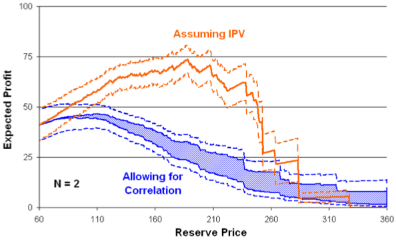


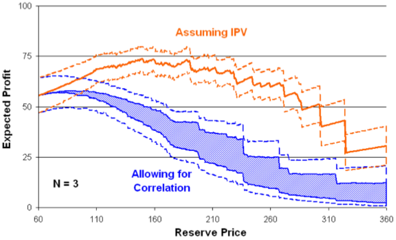
Specification test using bidder exclusion effect/Bulow-Klemperer insight

Example: Aradillas-Lopez, Gandhi, and Quint (2013)

Theorem

$$Rev(n = 2, r^{OPT}) \leq Rev(n = 3, r = 0)$$



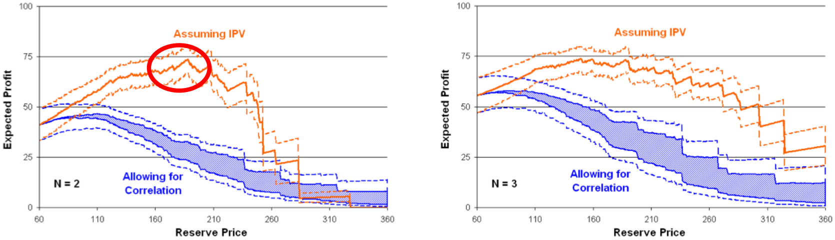


Specification test using bidder exclusion effect/Bulow-Klemperer insight

Example: Aradillas-Lopez, Gandhi, and Quint (2013)

Theorem

$$Rev(n=2, r^{OPT}) \leq Rev(n=3, r=0)$$

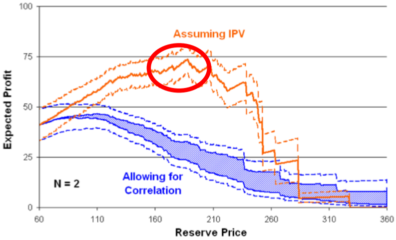


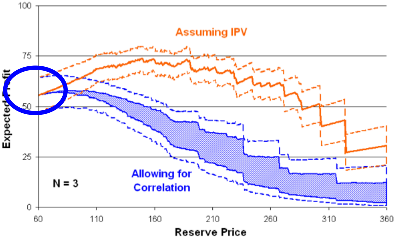
Specification test using bidder exclusion effect/Bulow-Klemperer insight

Example: Aradillas-Lopez, Gandhi, and Quint (2013)

Theorem

$$Rev(n = 2, r^{OPT}) \leq Rev(n = 3, r = 0)$$



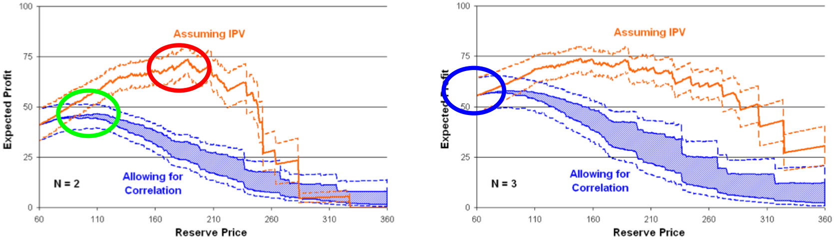


Specification test using bidder exclusion effect/Bulow-Klemperer insight

Example: Aradillas-Lopez, Gandhi, and Quint (2013)

Theorem

$$\color{red}{Rev(n = 2, r^{OPT})} \leq \color{blue}{Rev(n = 3, r = 0)}$$



Thank you