A multi-layer network perspective on systemic risk

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- **Main Research Areas**
  - **Economic Networks & Social Organizations**
    - e.g. ownership networks, R&D networks, financial networks, ...
    - e.g. online communities, OSS projects, animal societies, ...

- **Methodological Approach:** Data Driven Modeling
  - **economic databases**: ORBIS, Bloomberg, patent databases
  - **online data**: user interaction, communication records, blogs
What is the problem?

US banks failed during the crisis

Failed banks: 344
Losses: 642.3 bns of $
What is the problem?

Possible explanations:

- **Cascades**: spreading failure \(\Rightarrow\) domino
  - direct interaction: failure affects connected agents

- **Macroeconomic feedback**: indirect coupling \(\Rightarrow\) popcorn
  - no interaction, but externally driven toward critical state

Data: FDIC (Federal Deposit Insurance Corporation), 2011
Risk as endogeneous to the system

Systemic risk

- risk that a whole system comprised of many agents fails
- macroscopic property that emerges from the nonlinear interactions of agents and is amplified through macroscopic feedback

- complements exogeneous risk
- systems generate the conditions of their failure themselves
- failure of “the few” gets amplified
  ⇒ interaction
Bottom-up Approach

- Internal agent dynamics

\[ s_i(t + 1) = \Theta[\phi_i(t, s, A) - \theta_i] \]

- Frailty \( \phi_i(t) > 0 \): depends on neighbors (interaction matrix \( A \))
- Threshold \( \theta_i \): individual conditions (heterogeneity)

- Dynamics of \( \phi_i(t) \) depends on
  - Degree distribution \( p(k) \), distribution of load to neighbors

- Probabilistic approach: prediction of systemic risk \( \varrho(t) \)
Predicting systemic risk

**Mathematical network model**
- various degree distributions $p(k)$
- different threshold distributions $p(\theta)$
- finite/infinite networks
- different load distribution mechanisms

**Phase diagram of systemic risk**

- **Heterogeneity of agents matters!**
  - $\mu \to 0$: increasing global instability
  - $\sigma$: measure of *initial heterogeneity* in $\theta$;
  - small variations in initial conditions lead to complete failure
  - non-monotonous behavior: intermediate $\sigma$ most dangerous
The need to combine two perspectives

- **Micro:** *Socioeconomic perspective*
  - strategic behaviour of single agents’ $\Leftrightarrow$ network architecture

- **Macro:** *Physics/Computer science perspective*
  - statistical regularities of the network as a whole

- **Data-driven modeling:** infer interaction rules of agents

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Networks are constructed

- **aggregation over time**
  - network density depends on time window
  - importance of nodes changes
  - temporal ordering cannot be preserved
Networks are reconstructed

- aggregation over activity
  - direct interactions are often unknown
  - aggregation at institutional level
  - decomposition heuristics
    - co-appearence $\Rightarrow$ temporal network,
    - ranked activities $\Rightarrow$ weights for links
    - core-periphery structure

<table>
<thead>
<tr>
<th>BANK NAME</th>
<th>STATE</th>
<th>TOTAL ASSETS</th>
<th>TOTAL DERIVATIVES</th>
<th>TOTAL FUTURES (EXCH TR)</th>
<th>TOTAL OPTIONS (EXCH TR)</th>
<th>TOTAL FORWARDS (OTC)</th>
<th>TOTAL SWAPS (OTC)</th>
<th>TOTAL OPTIONS (OTC)</th>
<th>TOTAL CREDIT DERIVATIVES (OTC)</th>
<th>SPOT</th>
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<td>JPMORGAN CHASE BANK NA</td>
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Data: US Office of the Comptroller of the Currency, 1998/Q4 - 2012/Q4

Chair of Systems Design
www.sg.ethz.ch
Frank Schweitzer

Multi-layer network perspective on systemic risk
MFM Winter 2016 Meeting
Network approach
Stern School, NYU 28-29 Jan 2016
Interdependent Networks

- **inter-layer plus intra-layer** interactions
- **restricted access**: use *layer 0* to **control** *layer 1*
  - use *peripheral* nodes to control *central* nodes
  - **cost-efficient** strategy to influence the whole network

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Economics: A Multi-Layer Network

- **multiple interactions**
  - ownership/control
  - risk diversification
  - knowledge transfer

- **feedback within layers**
  - investments/participation
  - credit relations, OTC derivatives
  - R&D collaborations

- **feedback between layers**
  - ownership $\rightarrow$ failure risk
  - ownership $\rightarrow$ knowledge transfer
  - failure risk $\rightarrow$ knowledge transfer
Multi-layer structure of financial networks

Banking network of Mexico 30 September 2013

- (a) exposures from derivatives
- (b) securities, cross-holdings
- (c) foreign exchange exposures
- (d) deposits and loans
- (e) combined banking network

Banks colored according to their systemic impact $R_i$
Node-size represents banks total assets
Link-width is the exposure size between banks

http://multinets.io/

MULTINET.JS
A visualization framework for large, dynamic and multi-layered graphs developed by the Chair of Systems Design at ETH Zürich

FEATURES
- Navigation
- Sharing
- Styling
- Camera Options
- Snapshot

EXAMPLES
Systemic risk in a network of firms

- **dynamic state:** healthy: \( s_i(t) = 0 \), or failed: \( s_i(t) = 1 \)

\[
s_i(t + 1) = \Theta[\phi_i(t, s, A) - \theta_i]
\]

- **threshold:** \( \theta_i = \frac{C^i}{L^i} \)
  - ratio between *capital buffer* \( C^i \) and *liabilities* \( L^i \)

- **fragility:** depends on the fraction of failed neighbors
  - diversification mitigates the impact of a single neighbor

\[
\phi^i(k^i) = \frac{1}{k^i} \sum_{j \in \text{nb}(i)} s^j = \frac{n^i}{k^i} = \frac{\sum_{j \in \text{nb}(i)} s^j w^i}{L^i}
\]

- loss from failing neighbors divided by total liability
- firm's liability: \( L^i = \sum_{j \in \text{nb}(i)} w^i \)
- financial obligation to each neighbor: \( w^i = \frac{L^i}{k_i} \)

When is it beneficial to have two layers?

- **Strategic decision of firms**
  1. to operate in two *different* layers
    - core business: layer (0)
    - subsidiary business: layer (1)
    - different risk profiles $F_1(\theta^{(1)})$, $F_1(\theta^{(1)})$
  2. to *merge* businesses

- **compare** systemic risk
  - risk in *separate* layers vs *aggregated*
  - measure final *cascade size* in layer (0)

$$\varrho^{(0)} = \frac{1}{N} \sum_i s_i^{(0)}$$
Asymmetric feedback between layers

1. failure propagation $(0) \rightarrow (1)$
   - leads to a failure in $(1)$
   - $s_i^{(0)} = 1 \rightarrow s_i^{(1)} = 1$
   - failed nodes cannot recover
Asymmetric feedback between layers

1. **failure propagation** (0) → (1)
   - leads to a failure in (1)
   - $s_i^{(0)} = 1 \rightarrow s_i^{(1)} = 1$
   - failed nodes cannot recover

2. **failure propagation** (1) → (0)
   - leads to a threshold reduction in (0)
   - $\theta_i^{(0)} \rightarrow (1 - r_{10}) \theta_i^{(0)}$
   - may lead to subsequent failure in (0)

vary coupling strength $r_{10}$
How cascades propagate

Subsidiary Business

Core Business
How cascades propagate

Subsidiary Business

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How cascades propagate

Subsidiary Business

Core Business
How cascades propagate
How cascades propagate
How cascades propagate
How cascades propagate

Subsidiary Business

Core Business
Mathematical Model

Assumptions:

- each layer: random graph (ER network)
- network size $N \to \infty$, $C \to 0$
- node $i$: independently at random:
  - degrees $k_i^{(0)}$, $k_i^{(1)}$ from $p_0(k_i^{(0)})$, $p_1(k_i^{(1)})$
  - thresholds $\theta_i^{(0)}$, $\theta_i^{(1)}$ from $F_0(\theta_i^{(0)})$, $F_1(\theta_i^{(1)})$

Goal: calculate final cascade size $\varrho_0^*$

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Analytic Approach: Local tree approximation

- **Average** fraction of failed nodes: \( \varrho_0^* = \sum_{k_0} p_0(k_0) \mathbb{P}(s_0 = 1 | k_0) \)

- Prob. to fail because of neighb. failures in layer \((l)\):
  \[
  \varrho_{s,l}^* = \sum_{k_l} p_l(k_l) \sum_{n_l=0}^{k_l} B(n_l, k_l, \pi^*_l) F_l \left( \frac{n_l}{k_l} \right). 
  \]

- Failure prob. in \((l)\) given node’s degree:
  \[
  \mathbb{P}(s_l = 1 | k_l) = 
  \sum_{n_l=0}^{k_l} B(n_l, k_l, \pi^*_l) \mathbb{P} \left( s_l = 1 | k_l, n_l, \varrho_{s,1-l}^* \right). 
  \]

- Ability to withstand shocks:
  \[
  \mathbb{P} \left( s_l = 1 | k_l, n_l, \varrho_{s,1-l}^* \right) = 
  \left( 1 - \varrho_{s,1-l}^* \right) F_l \left( \frac{n_l}{k_l} \right) + \varrho_{s,1-l}^* F_l \left( \frac{n_l}{k_l} \right). 
  \]

- Fail. Prob. Neighb.:
  \[
  \pi_l^* = L(\pi_l^*):= \sum_{k_l} \frac{p_l(k_l) k_l}{z_l} \sum_{n_l=0}^{k_l-1} B(n_l, k_l - 1, \pi_l^*) \mathbb{P} \left( s_l = 1 | k_l, n_l, \varrho_{s,1-l}^* \right) 
  \]
  \[
  \text{where } z_l = \sum_{k_l} p_l(k_l) \cdot k_l. 
  \]
**Analytic Approach: Local tree approximation**

- **Average** fraction of failed nodes:
  \[
  \varrho_0^* = \sum_{k_0} p_0(k_0) \mathbb{P}(s_0 = 1 | k_0)
  \]

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  \left( \left( 1 - \varrho_{s,1-l}^* \right) F_l \left( \frac{n_l}{k_l} \right) + \varrho_{s,1-l}^* F_l \left( \frac{n_l}{k_l} / \left( 1 - r_{1-l,l} \right) \right) \right).
  \]

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  \[
  \pi_l^* = L(\pi_l^*) := \sum_{k_l} \frac{p_l(k_l) k_l}{z_l} \sum_{n_l=0}^{k_l-1} B(n_l, k_l - 1, \pi_l^*) \mathbb{P} \left( s_l = 1 | k_l, n_l, \varrho_{s,1-l}^* \right)
  \]
  where \( z_l = \sum_{k_l} p_l(k_l) \cdot k_l \).
**Result: Sharp regime shift**

- **Computer simulations/ analytical results**
  - *initial condition*: core business is safe
  - *vary*: threshold distribution of subsidiary layer: $\mu_1, \sigma_1$
  - **Emergence of large cascades** $\Rightarrow$ core business collapses
  - below a critical $\mu_1$: coupling leads to complete failure
nodes in aggregated layer

- **degree**: \( k_{\text{agg}} = k^{(0)} + k^{(1)} \)
  - better diversified, but higher connectivity
  \( \Rightarrow \) amplification of cascades

- **threshold**:
  \[ \theta_{\text{agg}} = \frac{\theta^{(0)} k^{(0)} + \theta^{(1)} k^{(1)}}{k^{(0)} + k^{(1)}} \]

- \( \theta = C/L \Rightarrow \) shared capital buffers
Is merging of businesses safer?

Layer (0): \( \rho_0^* \)  

Layer (1): \( \rho_1^* \)  

Aggregated: \( \rho_{\text{agg}}^* \)

\[ r_{10} = 0.1 \]

\[ r_{10} = 0.2 \]

No for small coupling \( r_{10} \).
Is merging of businesses safer?

$r_{10} = 0.4$

$r_{10} = 0.8$

Yes for stronger coupling $r_{10}$. 
Mergers not always decrease risk

Layer (0): $\varrho^*_0$

Layer (1): $\varrho^*_1$

Aggregated: $\varrho^{*\text{agg}}$

Influence of coupling $r_{10}$ and $\mu_1$ (small $\sigma_1$)

- $r_{10} \leq 0.2$: Disaggregation blocks cascade amplification!
- Increased systemic risk above critical coupling
Conclusions

1. **Systemic risk emerges**
   - bottom-up approach, mathematical framework \(\Rightarrow \rho(t) \rightarrow \rho^*\)

2. **Multi-layer network approach**
   - networks are (re)constructed \(\Rightarrow\) several pitfalls
   - interdependent networks \(\Rightarrow\) more pitfalls (intra/inter-layer links)

3. **Systemic risk in multi-layer networks**
   - split into core/subsidiary business \(\Rightarrow\) amplification of failure
   - critical coupling strength \(r_{10}\), critical risk profile \(\mu_1\)
   - sharp transition between no/full collapse
Conclusions

1. **Systemic risk emerges**
   - bottom-up approach, mathematical framework \( \varphi(t) \rightarrow \varphi^* \)

2. **Multi-layer network approach**
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Advice for risk managers

1. Understand the role of *couplings* between businesses!
2. Did you draw risk scenarios for split businesses from an *aggregated model*? Then you have **underestimated** your real risk!