Structural GARCH: The Volatility-Leverage Connection

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MFM Macroeconomic Fragility Fall 2013 Meeting
Leverage and Equity Volatility

- Crisis highlighted how leverage and equity volatility are tightly linked
- “Leverage Effect” has been around - e.g. Black (1976), Christie (1982) - but...
- A dynamic volatility model that incorporates leverage directly has remained elusive
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BAC Leverage and Realized Volatility

Date

1-Month Realized (Annualized) Volatility

Debt to Equity


0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2

0 10 20 30 40 50 60 70 80 90 100
GARCH-type model where equity volatility is amplified by leverage as in structural models of credit

Statistical test of how leverage affects volatility

Heteroskedastic asset returns from observed equity returns

The Leverage Effect and Asymmetric Volatility

Capital Shortfall in a Crisis
This Presentation

- GARCH-type model where equity volatility is amplified by leverage as in structural models of credit
- Statistical test of how leverage affects volatility
  - Heteroskedastic asset returns from observed equity returns
  - The Leverage Effect and Asymmetric Volatility
  - Capital Shortfall in a Crisis
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Theoretical Foundation
Under relatively weak assumptions on the vol process, structural models say $E_t = f(A_t, D_t, \sigma_{A,t}^f, \tau)$

- $A_t =$ market value of assets at time $t$
- $D_t =$ book value of debt at time $t$
- $\sigma_{A,t}^f =$ (t-forecast) asset volatility over the life of the debt, $\tau$

Taking derivatives,

$$\frac{dE_t}{E_t} = \Delta_t \cdot \frac{A_t}{E_t} \frac{dA_t}{A_t} + \frac{\partial f}{\partial \sigma_{A,t}^f} \cdot \frac{d\sigma_{A,t}^f}{E_t}$$

$\Delta_t = \partial f / \partial A_t$ is just our familiar $\Delta$ in option pricing
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For now, ignore the volatility term

Over the life of debt (e.g. 5 years), forecast long run asset volatility is virtually constant

We verify this later, and also estimate a model where we impose this

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Ignoring Long-Run Volatility Dynamics

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Vega Analysis
Ignoring Long-Run Volatility Dynamics

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» Vega Analysis
Equity Volatility as a Function of Leverage

- Assets follow: \( \frac{dA_t}{A_t} \approx 0 = \mu_A(t) dt + \sqrt{h_A(t)} dB(t) \)

- Straightforward to show:

\[
vol_t \left( \frac{dE_t}{E_t} \right) = \Delta_t \cdot \frac{A_t}{E_t} \times \sqrt{h_A(t)}
\]

- But we don’t observe \( A_t \), so invert call option formula:

\[
A_t \equiv g(E_t, D_t, \sigma^f_{A,t}, \tau) = f^{-1}(E_t, D_t, \sigma^f_{A,t}, \tau)
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Equity Volatility as a Function of Leverage

- Assets follow: 
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The Leverage Multiplier

- Substitute in and write volatility as a function of observed leverage:

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\text{vol}_t \left( \frac{dE_t}{E_t} \right) = \left[ \triangle_t \cdot g(E_t/D_t, 1, \sigma_{A,t}^f, \tau) \times \frac{D_t}{E_t} \right] \\
\times \text{vol}_t \left( \frac{dA_t}{A_t} \right) \\
= LM_t(D_t/E_t, \sigma_{A,t}^f, \tau) \times \text{vol}_t \left( \frac{dA_t}{A_t} \right)
\]

- Assumes pricing function is homogenous degree one in underlying and strike.

- We call \( LM_t \) the “leverage multiplier”
The Leverage Multiplier

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What Does the Leverage Multiplier Look Like?

Simple Case: Black-Scholes-Merton World $r = 0.03$; Varying Asset $\sigma, \tau$
The challenge is choosing the right functional form for $LM_t$

- Need a flexible function of leverage and long-run asset volatility

We modify Black-Scholes-Merton (BSM) functions:

$$LM_t(D_t/E_t, \sigma^f_{A,t}, \tau) = \left[ \Delta_t^{BSM} \times g^{BSM}\left(\frac{E_t}{D_t}, 1, \sigma^f_{A,t}, \tau\right) \times \frac{D_t}{E_t} \right]^{\phi}$$

$g^{BSM}(\cdot)$ is the inverse BSM call function. $\Delta_t^{BSM}$ is the BSM delta

$\phi \neq 1$ is the departure from the Merton model
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Our Specification

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Comment
Not the BSM world! Just using BSM functions...

- Goal is a flexible leverage multiplier - some function of leverage
- We simply use the BSM functions to build up our LM
- e.g. $g^{BSM}$ should not be interpreted as the “correct” $A_t/D_t$
What Does the Leverage Multiplier Look Like?

Our Specification ($\sigma_A = 0.15, r = 0.03, \tau = 5$)
Is it Reasonable?
Monte Carlo Exercise

- Is our specification plausible under SV and/or non-normality?
- Simulate risk-neutral asset returns as highly asymmetric GARCH (i.e. for risk-aversion) and symmetric GARCH
- Try two types of shocks: conditional normal and conditional $t$
- Calculate simulated leverage multiplier as function of leverage
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Leverage Multiplier with Stochastic Vol/Non-Normality

SV Parameters s.t. Unconditional Asset Volatility = 0.15. τ = 2, r = 0
In symmetric setting, making the tails longer via GARCH decreases LM for larger levels of debt

- Moving from normal to $t$-dist. errors amplifies this effect

- Volatility asymmetry makes asset returns negatively skewed $\Rightarrow$ shorter right tails $\Rightarrow$ increases LM

- $t$-dist. errors shortens right tail further, so increases LM

- Letting $\phi$ vary in our model captures all of these cases well

\[\therefore \text{Our LM specification is reasonable in SV/non-normal environment}\]
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Structural GARCH
The Full Recursive Model
Structural GARCH

\[ r_{E,t} = LM_{t-1} \times \sqrt{h_{A,t}} \times \varepsilon_{A,t} \]

\[ h_{A,t} \sim GJR(\omega, \alpha, \gamma, \beta) \]

\[ LM_{t-1} = \left[ \Delta_{t-1}^{BS} \times g^{BS} \left( E_{t-1}/D_{t-1}, 1, \sigma_{A,t-1}^f, \tau \right) \times \frac{D_{t-1}}{E_{t-1}} \right]^\phi \]

\[ \sigma_{A,t-1}^f = \sqrt{E_{t-1} \left[ \sum_{s=t}^{t+\tau} h_{A,s} \right]} \]

So parameter set is \( \Theta = (\omega, \alpha, \gamma, \beta, \phi) \)
Observations

- $\phi$ tunes our leverage multiplier
  - $\phi = 0$ is a vanilla GARCH. Leverage doesn’t affect equity vol
  - $\phi = 1$ is the classic Merton model
- Half-life of GARCH process means $\sigma_{A,t-1}^f$ is basically constant
- We re-estimate the model using a constant $\sigma_{A,t-1}^f$ ... results essentially unchanged
- Dividing equity returns by $LM_{t-1}$ gives daily asset returns
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Estimation Details

- QMLE, iterate over $\tau \in [1, 30]$
- Estimate both dynamic forecast and constant forecast, take best LL
- 88 financial firms
- $D_t$ is exponentially smoothed book value of debt
  - smoothing parameter $= 0.01$, so half-life of weights $\approx 70$ days
Estimation Results
Parameter Values
Cross-Sectional Summary of Estimated Parameters

| Parameter | Median  | Median t-stat | % with $|t| > 1.64$ |
|-----------|---------|---------------|----------------|
| $\omega$  | 1.0e-06 | 1.43          | 30.9           |
| $\alpha$  | 0.0442  | 3.16          | 85.2           |
| $\gamma$  | 0.0674  | 2.50          | 72.8           |
| $\beta$   | 0.9094  | 71.21         | 98.8           |
| $\phi$    | 0.9876  | 2.87          | 75.3           |

- Average $\tau = 8.28$
- Leverage matters
- BSM leverage multiplier does well
  - Schaefer and Strebulaev (2008)
Application: The Leverage Effect
Restating the Leverage Effect

- Equity volatility is negatively correlated with equity returns (i.e. volatility asymmetry)
- One explanation: financial leverage, e.g. Black (1976), Christie (1982)
- Second explanation: risk-premium effect, e.g. Schwert (1989)
- Which one is it? e.g. Bekaert and Wu (2000)
Structural GARCH and the Leverage Effect

- $\gamma$ parameter in GJR model is a measure of volatility asymmetry
- Structural GARCH models asset returns as GJR - effectively unlevers the firm
- Median $\gamma$ for asset returns is 0.0674
- Median $\gamma$ for equity returns is 0.0846

$\approx 23\%$ of so-called leverage effect comes from leverage
Structural GARCH and the Leverage Effect

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\[ \approx 23\% \text{ of so-called leverage effect comes from leverage} \]
More Tests

Higher Leverage and Higher Asymmetry Gap?

- Firms with more leverage should have larger \((\gamma_{E,i} - \gamma_{A,i})\)
- Run regression: \(\gamma_{E,i} - \gamma_{A,i} = a + b \times \frac{D}{E} + \text{error}_i\)

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Asset Asymmetry and Risk-Premia?

- Higher market betas should mean higher asset asymmetry
- Run two-stage regression:

Stage 1: \(r_{A,i} = c + \beta_{mkt,i}^A r_{mkt,t} + e_{i,t}\)

Stage 2: \(\gamma_{A,i} = e + f \times \beta_{mkt,i}^A + \epsilon_i\)

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Application: SRISK with Leverage Amplification
Acharya et. al (2012) and Brownlees and Engle (2012)

Three steps

1. GJR-DCC model using firm equity and market index returns
2. Expected firm equity return if market falls by 40% over 6 months \( \equiv \) LRMES
3. Combine LRMES with book value of debt to determine capital shortfall in a crisis

The crisis in this case is a 40% drop in the stock market index over 6 months
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The Role of Leverage?
Thought Experiment with Structural GARCH

- Firm experiences sequence of negative equity (asset) shocks
- Level of leverage goes up rapidly
- Leverage multiplier increases, equity vol amplification higher
- Painfully obvious in the crisis, so build into SRISK
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Asset Volatility or Leverage?

The Financial Crisis

![Graph showing annualized volatility and aggregated leverage multipliers from 2007 to 2010. The graph compares EVW Equity Vol Index and EVW Asset Vol Index, as well as the aggregated leverage multiplier over time.](image-url)
Asset Based Systemic Risk: Preliminary Numbers
Bank of America
LRMES: Full Sample

The image displays a graph with the date range from 01/03/2000 to 10/31/2011. The graph shows the Bank of America Marginal Expected Shortfall (in purple) and the Bank of America Structural Marginal Expected Shortfall (in orange) over time. The window options for analysis include 3m, 6m, 1y, 2y, 5y, and all. The V-Lab (2013) is also mentioned.
Bank of America
LRMES: 2006-2011
Citigroup
LRMES: 2006-2011
Citigroup
Capital Shortfall: 2006-2011
What’s Next
More Granular Debt Measurement

- Right now we use book value of debt
- We can decompose debt further. For example, short term vs long term:

\[ D = \theta_1 \times \text{LT Debt} + \theta_2 \times \text{ST Debt} + \theta_3 \times \text{Non-Debt Liabilities} \]

- \( \theta_1, \theta_2, \) and \( \theta_3 \) are now estimated parameters
Other Applications

- Endogenous Crisis Probability with Structural GARCH
- Estimation of Distance to Crisis
- Endogenous Capital Structure and Leverage Cycles
- Counter-cyclical Capital Regulation
Appendix
Ignoring the Vega Term

Can we ignore the vega term?

\[
\frac{dE_t}{E_t} = \Delta_t \cdot \frac{A_t}{E_t} \frac{dA_t}{A_t} + \frac{\partial f}{\partial \sigma^f_{A,t}} \cdot \frac{d\sigma^f_{A,t}}{E_t}
\]

Without ignoring it, the volatility of equity is:

\[
\text{var}_t \left( \frac{dE_t}{E_t} \right) = \left( \Delta_t \frac{A_t}{E_t} \right)^2 \text{var}_t \left( \frac{dA_t}{A_t} \right) + \left( \frac{\nu_t}{E_t} \right)^2 \text{var}_t \left( d\sigma^f_{A,t} \right) \\
+ 2 \left( \Delta_t \frac{A_t}{E_t} \right) \left( \frac{\nu_t}{E_t} \right) \sqrt{\text{var}_t \left( \frac{dA_t}{A_t} \right) \text{var}_t \left( d\sigma^f_{A,t} \right)} \times \rho_t \left( \frac{dA_t}{A_t}, d\sigma^f_{A,t} \right)
\]

We investigate the rough magnitudes of the additional terms
Magnitude of Volatility Terms

- Use Black-Scholes vega
- Use estimated asset volatility series to compute $d\sigma_{A,t}^f$
- Assume vol of vol and correlation with asset returns is constant
  - Use in-sample moments
  - Vol of vol = $4.9737e-4$
- Plot all terms that contribute to equity variance
Decomposition of Equity Variance: JPM

On average, LM term 12 times the size of vega terms