Discussion of
“Systemic Risk and Stability
in Financial Networks”
by Acemoglu, Ozdaglar, & Tahbaz-Salehi

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This Paper

- Provides a framework to think about how different financial networks may be more susceptible to systemic risk

- Two models:
  1. Exogenous Network Structure
  2. Endogenous Network Formation

- Two questions:
  1. How does the network structure affect financial stability?
  2. What externalities may occur in the equilibrium formation of networks?
An Exogenous Network Model of Contagion
A Financial Network Model of Contagion

- $n$ banks indexed by $j \in \{1, n\}$
- $y_{ij}$ is the amount bank $j$ owes bank $i$
- each bank also owes $v$ to an outside (senior) creditor
A Financial Network Model of Contagion

- $n$ banks indexed by $j \in \{1, n\}$
- $y_{ij}$ is the amount bank $j$ owes bank $i$
- each bank also owes $v$ to an outside (senior) creditor
- total liabilities of bank $j$

$$y_j + v \quad \text{where} \quad y_j = \sum_{i \neq j} y_{ij}$$

- The financial network is given by $\{y_{ij}\}$
- Restrict attention to “regular” networks:
  all banks have identical claims and liabilities

$$\forall i, \quad \sum_{j \neq i} y_{ij} = \sum_{j \neq i} y_{ji} = y \quad \text{for some } y$$
Examples: Ring Network and Complete Network

(a) The ring financial network

(b) The complete financial network
Shocks

- Bank $j$ receives returns $z_j \in \{a, a - \epsilon\}$
- $\epsilon \in (a - \nu, a)$ is a negative shock
  - $z_j = a$ if not hit with negative shock
  - $z_j \in (0, \nu)$ if hit with negative shock $\rightarrow$ will default
- $m$ banks get hit with negative shock. For now, let $m = 1$
Payments

- \( x_{js} \in [0, y_{js}] \) denotes repayment by bank \( s \) to bank \( j \)
- bank \( j \) total cash flow
  \[ z_j + \sum_{s \neq j} x_{js} \]
- bank \( j \) total liabilities
  \[ y_j + v \]
Seniority

\[ y(j) \]
\[ v \]

- Equity of bank \( j \)
- Bank creditors
- Outside investors

Cash flow

Figure:
Payment Equilibrium

- Equilibrium conditions for \( \{x_{ij}\} \)

\[
x_{ij} = \frac{y_{ij}}{y_j} \max \left\{ 0, \min \left\{ y_j, z_j - \nu + \sum_{s \neq j} x_{js} \right\} \right\}
\]

**Proposition**

*An equilibrium always exists and is (generically) unique*
In equilibrium, there are 3 types of banks:

- banks hit with the negative shock that default (zero equity)
- banks not hit with negative shock that default (zero equity)
- banks not hit with negative shock with positive equity
Measures of Financial Stability

• Stability
  \[
  \frac{1}{E[\#\text{defaults}]}
  \]

• Resilience
  \[
  \frac{1}{\max(\#\text{defaults})}
  \]

• these depend on \( m, \epsilon \), and the financial network structure

• let \( m = 1 \), consider these measures across different networks
Small Shock Regime

- Let $\epsilon^*$ denote some threshold

$$\epsilon^* = n(a - \nu)$$

total excess liquidity in the system

**Proposition**

Suppose $\epsilon < \epsilon^*$. Then $\exists \ y^*$ s.t. for $y > y^*$

(i) the ring network is the least resilient, least stable

(ii) the complete network is the most resilient, most stable
A Small Shock in the Ring Network

(a) The ring financial network
A Small Shock in the Ring Network

(a) The ring financial network
A Small Shock in the Complete Network

(b) The complete financial network
A Small Shock in the Complete Network

(b) The complete financial network
Large Shock Regime

- Let a $\delta$-connected network be one in which a collection $M \subset N$ has weak connections ($\delta$) to the rest of the network

Proposition

Suppose $\epsilon > \epsilon^*$ and $y > y^*$. Then

(i) the ring and complete networks are both the least resilient, least stable

(ii) for small enough $\delta$, any $\delta$-connected network
is strictly more stable, more resilient than the ring and complete
A Large Shock

(a) The ring financial network

(b) The complete financial network
Delta-Connected Network
A Large Shock in a Delta-Connected Network
Robust Yet Fragile

- Complete Network undergoes a phase transition
  - most stable → least stable
  - “Robust yet Fragile”

- If banks are not very connected, then clearly killing one bank with a large shock can’t propagate across system

- Does the Ring Network always have the most contagion?
Robust Yet Fragile

• Complete Network undergoes a phase transition
  • most stable $\rightarrow$ least stable
  • “Robust yet Fragile”

• If banks are not very connected, then clearly killing one bank with a large shock can’t propagate across system

• Does the Ring Network always have the most contagion?
  Answer: No
Multiple Shocks

Proposition

For $m > 1, \epsilon > \epsilon_m^*$ and intermediate values of $y$,

(i) the complete network is the least stable

(ii) the ring network is strictly more stable than the complete
Multiple Shocks in a Ring Network

(a) The ring financial network

(b) The ring financial network
Multiple Shocks in Ring: Alvarez-Barlevy (2013)

- For multiple shocks and $y$ not too large, senior creditors do more shock absorption in the ring than in the complete network.
  - The closer shocks are to each other, the larger the losses to senior creditors rather than to other banks.

- Alvarez-Barlevy (2013)
  - Consider $m$ random shocks in a Ring Network of $n$ banks.
  - Allocation is solution to discrete version of “circle covering problem”.
  - Well-studied geometric problem in applied probability (Stevens, 1939).
  - Bose-Einstein statistics $\rightarrow$ exact distribution of defaults.
What do we learn?

- Network structure clearly related to stability, resilience, systemic risk
- But can we say something systematic about network structure and contagion?

Empirical challenge for researchers/policy-makers
What do we learn?

- Network structure clearly related to stability, resilience, systemic risk.
- But can we say something systematic about network structure and contagion?
- This seems very difficult! Everything matters:
  - structure of the network
  - number and size of shocks ($m$ and $\epsilon$)
  - where these shocks are located
  - correlation of shocks (another component of systemic risk)
  - asymmetric networks?
- Must be careful in applying these results
- Empirical challenge for researchers/policy-makers
Welfare
Welfare Effects of Contagion

- All of the previous analysis can be done without considering welfare
  - Right now, welfare is independent of equilibrium allocation
  - Contagion only reshuffles resources across different claimants
Welfare Effects of Contagion

- In order for contagion to matter for welfare, you need some social surplus coming from positive equity banks.

- Suppose banks with positive equity receive long-term return $A$ but banks that default must prematurely liquidate project.

- Then welfare is decreasing in number of defaults:

\[ \text{Welfare} = (n - \#\text{defaults}) A + \text{const}. \]
Welfare Effects of Contagion

- In order for contagion to matter for welfare, you need some social surplus coming from positive equity banks.
- Suppose banks with positive equity receive long-term return $A$ but banks that default must prematurely liquidate project.
- Then welfare is decreasing in number of defaults.
  \[
  \text{Welfare} = (n - \text{#defaults}) A + \text{const}.
  \]
- Plausible assumption in order for contagion to be bad.
  \[\implies\text{Keeping as many banks as possible alive is good}\]
- Caveat: Suppose the banks closest to the bad shock bank also have bad investments/practices. Then welfare implications of these failing banks may be unclear.
Endogenous Network Formation
Endogenous Financial Network

- Now suppose banks make lending and borrowing decisions → endogenous formation of the financial network
- Is the equilibrium formation of networks efficient?
Endogenous Financial Network

• Now suppose banks make lending and borrowing decisions
  → endogenous formation of the financial network

• Is the equilibrium formation of networks efficient?

• The Game: take as given an exogenous opportunity network.
  1. All banks $i$ post interest rates $R_{ij}(\ell_{j1}, \ldots, \ell_{jn})$ for each $j$
  2. After observing posted contracts, each bank chooses contracts
  3. Given contracts, banks decide on how much to borrow $\ell_{ij}$

• Generates a financial network, with $y_{ij} = R_{ij}\ell_{ij}$
Summary of Main Results

- $R_{ij}(l_{j1}, \ldots, l_{jn})$ contingent on borrower’s lending behavior
- In 2-chain or 3-chain networks, the equilibrium is efficient
- Why? terms induce “right” behavior of borrower
Summary of Main Results

- $R_{ij}(\ell_{j1}, \ldots, \ell_{jn})$ contingent on borrower's lending behavior

- In 2-chain or 3-chain networks, the equilibrium is efficient
  - Why? terms induce “right” behavior of borrower

- If chains are longer, then equilibrium is not necessarily efficient
  - Externalities: over-lending, insufficiently dense networks

- Why? terms do not induce “right” behavior of borrower of borrower
Contracts and Incomplete Markets

- Why not contracts that are contingent on
  - who my borrower’s borrower lends to...
  - who my borrower’s borrower’s borrower lends to...
- Contracts contingent on the entire state of the network?
  - more contingencies to “complete the market”
- State-contingent debt (contingent on realization of shocks)?
Contracts and Incomplete Markets

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- Contracts contingent on the entire state of the network?
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- State-contingent debt (contingent on realization of shocks)?
- What are the underlying frictions that excludes these contracts?
  - Imperfect monitoring?
- Macro-prudential policy tools
  - capital requirements, leverage ratios
Conclusion

- Very Interesting Paper!
- Important work in understanding the relation between networks and systemic risk
- A lot of work to be done in this area, both theoretically and empirically
  - endogenous network formation
  - measuring/predicting systemic risk:
    - network structures, correlated shocks
  - how this should inform stress tests, macroprudential policy