Liquidity and Segmented Markets

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Talk Outline

- Define **Illiquid Markets:**

  *Illiquid Markets are markets in which asset values appear to deviate from fundamental values. Illiquid Markets are typically associated with lower volume, disintermediation, exit from trading.*

- Empirical Examples and Stylized Facts

- Theoretical Structures/Models of Trading Frictions

- Specific Models:
  - Eisfeldt (JF 2004): Adverse selection
  - Eisfeldt, Lustig, Zhang (WP 2017): Hedging Expertise
  - Atkeson, Eisfeldt, Weill (ECMA 2015): Search
  - Eisfeldt, Herskovic, Siriwardane (WP 2017): Networks
Fig. 4. Convertible debenture cheapness or richness. This figure displays the monthly median difference between the fundamental value of equity-sensitive convertible debentures and their traded prices during January 1990 through December 2010. We define equity-sensitive convertible debentures as convertibles with moneyness (ratio of issuer stock price to conversion price) greater than 0.65. Market prices are provided by Value Line Investment Surveys and various Wall Street investment banks. The fundamental or theoretical values of the convertible debentures are calculated using a finite difference model and input estimates (stock price, equity volatility, credit spread, and term structure of interest rates) corresponding to each convertible debenture on each date. On a given date, there are an average of 197 equity-sensitive bonds with a minimum of 39 (September 2002) and a maximum of 600 (June 2007). The minimum number of equity-sensitive bonds during the financial crisis was 158 in February 2009.
Fig. 7. Credit default swap (CDS)–corporate bond basis. This figure displays weekly CDS–corporate bond basis (in basis points) for high-yield issues (average of 204 issues per week) and investment-grade issues (average of 491 issues per week) during January 2005 through December 2010. A positive (negative) basis is when the implied spread from the CDS exceeds (is less than) the implied credit spread from the corporate bond. Data provided by J.P. Morgan.
Figure 2. Weighted Average TIPS–Treasury Mispricing in Basis Points. This figure plots the time series of the average TIPS–Treasury mispricing, measured in basis points, across the pairs included in the sample, where the average is weighted by the notional amount of the TIPS issue.

Tips-Treasury Basis: TIPS Cheapness
Fleckenstein, Longstaff, Lustig (2014)
Figure 4: **Short-Term Libor-Based Deviations from Covered Interest Rate Parity:** This figure plots the 10-day moving averages of the three-month Libor cross-currency basis, measured in basis points, for G10 currencies. The covered interest rate parity implies that the basis should be zero. One-hundred basis points equal one percent. The Libor basis is equal to $y_{t,t+n}^S - (y_{t,t+n}^L - \rho_{t,t+n})$, where $n = \text{three months}$, $y_{t,t+n}^S$ and $y_{t,t+n}^L$ denote the U.S. and foreign three-month Libor rates, and $\rho_{t,t+n} = \frac{1}{n}(f_{t,t+n} - s_t)$ denotes the forward premium obtained from the forward $f_{t,t+n}$ and spot $s_t$ exchange rates.

**CIRP Basis**

Du, Tepper, Verdelhan (2017)
ILLIQUIDITY EXAMPLES

[Graph showing illiquidity examples with dates from 1996 to 2013 and performance metrics meanperf and stdperf.]

Relative Value Asset Backed FI HF

Eisfeldt, Lustig, Zhang (2017)
ILLIQUIDITY EXAMPLES

Relative Value Asset Backed FI HFs
Eisfeldt, Lustig, Zhang (2017)
Modeling Illiquidity

- Direct Trading and Illiquidity
  - Transactions Costs
  - Adverse Selection
  - Moral Hazard
  - Search
  - Networks
  - Segmented Markets

- Intermediated Trade and Disintermediation
  - Moral Hazard
  - Search
  - Value at Risk Constraints
  - Networks (core periphery)
Models

- Two models consistent with:
  Increased heterogeneity, less trade.
  Larger deviations from fundamental (?) prices.
    - Eisfeldt (JF 2004):
      Adverse selection and macroeconomic fundamentals.
    - Eisfeldt, Lustig, Zhang (WP 2017):
      Financial modeling expertise and increases in risk.

- Two models of:
  Limited risk sharing from decentralized trade frictions.
    - Atkeson, Eisfeldt, Weill (ECMA (2015):
      Double continuum of banks, traders. Trade size limits.
    - Eisfeldt, Herskovic, Siriwardane (2017):
      Network model with cost of concentrated exposures.
Stylized Facts

- Profits larger from trading in illiquid markets.
  Profits large $\Rightarrow$ expect more entry or more trade?
- Cross sectional heterogeneity larger in illiquid markets.
  Heterogeneity large $\Rightarrow$ gains from trade larger?

Needed: Models which capture larger profits and larger dispersion in illiquid markets, despite free entry.
Endogenous Liquidity in Asset Markets
Eisfeldt JF 2004

- Adverse selection and liquidity “Paradoxical” relation to income shocks: Negative income shocks improve pooling.

- Basic idea in Eisfeldt (2004): Endogenous interaction between risk taking and non-adverse selection reasons for trade.

- Show: Adverse selection is worse when productivity is low, thus illiquidity is high. Illiquidity magnifies the effect of fundamentals shocks on investment and trading volume.
**Model: Preferences, Endowments, Technologies**

- **Preferences:** \( E \left[ \sum_{t=0}^{\infty} \beta^t (1 - \delta)^t \frac{c_t (1 - \sigma)}{1 - \sigma} \right] \)

- **Endowment:** constant endowment \( e \).

- **Technology:** Riskless Storage and Risky Projects

- **Claims Market:** Anonymous competitive market. Equilibrium claims price determined by the average quality of claims issued, \( P = \kappa p_H + (1 - \kappa) p_L \).

- **Liquidity Measures:** \( r_1 \) vs. \( r_2 \) and \( P \) vs. \( p_H \).
OVERLAPPING RISKY PROJECTS
**Individual’s Bellman Equation**

Define $z \equiv (w, y_o, q) = “Financial Position.”$

$$\nu(w, y_o, q) = \max_{\{c, x', y'_o\} \in \mathbb{R}_+^3, y_s \in [0, y_o]} \left\{ \frac{c^{(1-\sigma)}}{1 - \sigma} + \beta(1 - \delta)E [\nu(e + x' + (y_o - y_s)Y', y'_o, q')] | q] \right\}$$

subject to $c \leq w + y_s P - y'_o I - x'$

Laws of Motion for State Variables

- **Income:** $w' = e + x' + (y_o - y_s)Y'$, where
  $$Y' = \begin{cases} Y_H & \text{with probability } q_i \\ Y_L & \text{with probability } (1 - q_i) \end{cases} \quad \text{for } i \in \{L, H\}.$$

- **Ongoing Project Scale:** $y'_o = y'_o$

- **Ongoing Project Quality:**
  $$q' = \begin{cases} q_H & \text{with probability } q_0 \\ q_L & \text{with probability } (1 - q_0) \end{cases}$$
The Decision to Issue Claims

- Why do agents issue claims? Adverse selection reason and rebalancing reason; agents want to smooth consumption and diversify investment across vintages.

- Decompose the individual state space into selling and not selling regions. Describe how the number of claims sold within the selling region varies with the individual state.
Selling Region of State Space

Sell claims when current income low relative to risk exposure from ongoing projects.
Intuition: Liquidity increases with productivity

- Productivity \((E[Y] \text{ or } q)\) increases
  \(\Rightarrow\) risky investment opportunity improves.

- Agents optimally store less, initiate more risky projects.

- \(\Rightarrow\) the risky payoff often has a larger impact on current income.

- Agents more likely to have large scale ongoing projects at times when their completed projects fail

- \(\Rightarrow\) they are more likely to realize states within the selling region for high quality projects.

- Moreover, the selling region is larger when productivity is higher (investment opportunities better).

- As more claims to high quality projects are issued, \(P\) increases. The equilibrium \(P\) is the “fixed point” of these effects.
Conclusions:

- Adverse selection driven liquidity varies positively with productivity.
- Liquidity magnifies the effects of productivity on investment and volume.
- Key is interaction between risk taking and liquidity.

Future Directions:

- Interaction between risk taking and adverse selection
- Push diversification reason for selling more
- Consider intermediaries and adverse selection
- Adverse selection in decentralized markets
Complex assets/strategies:
  1. Persistent elevated excess returns ($\alpha$),
  2. High Sharpe Ratios,
  3. Low participation,

Despite free entry.

Our rational model:
  ▶ Complex assets expose investors to idiosync. risk
  ◀ Expertise $\Rightarrow$ better risk return tradeoff
  ◀ Sharpe ratios: market $\neq$ individual
  ▶ Expertise $\approx$ excess-capacity-like barrier to entry
PREFERENCES, ENDOWMENTS, TECHNOLOGIES

- Preferences: Measure one of investors, CRRA utility
  \[ u(c) = \frac{c^{1-\gamma}}{1-\gamma} \]

- Endowments:
  - Expertise \( x \) drawn from \( \lambda(x) \)
  - Wealth \( w \sim \phi(w|x) \) determined in equilibrium

- Technology:
  - Riskless Asset: perfectly elastic supply, return \( r_f \)
  - Complex Risky Asset: fixed supply, returns
    \[ dR_{i,t} - r_f \, dt = \underbrace{\alpha_t \, dt}_{\text{Mkt Clr}} + \underbrace{\sigma(x) \, dB_{i,t}}_{\text{\downarrow in } x} \]
  - Participation: entry, maintenance costs: \( f_{nx}w_t, f_{xx}w_t \)
\textbf{Long/Short Microfoundation}

\[ dR_{i,t} - r_f \, dt = \alpha \, dt + \sigma (x) \, dB_{i,t} \]

**Underlying asset**, returns:

\[ \frac{dF(t, s)}{F(t, s)} = [r_f + \alpha(s) + a(s)] \, dt + \sigma^F \, dB^F(t, s). \]

Each investor’s best per-unit **tracking portfolio returns**:

\[ \frac{dT_i(t, s)}{T_i(t, s)} = a(s) dt + \rho_i(x) \sigma^F \, dB^F(t, s) - \sigma^T(x) \, dB^T_i(t, s), \]

where \( dB^F(t, s) \perp dB^T_i(t, s) \). \( \rho_i(x) \) with \( \frac{\partial}{\rho_i(x)} \partial x > 0 \) represents dependence of tracking portfolio returns on fundamental risk.

Returns for the **complex net asset** evolve according to:

\[ dR_i(t, s) = \frac{dF(t, s)}{F(t, s)} - \frac{dT_i(t, s)}{T_i(t, s)} = [r_f + \alpha(s)] \, dt + \sigma(x) dB_i(t, s) \]

where:

\[ \sigma(x) dB_i(t, s) \equiv (1 - \rho_i(x)) \sigma^F dB^F(t, s) + \sigma^T(x) dB^T_i(t, s). \]
Returns for the complex net asset evolve according to:

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where

\[ \sigma(x) dB_i(t, s) \equiv (1 - \rho_i(x)) \sigma_F dB^F_i(t, s) + \sigma_T(x) dB^T_i(t, s). \]

At each level of expertise, half of investors over-hedge (type \( o \) with \( \rho_o(x) > 1 \)), and half under-hedge (type \( u \) with \( \rho_u(x) < 1 \)). That is,

\[ \frac{\rho_o(x) + \rho_u(x)}{2} = 1. \]

\( \Rightarrow \) no aggregate risk.
Bellman Equation Expert

\[ V^x(w_t, x) = \max_{c_t^x, \tau^n, \theta_t} E \left[ \int_t^{\tau^n} e^{-\rho t} u(c_t^x) \, dt + e^{-\rho \tau^n} V^n(w_t, x) \right] \]

s.t.

\[ dw_t = [w_t (r_f + \theta_t \alpha) - c_t^x - f_{xx} w_t] \, dt + w_t \theta_t \sigma(x) \, dB_t \]

\[ R_t - r_f \, dt = \alpha \, dt + \sigma(x) \, dB_t \]

\[ \text{Mkt Clr} \quad \downarrow \text{in x} \]
Participation Threshold for Expertise

\[ \frac{1}{2\gamma} \left[ \frac{\alpha^2}{\sigma^2(x)} \right] \geq f_{xx} \implies \text{Participate if } x > x \]

Return compensation vs. risk exceeds maintenance cost
Value and Policy Functions

\[ V^x(w_t, x) = y^x(x) \frac{w_t^{1-\gamma}}{1 - \gamma}, \]

Consume, save constant fraction of wealth.

Portfolio allocation: constant fraction, increasing in \( x \):

\[ \theta_t(x) = \frac{\alpha}{\gamma \sigma^2(x)}. \]
A **Stationary Equilibrium** consists of a market clearing $\alpha$, policy functions for all investors, and a stationary distribution over investor types $i \in \{x, n\}$, expertise levels $x$, and wealth shares $z$, $\phi(i, z, x, t)$, such that given an initial wealth distribution, an expertise distribution $\lambda(x)$, and parameters $\{\gamma, \rho, S, r_f, f_{nx}, f_{xx}, \sigma_v\}$ the economy satisfies:

1. Investor optimality: participation, consumption/savings, portfolio choice.

2. Market clearing: $I = \int_{x > x} \lambda(x) I(x) \, dx = S$,

   $$I(x) = \frac{\alpha}{\gamma \sigma^2(x)} z_{\min} \left(1 + \frac{1}{\beta(x) - 1}\right).$$

   portfolio choice(x) wealth share(x)

3. The distribution over all individual state variables is stationary.
Wealth Distribution(s): Kolmogorov FE

Stationary Distribution ⇒ \( \partial_t \phi^x(z, x, t) = 0 \):

\[
0 = -\partial_z \left[ \left( \frac{r_f - f_{xx} - \rho}{\gamma} + \frac{(\gamma + 1) \alpha^2}{2\gamma^2\sigma^2(x)} - g(\bar{x}) \right) \phi^x(z, x) \right] \\
+ \frac{1}{2} \partial_{zz} \left[ \left( z \frac{\alpha}{\gamma\sigma(x)} \right)^2 \phi^x(z, x) \right]
\]

Technically: Wealth shares w/ reflecting barrier (vs. Poisson death).

The stationary distribution of wealth \( \phi^x(z, x) \) is **Pareto** for each \( x \):

\[
\phi(z, x) \propto Cz^{-\beta(x)-1}.
\]
Distribution(s) of Wealth

The stationary distribution of wealth $\phi^x(z, x)$ is Pareto for each $x$:

$$\beta (x) = \left( \gamma + \frac{z_{\min}/\bar{z}}{1 - z_{\min}/\bar{z}} \right) \frac{\sigma^2(x)}{\sigma^2(\bar{x})} - \gamma > 1$$

effective var ratio

Tail parameter $\beta$: Lower $\beta \Rightarrow$ slower decay and fatter upper tail.

- Effective var ratio $\downarrow$ with expertise $\Rightarrow$

- $\beta \downarrow$ in $x$, higher expertise has fatter wealth tail
Comparative Statics: Asset Complexity

Complexity $\equiv$ Total Volatility
before expertise applied

Total Volatility $= \sigma_\nu$

Effective Volatility $\sigma(x) \equiv \sigma(x, \sigma_\nu)$
Results, Future Directions

Conclusions:

▶ Excess returns increase in complexity.
▶ If expertise and complexity are complementary:
  ▶ Participation decreases with complexity
  ▶ Market level equilibrium average Sharpe Ratios increase with complexity

Future Directions:

▶ Slow moving capital in complex asset markets
▶ New risks vs. old risks
▶ Endogenous supply of risk (financial innovation)
**Stationary Distribution Wealth Shares**

Define ratio $z(t, s)$ of individual wealth to the mean wealth of agents with highest expertise:

$$z(t, s) \equiv \frac{w(t, s)}{\mathbb{E}[w | \bar{x}(t, s)]}.$$

LoM mean wealth of agents with expertise $x$ is:

$$\frac{d\mathbb{E}[w | x(t, s)]}{\mathbb{E}[w | x(t, s)]} \equiv [g(x)] \, dt,$$

Define $g(\bar{x}) \equiv \sup_x g(x)$. Then, for any individual investor,

$$\frac{dz(t, s)}{z(t, s)} = \left( \frac{r_f - f_{xx} - \rho}{\gamma} + \frac{(\gamma + 1)\alpha^2(t, s)}{2\gamma^2\sigma^2(x)} - g(\bar{x}) \right) \, dt + \frac{\alpha(t, s)}{\gamma\sigma(x)} dB(t, s),$$

which has a negative drift, or growth rate.
Entry and Exit in OTC Markets

Atkeson, Eisfeldt, Weill ECMA 2015

- Parsimonious equilibrium model of OTC derivatives market from primitives with:
  - Incentives to share risk, make intermediation profits
  - 2 key frictions: Entry cost, Line limits
- Positive: OTC frictions ⇒ observed market structure?
  - Bilateral trade patterns:
    Linkages between banks, and price dispersion
  - Entry patterns:
    Why do large banks become intermediaries?
    Why do middle-sized banks become customers?
- Normative: Can planner/policy maker do better?
  Entry: Large banks may participate too much
  Exit: And they may also exit too quickly
Double continuum of agents: Banks, Traders

Full risk sharing within banks of size $S \sim f(S)$

CARA preferences

Endowed pre-trade exposure to normal default risk

Trade CDS to share risk

Subject to entry cost and per-trader trade size limits
CDS Trading Network: Nearly 1,000 nodes in a core-periphery network
Fact 1: Sparse trading network
FACT 2: STATIC TRADING NETWORK

- **Degree Distribution**
  - Feb 2010: Lower Quartile, Median, Upper Quartile, Mean
  - Aug 2010: Lower Quartile, Median, Upper Quartile, Mean
  - Feb 2011: Lower Quartile, Median, Upper Quartile, Mean
  - Aug 2011: Lower Quartile, Median, Upper Quartile, Mean
  - Feb 2012: Lower Quartile, Median, Upper Quartile, Mean
  - Aug 2012: Lower Quartile, Median, Upper Quartile, Mean
  - Feb 2013: Lower Quartile, Median, Upper Quartile, Mean
  - Aug 2013: Lower Quartile, Median, Upper Quartile, Mean

- **Eigenvector Centrality**
  - Feb 2010: Lower Quartile, Median, Upper Quartile, Mean
  - Aug 2010: Lower Quartile, Median, Upper Quartile, Mean
  - Feb 2011: Lower Quartile, Median, Upper Quartile, Mean
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  - Aug 2013: Lower Quartile, Median, Upper Quartile, Mean
FACT 3: TIME SERIES OF AVERAGE PRICE DISPERSION (EW)
Model

- n agents
- Pre-trade exposures: $w_i$ units of the underlying asset
- Asset is risky with payment given by $1 - D$
  
  $\mathbb{E}[D] = \mu$ and $\mathbb{V}[D] = \sigma^2$

- Agents trade CDS contracts with each other that pays $D$
- Network of trading connections $G$ (n by n matrix)
  
  $g_{ij} = \begin{cases} 
  1 & i \text{ and } j \text{ can trade} \\
  0 & i \text{ and } j \text{ cannot trade}
  \end{cases}$

- Agents take prices as given and choose how much to sell to each counterparty
- Risk averse agents and also averse to counterparty risk
Network example

\[ G = \begin{bmatrix}
1 & 1 & 1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 \\
\end{bmatrix} \]


**Model**

Mean-variance preferences (CARA/Normal):

\[
\max_{z_i, \{\gamma_{ij}\}_{j=1}^n} w_i(1 - \mu) + \sum_{j=1}^n \gamma_{ij}(R_{ij} - \mu) - \frac{\alpha}{2} (w_i + z_i)^2 \sigma^2 - \frac{\phi}{2} \sum_{j=1}^n \gamma_{ij}^2 \\
\text{s.t. } z_i = \sum_{j=1}^n \gamma_{ij} \quad \text{and} \quad \gamma_{ij} = 0 \text{ if } g_{ij} = 0,
\]

- \(\gamma_{ij}\) \equiv \text{how much } i \text{ sells to } j; \ z_i \equiv \text{net position in the CDS market}
- \(\alpha\) \equiv \text{risk aversion}; \(\phi\) \equiv \text{counterparty risk aversion, convex cost}

Symmetric prices \(R_{ij} = R_{ji}\) \ \forall i, j

Clearing conditions \(\gamma_{ij} + \gamma_{ji} = 0\) \ \forall i, j

Network \(G\): exogenous and fixed over time
Equilibrium Intuition

- First-order conditions

\[ R_{ij} - \mu = \phi \gamma_{ij} + \alpha (w_i + z_i) \sigma^2 \]

MB of selling CDS  \hspace{2cm} MC of trading with j  \hspace{2cm} MC of risk exposure

- Shadow prices of insurance

\[ \hat{z}_i = \alpha (\omega_i + z_i) \sigma^2 \]

- Prices are an average of shadow prices of insurance

\[ R_{ij} - \mu = \frac{\hat{z}_i + \hat{z}_j}{2} \]

contract premium

- Bilateral exposures are driven by relative need for insurance

\[ \gamma_{ij} = \frac{\hat{z}_j - \hat{z}_i}{2\phi} \]
Equilibrium Intuition

- Agent $i$’s shadow price of insurance depends on
  (I) $i$’s initial exposure
  (II) neighbors’ shadow prices
- Recursive formulation

\[ \hat{z}_i = (1 - \delta_i)\alpha\sigma^2\omega_i + \delta_i \sum_j g_{ij}\hat{z}_j \]

where \( \delta_i = \left(1 + \frac{2\phi}{K_i\alpha\sigma^2}\right)^{-1} < 1 \)

- Shadow prices depend on all path-connected agents

\[ \hat{z}_i = (1 - \delta_i)\alpha\sigma^2\omega_i + \sum_j \delta_i g_{ij}(1 - \delta_j)\alpha\sigma^2\omega_j + \sum_j \delta_i g_{ij} \sum_s \delta_j g_{js}(1 - \delta_s)\alpha\sigma^2\omega_s + \ldots \]
How do φ and α affect trading patterns?

- φ ∼ cost of bilateral trading
- α ∼ cost of having the wrong total exposure

- Counterparty relative to overall risk aversion: φ/α
  - Distribution of bilateral and net exposures: z’s and γ’s
  - High φ/α
    - Less trade: low z’s, γ’s
    - Lowers XS dispersion of gross and net positions
    - Increases XS dispersion prices

- Overall risk aversion: α
  - Magnitude of CDS premium: holding φ/α constant
  - High α
    - Increases R’s
    - Increases XS dispersion prices
Results and Future Directions

- Estimate $\phi$
- Dealer-Dealer versus Dealer-Customer Transactions
- Trading profits and risk reallocation
- Price dispersion
  - Counterparty risk
  - Bid-ask spread
  - Market segmentation
  - Relationships
- CDS Bond Basis
- Systemically Important Financial Institutions
Talk Outline: Wrapping Up

- Define **Illiquid Markets**:

  *Illiquid Markets are markets in which asset values appear to deviate from fundamental values. Illiquid Markets are typically associated with lower volume, disintermediation, exit from trading.*

- Empirical Examples and Stylized Facts

- Theoretical Structures/Models of Trading Frictions

- Specific Models:
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