Systemic stress-testing: fire sales, endogenous risk and price-mediated contagion

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Channels of loss amplification in the financial system

1. Counterparty Risk: balance sheet contagion through asset devaluation

2. Funding channel: balance sheet contagion through withdrawal of funding (bank runs by depositors, institutional bank runs by lenders)

3. **Feedback effects from fire sales**: loss contagion through mark-to-market losses in common asset holdings

Research on financial networks and their use in macroprudential regulation has focused on direct contagion mechanisms (1+2). Regulatory measures have focused on 1 (large exposure limits, central clearing, CVA, ring-fencing) or 2 (LCR, NSFR).
Bank stress tests

- Stress tests assume ‘passive’ behavior by banks. 
  BCBS 2015: “Stress tests conducted by bank supervisors still lack a genuine macro-prudential component” .. “endogenous reactions to initial stress and feedback effects” are missing.

- How do financial institutions react when faced with stress?

- Market stress can lead financial institutions to unwind positions (constrained by capital, liquidity, leverage...): 
  - evidence from banks ‘living wills’: (Credit Suisse Report, 2015): “If we are unable to raise needed funds in the capital markets (...), we may need to liquidate unencumbered assets to meet our liabilities [...] at depressed prices, which in either case could adversely affect our results of operations and financial condition.”
A framework for systemic stress testing with endogenous effects

We build on previous theoretical work on fire sales (Shleifer 2010, Coval & Stafford 2007, Ellul et al 2011, Kyle & Xiong 2005, Cont & Wagalath 2013) and recent empirical studies (Greenwood et al 2013, Eisenbach-Duarte 2014) to construct an operational framework for quantifying fire sales spillovers and incorporating it in a system-wide stress test for financial institutions.

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Systemic stress testing with endogenous effects

Ingredients:

1. Portfolio holdings: holdings of financial institutions by asset class: \( N \) institutions, \( K \) illiquid asset classes, \( M \) marketable asset classes \( \rightarrow N \times (M + K) \) portfolio matrix (network)

2. Portfolio constraints: capital ratio, leverage ratio, liquidity ratio,... \( \rightarrow \) range of admissible portfolios ("safety zone").

3. Reaction function: reaction of a bank when its portfolio exits the admissible region (deleveraging/ rebalancing)

4. Market impact: market prices react to portfolio rebalancing

5. Mark-to-market accounting: transmits market impact to all institutions \( \rightarrow \) may lead to feedback if market losses large
Balance sheets: illiquid vs marketable assets

<table>
<thead>
<tr>
<th>Illiquid assets</th>
<th>Marketable assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential mortgage exposures</td>
<td>Corporate bonds</td>
</tr>
<tr>
<td>Commercial real estate exposure</td>
<td>Sovereign debt</td>
</tr>
<tr>
<td>Retail exposures: Revolving credits, SME, Other</td>
<td>Derivatives</td>
</tr>
<tr>
<td>Indirect sovereign exposures in the trading book</td>
<td></td>
</tr>
<tr>
<td>Defaulted exposures</td>
<td>Institutional client exposures: interbank, CCPs,...</td>
</tr>
<tr>
<td>Residual exposures</td>
<td></td>
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</tbody>
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Table: Stylized representation of asset classes in bank balance sheets. (Data: European Banking Authority 2011)
• Illiquid holdings of institution $i$: $\Theta^i := \sum_{\kappa = 1}^{K} \Theta^i_{\kappa}$.
• Marketable Securities held by $i$: $\Pi^i := \sum_{\mu = 1}^{M} \Pi^i_{\mu}$.
• Equity (Tier 1 capital): $C^i$
• Financial institutions are subject to various one-sided portfolio constraints: leverage ratio, capital ratio, liquidity ratio.
• Leverage ratio of $i$:
  $$\lambda^i = \frac{\text{Assets}(i)}{C^i} = \frac{\Theta^i + \Pi^i}{C^i} \leq \lambda_{\text{max}}$$
• Capital ratio of $i$:
  $$\lambda^i = \frac{RWA(i)}{C^i} = \frac{\sum w_\kappa \Theta^i_{\kappa} + \sum \mu \Pi^i_{\mu} w_\mu}{C^i} \leq R_{\text{max}}$$
  Basel III: $\lambda_{\text{max}} = 33$, $R_{\text{max}} = 12.5 = 1/0.08$
• Banks maintain a capital/liquidity buffer (slightly) above the regulatory requirements $\rightarrow$ target leverage ratio $\lambda^i_b < \lambda_{\text{max}}$, target capital ratio $R^i < R_{\text{max}}$. 
Deleveraging

- Observation: when portfolio constraints are breached following a loss in asset values, financial institutions **deleverage** their portfolio by selling some assets in order to comply with the portfolio constraint.
Deleveraging

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**Deleveraging assumption**: if following a loss $L^i$ in asset values, leverage of bank $i$ exceeds constraint,

$$\lambda^i = \frac{\Theta^i + \Pi^i - L^i}{C^i - L^i} > \lambda_{\text{max}}$$

bank deleverages by selling a proportion $\Gamma^i \in [0, 1]$ of assets in order to restore a leverage ratio $\lambda^i_b \leq \lambda_{\text{max}}$:

$$\frac{(1 - \Gamma^i)\Pi^i + \Theta^i - L^i}{C^i - L^i} = \lambda^i_b \leq \lambda_{\text{max}} \implies \Gamma^i = \frac{C^i(\lambda^i - \lambda^i_b)}{\Pi^i} \mathbf{1}_{\lambda^i > \lambda_{\text{max}}}$$. 

Systemic Stress Testing and price-mediated contagion

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Deleveraging in response to a loss

**Figure:** Percentage of marketable asset deleveraged in response to a shock to assets (circles) for a leverage constraint of 20. Leverage targeting (dotted blue) would lead to a linear response.
Market impact and Feedback effects

Total liquidation in asset $\mu$ at $k$-th round: $q^\mu = \sum_{j=1}^{N} \Pi_{k}^{j,\mu} \Gamma_{k+1}^{j}$

Market impact: $\frac{\Delta S^\mu}{S^\mu} = -\psi_{\mu}(q^\mu)$,

Impact/ inverse demand function: $\psi_{\mu} > 0, \psi'_{\mu} > 0, \psi_{\mu}(0) = 0$. 
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Price move at each iteration of fire sales:

$S_{k+1}^\mu = S_k^\mu \left(1 - \Psi^\mu \left(\sum_{j=1}^{N} \Pi_{k}^{j,\mu,}\Gamma_{k+1}^{j}\right)\right)$,
Market impact and Feedback effects

Total liquidation in asset $\mu$ at k-th round: $q^{\mu} = \sum_{j=1}^{N} \prod_{k}^{j} \Gamma_{k+1}^{j}$

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$\prod_{k+1}^{i,\mu} = \left(1 - \Gamma_{k+1}^{i}\right)$

Non-liquidated assets

$\prod_{k}^{i,\mu} \left(1 - \psi_{\mu} \left(\sum_{j=1}^{N} \prod_{k}^{j} \Gamma_{k+1}^{j}\right)\right)$

Price impact on remaining holdings
**Figure:** Portfolio constraints define a set of admissible portfolios. A large loss may take the portfolio outside this set, in which case banks deleverage in order to revert to this set.
Portfolio overlaps as drivers of loss contagion

When market impact is linear $\Psi_\mu(x) = x/D_\mu$ (where $D_\mu =$ market depth) the mark-to-market loss of $i$ resulting from fire sales is

$$L^i \approx \sum_{j=1}^N \sum_{\mu=1}^M \frac{\Pi^{i\mu} \Pi^{j\mu}}{D_\mu} \Omega_{ij} \Gamma^j = \sum_{j=1}^N \Omega_{ij} \Gamma^j,$$

where $\Omega_{ij}$ is the **liquidity weighted overlap** between portfolios $i$ and $j$ (Cont & Wagalath 2013):

$$\Omega_{ij} = \sum_{\mu=1}^M \frac{\Pi^{i\mu} \Pi^{j\mu}}{D_\mu}$$

$D_\mu =$ market depth for asset $\mu$

$\Omega_{ij} =$ exposure of marketable assets of $i$ to 1% deleveraging by $j$.  
$\Rightarrow$ loss contagion = contagion process on network defined by $[\Omega_{ij}]$. 

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The EU indirect contagion network
Portfolio overlaps across EU banks (EBA 2011)
2nd round overlaps across EU banks (EBA 2011)

Distribution of elements of $\Omega^2$ representing 2nd round spillover effects.
Market impact function
Market impact function and market depth

The impact of a total distressed liquidation volume $q$ is modelled by a *level-dependent market impact function*

\[
\Psi_{\mu}(q, S) = \left(1 - \frac{B_{\mu}}{S}\right) \left(1 - \exp \left(-\frac{q}{\delta_{\mu}}\right)\right),
\]

- $S \geq B_{\mu}$ is the price-floor
- $\text{ADV}$: average daily volume,
- $\sigma_{\mu}$: daily volatility
- $c \approx 0.25$, a coefficient to make $\Psi_{\mu}$ consistent with empirical estimates of the linear impact model for small volumes $q$.
- $\tau$ is the liquidation horizon
Market impact function and market depth

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where

\[
\delta_{\mu} = c \frac{ADV_{\mu}}{\sigma_{\mu}} \sqrt{\tau},
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Estimated market depth

Percent

Market depth (EUR)

$10^8$ $10^9$ $10^{10}$ $10^{11}$ $10^{12}$ $10^{13}$ $10^{14}$ $10^{15}$

Price-mediated contagion  Fire sales  Systemic stress test  Indirect exposures  Monitoring exposure to fire sales

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Stress scenarios

- A stress scenario is defined by a vector $\epsilon \in [0, 1]^K$ whose components $\epsilon_\kappa$ are the percentage shocks to asset class $\kappa$.
- Initial/Direct loss of portfolio $i$: $L_0^i(\epsilon) = \epsilon_i \Pi_i = \sum_\kappa \Pi^{i_\kappa} \epsilon_\kappa$
- Gradual increase of the shock size $\epsilon_\kappa$ from 0% to 20%.
- As an illustration we consider the following stress scenarios:
  1. Spanish residential and commercial real estate losses
  2. Northern Europe residential losses
  3. Southern Europe commercial real estate losses
  4. Eastern Europe commercial real estate losses
Fire sales losses

- Leverage targeting
- Threshold model
- Threshold $\lambda_b < \lambda_{m\text{ax}}$
Fire sales losses
Fire sales losses and market depth

![Graph showing the relationship between fire sales losses and market depth scaling factor versus shock size percentage.](image-url)
Endogenous losses modify stress test outcomes

![Chart showing the relationship between market depth scaling factor, shock size (in %), and number of banks. The chart indicates how endogenous losses affect stress test outcomes.](image-url)
Failures due to illiquidity and insolvency

**Figure:** The model allows to distinguish between failures due to insolvency (negative equity - left) and failures due to illiquidity (zero liquid assets - right).
Indirect exposures

Consider two institutions (A) and (B).

- A and B hold a common financial asset (1). A holds an illiquid asset (2) that B does not hold. Notional exposure of B to 2 is zero.

- However, in the event of a large shock to the value of the illiquid asset (2), A may be forced to sell some of its financial assets, pushing down its market price, resulting in a market loss for the bank B.

- So: B experiences a loss following a large shock to the illiquid asset: B has an (indirect) exposure to an asset it does not hold!

- Magnitude of this indirect exposure is directly linked to the overlap between B and institutions holding this asset.

- Large diversified institutions increase overlaps across system and become nodes for price-mediated contagion.
Indirect exposures

The effective exposure of institution $i$ to asset class $\kappa$ is given by

$$E^{i,\kappa}(\epsilon_\kappa) := \frac{\text{Loss}(i, \epsilon_\kappa)}{\epsilon_\kappa} = \Theta^{i,\kappa} + \frac{F\text{Loss}(i, \epsilon_\kappa)}{\epsilon_\kappa},$$

where $F\text{Loss}(i, \epsilon_\kappa)$ is the total fire sales loss that $i$ suffers as a result to the shock $\epsilon_\kappa$ to asset class $\kappa$. 

\begin{itemize}
  \item Notional exposure
  \item Indirect exposure
\end{itemize}
Indirect exposures

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where $F\text{Loss}(i, \epsilon_\kappa)$ is the total fire sales loss that $i$ suffers as a result to the shock $\epsilon_\kappa$ to asset class $\kappa$. The effective exposure is scenario dependent and accounts for losses that $i$ would suffer in a stress scenario.

→ it reflects the network dependent (and actual!) risk of $i$’s portfolio.
Indirect exposures: how large are they?

**Figure:** Losses of HSBC and Banco Santander as a function of depreciation of Spanish housing sector.
Figure: Indirect exposures of UK banks to Spanish housing sector, 2011.
Monitoring
Portfolio overlaps as drivers of Indirect contagion

When market impact is linear, the mark-to-market loss of $i$ resulting from fire sales is given by

$$L^i \approx \sum_{j=1}^{N} \sum_{\mu=1}^{M} \frac{\prod_{i=\mu} \prod_{j=\mu}}{D_{\mu}} \Gamma^j = \sum_{j=1}^{N} \Omega_{ij} \Gamma^j,$$

where $\Omega_{ij}$ is the liquidity weighted overlap between portfolios $i$ and $j$ (Cont & Wagalath 2013).

Thus: price mediated contagion can be modeled as a contagion process on a network whose nodes are financial institutions and whose links are weighted with liquidity weighted overlaps.
European banking system: indirect contagion network
**Figure:** European banking system: Principal component analysis of liquidity-weighted overlaps. Source: EBA (public)
Liquidity weighted overlaps: 1st principal component

Figure: European banking system: Liquidity weighted overlaps. Source: EBA (public)
Indirect Contagion Index

The principal eigenvector $U = (U_i, i = 1 \ldots N)$ corresponding to the largest eigenvalue of the matrix of liquidity-weighted overlaps provides a measure of (eigenvector) centrality of the node $i$ in the indirect contagion network.

**Definition (Indirect Contagion Index (ICI))**

We define the **Indirect Contagion Index (ICI)** of a financial institution $i$ as its component $U_i$ in the (normalized) principal eigenvector of the matrix of liquidity weighted portfolio overlaps:

$$ICI(i) = U_i$$
Indirect Contagion Index as a measure of exposure to fire sales loss

Figure: Regression of log($FLoss_i$) on log($ICI_i$) for a 13% shock at estimated market depth. $R^2 = 0.89$. Source: EBA (public) & authors calculations.

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Comparison to "leverage targeting" models
Response functions

Figure: Leverage targeting response function (dashed) and two variants of the threshold model (full and circles) response functions.
Fire sales losses and market depth

![Diagram showing the relationship between fire-sales loss (% of total bank equity) and shock size (%) with market depth scaling factor.](image)
Fire sales losses and market depth
Distribution of fire sales losses

Figure: Fire sales loss for different scenarios and different model combinations.
Fire sales losses and market depth

fire_sales_loss_percentage_total_loss.png
Can the effect of price-mediated contagion be mimicked by scaling up macro shocks in stress tests?

No: scaling up the macro shocks can replicate the average bank loss but not the cross-sectional distribution of losses across banks.
Summary

Quantitative model for fire sales spillovers in a network of institutions with common asset holdings subject to *one-sided* portfolio constraints:

- **Tipping point:** Existence of critical macro shock level beyond which fire sales are triggered and significant contagion occurs. In EU banks: threshold large – but not extreme.
Summary

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- **Fire sales losses:** Even with optimistic estimates of market depth, fire sales losses can amount to over 20% of system bank equity. This is significant enough to change the outcome of stress tests.
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Quantitative model for fire sales spillovers in a network of institutions with common asset holdings subject to one-sided portfolio constraints:

• **Tipping point:** Existence of critical macro shock level beyond which fire sales are triggered and significant contagion occurs. In EU banks: threshold large – but not extreme.

• **Fire sales losses:** Even with optimistic estimates of market depth, fire sales losses can amount to over 20% of system bank equity. This is significant enough to change the outcome of stress tests.

• **Heterogeneity of bank losses:** The cross sectional distribution of losses due to fire sales cannot be replicated by simply applying a larger initial macro-shock to all assets or banks.
Summary

• **Illiquidity and insolvency:** Our model allows to distinguish between failures due to insolvency and defaults due to illiquidity. Ignoring failures due to illiquidity may lead to a severe underestimation of the extent of contagion.

• **Indirect exposures:** Our model leads to a quantifiable notion of *indirect* exposure to an asset class. EU banks are shown to have significant exposure to housing markets in other European countries.
  → Calls for a re-thinking of macro-prudential regulation at the national level.

• **Indirect contagion index:** Liquidity-weighted overlaps lead to a bank-level indicator that may be used for monitoring and for quantifying the contribution (and vulnerability) of a financial institution to price-mediated contagion;
Implications for macroprudential supervision and policy

• Incorporating bank reactions greatly alters the outcome of the stress tests

• Capital adequacy should be examined in the light of systemic stress tests incorporating such endogenous effects and contagion mechanisms

• Fire sales and the resulting price-mediated contagion leads to significant **indirect exposures** across sectors and countries. Systemic stress tests allow to evaluate these indirect exposures.

• **Disseminating indirect exposures** can help financial institutions manage and internalize this risk.

• Most failures occurs through illiquidity, not insolvency: suspension of mark-to-market accounting for illiquid assets does not necessarily help this.
References

