Retirement, Home Production, and Labor Supply Elasticities

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Recent Developments in the Economics of Home Production and Nonmarket Work
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Outline of discussion

• **Key insight**: data on the change in (i) time allocation and (ii) consumption expenditures between work and retirement informative about key elasticities
Outline of discussion

• **Key insight**: data on the change in (i) time allocation and (ii) consumption expenditures between work and retirement informative about key elasticities

• Outline:
  
  ▶ **Summary of methodology and result**
  
  ▶ **A remark on calibration**
  
  ▶ **Methodology is more broadly applicable than you may think**
  
  ▶ **Methodology is more robust than you may think**
  
  ▶ **The household?**

Violante, Discussion of Rogerson-Wallenius: "Retirement, Home Production, and Labor Supply Elasticities"
Summary

\[ \max \{g_w, g_r, h_w, h_r\} \quad u(c_w) + \frac{\alpha}{1 - \frac{1}{\gamma}} (1 - \overline{h} - h_w)^{1 - \frac{1}{\gamma}} + u(c_r) + \frac{\alpha}{1 - \frac{1}{\gamma}} (1 - h_r)^{1 - \frac{1}{\gamma}} \]

s.t.

\[ c_t = \left[ a g_t^{1 - \frac{1}{\eta}} + (1 - a) h_t^{1 - \frac{1}{\eta}} \right]^{\eta}_{\eta - 1} \quad t = w, r \]

\[ g_w + g_r = \bar{w}h + b \]
Summary

\[
\max_{\{g_w, g_r, h_w, h_r\}} u(c_w) + \frac{\alpha}{1 - \frac{1}{\gamma}} (1 - \bar{h} - h_w)^{1 - \frac{1}{\gamma}} + u(c_r) + \frac{\alpha}{1 - \frac{1}{\gamma}} (1 - h_r)^{1 - \frac{1}{\gamma}}
\]

s.t.

\[
c_t = \left[ a g_t^{1 - \frac{1}{\eta}} + (1 - a) h_t^{1 - \frac{1}{\eta}} \right]^{\frac{\eta}{\eta - 1}}
\]

\[
g_w + g_r = w\bar{h} + b
\]

• Take FOC’s, rearrange, and obtain the elasticity ratio formula

\[
\frac{\gamma}{\eta} = \frac{\log \left( \frac{1 - h_r}{1 - h - h_w} \right)}{\log \left( \frac{h_r}{h_w} \right) - \log \left( \frac{g_r}{g_w} \right)}
\]
\[ \frac{\gamma}{\eta} = \log \left( \frac{1-h_r}{1-h-h_w} \right) \left( \frac{h_r}{h_w} \right) - \log \left( \frac{g_r}{g_w} \right) \]

- **Parameterization:**
  - \( g_r / g_w = 0.90 \leftrightarrow \) consumption exp. drop at retirement
  - \( h_w = 0.15 \leftrightarrow \) hours of HP for empl. males 60-64 (15 per week)
  - \( \bar{h} = 0.42 \leftrightarrow \) hours of MP for FT empl. males (42 per week)
  - \( h_r = 0.234 \leftrightarrow \) fraction of additional time available at retirement
    \[
    \left( \frac{h_r-h_w}{h} \right) \text{ devoted to home production (20%)}
    \]
Summary

\[
\frac{\gamma}{\eta} = \frac{\log \left( \frac{1-h_r}{1-h-h_w} \right)}{\log \left( \frac{g_r}{h_w} \right) - \log \left( \frac{g_r}{g_w} \right)} = \frac{\log \left( \frac{1-0.234}{1-0.42-0.15} \right)}{\log \left( \frac{0.234}{0.15} \right) - \log (0.90)} = 1.05
\]

- Parameterization:
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\gamma \eta = \frac{\log \left( \frac{1-h_r}{1-h-h_w} \right)}{\log \left( \frac{h_r}{h_w} \right) - \log \left( \frac{g_r}{g_w} \right)} = \frac{\log \left( \frac{1-0.234}{1-0.42-0.15} \right)}{\log \left( \frac{0.234}{0.15} \right) - \log (0.90)} = 1.05
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  - \( h_r = 0.234 \leftrightarrow \) fraction of additional time available at retirement \((\frac{h_r-h_w}{h})\) devoted to home production (20%)
A remark on the parameterization

\[
\gamma = \frac{\log \left( \frac{1-h_r}{1-h-h_w} \right)}{\log \left( \frac{h_r}{h_w} \right) - \log \left( \frac{g_r}{g_w} \right)} = \frac{\log \left( \frac{1-0.20}{1-0.26-0.15} \right)}{\log \left( \frac{0.20}{0.15} \right) - \log (0.90)} = 0.75
\]

- **Parameterization:**
  - \( g_r / g_w = 0.90 \leftrightarrow \text{consumption exp. drop at retirement} \)
  - \( h_w = 0.15 \leftrightarrow \text{hours of HP for empl. males 60-64 (15 per week)} \)
  - \( \bar{h} = 0.26 \leftrightarrow \text{hours of MP for empl. males 60-64 (26 per week)} \)
  - \( h_r = 0.20 \leftrightarrow \text{fraction of additional time available at retirement} \)
  \( \left( \frac{h_r-h_w}{h} \right) \text{ devoted to home production (20%)} \)
The key insight is broader

- Change in (i) time allocation and (ii) consumption exp. between work and retirement is informative about labor supply elasticity
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- Change in (i) time allocation and (ii) consumption exp. between work and retirement is informative about labor supply elasticity

\[
\frac{\gamma}{\eta} = \frac{\log \left( \frac{1-h_2}{1-h-h_1} \right)}{\log \left( \frac{h_2}{h_1} \right) - \log \left( \frac{g_2}{g_1} \right)}
\]
The key insight is broader

• Change in (i) time allocation and (ii) consumption exp. between work and retirement is informative about labor supply elasticity

\[ \frac{\gamma}{\eta} = \frac{\log \left( \frac{1-h_2}{1-h-h_1} \right)}{\log \left( \frac{h_2}{h_1} \right) - \log \left( \frac{g_2}{g_1} \right)} \]

• Other sources of large “exogenous” variation in time allocation:
  
  ► Transition from education into work
  
  ► Transition from childless household to household with children
  
  ► Transition from employment to unemployment
Transition from employment into unemployment

\[
\frac{\gamma}{\eta} = \frac{\log \left( \frac{1-h_u}{1-h-h_w} \right)}{\log \left( \frac{h_u}{h_w} \right) - \log \left( \frac{g_u}{g_w} \right)}
\]
Transition from employment into unemployment

\[
\frac{\gamma}{\eta} = \frac{\log \left( \frac{1-h_u}{1-h-h_w} \right)}{\log \left( \frac{h_u}{h_w} \right) - \log \left( \frac{g_u}{g_w} \right)}
\]

- \( g_u/g_w = 0.90 \leftrightarrow \) consumption exp. drop after job loss
- \( h_w = 0.10 \leftrightarrow \) hours of HP for FT empl. male (10 per week)
- \( \bar{h} = 0.42 \leftrightarrow \) hours of MP for FT empl. male (42 per week)
- \( h_u = 0.23 \leftrightarrow \) fraction of additional time available to the unemployed \( \left( \frac{h_u-h_w}{h_w} \right) \) devoted to home production (30%)

Transition from employment into unemployment

\[
\frac{\gamma}{\eta} = \frac{\log \left( \frac{1-h_u}{1-h-h_w} \right)}{\log \left( \frac{h_u}{h_w} \right) - \log \left( \frac{g_u}{g_w} \right)} = \frac{\log \left( \frac{1-0.23}{1-0.42-0.10} \right)}{\log (0.23) - \log (0.90)} = 0.5
\]

- \( g_r/g_w = 0.90 \) ↔ consumption exp. drop after job loss
- \( h_w = 0.10 \) ↔ hours of HP for FT empl. male (10 per week)
- \( h = 0.42 \) ↔ hours of MP for FT empl. male (42 per week)
- \( h_u = 0.226 \) ↔ fraction of additional time available to the unemployed \( \left( \frac{h_u-h_w}{h_w} \right) \) devoted to home production (30%)
Robustness to individual heterogeneity

• Allow individual heterogeneity in \( \{u_i, \alpha_i, a_i, w_i, b_i\} \)

• Assume heterogeneity fixed over time
Robustness to individual heterogeneity

- Allow individual heterogeneity in \( \{u_i, \alpha_i, a_i, w_i, b_i\} \)

- Assume heterogeneity fixed over time

\[
\max_{\{g_{it}, h_{it}\}} u_i(c_{iw}) + \frac{\alpha_i}{1 - \frac{1}{\gamma}} (1 - \bar{h} - h_{iw})^{1-\frac{1}{\gamma}} + u_i(c_{ir}) + \frac{\alpha_i}{1 - \frac{1}{\gamma}} (1 - h_{ir})^{1-\frac{1}{\gamma}}
\]

s.t.

\[
c_{it} = \left[ a_i g_{it}^{\frac{1}{\eta}} + (1 - a_i) h_{it}^{\frac{1}{\eta}} \right] \frac{\eta}{\eta - 1} \quad t = w, r
\]

\[
g_{iw} + g_{ir} = w_i \bar{h} + b_i \quad (\mu_i)
\]
Robustness to individual heterogeneity

• Ratio of FOCs with respect to \( \{g_{iw}, g_{ir}\} \)

\[
\frac{u_i'(c_{iw}) c_{iw}^{\eta-1}}{u_i'(c_{ir}) c_{ir}^{\eta-1}} \cdot \frac{1}{\left(\frac{\mu_i}{a_i}\right)^{\frac{1}{\eta}}} g_{iw}^{\eta} \cdot \frac{1}{g_{ir}^{\eta}} = g_{iw}^{\eta} = g_{ir}^{\eta}
\]

• Ratio of FOCs with respect to \( \{h_{iw}, h_{ir}\} \)

\[
\frac{u_i'(c_{iw}) c_{iw}^{\eta-1}}{u_i'(c_{ir}) c_{ir}^{\eta-1}} \cdot \frac{\alpha_i}{1-a_i} \cdot \frac{1}{\left(1-h-h_{iw}\right)^{\frac{1}{\gamma}}} h_{iw}^{\eta} \cdot \frac{1}{h_{ir}^{\eta}} \cdot = \frac{1}{\left(1-h-h_{ir}\right)^{\frac{1}{\gamma}}} h_{iw}^{\eta} = h_{ir}^{\eta}
\]

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Robustness to individual heterogeneity

- Not robust if heterogeneity in $\{\alpha_{it}, a_{it}\}$ is time-varying
Robustness to individual heterogeneity

- Not robust if heterogeneity in \( \{ \alpha_{it}, a_{it} \} \) is time-varying

- Ratio of FOCs with respect to \( \{ g_{iw}, g_{ir} \} \)

\[
\frac{u'_i(c_{iw}) c_{iw}^{\eta - 1}}{u'_i(c_{ir}) c_{ir}^{\eta - 1}} = \frac{\left( \frac{\mu_i}{\alpha_{iw}} \right) g_{iw}^{\eta}}{\left( \frac{\mu_i}{\alpha_{ir}} \right) g_{ir}^{\eta}}
\]

- Ratio of FOCs with respect to \( \{ h_{iw}, h_{ir} \} \)

\[
\frac{u'_i(c_{iw}) c_{iw}^{\eta - 1}}{u'_i(c_{ir}) c_{ir}^{\eta - 1}} = \frac{\alpha_{iw}}{1-a_{iw}} \cdot \frac{h_{iw}^{\eta}}{(1-h-h_{iw})^{\frac{1}{\gamma}}}
\]

\[
\frac{\alpha_{ir}}{1-a_{ir}} \cdot \frac{h_{ir}^{\eta}}{(1-h-h_{ir})^{\frac{1}{\gamma}}}
\]
Family

- Individuals belong to families *(elderly belong to couples)*

- *Collective model* of the family
Family

- Individuals belong to families (elderly belong to couples)

- **Collective model** of the family

\[
\max \{G_t, h_{it}, c_{it}\} \sum_{t=w,r} \sum_{i=1,2} \pi_{it} \left[ u(c_{it}) + \frac{\alpha}{1 - \frac{1}{\gamma}} (1 - \bar{h}_t - h_{it})^{1-\frac{1}{\gamma}} \right]
\]

s.t.

\[
C_t = \left[ aG_t^{1-\frac{1}{\eta}} + (1 - a) (h_{1t} + h_{2t})^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}
\]

\[
C_t = \sum_{i=1,2} c_{it}
\]

\[
\sum_{i=1,2} (w_i \bar{h}_t + b_i) = G_w + G_r
\]

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Family

- Individuals belong to families *(elderly belong to couples)*

- **Collective model** of the family

\[
\max_{\{G_t, h_{it}, c_{it}\}} \sum_{t=w,r} \sum_{i=1,2} \pi_{it} \left[u(c_{it}) + \frac{\alpha}{1 - \frac{1}{\gamma}} \left(1 - \bar{h}_t - h_{it}\right)^{1 - \frac{1}{\gamma}}\right]
\]

s.t.

\[
C_t = \left[aG_t^{1 - \frac{1}{\eta}} + (1 - a) (h_{1t} + h_{2t})^{1 - \frac{1}{\eta}}\right]^{\frac{\eta}{\eta - 1}}
\]

\[
C_t = \sum_{i=1,2} c_{it}
\]

\[
\sum_{i=1,2} \left(w_i \bar{h}_t + b_i\right) = G_w + G_r
\]

- **Elasticity-ratio formula** does not depend on Pareto weights \(\pi_{it}\)

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Family

• New elasticity-ratio formula:

\[
\frac{\gamma}{\eta} = \frac{\log \left( \frac{1-h_{1r}}{1-h-h_{1w}} \right)}{\log \left( \frac{h_{1r}+h_{2r}}{h_{1w}+h_{2w}} \right) - \log \left( \frac{G_r}{G_w} \right)}
\]
Family

- New elasticity-ratio formula:

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\frac{\gamma}{\eta} = \frac{\log \left( \frac{1-h_{1r}}{1-h-h_{1w}} \right)}{\log \left( \frac{h_{1r}+h_{2r}}{h_{1w}+h_{2w}} \right) - \log \left( \frac{G_r}{G_w} \right)}
\]

- Now, home-production time of both spouses shows up in formula

- For married couples, basically no change in total HP time from age 60-64 to 65+ for the couple

Family

• New elasticity-ratio formula:

\[
\frac{\gamma}{\eta} = \frac{\log \left( \frac{1-h_1r}{1-h-h_1w} \right)}{\log \left( \frac{h_{1r}+h_{2r}}{h_{1w}+h_{2w}} \right) - \log \left( \frac{G_r}{G_w} \right)} = \frac{\log \left( \frac{1-0.20}{1-0.26-0.15} \right)}{\log (1) - \log (0.90)} = 2.9
\]

• Now, home-production time of both spouses shows up in formula

• For married couples, basically no change in total HP time from age 60-64 to 65+ for the couple