Optimal Financial Transaction Taxes

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Motivation

- Should financial transactions be taxed?
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- Verbal arguments
  - Tobin 72
  - Stiglitz 89, Summers and Summers 89, Ross 89
- Little formal analysis
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- Little formal analysis
- This paper: welfare-theoretic analysis of transaction taxes
Preview: 3 roles of financial markets

Exchange
Economy

Fundamental Trading
Preview: 3 roles of financial markets

- Exchange
- Economy

Risk Sharing/Transfer
- Life Cycle
- Informed Trading

Fundamental Trading

[OBPS]

τ∗ > 0

"q-theory"

Tax/Subsidy τ∗ ≷ 0

Information aggregation ⇒ Irrelevance results

Information acquisition ⇒ Less information acquired

"Trading Costs and Informational Efficiency"
Davila/Parlatore (2015)
Preview: 3 roles of financial markets

Exchange Economy

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Fundamental Trading

Non-fundamental Trading

"q-theory"

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Tax $\tau^* > 0$

Tax/Subsidy $\tau^* \gtrless 0$

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Preview: 3 roles of financial markets

- Exchange
- Economy
- Risk Sharing/Transfer
  - Life Cycle
  - Informed Trading
- Fundamental Trading
- Non-fundamental Trading
  - Speculation/Betting
  - Belief Disagreement

Davila/Parlatore (2015)

"Trading Costs and Informational Efficiency"
Preview: 3 roles of financial markets

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Mathematical notation:

\[ \tau^* > 0 \]
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"Trading Costs and Informational Efficiency"

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Preview: 3 roles of financial markets

- Non-fundamental Trading
  - Tax: $\tau^* > 0$
- Fundamental Trading
  - Information aggregation $\Rightarrow$ Irrelevance results
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- Production
  - "q-theory"

[OBPS]
Preview: 3 roles of financial markets

- Exchange
- Economy
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Non-fundamental Trading

[OBPS] Tax $\tau^* > 0$

Fundamental Trading

Tax/Subsidy $\tau^* \geq 0$

Production

"q-theory"
Preview: 3 roles of financial markets

1. **Exchange**
   - Fundamental Trading
   - Risk Sharing/Transfer
   - Life Cycle
   - Informed Trading

2. **Speculation/Betting**
   - Non-fundamental Trading
   - Belief Disagreement

3. **Tax**
   - Taxation
   - Tax/Subsidy

Production

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Roadmap

1. Baseline model: static exchange economy
   - Positive analysis
   - Normative analysis (main results)
Roadmap

1. Baseline model: static exchange economy
   - Positive analysis
   - **Normative** analysis (**main results**)  
2. Extensions
   - Static model (several)
   - Dynamics
   - Production
3. Conclusion

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Baseline model: CARA-Normal (Lintner 69)

- Single trading stage, $t = \{1, 2\}$
- Distribution of investors $F(i)$ - CARA utility $A_i$
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- Single trading stage, \( t = \{1, 2\} \)
- Distribution of investors \( F(i) \) - CARA utility \( A_i \)
- Two assets - Unrestricted portfolios
  1. Riskless asset: Gross rate \( R = 1 \) - Elastic supply - Irrelevant endowment
  2. Risky asset: Price \( P_1 > 0 \) - Fixed supply \( Q \) - Pays dividend \( D \)
     - Initial position \( X_{0i} \) - Choose \( X_{1i} \)
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- (True) Dividend distribution
  $$D \sim N(\mathbb{E}[D], \text{Var}[D])$$
- Heterogeneous dogmatic beliefs (disagreement about means)
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- Stochastic endowment, $E_{2i}$, correlated with $D$, $\text{Cov}[E_{2i}, D] \neq 0$
- Four reasons to trade
  1. [Fundamental] Different hedging needs $\text{Cov}[E_{2i}, D]$
  2. [Fundamental] Different risk aversion $A_i$
  3. [Fundamental] Different initial conditions $X_{0i}$
  4. [Non-fundamental] Different beliefs $\mathbb{E}_i[D]$
Policy instrument

- **Linear anonymous tax**: single instrument
  - Paid by buyers and sellers on the dollar value of the transaction
  - Revenue: \(2\tau P_1 |\Delta X_{1i}|\)

- Assumption: No tax avoidance

- Lump-sum rebate: \(T_{1i} = \tau P_1 |\Delta X_{1i}|\) (for simplicity)

- Ex-ante lump-sum transfers (Kaldor/Hicks: focus on efficiency)

- CARA utility:
  \[U_i(W_{2i}) = -e^{-A_i W_{2i}}\max X_{1i} E_i[U_i(W_{2i})] \Rightarrow \max X_{1i} E_i[W_{2i}] - A_i^2 \text{Var}[W_{2i}]\]

- Return/Budget constraint:
  \[W_{2i} = E_{2i} + X_{1i}D + \left(X_0P_1 - X_{1i}P_1 - \tau P_1 |\Delta X_{1i}| + T_{1i}\right)\]
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  $$\max_{X_{1i}} \mathbb{E}_i [U_i (W_{2i})] \Rightarrow \max_{X_{1i}} \mathbb{E}_i [W_{2i}] - \frac{A_i}{2} \text{Var} [W_{2i}]$$
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\]

- Return/Budget constraint:

\[
W_{2i} = E_{2i} + X_{1i} D + \left( X_{0i} P_1 - X_{1i} P_1 - \tau P_1 |\Delta X_{1i}| + T_{1i} \right) \\
\text{Tax/Rebate}
\]
Investors’ problem: solution when $\tau = 0$

$$X_{1i} = \frac{\mathbb{E}_i[D] - A_i \text{Cov}[E_{2i}, D] - P_1}{A_i \text{Var}[D]}$$
Investors’ problem: Inaction + Dampening

\[ X_{1i} = \begin{cases} 
\frac{E_i[D] - A_i \text{Cov}[E_{2i}, D] - P_1(1+\tau)}{A_i \text{Var}[D]} ; & \Delta X_{1i} > 0 \quad \text{Buyer} \\
X_{0i} ; & \Delta X_{1i} = 0 \quad \text{No Trade} \\
\frac{E_i[D] - A_i \text{Cov}[E_{2i}, D] - P_1(1-\tau)}{A_i \text{Var}[D]} ; & \Delta X_{1i} < 0 \quad \text{Seller} 
\end{cases} \]
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X_{1i} = \begin{cases} 
\frac{\mathbb{E}_i[D] - A_i \text{Cov}[E_{2i}, D] - P_1 (1+\tau)}{A_i \text{Var}[D]} ; & \Delta X_{1i} > 0 \text{ Buyer} \\
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\end{cases}
\]

\[
\Delta X_{1i} \equiv X_{1i} - X_{0i} \text{ (Net Change in Asset Holdings)}
\]

\[
\Delta X_{1i} > 0 \text{ (Buyer)} \quad \Delta X_{1i} < 0 \text{ (Seller)}
\]
Investors’ problem: Inaction + Dampening

\[ X_{1i} = \begin{cases} \frac{E_i[D] - A_i \text{Cov}[E_{2i}, D] - P_1 (1 + \tau)}{A_i \text{Var}[D]} ; & \Delta X_{1i} > 0 \quad \text{Buyer} \\ X_{0i} ; & \Delta X_{1i} = 0 \quad \text{No Trade} \\ \frac{E_i[D] - A_i \text{Cov}[E_{2i}, D] - P_1 (1 - \tau)}{A_i \text{Var}[D]} ; & \Delta X_{1i} < 0 \quad \text{Seller} \end{cases} \]

\[ \Delta X_{1i} \equiv X_{1i} - X_{0i} \quad \text{(Net Change in Asset Holdings)} \]

- Convex problem
Equilibrium

- Standard equilibrium definition - Market clearing $\int X_1i \, dF(i) = Q$
Equilibrium

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- Equilibrium price $P_1$
Equilibrium

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Positive Results:

- **Lemma 1**: \( \frac{dP_1}{d\tau} \) can be positive/negative/zero
- **Lemma 2**: \( \frac{dX_{1i}}{d\tau} \) is negative for buyers (positive for sellers)

\[
\frac{dX_{1i}}{d\tau} = \frac{\partial X_{1i}}{\partial \tau} + \frac{\partial X_{1i}}{\partial P_1} \frac{dP_1}{d\tau}
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Equilibrium

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\]
- Aligned with empirical evidence
Normative analysis: Welfare criterion

- How to assess welfare with heterogeneous beliefs?
Normative analysis: Welfare criterion

• How to assess welfare with heterogeneous beliefs?
• My approach:
  1. Solve planner’s problem using a **single** belief $\mathbb{E}[D]$
Normative analysis: Welfare criterion

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  1. Solve planner’s problem using a **single** belief $\mathbb{E}[D]$
  2. Characterize conditions under which the solution to the planner’s problem is **independent** (!) of the belief chosen

Paternalism?

1. Philosophy - Does the planner respect investors’ beliefs? No
2. Constrained Efficiency - Does the planner need to know more than the investors? Not always. No informational advantage

• Different from welfare criteria papers (complementary)
  • BSX15: convex combination of beliefs
  • BCEST15: worst case scenarios over set of possible beliefs
  • GSS14: axiomatic

• Behavioral Welfare Economics
  • O’Donoghue-Rabin, Chetty, Farhi-Gabaix, Campbell 2016, etc
Normative analysis: Welfare criterion

- How to assess welfare with heterogeneous beliefs?
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Normative analysis: Planner’s problem

- Social welfare $V(\tau)$ - Pareto frontier - Welfare weights $\lambda_i$

\[
V(\tau) = \int \lambda_i V_i dF(i) \quad \text{with} \quad V_i \equiv \mathbb{E} [U_i (X_{1i})]
\]
Normative analysis: Planner’s problem

- Social welfare $V(\tau)$ - Pareto frontier - Welfare weights $\lambda_i$

$$V(\tau) = \int \lambda_i V_1 dF(i) \quad \text{with} \quad V_i \equiv \mathbb{E}[U_i(X_{1i})]$$

1. $X_{1i}$ chosen by investors
2. Planner uses $\mathbb{E}$, instead of $\mathbb{E}_i$
Marginal tax change

Proposition 1a: General case

\[
\frac{dV}{d\tau} = \int \lambda_i \frac{dV_i}{d\tau} dF (i)
\]
Marginal tax change

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\]

\[
\frac{dV_i}{d\tau} = \mathbb{E} \left[ U'_i(W_{2i}) \right] \quad \frac{d\hat{V}_i}{d\tau}
\]

- Expected Marginal Utility
- Change in Certainty Equivalent
Marginal tax change

Proposition 1a: General case

\[
\frac{dV}{d\tau} = \int \lambda_i \mathbb{E} \left[ U_i' (W_{2i}) \right] \frac{d\hat{V}_i}{d\tau} dF (i)
\]
Marginal tax change

Proposition 1a: General case

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\[
\frac{d\hat{V}_i}{d\tau} = \begin{bmatrix}
\mathbb{E} [D] - \mathbb{E}_i [D] + \text{Belief distortion} \\
\text{Fundamental distortion}
\end{bmatrix} \begin{bmatrix}
\frac{dX_{1i}}{d\tau} - \text{Terms-of-trade}
\end{bmatrix}
\]

\[
\text{sgn} (\Delta X_{1i}) P_1 \tau
\]
Marginal tax change

Proposition 1a: General case

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\[ \frac{d\hat{V}_i}{d\tau} = \begin{cases} \mathbb{E} [D] - \mathbb{E}_i [D] + \text{Belief distortion} & \text{Belief distortion} \\ \text{Fundamental distortion} & \text{Fundamental distortion} \\ \frac{dX_{1i}}{d\tau} - \Delta X_{1i} \frac{dP_1}{d\tau} & \text{Terms-of-trade} \end{cases} \]
Marginal tax change

Proposition 1a: General case

\[
\frac{dV}{d\tau} = \int \lambda_i \mathbb{E} \left[ U_i'(W_{2i}) \right] \frac{d\hat{V}_i}{d\tau} dF(i)
\]

\[
\frac{d\hat{V}_i}{d\tau} = \left[ \underbrace{\mathbb{E}[D] - \mathbb{E}_i[D]}_{\text{Belief distortion}} + \underbrace{\text{sgn}(\Delta X_{1i}) P_1 \tau}_{\text{Fundamental distortion}} \right] \frac{dX_{1i}}{d\tau} - \underbrace{\Delta X_{1i} \frac{dP_1}{d\tau}}_{\text{Terms-of-trade}}
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\end{bmatrix} \frac{dX_{1i}}{d\tau} - \Delta X_{1i} \frac{dP_1}{d\tau} \text{Terms-of-trade}
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Marginal tax change

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\]
Marginal tax change

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\]
Marginal tax change

**Proposition 1a: General case**

\[
\frac{dV}{d\tau} = \int \lambda_i \mathbb{E}[U_i'(W_{2i})] \left[ \mathbb{E}[D] - \mathbb{E}_i[D] + \text{sgn}(\Delta X_i) P_1 \right] \left. \frac{dX_{1i}}{d\tau} - \Delta X_i \frac{dP_1}{d\tau} \right| dF(i)
\]

- **Assumption [NR]: No Redistribution**

\[
\lambda_i \mathbb{E}[U_i'(W_{2i})] \text{ is constant } \forall i
\]
Marginal tax change

Proposition 1a: General case

\[ \frac{dV}{d\tau} = \int \lambda_i \mathbb{E} \left[ U_i' (W_{2i}) \right] \left\{ \mathbb{E} [D] - \mathbb{E}_i [D] + \text{sgn} (\Delta X_{1i}) P_1 \tau \right\} dX_{1i} \frac{d}{d\tau} - \Delta X_{1i} \frac{dP_1}{d\tau} \right\} dF (i) \]

\[ \text{Welfare weight} \]
\[ \text{Belief distortion} \]
\[ \text{Fundamental distortion} \]
\[ \text{Terms-of-trade} \]

- **Assumption [NR]: No Redistribution**

\[ \lambda_i \mathbb{E} \left[ U_i' (W_{2i}) \right] \text{ is constant } \forall i \]

- Kaldor/Hicks efficiency ⇒ Quasilinearity
- Maximization of certainty equivalents (ex-ante transfers)
Marginal tax change

Proposition 1b: [NR] holds

When [NR] holds:

\[
\frac{dV}{d\tau} = \int \left[ \left( \mathbb{E} [D] - \mathbb{E}_i [D] \right) + \text{sgn} (\Delta X_{1i}) P_1 \tau \right] \frac{dX_{1i}}{d\tau} - \Delta X_{1i} \frac{dP_1}{d\tau} \right] \right] dF (i)
\]
Marginal tax change

Proposition 1b: [NR] holds

When [NR] holds:

\[
\frac{dV}{d\tau} = \int \left[ (\mathbb{E}[D] - \mathbb{E}_i[D]) + \text{sgn}(\Delta X_{1i}) P_1 \tau \frac{dX_{1i}}{d\tau} - \Delta X_{1i} \frac{dP_1}{d\tau} \right] dF(i)
\]

- Market clearing implies

\[
\int \Delta X_{1i} dF(i) = 0
\]
Marginal tax change

**Proposition 1b: [NR] holds**

When [NR] holds:

\[
\frac{dV}{d\tau} = \int \left[ (\mathbb{E} [D] - \mathbb{E}_i [D]) + \text{sgn} (\Delta X_{1i}) P_1 \tau \frac{dX_{1i}}{d\tau} \right] dF (i)
\]

- **Terms-of-trade** drop out
Marginal tax change

Proposition 1b: [NR] holds

When [NR] holds:

\[
\frac{dV}{d\tau} = \int \left[ (\mathbb{E}[D] - \mathbb{E}_i[D]) + \text{sgn}(\Delta X_{1i}) P_1 \tau \frac{dX_{1i}}{d\tau} \right] dF(i)
\]

- Market clearing implies

\[
\int \frac{dX_{1i}}{d\tau} dF(i) = 0
\]
Marginal tax change

Proposition 1b: [NR] holds

When [NR] holds:

\[
\frac{dV}{d\tau} = \int \left[ -E_i [D] + \text{sgn} (\Delta X_{1i}) P_1 \tau \right] \frac{dX_{1i}}{d\tau} dF (i)
\]

- Planner’s belief drops out
  - Identical optimal policy for any belief (!)
  - Consistency
Marginal tax change

Proposition 1b: [NR] holds

When [NR] holds:

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\frac{dV}{d\tau} = \int \left[ -E_i[D] + \text{sgn}(\Delta X_{1i}) P_1\tau \right] \frac{dX_{1i}}{d\tau} dF(i)
\]

- **Planner’s belief** drops out
  - Identical optimal policy for any belief (!)
  - Consistency

- **Key assumptions**
  1. No redistribution
  2. Fixed supply
Marginal tax change

Proposition 1c: Sign around $\tau = 0$

When [NR] holds:

$$\frac{dV}{d\tau} = \int \left[ -E_i[D] + \text{sgn}(\Delta X_{1i}) P_1 \tau \right] \frac{dX_{1i}}{d\tau} \, dF(i)$$
Marginal tax change

**Proposition 1c: Sign around** $\tau = 0$

When $[\text{NR}]$ holds:

$$\frac{dV}{d\tau} \bigg|_{\tau=0} = \int -\mathbb{E}_i[D] \frac{dX_{1i}}{d\tau} dF(i)$$

- **Assumption $[\text{OBPS}]$:** Optimists Buyers/Pessimists Sellers
  $$\text{Cov}_F(E_i[D], dX_{1i} \bigg|_{\tau=0}) < 0$$

- Two justifications: disagreement drives trading
  1. Theoretical
     $$X_{1i} = f_1(E_i[D]) + f_2(\text{Cov}[E_2i, D], A_i, X_{0i})$$
  2. Empirical
Marginal tax change

Proposition 1c: Sign around $\tau = 0$

When [NR] holds:

$$\left. \frac{dV}{d\tau} \right|_{\tau=0} = \int -E_i[D] \left. \frac{dX_{1i}}{d\tau} dF (i) \right. = -\text{Cov}_F \left( E_i[D], \left. \frac{dX_{1i}}{d\tau} \right|_{\tau=0} \right)$$
Marginal tax change

**Proposition 1c: Sign around \( \tau = 0 \)**

When \([\text{NR}]\) holds:

\[
\left. \frac{dV}{d\tau} \right|_{\tau=0} = \int -\mathbb{E}_i [D] \frac{dX_{1i}}{d\tau} dF(i) = -\text{Cov}_F \left( \mathbb{E}_i [D], \left. \frac{dX_{1i}}{d\tau} \right|_{\tau=0} \right)
\]

- **Assumption [OBPS]: Optimists Buyers/Pessimists Sellers**

\[
\text{Cov}_F \left( \mathbb{E}_i [D], \left. \frac{dX_{1i}}{d\tau} \right|_{\tau=0} \right) < 0
\]
Marginal tax change

**Proposition 1c: Sign around \( \tau = 0 \)**

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- **Two justifications: disagreement drives trading**
  1. **Theoretical** - \( X_{1i} = f_1(E_i[D]) + f_2(\text{Cov}[E_{2i}, D], A_i, X_{0i}) \)
  2. **Empirical**
Marginal tax change

Proposition 1c: Sign around $\tau = 0$

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$$\left. \frac{dV}{d\tau} \right|_{\tau=0} = \int -E_i[D] \frac{dX_{1i}}{d\tau} dF(i) = -\text{Cov}_F \left( E_i[D], \left. \frac{dX_{1i}}{d\tau} \right|_{\tau=0} \right)$$

- Assumption [OBPS]: Optimists Buyers/Pessimists Sellers

$$\text{Cov}_F \left( E_i[D], \left. \frac{dX_{1i}}{d\tau} \right|_{\tau=0} \right) < 0$$
Marginal tax change

Proposition 1c: Sign around $\tau = 0$

When [NR] and [OBPS] hold:

$$\frac{dV}{d\tau} \bigg|_{\tau=0} > 0$$
Marginal tax change

**Proposition 1c: Sign around $\tau = 0$**

When $[\text{NR}]$ and $[\text{OBPS}]$ hold:

\[
\left. \frac{dV}{d\tau} \right|_{\tau=0} > 0
\]

- Intuition: start from $\tau = 0$, $\uparrow \tau \Rightarrow$ Less trading
  - Less Fundamental trading (2nd order loss)
  - Less Non-fundamental trading (1st order gain)
Marginal tax change

**Proposition 1c: Sign around \( \tau = 0 \)**

When [NR] and [OBPS] hold:

\[
\left. \frac{dV}{d\tau} \right|_{\tau=0} > 0
\]

- Intuition: start from \( \tau = 0 \), \( \uparrow \tau \Rightarrow \) Less trading
  - Less Fundamental trading (2nd order loss)
  - Less Non-fundamental trading (1st order gain)
- \( \tau^* \) can be negative (a subsidy) if [OBPS] doesn’t hold
Optimal tax $\tau^*$

Proposition 2: Optimal tax

1. When [NR] and [OBPS] hold, $\tau^* > 0$
Optimal tax $\tau^*$

### Proposition 2: Optimal tax

1. When [NR] and [OBPS] hold, $\tau^* > 0$
2. When [NR] holds, the optimal tax is given by:

$$\tau^* = \frac{\Omega_B - \Omega_S}{2}$$
Proposition 2: Optimal tax

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- With $\Omega_B \equiv \int_{i \in B} \omega_i^B \frac{E_i[D]}{P_1} dF(i)$, equivalently $\Omega_S$
Proposition 2: Optimal tax

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- **Sufficient statistics**
  1. Beliefs (Pigovian principle)
  2. Equilibrium portfolio derivatives (drop out with symmetry)
Proposition 2: Optimal tax

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- **Sufficient statistics**
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- Alternative implementation: volume as intermediate target
Optimal tax $\tau^*$

**Proposition 2: Optimal tax**

1. When [NR] and [OBPS] hold, $\tau^* > 0$
2. When [NR] holds, the **optimal tax** is given by:

$$\tau^* = \frac{\Omega_B - \Omega_S}{2}$$

- With $\Omega_B \equiv \int_{i \in \mathcal{B}} \omega_i B \mathbb{E}_i[D] dF(i)$, equivalently $\Omega_S$

- **Sufficient statistics**
  1. Beliefs (Pigovian principle)
  2. Equilibrium portfolio derivatives (drop out with symmetry)

- Alternative implementation: volume as intermediate target
- Measurement
  - *Recovering heterogenous beliefs*, Borovicka/Davila (in progress)
Numerical examples

1. **First Example**: Only disagreement trading
2. **Second Example**: All cases
Numerical examples

1. **First Example**: Only disagreement trading

2. **Second Example**: All cases

   - **Simplifications**
     - Identical risk aversion $A_i = 1$
     - Identical initial positions $X_{0i} = Q = 1$

   - **Trading motives**
     - [Non-fundamental] Different beliefs $E_i[D]$
     - [Fundamental] Different risk-sharing needs $Cov[E_{2i}, D]$
Example 1: Only disagreement trading

\[ \mathbb{E} [D] = 100 \text{ and } \text{Var} [D] = 16 \]
Example 1: Only disagreement trading

\[ \mathbb{E}[D] = 100 \text{ and } \text{Var}[D] = 16 \]

- Optimists
  \[ \mathbb{E}_H[D] = 106 \quad \text{Cov}[E_{2i}, D] = 0 \quad \text{Optimistic Buyers (50\%)} \]

- Pessimists
  \[ \mathbb{E}_L[D] = 96 \quad \text{Cov}[E_{2i}, D] = 0 \quad \text{Pessimistic Sellers (50\%)} \]
Example 1: Only non-fundamental trading - $\tau^* = 5.98\%$ - Gain $0.86\%$
Example 3: Optimists/Pessimists/Fundamental investors

\[ \mathbb{E} [D] = 100 \text{ and } \operatorname{Var} [D] = 16 \]
Example 3: Optimists/Pessimists/Fundamental investors

\[ \mathbb{E}[D] = 100 \text{ and } \text{Var}[D] = 16 \]

Optimists
\[ \mathbb{E}_H[D] = 106 \]

Correct
\[ \mathbb{E}[D] = 100 \]

Pessimists
\[ \mathbb{E}_L[D] = 96 \]
Example 3: Optimists/Pessimists/Fundamental investors

$$\mathbb{E}[D] = 100$$ and $$\text{Var}[D] = 16$$

---

Optimists

$$\mathbb{E}_H[D] = 106$$

$$\text{Cov}[E_{2i}, D] = 0$$  Optimistic Buyers (30%)

Correct

$$\mathbb{E}[D] = 100$$

Pessimists

$$\mathbb{E}_L[D] = 96$$
Example 3: Optimists/Pessimists/Fundamental investors

\[ \mathbb{E}[D] = 100 \text{ and } \text{Var}[D] = 16 \]

---

Optimists
\[ \mathbb{E}_H[D] = 106 \]
\[ \text{Cov}[E_{2i}, D] = 0 \text{ Optimistic Buyers (30\%)} \]

Correct
\[ \mathbb{E}[D] = 100 \]

Pessimists
\[ \mathbb{E}_L[D] = 96 \]
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Example 3: Optimists/Pessimists/Fundamental investors

\[ \mathbb{E} [D] = 100 \text{ and } \text{Var} [D] = 16 \]

---

Optimists
\[ \mathbb{E}_H [D] = 106 \]
\[ \text{Cov} [E_{2i}, D] = 0 \quad \text{Optimistic Buyers (30\%)} \]

Correct
\[ \mathbb{E} [D] = 100 \]
\[ \text{Cov} [E_{2i}, D] < 0 \quad \text{Correct Buyers (20\%)} \]

Pessimists
\[ \mathbb{E}_L [D] = 96 \]
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  - \( \mathbb{E}_{H}[D] = 106 \)
  - \( \text{Cov}[E_{2i}, D] = 0 \) **Optimistic Buyers** (30%)
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Example 3: Optimists/Pessimists/Fundamental investors

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  - \( \text{Cov}[E_{2i}, D] > 0 \) **Optimistic Sellers** (20%)

- **Correct**
  \[ \mathbb{E}[D] = 100 \]
  - \( \text{Cov}[E_{2i}, D] = 0 \) **Pessimistic Sellers** (30%)

- **Pessimists**
  \[ \mathbb{E}_L[D] = 96 \]
  - \( \text{Cov}[E_{2i}, D] = 0 \) **Pessimistic Sellers** (30%)
Example 3: 35% Non-fundamental trading - $\tau^* = 2.01\%$ - Gain 0.11\%
Remarks

\[ \tau^* = \frac{\int \mathbb{E}_i[D] \frac{dX_{1i}}{d\tau} dF(i)}{\int \text{sgn}(\Delta X_{1i}) \frac{dX_{1i}}{d\tau} dF(i)} = \frac{\Omega_B - \Omega_S}{2} \]

- **Remark 1:** Investors who stop trading are *inframarginal* for \( \tau^* \)
  - **Meaningful non-convexity**

- **Remark 2:** Harberger 64 revisited (money metric respecting beliefs)

- **Upper bound on welfare loss:**
  \[ L(\tau) = 2 \tau P_1 \int_{i \in B} dX_{1i} d\tau dF(i) \]

- **Remark 3:** Allocation changes (volume) determine social welfare
  - Intuition: price changes are only redistributional

- **Remark 4:** Derivatives \( \frac{dX_{1i}}{d\tau} \) appear because of second-best problem

- **Diamond 73**
Remarks

\[ \tau^* = \frac{\int \frac{\mathbb{E}_i[D]}{P_1} \frac{dX_{1i}}{d\tau} dF(i)}{\int \text{sgn}(\Delta X_{1i}) \frac{dX_{1i}}{d\tau} dF(i)} = \frac{\Omega_B - \Omega_S}{2} \]

- **Remark 1:** Investors who stop trading are *infra marginal* for \( \tau^* 
  - **Meaningful non-convexity** [Figure]

- **Remark 2:** Harberger 64 revisited (money metric respecting beliefs)
  - Upper bound on welfare loss: \( \mathcal{L}(\tau) = 2\tau P_1 \int_{i \in \mathcal{B}} \frac{dX_{1i}}{d\tau} dF(i) \)
  - Change in volume

- **Remark 3:** Allocation changes (volume) determine social welfare
  - Intuition: price changes are only redistributional

- **Remark 4:** Derivatives \( \frac{dX_{1i}}{d\tau} \) appear because of second-best problem
  - Diamond 73
Remarks

\[
\tau^* = \frac{\int \mathbb{E}_i[D] \frac{dX_{1i}}{d\tau} dF(i)}{\int \text{sgn}(\Delta X_{1i}) \frac{dX_{1i}}{d\tau} dF(i)} = \frac{\Omega_B - \Omega_S}{2}
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  - Diamond 73
Remarks

\[ \tau^* = \frac{\int \mathbb{E}_i[D] \frac{dX_{1i}}{d\tau} dF(i)}{\int \text{sgn}(\Delta X_{1i}) \frac{dX_{1i}}{d\tau} dF(i)} = \frac{\Omega_B - \Omega_S}{2} \]

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• **Remark 3:** Allocation changes (volume) determine social welfare
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  • Diamond 73
Extensions

1. **Multiple \((J)\) risky assets:** weighted average

\[
\tau^* = \sum_{j=1}^{J} \omega_j \tau_j^*
\]
Extensions

1. **Multiple ($J$) risky assets:** weighted average

   $$\tau^* = \sum_{j=1}^{J} \omega_j \tau_j^*$$

2. **Preexisting trading costs:** $\tau^*$ formula unchanged, as long as they are a compensation for the use of economic resources
Extensions

1. **Multiple \( (J) \) risky assets**: weighted average
\[
\tau^* = \sum_{j=1}^{J} \omega_j \tau_j^*
\]

2. **Preexisting trading costs**: \( \tau^* \) formula unchanged, as long as they are a compensation for the use of economic resources

3. **Portfolio constraints**: modeled as \( g(P_1) \leq X_{1i} \leq \bar{g}(P_1) \)
   - \( \tau^* \) formula unchanged if price independent (**short-sale constraints**)
   - Corrected formula if price dependent (**borrowing constraints**)
1. **Multiple \((J)\) risky assets:** weighted average

\[\tau^* = \sum_{j=1}^{J} \omega_j \tau_j^*\]

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   - \(\tau^*\) formula unchanged if price independent (**short-sale constraints**)
   - Corrected formula if price dependent (**borrowing constraints**)

4. **Asymmetric taxes/Multiple instruments:**
   - First-best requires investor specific taxes:

\[\tau_i^* = \text{sgn} (\Delta X_{1i}) \frac{F - \mathbb{E}_i [D]}{P_1}, \quad F \in \mathbb{R}\]
Extension: Dynamics

- **General dynamic model**: arbitrary utility/general disagreement
Extension: Dynamics

- General dynamic model: arbitrary utility/general disagreement
  1. Approximation
    - With constant marginal utility, the optimal CARA + Normal tax is recovered (Arrow-Pratt)
Extension: Dynamics

- **General dynamic model**: arbitrary utility/general disagreement
  1. **Approximation**
     - With constant marginal utility, the optimal CARA+Normal tax is recovered (Arrow-Pratt)
  2. **High frequency investors are more affected**
     - Intuition: forward-looking behavior
     - Dynamics only modifies weights
2. **Production**: new first-order effect
   - Introduce producer who can vary $S_{1k}$
   - New decision: how many trees to plant
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   - Introduce producer who can vary $S_{1k}$
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\int \left( \mathbb{E} [D] - \mathbb{E}_i [D] \right) \frac{dX_{1i}}{d\tau} dF (i) = - \text{Cov}_F \left[ \mathbb{E}_i [D], \frac{dX_{1i}}{d\tau} \right] \quad + \quad \left( \mathbb{E} [D] - \mathbb{E}_F \mathbb{E}_i [D] \right) \frac{dS_{1k}}{d\tau}
\]

- Belief dispersion
- Aggregate distortion $\times$ Investment response
2. **Production**: new first-order effect

- Introduce producer who can vary $S_{1k}$
- New decision: how many trees to plant

$$
\int (\mathbb{E}[D] - \mathbb{E}_i[D]) \frac{dX_{1i}}{d\tau} dF(i) = - \text{Cov}_F \left[ \mathbb{E}_i[D], \frac{dX_{1i}}{d\tau} \right] + (\mathbb{E}[D] - \mathbb{E}_F[\mathbb{E}_i[D]]) \frac{dS_{1k}}{d\tau}
$$

**Belief dispersion**

**Aggregate distortion \times Investment response**
2. **Production**: new first-order effect
   - Introduce producer who can vary $S_{1k}$
   - New decision: how many trees to plant

\[
\int (\mathbb{E}[D] - \mathbb{E}_i[D]) \frac{dX_{1i}}{d\tau} dF(i) = -\text{Cov}_F \left[ \mathbb{E}_i[D], \frac{dX_{1i}}{d\tau} \right] + (\mathbb{E}[D] - \mathbb{E}_F[\mathbb{E}_i[D]]) \frac{dS_{1k}}{d\tau}
\]

Belief dispersion

Aggregate distortion $\times$

Investment response
Broader price-theoretic agenda in Macro-Finance

• Three examples
  1. Optimal Bankruptcy Exemptions
     • How much should bankrupt borrowers keep?
  2. Optimal Deposit Insurance
     • What is the optimal level of deposit insurance coverage?
  3. Fire Sales Externalities
     • Which variables determine welfare losses associated with price changes in economies with financial frictions?
Conclusion

• **This paper:** microfounded welfare analysis of FTT
  • Different welfare effects on fundamental vs non-fundamental trading
  • Irrelevance of planner’s belief
Conclusion

- **This paper:** microfounded welfare analysis of FTT
  - Different welfare effects on fundamental vs non-fundamental trading
  - Irrelevance of planner’s belief

- **Practical implications:**
  1. **Belief dispersion:** force towards **positive tax**
  2. With **production:** wrong average beliefs needed
1. **Tobin Tax:**
   - **Proposals:** Tobin 72/78, Summers-Summers 89, Stiglitz 89
   - **Empirical:** Campbell-Froot 94, Habermeier-Kirilenko 03
   - **Theory:** Subrahmanyam 98, Dow-Rahi 00, Buss et al 14, Adam et al 14


3. **Belief Disagreement:** Lintner 69, Miller 77, Harrison-Kreps 78, Scheinkman-Xiong 03, Hong-Stein 03, Geanakoplos 09, Simsek 12,13

4. **Behavioral Welfare Economics:**
   - **Belief Disagreement:** Morris 95, *Brunnermeier-Simsek-Xiong 12*, Blume-Cogley-Easley-Sargent-Tsyrennikov 13
   - **O’Donoghue-Rabin 06, Bernheim-Rangel 11**

5. **Information Diffusion/Acquisition:** Grossman-Stiglitz 80, Diamond-Verrechia 81, many others
Identifying beliefs - Volume as intermediate target

• Volume decomposition:

\[ \int_{i \in B} \Delta X_{1i} dF (i) = \Theta_F + \Theta_{NF} - \Theta_T, \]

- Volume
- Fundamental
- Non-fundamental
- Tax induced reduction
Identifying beliefs - Volume as intermediate target

• Volume decomposition:

\[
\int_{i \in B} \Delta X_1 i dF(i) = \underbrace{\Theta_F}_{\text{Volume}} + \underbrace{\Theta_{NF}}_{\text{Fundamental}} - \underbrace{\Theta_{\tau}}_{\text{Non-fundamental}} - \underbrace{\Theta_{\tau}}_{\text{Tax induced reduction}}
\]

• Alternative implementation (conditions needed):

\[
\Phi_{NF} = \Phi_{\tau} \iff \tau^*
\]
Identifying beliefs - Volume as intermediate target

- Volume decomposition:

\[ \int_{i \in B} \Delta X_1 dF(i) = \Theta_F + \Theta_{NF} - \Theta_{\tau} \]

- Fundamental, Non-fundamental, Tax induced reduction

- Alternative implementation (conditions needed):

\[ \Phi_{NF} = \Phi_{\tau} \iff \tau^* \]
Identifying beliefs - Volume as intermediate target

- Fundamental

\[
2\Omega_F = \int_S \left( \frac{\text{Cov}[E_{2i}, D]}{\text{Var}[D]} + X_{0i} + \frac{P_1}{A_i \text{Var}[D]} \right) dF(i)
- \int_B \left( \frac{\text{Cov}[E_{2i}, D]}{\text{Var}[D]} + X_{0i} + \frac{P_1}{A_i \text{Var}[D]} \right) dF(i)
\]
Identifying beliefs - Volume as intermediate target

- **Fundamental**
  \[
  2\Omega_F = \int_S \left( \frac{\text{Cov}[E_{2i}, D]}{\text{Var}[D]} + X_{0i} + \frac{P_1}{A_i \text{Var}[D]} \right) dF(i)
  \]
  \[
  - \int_B \left( \frac{\text{Cov}[E_{2i}, D]}{\text{Var}[D]} + X_{0i} + \frac{P_1}{A_i \text{Var}[D]} \right) dF(i)
  \]

- **Non-Fundamental**
  \[
  2\Omega_{NF} = \int_B \frac{\mathbb{E}_i[D]}{A_i \text{Var}[D]} dF(i) - \int_S \frac{\mathbb{E}_i[D]}{A_i \text{Var}[D]} dF(i)
  \]
Identifying beliefs - Volume as intermediate target

- **Fundamental**

\[
2\Omega_F = \int_S \left( \frac{\text{Cov} [E_{2i}, D]}{\text{Var} [D]} + X_0 + \frac{P_1}{A_i \text{Var} [D]} \right) dF (i) \\
- \int_B \left( \frac{\text{Cov} [E_{2i}, D]}{\text{Var} [D]} + X_0 + \frac{P_1}{A_i \text{Var} [D]} \right) dF (i)
\]

- **Non-Fundamental**

\[
2\Omega_{NF} = \int_B \frac{\mathbb{E}_i [D]}{A_i \text{Var} [D]} dF (i) - \int_S \frac{\mathbb{E}_i [D]}{A_i \text{Var} [D]} dF (i)
\]

- **Tax**

\[
2\Omega_{\tau} = \tau \left( \int_B \frac{P_1}{A_i \text{Var} [D]} dF (i) + \int_S \frac{P_1}{A_i \text{Var} [D]} dF (i) \right)
\]
General dynamic model

Environment

• Investors solve

\[
\max_{C_{ti}, X_{ti}, Y_{ti}} \mathbb{E}_i \left[ \sum_{t=1}^{T} \beta^{t-1} U_i (C_{ti}) \right]
\]
General dynamic model

Environment

- Investors solve

\[
\max_{C_{ti}, X_{ti}, Y_{ti}} \mathbb{E}_i \left[ \sum_{t=1}^{T} \beta^{t-1} U_i (C_{ti}) \right]
\]

- Subject to

\[
C_{ti} = E_{ti} + X_{t-1i} (P_t + D_t) - X_{ti} P_t - \tau P_t |\Delta X_{ti}| + T_{ti} + RY_{t-1i} - Y_{ti}
\]

Beliefs:

- \(Z_{ti}\) ≡ Radon-Nikodym at each node/state
  - Can be stochastic but cannot depend on endogenous variables
  - \(E_{ti}\) and \(D_t\) arbitrary distributions

Planner

- Single linear tax - Commitment
  - No need to solve (hard) problem
General dynamic model

Environment

• Investors solve

\[
\max_{C_{ti}, X_{ti}, Y_{ti}} \mathbb{E}_i \left[ \sum_{t=1}^{T} \beta^{t-1} U_i (C_{ti}) \right]
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General dynamic model

Environment

• Investors solve

$$\max_{C_{ti}, X_{ti}, Y_{ti}} \mathbb{E}_i \left[ \sum_{t=1}^{T} \beta^{t-1} U_i (C_{ti}) \right]$$

• Subject to

$$C_{ti} = E_{ti} + X_{t-1i} (P_t + D_t) - X_{ti} P_t - \tau P_t |\Delta X_{ti}| + T_{ti} + RY_{t-1i} - Y_{ti}$$

• Beliefs: $Z_{ti} \equiv$ Radon-Nikodym at each node/state
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General dynamic model

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\[
\max_{C_{ti}, X_{ti}, Y_{ti}} \mathbb{E}_i \left[ \sum_{t=1}^{T} \beta^{t-1} U_i (C_{ti}) \right]
\]

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\[
C_{ti} = E_{ti} + X_{t-1i} (P_t + D_t) - X_{ti} P_t - \tau P_t |\Delta X_{ti}| + T_{ti} + RY_{t-1i} - Y_{ti}
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General dynamic model: Three takeaways

1. **Approximation**: When marginal utilities are constant, the optimal CARA+Normal tax is recovered
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\[
\tau^* = \frac{\mathbb{E}\left[\sum_{t=1}^{T} \beta^t \int \mathbb{E}_{ti} \left[ D_{t+1} + P_{t+1} \right] \frac{dX_{ti}}{d\tau} dF (i) \right]}{\mathbb{E}\left[\sum_{t=1}^{T} \beta^t \int P_t \text{sgn} (\Delta X_{ti}) (1 - \kappa_{ti}) \frac{dX_{ti}}{d\tau} dF (i) \right]}
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   - Truth is needed to weight nodes
1. **Approximation**: When marginal utilities are constant, the optimal CARA+Normal tax is recovered

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- \( \kappa_{ti} \equiv \mathbb{E}_t \left[ \frac{P_{t+1}}{P_t} \text{sgn} (\Delta X_{ti}) \text{sgn} (\Delta X_{t+1i}) \right] \) (Forward looking)
General dynamic model: Three takeaways

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General dynamic model: Three takeaways

1. **Approximation**: When marginal utilities are constant, the optimal CARA+Normal tax is recovered
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2. **Lower tax with flip-floppers**: the optimal tax can be written as

   \[
   \tau^* = \sum_{t=1}^{T} \mathbb{E} [w_t f_t \tau_t^*]
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   - Intuition: tax more powerful with forward-looking investors
     - Larger tax needed to correct for persistent disagreement (ineffective for bubbles)
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   \[ \tau_{\text{dynamic}} \approx \frac{1}{2} \tau_{\text{static}} \]
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   - Tobin/Keynes - Price volatility - **Wrong** argument
   - Dynamic Harberger / Incomplete markets / Production
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Three takeaways: covariance

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3. **Price covariance matters/not variance**
   - Tobin/Keynes - Price volatility - **Wrong** argument

\[ \tau^* = \sum_{t=1}^{T} \mathbb{E}\left[ \mathbb{E}\left[ Z_i U'_i(C_t) \Delta X_t \right] \right] \frac{dP_t}{d\tau} \]

\[ \tau^* \text{ is positive when } \text{Cov\left[ \text{Cov}\left[ F_i Z_i U'_i(C_t), \Delta X_t \right] dP_t d\tau \right] < 0 \]

Hard to disentangle insurance from redistribution
Three takeaways: covariance

3. Price covariance matters/not variance

- Tobin/Keynes - Price volatility - \textbf{Wrong} argument
- Incomplete markets
- Dynamic Harberger - Assume $\lambda_i = 1$ and $\beta = 1$

$$
\tau^* = \frac{\sum_{t=1}^{T} \mathbb{E} \left[ \mathbb{E}_F \left[ Z_i U_i' (C_{ti}) \Delta X_{ti} \right] \frac{dP_t}{d\tau} \right]}{\sum_{t=1}^{T} \mathbb{E} \left[ \mathbb{E}_F \left[ \int \xi_{ti} \frac{dX_{ti}}{d\tau} \right] \right]}
$$
Three takeaways: covariance

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$$

- $\tau^*$ is positive when

$$
\text{Cov} \left[ \text{Cov}_F \left[ Z_i U'_i (C_{ti}), \Delta X_{ti} \right], \frac{dP_t}{d\tau} \right] < 0
$$
Three takeaways: covariance

3. Price covariance matters/not variance

- Tobin/Keynes - Price volatility - **Wrong** argument
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$$
\tau^* = \frac{\sum_{t=1}^{T} E \left[ E_F \left[ Z_i u_i' (C_{ti}) \Delta X_{ti} \right] \frac{dP_t}{d\tau} \right]}{\sum_{t=1}^{T} E \left[ E_F \left[ \int \xi_{ti} \frac{dX_{ti}}{d\tau} \right] \right]}
$$

- $\tau^*$ is positive when

$$
\text{Cov} \left[ \text{Cov}_F \left[ Z_i u_i' (C_{ti}) , \Delta X_{ti} \right] , \frac{dP_t}{d\tau} \right] < 0
$$

- Hard to disentangle insurance from redistribution
Production (q-theory)

- **Producers** - indexed by $k$

\[
\text{Output: } D(Q + S_1^k)
\]

\[
\text{Solve max } C_1^k, C_2^k, S_1^k \quad \text{s.t.} \quad C_1^k + C_2^k = E_1^k + E_2^k + P_s S_1^k - \Phi(S_1^k)
\]

\[
\text{Production} \quad \text{Optimality conditions}
\]

\[
U_k'(C_1^k) = E[U_k'(C_2^k)]
\]

\[
\text{Euler} \quad P_s = \Phi'(S_1^k)
\]

Supply

\[
\text{Social Welfare} \quad V(\tau) = \int \lambda_i V_i dF(i) + \lambda_k V_k dV_k d\tau = E[U_k'(C_2^k)] dP_1 d\tau S_1^k
\]

\[
\text{Only terms-of-trade}
\]
Production (q-theory)

- **Producers** - indexed by $k$
  - Produce shares (trees) at a convex cost: $\Phi(S_{1k})$
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  - Solve
    \[
    \max_{C_{1k}, C_{2k}, S_{1k}} U_k(C_{1k}) + \mathbb{E}[U_k(C_{2k})]
    \]
Production (q-theory)

- **Producers** - indexed by \( k \)
  - Produce shares (trees) at a convex cost: \( \Phi (S_{1k}) \)
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    \[
    \max_{C_{1k}, C_{2k}, S_{1k}} U_k (C_{1k}) + \mathbb{E} [U_k (C_{2k})]
    \]

    \[
    \text{s.t. } C_{1k} + C_{2k} = E_{1k} + E_{2k} + P^s_{1} S_{1k} - \Phi (S_{1k})
    \]

- **Supply**
  - Social Welfare
    
    \[
    V (\tau) = \int \lambda_i V_i dF (i) + \lambda_k V_k dV_k d\tau = \mathbb{E} [U_k (C_{2k})] dP^1 d\tau S_{1k}
    \]

- **Only terms-of-trade
Production (q-theory)

- **Producers** - indexed by $k$
  - Produce shares (trees) at a convex cost: $\Phi(S_{1k})$
  - Output: $D(Q + S_{1k})$
  - Solve
    \[
    \max_{C_{1k}, C_{2k}, S_{1k}} \quad U_k(C_{1k}) + \mathbb{E}[U_k(C_{2k})]
    \]
    \[\text{s.t. } C_{1k} + C_{2k} = E_{1k} + E_{2k} + P_s S_{1k} - \Phi(S_{1k})\]

- **Optimality conditions**
  \[U'_k(C_{1k}) = \mathbb{E}[U'_k(C_{2k})] \quad \text{Euler} \]
  \[P_s = \Phi'(S_{1k}) \quad \text{Supply} \]
Production (q-theory)

- **Producers** - indexed by $k$
  - Produce shares (trees) at a convex cost: $\Phi(S_{1k})$
  - Output: $D(Q + S_{1k})$
  - Solve

\[
\max_{C_{1k}, C_{2k}, S_{1k}} U_k(C_{1k}) + \mathbb{E}[U_k(C_{2k})]
\]

\[
\text{s.t. } C_{1k} + C_{2k} = E_{1k} + E_{2k} + P_1S_{1k} - \Phi(S_{1k})
\]

- Optimality conditions
  - $U'_k(C_{1k}) = \mathbb{E}[U'_k(C_{2k})]$ **Euler**
  - $P_1^s = \Phi'(S_{1k})$ **Supply**

- Social Welfare
  \[
  V(\tau) = \int \lambda_i V_i dF(i) + \lambda_k V_k
  \]
Production (q-theory)

- **Producers** - indexed by \( k \)
  - Produce shares (trees) at a convex cost: \( \Phi (S_{1k}) \)
  - Output: \( D (Q + S_{1k}) \)
  - Solve

\[
\max_{C_{1k}, C_{2k}, S_{1k}} U_k (C_{1k}) + \mathbb{E} [U_k (C_{2k})]
\]

\[
\text{s.t. } C_{1k} + C_{2k} = E_{1k} + E_{2k} + P_1^s S_{1k} - \Phi (S_{1k})
\]

- **Optimality conditions**

\[
U_k' (C_{1k}) = \mathbb{E} [U_k' (C_{2k})]
\]

\[
P_1^s = \Phi' (S_{1k})
\]

- **Supply**

\[
V (\tau) = \int \lambda_i V_i dF (i) + \lambda_k V_k
\]

\[
\frac{dV_k}{d\tau} = \mathbb{E} [U_k' (C_{2k})] \frac{dP_1}{d\tau} S_{1k}
\]
Production (q-theory)

- **Producers** - indexed by $k$
  - Produce shares (trees) at a convex cost: $\Phi(S_{1k})$
  - Output: $D(Q + S_{1k})$
  - Solve

$$\max_{C_{1k},C_{2k},S_{1k}} U_k(C_{1k}) + \mathbb{E}[U_k(C_{2k})]$$

s.t. $C_{1k} + C_{2k} = E_{1k} + E_{2k} + P_1^S S_{1k} - \Phi(S_{1k})$

- **Optimality conditions**
  - $U_k'(C_{1k}) = \mathbb{E}[U_k'(C_{2k})]$ Euler
  - $P_1^S = \Phi'(S_{1k})$ Supply

- **Social Welfare**

$$V(\tau) = \int \lambda_i V_i dF(i) + \lambda_k V_k$$

$$\frac{dV_k}{d\tau} = \mathbb{E}[U_k'(C_{2k})] \frac{dP_1^1}{d\tau} S_{1k}$$

- **Only terms-of-trade**
Production

- Optimal tax under [NR]

\[
\tau^* = \frac{\int (\mathbb{E}[D] - \mathbb{E}_i[D]) \frac{dX_{1i}}{d\tau} dF(i)}{-P_1 \int \text{sgn}(\Delta X_{1i}) \frac{dX_{1i}}{d\tau} dF(i)}
\]
Production

- Optimal tax under [NR]

\[ \tau^* = \frac{\int (\mathbb{E}[D] - \mathbb{E}_i[D]) \frac{dX_{1i}}{d\tau} dF(i)}{-P_1 \int \text{sgn}(\Delta X_{1i}) \frac{dX_{1i}}{d\tau} dF(i)} \]

- \[ \int \frac{dX_{1i}}{d\tau} dF(i) = \frac{dS_{1k}}{d\tau} \geq 0 \], exchange economy \[ \frac{dS_{1k}}{d\tau} = 0 \]
Production

- Optimal tax under [NR]

\[ \tau^* = \int (\mathbb{E}[D] - \mathbb{E}_i[D]) \frac{dX_{1i}}{d\tau} dF(i) \]

- Numerator (first-order effects)

\[ \int (\mathbb{E}[D] - \mathbb{E}_i[D]) \frac{dX_{1i}}{d\tau} dF(i) = -\text{Cov}_F \left[ \mathbb{E}_i[D], \frac{dX_{1i}}{d\tau} \right] + (\mathbb{E}[D] - \mathbb{E}_F[\mathbb{E}_i[D]]) \frac{dS_{1k}}{d\tau} \]

Belief dispersion

Aggregate distortion $\times$

Investment response
Production

• Optimal tax under [NR]

$$\tau^* = \int (\mathbb{E} [D] - \mathbb{E}_i [D]) \frac{dX_{1i}}{d\tau} dF (i) - P_1 \int \text{sgn} (\Delta X_{1i}) \frac{dX_{1i}}{d\tau} dF (i)$$

• Numerator (first-order effects)

$$\int (\mathbb{E} [D] - \mathbb{E}_i [D]) \frac{dX_{1i}}{d\tau} dF (i) = - \text{Cov}_F \left[ \mathbb{E}_i [D], \frac{dX_{1i}}{d\tau} \right] + (\mathbb{E} [D] - \mathbb{E}_F [\mathbb{E}_i [D]]) \frac{dS_{1k}}{d\tau}$$

Belief dispersion
Aggregated distortion $\times$
Investment response
Production

• Optimal tax under [NR]

\[ \tau^* = \int (\mathbb{E}[D] - \mathbb{E}_i[D]) \frac{dX_{1i}}{d\tau} dF(i) \]
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\[ \int (\mathbb{E}[D] - \mathbb{E}_i[D]) \frac{dX_{1i}}{d\tau} dF(i) = - \text{Cov}_F \left[ \mathbb{E}_i[D], \frac{dX_{1i}}{d\tau} \right] + \left( \mathbb{E}[D] - \mathbb{E}_F[\mathbb{E}_i[D]] \right) \frac{dS_{1k}}{d\tau} \]

Belief dispersion

 Aggregate distortion × Investment response
Production

- Optimal tax under [NR]

\[
\tau^* = \frac{\int (E[D] - E_i[D]) \frac{dX_{1i}}{d\tau} dF(i)}{-P_1 \int \text{sgn}(\Delta X_{1i}) \frac{dX_{1i}}{d\tau} dF(i)}
\]

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\[
\int (E[D] - E_i[D]) \frac{dX_{1i}}{d\tau} dF(i) = -\text{Cov}_F \left[ E_i[D], \frac{dX_{1i}}{d\tau} \right] + (E[D] - E_F[E_i[D]]) \frac{dS_{1k}}{d\tau}
\]

  - Belief dispersion
  - Aggregate distortion \times Investment response

- A decomposition

\[
\tau^* = \omega \tau^*_{\text{exchange}} + (1 - \omega) \tau^*_{\text{production}},
\]
Production

- Optimal tax under [NR]

\[ \tau^{*} = \frac{\int (\mathbb{E}[D] - \mathbb{E}_{i}[D]) \frac{dX_{1i}}{d\tau} dF(i) - P_{1} \int \text{sgn}(\Delta X_{1i}) \frac{dX_{1i}}{d\tau} dF(i)}{\mathbb{Cov}_{F}[\mathbb{E}_{i}[D], \frac{dX_{1i}}{d\tau}]} + (\mathbb{E}[D] - \mathbb{E}_{F}[\mathbb{E}_{i}[D]]) \frac{dS_{1k}}{d\tau} \]

• Numerator (first-order effects)

\[ \int (\mathbb{E}[D] - \mathbb{E}_{i}[D]) \frac{dX_{1i}}{d\tau} dF(i) = -\mathbb{Cov}_{F}[\mathbb{E}_{i}[D], \frac{dX_{1i}}{d\tau}] + (\mathbb{E}[D] - \mathbb{E}_{F}[\mathbb{E}_{i}[D]]) \frac{dS_{1k}}{d\tau} \]

- A decomposition

\[ \tau^{*} = \omega \tau_{\text{exchange}}^{*} + (1 - \omega) \tau_{\text{production}}^{*} \]

- Sign of \( \tau_{\text{production}}^{*} \)?
Production

- Optimal tax under [NR]

\[ \tau^* = \frac{\int (E[D] - E_i[D]) \frac{dX_{1i}}{d\tau} dF(i)}{-P_1 \int \text{sgn} (\Delta X_{1i}) \frac{dX_{1i}}{d\tau} dF(i)} \]

- Numerator (first-order effects)

\[ \int (E[D] - E_i[D]) \frac{dX_{1i}}{d\tau} dF(i) = -\text{Cov}_F \left[ E_i[D], \frac{dX_{1i}}{d\tau} \right] + (E[D] - E_F[E_i[D]]) \frac{dS_{1k}}{d\tau} \]

Belief dispersion

Aggregate distortion \times Investment response

- A decomposition

\[ \tau^* = \omega \tau^*_\text{exchange} + (1 - \omega) \tau^*_\text{production} \]

- Sign of \( \tau^*_\text{production} \)? Perhaps positive \( \Rightarrow \tau^* > 0 \)
Production

- Optimal tax under $[NR]$

\[ \tau^* = \int (\mathbb{E}[D] - \mathbb{E}_i[D]) \frac{dX_{1i}}{d\tau} dF(i) \]

\[ - P_1 \int \text{sgn} (\Delta X_{1i}) \frac{dX_{1i}}{d\tau} dF(i) \]

- Numerator (first-order effects)

\[ \int (\mathbb{E}[D] - \mathbb{E}_i[D]) \frac{dX_{1i}}{d\tau} dF(i) = - \text{Cov}_F \left[ \mathbb{E}_i[D], \frac{dX_{1i}}{d\tau} \right] + (\mathbb{E}[D] - \mathbb{E}_F[\mathbb{E}_i[D]]) \frac{dS_{1k}}{d\tau} \]

  - Belief dispersion
  - Aggregate distortion $\times$ Investment response

- A decomposition

\[ \tau^* = \omega \tau^*_\text{exchange} + (1 - \omega) \tau^*_\text{production} \]

- Sign of $\tau^*_\text{production}$? Perhaps positive $\Rightarrow \tau^* > 0$

\[ \text{sgn} (\tau^*_\text{production}) = \text{sgn} \left( \left( \hat{P}_1 - P_1 \right) \frac{dP_1}{d\tau} \right) \]
Production

- Optimal tax under [NR]

\[ \tau^* = \frac{\int (\mathbb{E}[D] - \mathbb{E}_i[D]) \frac{dX_{1i}}{d\tau} dF(i)}{-P_1 \int \text{sgn}(\Delta X_{1i}) \frac{dX_{1i}}{d\tau} dF(i)} \]

- Numerator (first-order effects)

\[ \int (\mathbb{E}[D] - \mathbb{E}_i[D]) \frac{dX_{1i}}{d\tau} dF(i) = -\text{Cov}_F \left[ \mathbb{E}_i[D], \frac{dX_{1i}}{d\tau} \right] + (\mathbb{E}[D] - \mathbb{E}_F[\mathbb{E}_i[D]]) \frac{dS_{1k}}{d\tau} \]

  - Belief dispersion
  - Aggregate distortion \times Investment response

- A decomposition

\[ \tau^* = \omega \tau_{\text{exchange}}^* + (1 - \omega) \tau_{\text{production}}^* \]

- Sign of \( \tau_{\text{production}}^* \)?  \textbf{Perhaps positive} \( \Rightarrow \tau^* > 0 \)

\[ \text{sgn}(\tau_{\text{production}}^*) = \text{sgn} \left( \left( \hat{P}_1 - P_1 \right) \frac{dP_1}{d\tau} \right) \]
Production

- Optimal tax under [NR]

\[
\tau^* = \frac{\int (\mathbb{E} [D] - \mathbb{E}_i [D]) \frac{dX_{1i}}{d\tau} dF (i)}{-P_1 \int \text{sgn} (\Delta X_{1i}) \frac{dX_{1i}}{d\tau} dF (i)} - P_1 \int \text{sgn} (\Delta X_{1i}) \frac{dX_{1i}}{d\tau} dF (i)
\]

- Numerator (first-order effects)

\[
\int (\mathbb{E} [D] - \mathbb{E}_i [D]) \frac{dX_{1i}}{d\tau} dF (i) = -\text{Cov}_F \left[ \mathbb{E}_i [D], \frac{dX_{1i}}{d\tau} \right] + (\mathbb{E} [D] - \mathbb{E}_F [\mathbb{E}_i [D]]) \frac{dS_{1k}}{d\tau}
\]

Belief dispersion

Aggregate distortion \times Investment response

- A decomposition

\[
\tau^* = \omega \tau^*_\text{exchange} + (1 - \omega) \tau^*_\text{production},
\]

- Sign of \(\tau^*_\text{production}\)? 

Perhaps positive \(\Rightarrow \tau^* > 0\)

\[
\text{sgn} (\tau^*_\text{production}) = \text{sgn} \left( \left( \hat{P}_1 - P_1 \right) \frac{dP_1}{d\tau} \right)
\]
Production

- Optimal tax under [NR]

\[
\tau^* = \frac{\int (\mathbb{E}[D] - \mathbb{E}_i[D]) \frac{dX_{1i}}{d\tau} dF(i)}{-P_1 \int \text{sgn}(\Delta X_{1i}) \frac{dX_{1i}}{d\tau} dF(i)}
\]

- Numerator (first-order effects)

\[
\int (\mathbb{E}[D] - \mathbb{E}_i[D]) \frac{dX_{1i}}{d\tau} dF(i) = -\text{Cov}_F \left[ \mathbb{E}_i[D], \frac{dX_{1i}}{d\tau} \right] + (\mathbb{E}[D] - \mathbb{E}_F[\mathbb{E}_i[D]]) \frac{dS_{1k}}{d\tau}
\]

- A decomposition

\[
\tau^* = \omega \tau^*_\text{exchange} + (1 - \omega) \tau^*_\text{production}
\]

- Sign of \( \tau^*_\text{production} \)? **Perhaps positive** \(\Rightarrow \tau^* > 0\)

\[
\text{sgn} \left( \tau^*_\text{production} \right) = \text{sgn} \left( \left( \hat{P}_1 - P_1 \right) \frac{dP_1}{d\tau} \right) > 0
\]
Hayekian production

- Environment
Hayekian production

- Environment
  - $\pi_I$ Informed investors - Observe $\theta$
Hayekian production

- Environment
  - $\pi_I$ Informed investors - Observe $\theta$
  - $\pi_U = 1 - \pi_I$ Uninformed investors - Do not update from prices
Hayekian production

- **Environment**
  - $\pi_I$ Informed investors - Observe $\theta$
  - $\pi_U = 1 - \pi_I$ Uninformed investors - Do not update from prices
  - Firm manager - Chooses production given prices

\[\Pi = \alpha + \theta + \beta(\theta^2 - (\theta - k^*)^2), \beta \geq 0\]

- **Optimal investment**
  \[k^* = \mathbb{E}[\theta | P_1]\]

- **Assumptions**
  1. Informed investors do not internalize effect in production
  2. Uninformed investors do not learn

There is no trade in equilibrium when $\theta(\tau) \leq \theta \leq \theta(\tau)$
Hayekian production

• Environment
  • $\pi_I$ Informed investors - Observe $\theta$
  • $\pi_U = 1 - \pi_I$ Uninformed investors - Do not update from prices
  • Firm manager - Chooses production given prices

• Dividend $\Pi = D + \theta + \beta \left( \theta^2 - (\theta - k^*)^2 \right)$, $\beta \geq 0$
Hayekian production

- **Environment**
  - $\pi_I$ Informed investors - Observe $\theta$
  - $\pi_U = 1 - \pi_I$ Uninformed investors - Do not update from prices
  - Firm manager - Chooses production given prices

- Dividend $\Pi = D + \theta + \beta \left( \theta^2 - (\theta - k^*)^2 \right)$, $\beta \geq 0$

- Optimal investment: $k^* = \mathbb{E} [\theta | P_1]$
Hayekian production

- **Environment**
  - $\pi_I$ Informed investors - Observe $\theta$
  - $\pi_U = 1 - \pi_I$ Uninformed investors - Do not update from prices
  - Firm manager - Chooses production given prices

- Dividend $\Pi = D + \theta + \beta \left( \theta^2 - (\theta - k^*)^2 \right)$, $\beta \geq 0$

- Optimal investment: $k^* = \mathbb{E} [\theta | P_1]$

- **Assumptions**
  1. Informed investors do not internalize effect in production

\[\theta(\tau)\]
Hayekian production

- **Environment**
  - $\pi_I$ Informed investors - Observe $\theta$
  - $\pi_U = 1 - \pi_I$ Uninformed investors - Do not update from prices
  - Firm manager - Chooses production given prices

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Hayekian production

- **Environment**
  - $\pi_I$ Informed investors - Observe $\theta$
  - $\pi_U = 1 - \pi_I$ Uninformed investors - Do not update from prices
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- Dividend $\Pi = D + \theta + \beta \left( \theta^2 - (\theta - k^*)^2 \right)$, $\beta \geq 0$
- Optimal investment: $k^* = \mathbb{E}[\theta|P_1]$
- **Assumptions**
  1. Informed investors do not internalize effect in production
  2. Uninformed investors do not learn
- There is no trade in equilibrium when $\bar{\theta}(\tau) \leq \theta \leq \bar{\theta}(\tau)$

\[\begin{tabular}{c|c|c}
Sell & No Trade & Buy \\
$\theta(\tau)$ & 0 & $\bar{\theta}(\tau)$
\end{tabular}\]
Hayekian production

- Marginal tax change

\[
\frac{dV}{d\tau} = \mathbb{E}_\theta \left[ \frac{dV}{d\tau} \bigg| \theta \right] + \Psi(\tau) < 0
\]

- Even locally first order \( \Psi(\tau) |_{\tau=0} < 0 \)
- Learning externality
- Optimal tax \( \tau^* = \tau^*_{\text{no-info}} \) \( + \tau^*_{\text{information}} \)

If \( \tau^*_{\text{no-info}} = 0 \), optimal policy is subsidy, not laissez-faire
Hayekian production

- Marginal tax change

\[
\frac{dV}{d\tau} = E_\theta \left[ \frac{dV}{d\tau} \right]_\theta + \Psi(\tau)
\]

Where \( \Psi(\tau) \) captures distortions in production efficiency

\[
\Psi(\tau) \equiv \left( V^T(\theta) - V^{NT}(\theta) \right) \phi_{Var[\theta]}(\theta) \left( \frac{d\theta(\tau)}{d\tau} \right)_{>0} + \left( V^{NT}(\bar{\theta}) - V^T(\bar{\theta}) \right) \phi_{Var[\theta]}(\bar{\theta}) \left( \frac{d\bar{\theta}(\tau)}{d\tau} \right)_{<0} + \left( V^{NT}(\theta) - V^T(\theta) \right) \phi_{Var[\theta]}(\theta) \left( \frac{d\theta(\tau)}{d\tau} \right)_{<0} + \left( V^T(\theta) - V^{NT}(\theta) \right) \phi_{Var[\theta]}(\theta) \left( \frac{d\theta(\tau)}{d\tau} \right)_{>0}
\]
Hayekian production

- Marginal tax change

\[
\frac{dV}{d\tau} = \mathbb{E}_\theta \left[ \frac{dV}{d\tau} \right]_{\theta} + \Psi(\tau) < 0
\]

Where \( \Psi(\tau) \) captures distortions in production efficiency

\[\Psi(\tau) \equiv \left( V^T(\theta) - V^{NT}(\theta) \right) \phi_{\text{Var}[\theta]}(\tilde{\theta}) \frac{d\theta(\tau)}{d\tau} + \left( V^{NT}(\tilde{\theta}) - V^T(\tilde{\theta}) \right) \phi_{\text{Var}[\theta]}(\tilde{\theta}) \frac{d\tilde{\theta}(\tau)}{d\tau} > 0\]
Hayekian production

- Marginal tax change

\[
\frac{dV}{d\tau} = E_\theta \left[ \frac{dV}{d\tau} \bigg| \theta \right] + \Psi(\tau) < 0
\]

Where \( \Psi(\tau) \) captures distortions in production efficiency

\[
\Psi(\tau) \equiv \left( V^T(\theta) - V^{NT}(\theta) \right) \phi_{\text{Var}[\theta]}(\theta) \frac{d\theta(\tau)}{d\tau} < 0 + \left( V^{NT}(\bar{\theta}) - V^T(\bar{\theta}) \right) \phi_{\text{Var}[\theta]}(\bar{\theta}) \frac{d\bar{\theta}(\tau)}{d\tau} > 0
\]
Hayekian production

- Marginal tax change

\[
\frac{dV}{d\tau} = \mathbb{E}_\theta \left[ \left. \frac{dV}{d\tau} \right|_\theta \right] + \Psi(\tau) < 0
\]

Where \( \Psi(\tau) \) captures distortions in production efficiency

\[
\Psi(\tau) \equiv \left( V^T(\theta) - V^{NT}(\theta) \right) \phi_{\text{Var}[\theta]}(\theta) \frac{d\theta(\tau)}{d\tau} < 0 + \left( V^{NT}(\bar{\theta}) - V^T(\bar{\theta}) \right) \phi_{\text{Var}[\theta]}(\bar{\theta}) \frac{d\bar{\theta}(\tau)}{d\tau} > 0
\]

- Even locally first order \( \Psi(\tau)|_{\tau=0} < 0 \)
Hayekian production

- Marginal tax change

$$\frac{dV}{d\tau} = \mathbb{E}_\theta \left[ \left. \frac{dV}{d\tau} \right| \theta \right] + \Psi(\tau) < 0$$

Where $\Psi(\tau)$ captures distortions in production efficiency

$$\Psi(\tau) \equiv \left( V^T(\theta) - V^{NT}(\theta) \right) \phi_{\text{Var}[\theta]}(\theta) \frac{d\theta}{d\tau} + \left( V^{NT}(\bar{\theta}) - V^T(\bar{\theta}) \right) \phi_{\text{Var}[\theta]}(\bar{\theta}) \frac{d\bar{\theta}}{d\tau}$$

- Even locally first order $\Psi(\tau)|_{\tau=0} < 0$ - Learning externality
Hayekian production

- Marginal tax change

\[ \frac{dV}{d\tau} = \mathbb{E}_\theta \left[ \frac{dV}{d\tau} \bigg| \theta \right] + \Psi(\tau) \]

Where \( \Psi(\tau) \) captures distortions in production efficiency

\[ \Psi(\tau) \equiv \left( V^T(\theta) - V^{NT}(\theta) \right) \phi_{\text{Var}[\theta]}(\theta) \frac{d\theta(\tau)}{d\tau} + \left( V^{NT}(\bar{\theta}) - V^T(\bar{\theta}) \right) \phi_{\text{Var}[\theta]}(\bar{\theta}) \frac{d\bar{\theta}(\tau)}{d\tau} \]

- Even locally first order \( \Psi(\tau)|_{\tau=0} < 0 \) - Learning externality

- Optimal tax

\[ \tau^* = \tau^*_{\text{no-info}} + \tau^*_{\text{information}} \]

\[ > 0 \quad < 0 \quad \geq 0 \]
Hayekian production

- Marginal tax change

\[
\frac{dV}{d\tau} = \mathbb{E}_\theta \left[ \frac{dV}{d\tau} \bigg|_{\theta} \right] + \Psi(\tau) < 0
\]

Where \( \Psi(\tau) \) captures distortions in production efficiency

\[
\Psi(\tau) \equiv \left( V^T(\theta) - V^{NT}(\theta) \right)^{\phi_{\text{Var}[\theta]}(\theta)} \frac{d\theta(\tau)}{d\tau} + \left( V^{NT}(\overline{\theta}) - V^T(\overline{\theta}) \right)^{\phi_{\text{Var}[\theta]}(\overline{\theta})} \frac{d\overline{\theta}(\tau)}{d\tau} > 0
\]

- Even locally first order \( \Psi(\tau)\big|_{\tau=0} < 0 \) - Learning externality

- Optimal tax

\[
\tau^* = \tau^*_{\text{no-info}} + \tau^*_{\text{information}} > 0 < 0 \geq 0
\]

- If \( \tau^*_{\text{no-info}} = 0 \), optimal policy is subsidy, not laissez-faire
Extra Derivations

\[ \hat{V}_i = (\mathbb{E}[D] - A_i \text{Cov}[E_{2i}, D] - P_1) X_{1i} + P_1 X_{0i} - \frac{A_i}{2} \text{Var}[D] (X_{1i})^2 \]

\[ \frac{d\hat{V}_i}{d\tau} = (\mathbb{E}[D] - A_i \text{Cov}[E_{2i}, D] - P_1 - A_i X_{1i} \text{Var}[D]) \frac{dX_{1i}}{d\tau} - \Delta X_{1i} \frac{dP_1}{d\tau} \]

\[ \frac{dV_i}{d\tau} = [(\mathbb{E}[D] - \mathbb{E}_i[D]) + \text{sgn}(\Delta X_{1i}) P_1 \tau] \frac{dX_{1i}}{d\tau} - \Delta X_{1i} \frac{dP_1}{d\tau} \]
Example 3: 35% Non-fundamental trading - $\tau^* = 2.01\%$ - Gain 0.11%
Intuition on $\frac{dP_1}{d\tau}$