Sovereign Default: The Role of Expectations.

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• Can sovereign debt crisis can be the result of self-fulfilling equilibria? (Calvo 1988)

• High interest rates imply higher probability of default which in turn feeds-back into high interest rates.

• Argentina 2001? Currency board. Average rate was 10%. (93-98) Average debt to GDP ratio was 30%.

• Same interest payment than a 3& rate (the US bond) and 100 % of debt to GDP ratio.

• Southern Europe 2012? No default with debt to GDP ratios of 80% for Spain and over 100% for Italy and Portugal.

• ECB intervened in September 2012: Outright Monetary Transactions (OTM).
• This paper


2. Evaluates the effect of policy by an agent with deep pockets (IMF or ECB?).

3. Shows (in an infinite period model) how a sunspot can be used to replicate behavior of spreads in Italy and Spain during a sovereign debt crisis.
Plan

- Two period model.

- The price of the bond (or the interest rate) as a function of the "debt".

- Discussion of equilibria. Multiplicity.

- A sunspot in the infinite period model: engineering a sovereign bond crisis.

- Skepticism.
A two period model

• Two-period, endowment economy populated by a representative agent

\[ U(c_1) + \beta E_1[U(c_2)] \]

• The endowment is 1 in the first period.

• Second period endowment \( y' \in [1, Y] \), with density \( f(y') \) and cdf \( F(y') \).

• In period one, the agent can borrow in a non contingent bond in international financial markets.
• The risk neutral gross international interest rate is $R^*$ (the price of the bond $q^* = 1/R^*$).

• At $t = 2$, after observing $y'$, the representative agent decides either to pay the debt or default.

• If default, consumption in second period is 1.
• The representative agent takes into account the effect of the level of debt on interest rates.

• Lenders offer a schedule of interest rates as a function of the level of debt.

• There are two classes of schedules consistent with zero profits.
• In one, we guess that \( q \) depends on the resources obtained in the first period, \( b' \). (Calvo schedules)

• In this case, the representative agent will default if and only if \( U(y' - b' \frac{1}{q}) \leq U(1) \), or

\[
y' \leq 1 + b' \frac{1}{q}
\]

and the probability of default is given by \( F \left[ 1 + b' \frac{1}{q} \right] \).
• In the other class, we guess \( q \) depends on the resources paid (if no default) in the second period that we denote by \( b^{q'} \). (Arellano schedules).

• Expressed in terms of \( b^{q'} \), the condition for the threshold is written as

\[
y' \leq 1 + b^{q'}
\]

and the probability of default is given by \( F [1 + b^{q'}] \).
• Investors are risk neutral, so arbitrage in international capital markets implies

\[
\frac{1}{q^*} = \frac{1}{q(b')} \left[ 1 - F \left( 1 + b' \frac{1}{q(b')} \right) \right].
\]  
(1)

for the schedule \( R(b') \), or

\[
R^* = \frac{1}{q(b'^q)} \left[ 1 - F \left( 1 + b'^q \right) \right],
\]  
(2)

for the schedule \( q(b'^q) \).

• This verifies that the schedule depends on \( b' \) in the first case and on \( b'^q \) on the second.
• Note that \( R(z) = \frac{1}{q(z)} \), for \( z \in \{b^{ql}, b'\} \), so the choice between \( R \) or \( q \) is irrelevant.

• What matters, is if the schedule is defined over the variable \( b^{ql} \) (the value of the debt at maturity) or \( b' \) (the value of debt when issued).

• This depends on which one foreign lenders coordinate upon, not on any decision taken by the small open economy.

• The optimal choice of debt depends on which schedule the representative agent faces. We will discuss this in the context of a specific example below.
**Equilibrium**  An equilibrium is a schedule $q(b')$ (or $q(b^{q'})$) and a point in the schedule $(b'^*, q^*)$ (or $(b^{q'}^*, q^*)$) such that:

1. Given the schedule, $b'$ (or $b^{q'}$) maximizes utility

2. The schedule solves the functional equation (1) (or (2))

3. $q^* = q(b'^*)$ (or $q^* = q(b^{q'}^*)$).
Calvo schedules \( q(b') \)

- Let
  
  \[
  h \left( \frac{1}{q}; b' \right) = \frac{1}{q} \left[ 1 - F \left( 1 + b' \frac{1}{q} \right) \right].
  \]

- For \( \frac{1}{q} = 0 \), \( h(0; b') = 0 \). With a bounded support, for \( \frac{1}{q} \) such that
  \( 1 + b' \frac{1}{q} \geq Y \), then \( h \left( \frac{1}{q}; b' \right) = 0 \).

- For standard distributions, the function \( h \left( \frac{1}{q}; b' \right) \) is concave, so that there are at most two solutions of \( \frac{1}{q^*} = h \left( \frac{1}{q}; b' \right) \).
\( h(R) \)

- \( b = 2.2, \gamma = 0 \)
- \( b = 2.6, \gamma = 0 \)
- \( R^* = 1.04 \)
Arellano Schedules

- The model is also consistent with an alternative class of schedules that have the contrasting feature that the equilibrium is unique.

- These are the schedules that Arellano (08) considers.

- A schedule now is $q(b^{b'})$, so that the price of a bond in the first period, $q$, is a function of the bonds to be paid in the second period, $b^{q'}$.

- It must satisfy

$$
\frac{1}{q^*} = \frac{1}{q(b^{q'})} \left[ 1 - F(1 + b^{q'}) \right]
$$
• This schedule is increasing in $b^{q'}$ in all the support.

• But this is the same schedule as before, with only a change of variables.

• So while this schedule $q \left( \frac{b^{b'}}{q} \right)$ is increasing, it includes the high rate, decreasing schedule for $\frac{1}{q} (b')$.

• In particular high values of $b^{q'}$ can be associated with low values of $b'$, and high values of $\frac{1}{q}$.
The two schedules \( q(b^q) \) and \( \frac{1}{q}(b') \) are solutions of the same functional equation with just a change of variables.

But it makes a big difference whether the representative agent in the small open economy faces one schedule or the other.

Once offered the increasing schedule, \( q(b^q) \), the representative agent will be able to pick a point on the schedule.

By choosing \( b^q \), the probability of default will be pinned down.

That way the country can avoid the default probabilities associated with the high rate, high probability outcome.
Fragility of the decreasing Calvo schedule

• Consider a perturbation of a point \((\frac{1}{q}, \hat{b})\) in the the schedule \(\frac{1}{q}(\frac{Y-1}{4q^*}, 2)\) that consists of the same interest rate, but a slightly lower value for the debt \((\frac{1}{q}, \hat{b} - \varepsilon)\).

• At the point \((\frac{1}{q}, \hat{b} - \varepsilon)\), the interest rate is the same as in \((\frac{1}{q}, \hat{b})\), but the debt lower, so the probability of default is also lower.

• Thus, profits for the lenders are higher than at \((\frac{1}{q}, \hat{b})\), where profits are zero: Individual investor would cut down rates.

• A reasonable refinement would restore uniqueness?
A distribution with normal and disaster times

• Consider two independent random variables, $y^1$ and $y^2$, both normal with different mean, $\mu^1$ and $\mu^2$, respectively, and the same standard deviation, $\sigma$.

• Now, let the endowment in the second period $y'$ be equal to $y^1$ with probability $p$, and equal to $y^2$ with probability $1 - p$.

• If the two means, $\mu^1$ and $\mu^2$, are sufficiently apart, then $h(b', \frac{1}{q})$ has four solutions, for some values of the debt.
$h(R)$

- $b = 2.0556$
- $b = 2.5859$
- $b = 2.9394$
- $R^* = 1.04$
Perturbing the uniform distribution

• Is that bimodal distribution empirically plausible? No.

• But the answer can be yes if the debt level is high enough.

• Consider a perturbation $g(y')$ of the uniform distribution, so that the
density would be

$$f(y') = \frac{1}{Y - 1} + \gamma g(y'), \text{ with } \int_1^Y g(y') dy' = 0.$$

• In particular let $g(y') = \sin k y'$, with $k = \frac{2\pi}{Y-1} N$, where $N$ is a natural number.
• If $N = 0$, the distribution is uniform, so there is a single increasing schedule.

• If $N = 1$, there is a single full cycle added to the uniform distribution.

• The amplitude of the cycle (relative to the uniform distribution) is controlled by the parameter $\gamma$.

• The number of full cycles of the $\sin ky'$ function added to the uniform is given by $N$. 
\[ h(R) \]

- \( b = 2.2, \gamma > 0 \)
- \( b = 2.2, \gamma = 0 \)
- \( b = 2.9, \gamma > 0 \)
- \( b = 2.9, \gamma = 0 \)
- \( R^* = 1.04 \)
• As $\gamma \to 0$, so does the perturbation.

• Given a value for $\gamma$, the closer the debt to its maximum value, the larger the degree of multiplicity.

• The function

$$\frac{1}{q} - \frac{1}{q^*} \left[ 1 - \frac{1 + b'1}{q} - \gamma \sin kb'1 \right] = 0$$

has more than two zeros for $\frac{1}{q}$, for $\gamma$ that can be made arbitrarily small, as long as $b'$ is close enough to $b^{Max}$.

• if $\gamma$ is small, it may take a very long series to identify it in the data.
Policy

- Consider an agent with deep pockets.

- Assume it offers to lend to the country, at a policy rate \( \left( \frac{1}{q} \right)^P \), any amount lower than or equal to a maximum level \( b^P \).

- Then, given any schedule offered by the foreign lenders to the country \( \frac{1}{q}(b', .) \), the schedule faced by the representative agent is given by

\[
\frac{1}{q}(b') = \min \left\{ \frac{1}{q}P, \frac{1}{q}(b', .) \right\}
\]

so there cannot be an equilibrium with an interest rate larger than \( \frac{1}{q}P \).
• If well designed, the amount borrowed from the large lender is zero.

• The level of lending offered has to be limited.
The infinite period model: Numerical exploration

- The endowment $y$ has bounded support, given by $[y_{\text{min}}, y_{\text{max}}] \subset \mathbb{R}_+$ and follows a Markov process with distribution $F(y' | y)$.

- We let the value after default be $V^{aut} = \frac{U(y^d)}{1-\beta}$.

- In here we explore interest rate schedules for $q$ the depend on $b'$. (Calvo)

- They will also depend on $y$, and $s$, which is a sunspot variable that selects one of the multiple schedules for the interest rate.
• Our exploration will be based on the bimodal distribution we studied above. (at most two increasing solutions?)

• Thus, \( s = 1, 2 \), with transition probabilities, \( p_{11} = p_{22} = p \).

• The value for the representative agent, after deciding not to default, is given by value functions \( V \), and schedules \( \frac{1}{q} \), satisfying
\[ V(\omega, y, 1) = \max_{c, b', \omega'} \left\{ U(c) + \beta \mathbb{E}_{y'} \left[ \begin{array}{c} p \max \{ V(\omega', y', 1), V^{aut} \} \\ + (1 - p) \max \{ V(\omega', y', 2), V^{aut} \} \end{array} \right] \right\} \]

subject to

\[
\begin{align*}
c & \leq \omega + b' \\
\omega' & = y' - b' \frac{1}{q} (b', y, 1) \\
b' & \leq b
\end{align*}
\]
and

\[
V(\omega, y, 2) = \max_{c, b', \omega'} \left\{ U(c) + \beta \mathbb{E} y' \right. \\
\left. \quad \left[ p \max \left\{ V(\omega', y', 2), V^{aut} \right\} \\
\quad \quad + (1 - p) \max \left\{ V(\omega', y', 1), V^{aut} \right\} | y \right] \right\}
\]

subject to

\[
\begin{align*}
c &\leq \omega + b' \\
\omega' &= y' - \frac{1}{q} (b', y, 2) \\
b' &\leq b
\end{align*}
\]
Conditions the schedules must satisfy

- In state 1, investors offer the schedule \( \frac{1}{q}(b', y, 1) \).

- The following period the state is 1 with probability \( p \), and 2 with probability \( 1 - p \).

- If in state \( s \), the threshold for default is defined by

\[
V^{aut} = V \left( \omega, y(\omega, y, s), s \right).
\]

Then

\[
\frac{1}{q^*} = \frac{1}{q} (b', y, 1)[p[1 - F(y(\omega', y', 1)|y)] + (1 - p)[1 - F(y(\omega', y', 2)|y)]
\]
• Similarly, in state 2, investors offer the schedule $\frac{1}{q}(b', y, 2)$.

• Then, the arbitrage condition must be written as

$$\frac{1}{q^*} = \frac{1}{q}(b', y, 2)[p[1 - F(y(\omega', y', 2)|y)] + (1 - p)[1 - F(y(\omega', y', 1)|y)]]$$
Equilibrium

An equilibrium is given by functions

\[ V(\omega, y, s), c(\omega, y, s), b'(\omega, y, s), \frac{1}{q}(b'(\omega, y, s), y, s), y(\omega, y, s) \]

such that,

1. given \( V(\omega, y, s), y(\omega, y, s) \) solves the threshold equation.

2. given \( \frac{1}{q}(b'(\omega, y, s), y, s), V(\omega, y, s), c(\omega, y, s), b'(\omega, y, s) \) solve the DP problems.

3. The arbitrage conditions and the law of motion for \( \omega \) are satisfied.