Higher-Order Dynamics in Asset-Pricing Models with Recursive Preferences

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Asset-Pricing Models

Asset pricing models have become increasingly complex over the last three decades


Modern generation: Bansal and Yaron (2004) or Hansen, Heaton, and Li (2008)

Highly nonlinear preferences structures, complex exogenous specifications

Complexity requires numerical approximations for tractability
Standard Solution Methods


Finite-state Markov chain replaces continuous state space
Guvenen (2009) and many others

**Log-linearization** methods: Judd (1996), Bansal and Yaron (2004)

Linear approximation of policy and price functions
Bansal, Kiku, Yaron (2010), Constantinides and Ghosh (2011),
Bansal et al. (2014), Beeler and Campbell (2012)

Methods are well suited for the early models and provide
(more or less) accurate and robust solutions
Motivation

But: Models are now so much more complex,
still the same solution methods are applied

By construction: Log-linear approximation misses
higher-order effects

Question: Does it matter?

Purpose: Solve models accurately using projection methods,
analyze the influence of higher order dynamics
Outline

Motivation

Introductory Example

Projection Method

Long-Run Risks Model

Conclusion
Standard Asset Pricing Framework

Euler equation for asset with price $P_t$ that pays dividend $D_t$

$$P_t = E_t \left\{ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left( P_{t+1} + D_{t+1} \right) \right\}$$

Assumption: dividends are the only source of income ($C_t = D_t$)

$$\frac{P_t}{C_t} = E_t \left\{ \beta \left( \frac{C_{t+1}}{C_t} \right)^{1-\gamma} \left( \frac{P_{t+1}}{C_{t+1}} + 1 \right) \right\}$$

Log-consumption growth $\Delta c_{t+1} = \log(\frac{c_{t+1}}{c_t})$ is an AR(1) process

$$\Delta c_{t+1} \equiv x_{t+1} = (1 - \rho) \mu + \rho x_t + \epsilon_{t+1}, \quad \epsilon_{t+1} \sim N(0, \sigma^2)$$
State-Space Representation

Log-consumption growth $\Delta c = x$ is state variable

Assumption: p-c ratio $\frac{P_t}{C_t}$ is a function of the state $x$, $y(x)$,

$$y(x) = \mathbb{E} \left\{ \beta \left( e^{x'} \right)^{1-\gamma} (y(x') + 1) \middle| x \right\}$$

$$= \mathbb{E} \left\{ h(x') \middle| x \right\}$$

with $x'$ denoting the state in the next period

Projection, Step I

Approximation of the p-c ratio $y(x)$ by a linear combination of basis functions

Example: $y(x) = \alpha_0 + \alpha_1 x$
Gaussian Quadrature

Gauss-Hermite Quadrature

\[
\int_{-\infty}^{+\infty} e^{-z^2} f(z) dz \approx \sum_{i=1}^{n} \omega_i f(z_i)
\]

with the Gauss-Hermite quadrature nodes \(z_i\) and weights \(\omega_i\)

Change of variable for normally distributed variables \(z \sim N(a, b^2)\)

\[
E(h(z)) \approx \frac{1}{\sqrt{\pi}} \sum_{i=1}^{n} \omega_i h(\sqrt{2}bz_i + a)
\]

In our example: \(x'|x \sim N((1 - \rho)\mu + \rho x, \sigma^2)\)

\[
E \{ h(x')|x \} \approx \frac{1}{\sqrt{\pi}} \sum_{i=1}^{n} \omega_i h(\sqrt{2}\sigma z_i + (1 - \rho)\mu + \rho x)
\]
Solving the Integral

Define $x_i'(x) \equiv \sqrt{2}\sigma z_i + (1 - \rho)\mu + \rho x$, so for the linear example

$$\alpha_0 + \alpha_1x = \frac{1}{\sqrt{\pi}} \sum_{i=1}^{n} \omega_i \left( \beta e^{(1-\gamma)x_i'(x)} \right) (\alpha_0 + \alpha_1x_i'(x) + 1)$$

Note that $x_i'(x)$ and hence $A_i(x)$ are known for a given value of $x$

Define the residual function as

$$\hat{F}(x, \alpha) = \alpha_0 + \alpha_1x - \frac{1}{\sqrt{\pi}} \sum_{i=1}^{n} A_i(x) (\alpha_0 + \alpha_1x_i'(x) + 1)$$

which is linear in the coefficients $\alpha = [\alpha_0, \alpha_1]$
Applying the Projection

Projection, Step II:
Collocation conditions: \( \hat{F}(x_j, \alpha) = 0 \) for \( j = 1, 2 \)

Conditions lead to 2 linear equation and 2 unknowns

\[
\begin{pmatrix}
1 - \frac{1}{\sqrt{\pi}} \sum_i A_i(x_1) & x_1 - \frac{1}{\sqrt{\pi}} \sum_i A_i(x_1)x'_i(x_1) \\
1 - \frac{1}{\sqrt{\pi}} \sum_i A_i(x_2) & x_2 - \frac{1}{\sqrt{\pi}} \sum_i A_i(x_2)x'_i(x_2)
\end{pmatrix}
\begin{pmatrix}
\alpha_0 \\
\alpha_1
\end{pmatrix}
\]

\[
= \begin{pmatrix}
\frac{1}{\sqrt{\pi}} \sum_i A_i(x_1) \\
\frac{1}{\sqrt{\pi}} \sum_i A_i(x_2)
\end{pmatrix}
\]
Example

Parameter values $\gamma = 5, \beta = 0.95, \mu = 0.02, \rho = 0.8, \sigma = 0.0205$

Approximate solution function within $\pm m$ standard deviations around the unconditional mean of the underlying process

With $m = 2$ we obtain $x_{min} = -0.0484$ and $x_{max} = 0.0884$

For the degree–1 approximation with 2 nodes, $x_1 = x_{min}$ and $x_2 = x_{max}$ and the system of equations becomes

$$
\begin{pmatrix}
-0.0482 & -0.0132 \\
0.4365 & 0.0450
\end{pmatrix}
\begin{pmatrix}
\alpha_0 \\
\alpha_1
\end{pmatrix} =
\begin{pmatrix}
1.8207 \\
1.3360
\end{pmatrix}
$$

\[\text{11/36}\]
Results Degree 1

Degree 1 Approximation, Maximum Absolute Error: 10.1072

Closed-form solution by Burnside (1998)
Results Degree 2

Degree 2 Approximation, Maximum Absolute Error: 2.5817
Results Degree 5

Degree 5 Approximation, Maximum Absolute Error: 0.1429
Results Degree 10

Degree 10 Approximation, Maximum Absolute Error: 5.1e-5
General Projection Approach

Projection methods in economics, Judd (1992), Chen, Cosimano and Himonas (2014)

Functional equation

\[(Gz)(x) = 0,\]

with

- the variable \(x\) in a (state) space \(X \subset \mathbb{R}^l, \ l \geq 1\)
- an **unknown solution function**, \(z : X \rightarrow \mathbb{R}^m\)
- the given function \(G\) is a continuous mapping between two function spaces

Projection Method, Step I:
Approximate term \(z(x)\) on its domain \(X\) by a linear combination of basis functions

\[\hat{z}(x; \alpha) = \sum_{k=0}^{n} \alpha_k \Lambda_k(x)\]
Residual Function

Residual function $\hat{F}(x; \alpha)$ is the error in the original equation,

$$\hat{F}(x; \alpha) = (G\hat{z})(x; \alpha)$$

Projection Method, Step II:

Coefficients $\alpha$ are chosen by imposing certain conditions on the residual function (projection conditions), so that $\hat{F}(x; \alpha)$ close to zero

Different projection methods differ in how “close to zero” is defined

This paper: Collocation and Galerkin Projection
Projection Conditions

Collocation Method:

\[ \hat{F}(x_k; \alpha) = 0, \ k = 0, 1, \ldots, n \]

with \( n + 1 \) distinct nodes \( \{x_k\}_{k=0}^n \)

Collocation requires solving a nonlinear system of equations

Galerkin Method:

\[ \int_X \hat{F}(x; \alpha) \Lambda_k(x) \frac{1}{\sqrt{1 - x^2}} \, dx = 0, \ k = 0, 1, \ldots, n \]

for domain \([-1, 1]\)

Galerkin projection requires numerical integration and solving a nonlinear system of equations
Application to Asset Pricing

Asset pricing model with recursive preferences

Wealth-Euler equation

\[ E_t \left[ \delta^\theta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} R_{w,t+1}^\theta \right] = 1 \]

with \( R_{w,t+1} \) return on claim to aggregate consumption

In state-space representation (lower case letters = log of variables)

\[ E \left[ \exp \left( \theta \log \delta - \frac{\theta}{\psi} \Delta c(x'|x) + \theta r_{w}(x'|x) \right) \bigg| x \right] = 1, \quad \forall x \]

with \( \Delta c(x'|x) = c(x') - c(x) \)
Functional Equation

Unknown solution function:
\[ z_w(x) = \log \left( \frac{W(x)}{C(x)} \right) \]

State-dependent log return of aggregate consumption claim
\[ r_w(x'|x) = z_w(x') - \log \left( e^{z_w(x)} - 1 \right) + \Delta c(x'|x) \]

Functional equation
\[ 0 = \int_X \left[ \exp \left( \theta \left( \log \delta + \left( 1 - \frac{1}{\psi} \right) \Delta c(x'|x) + z_w(x') - \log \left( e^{z_w(x)} - 1 \right) \right) \right) - 1 \right] \, df_x \]
Approximate Solution Functions

Residual function

\[ \hat{F}_w(x; \alpha_w) = \int_X \left[ \exp \left( \theta \left( \log \delta + (1 - \frac{1}{\psi}) \Delta c(x'|x) + \hat{z}_w(x'; \alpha_w) - \log \left( e^{\hat{z}_w(x'; \alpha_w)} - 1 \right) \right) \right) - 1 \right] df_x \]

Projection methods determine unknown solution coefficients \( \alpha_w \)

Approximate state-dependent wealth-consumption ratio \( \hat{z}_w(x; \alpha_w) \)

Approximation of return of aggregate consumption claim

\[ \hat{r}_w(x'|x; \alpha_w) = \hat{z}_w(x'; \alpha_w) - \log \left( e^{\hat{z}_w(x'; \alpha_w)} - 1 \right) + \Delta c(x'|x) \]
Return of any Asset

Euler equation

\[ E_t \left[ \delta^\theta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} R_{w,t+1}^{\theta-1} R_{i,t+1} \right] = 1 \]

with \( R_{i,t+1} \) (gross) return of asset \( i \)

In state-space representation and with logs

\[ E \left[ \exp \left( \theta \log \delta - \frac{\theta}{\psi} \Delta c(x'|x) + (\theta - 1)r_w(x'|x) + r_i(x'|x) \right) \bigg| x \right] = 1 \]
Residual for Asset $i$

Unknown solution function:
\[
\log \text{price dividend ratio } z_i(x) = \log \left( \frac{P(x)}{D(x)} \right)
\]

State-dependent log return of asset $i$
\[
r_i(x' | x) = \log \left( e^{z_i(x')} + 1 \right) - z_i(x) + \Delta d_i(x' | x)
\]

Residual function
\[
\hat{F}_i(x; \alpha_i) = \int_X \left[ \exp \left( \theta \log \delta - \frac{\theta}{\psi} \Delta c(x' | x) + (\theta - 1) \hat{r}_w(x' | x; \alpha_w) \right) 
+ \log \left( e^{\hat{z}_i(x'; \alpha_i)} + 1 \right) - \hat{z}_i(x; \alpha_i) + \Delta d_i(x' | x) \right] - 1 \right] d\mu_X
\]
Projection Methods ... Again

Coefficients $\alpha_w$ and function $\hat{r}_w(x'|x; \alpha_w)$ previously computed

Projection methods determine unknown solution coefficients $\alpha_i$

Approximate state-dependent price-dividend ratio $\hat{z}_i(x; \alpha_i)$

Approximation of log return of asset $i$,

$$\hat{r}_i(x'|x; \alpha_w) = \log \left( e^{\hat{z}_i(x; \alpha_i)} + 1 \right) - \hat{z}_i(x; \alpha_i) + \Delta d_i(x'|x)$$
Risks for the Long Run

Highly influential asset pricing model by Bansal and Yaron (2004)

Growth rates have random but highly persistent long-run shocks

Conditional variance of growth rates is stochastic

Model has two state variables

long-run component of growth, \( x_t \)
conditional variance level, \( \sigma_t^2 \)

Calibration of Bansal, Kiku, and Yaron (2012) increases influence of volatility channel versus the long-run risks channel compared to original Bansal–Yaron (2004) calibration
Model Specification

\[ \Delta c_{t+1} = \mu_c + x_t + \sigma_t \eta_{t+1} \]
\[ x_{t+1} = \rho x_t + \phi_e \sigma_t e_{t+1} \]
\[ \sigma^2_{t+1} = \bar{\sigma}^2(1 - \nu) + \nu \sigma_t^2 + \sigma_\omega \omega_{t+1} \]
\[ \Delta d_{t+1} = \mu_d + \Phi x_t + \phi \sigma_t u_{t+1} + \pi \sigma_t \eta_{t+1} \]

\[ \eta_{t+1}, e_{t+1}, \omega_{t+1}, u_{t+1} \sim i.i.d. \ N(0, 1). \]

<table>
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<tr>
<th></th>
<th>(\mu_c)</th>
<th>(\rho)</th>
<th>(\phi_e)</th>
<th>(\bar{\sigma})</th>
<th>(\nu)</th>
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## Relative Errors and Running Times

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<td>$E (w_t - c_t)$</td>
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<td>2.0e-8</td>
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### Relative Errors of Annualized Moments

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### Annualized Moments (in %)

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<th>$\sigma(p_t - d_t)$</th>
<th>$E(r_t^m)$</th>
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<th>$E(r_t^f)$</th>
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<td>2.01</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>3.44</td>
<td>0.37</td>
<td>5.21</td>
<td>19.58</td>
<td>1.34</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>3.33</td>
<td>0.27</td>
<td>5.50</td>
<td>19.57</td>
<td>1.32</td>
<td>1.27</td>
</tr>
</tbody>
</table>
Approximation Errors

\[ w_t - c_t \]

\[ x \times 10^{-4} \]

\[ p_t - d_t \]

\[ x \times 10^{-4} \]
Errors: First Derivative

\[
\frac{\partial (w_t - c_t)}{\partial x}
\]

\[
\frac{\partial (p_t - d_t)}{\partial x}
\]

\[
\frac{\partial (w_t - c_t)}{\partial \sigma^2}
\]

\[
\frac{\partial (p_t - d_t)}{\partial \sigma^2}
\]

Motivation

Introductory Example

Projection Method

Long-Run Risks Model

Conclusion
Errors: Second Derivative
Sensitivity of the Approximation Errors

![Graphs showing sensitivity of the approximation errors with respect to different parameters.](image-url)
Sensitivity of the Approximation Errors

\[ \sigma(p_t - d_t) \]

\( \gamma \)

\( \rho \)

\( \psi \)

Collocation
BY Log-Lin

\[ \sigma(p_t - d_t) \]

\( \gamma \)

\( \rho \)

\( \psi \)
Conclusion: Log-linearization

Recent asset pricing models feature interplay of long run risks, stochastic volatility, and recursive preferences.

Features can result in highly nonlinear relationships between equilibrium quantities and the state variables.

Log-linearization, common solution approach, cannot capture nonlinear relationships.

Overestimation of equity premium by more than 100 basis points.
Projection methods prove to be highly accurate even with low degree approximation while requiring only slightly longer computation times.

Increasing complexity of asset-pricing models requires numerical solution techniques, such as projection methods, that can capture nonlinear dynamics and are robust to changes in model specification.