Lifetime-Laffer Curves and the Eurozone Crisis

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Motivation

- Eurozone crisis: Key features
  1. Sentiments seemed to play role (OMT)
     - Not liquidity Bocola and Dovis (2015)
  2. Borrowing into high spreads $\rightarrow$ Debt-to-GDP exploded
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  1. Parsimonious model to generate such crises
     - Multiplicity of financing trajectories
     - Driven by lack of commitment to future behavior
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  2. Calibrated example/Quantitative relevance
     - Could be responsible for more than 380 basis points (84.6%) of average spread for Ireland
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  3. Policy prescriptions
  4. Extensions
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- This paper: Impose commitment in terminal periods, but not in initial
  - Work in progress: Commitment in all periods
Model Outline: Sovereign

- 3-period model: \( t = 0, 1, 2 \); only risk is default in period 2
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  - Financing trajectory, $< b_1, b_2 >$, endogenous
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- **Budget Condition**

\[
\begin{align*}
  d_0 &= q_0(b_1 - b_0) \\
  d_1 &= q_1(b_2 - b_1)
\end{align*}
\]
Model Outline: Lenders

- Default risk in period 2: $g : \mathcal{R} \rightarrow [0, 1]$
  - $g$ is increasing, continuous, differentiable, and convex up to $\bar{b} < \infty$ s.t.
    $$g(\bar{b}) = 1$$
  - and equals one thereafter
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    and equals one thereafter
- Lenders are risk-neutral, deep-pocketed, price against risk-free \( R \)
- Implies No-Arbitrage Condition

\[
q_0 = \frac{\hat{B}}{R^2}[1 - g(b_2)]
\]
\[
q_1 = \frac{\hat{B}}{R}[1 - g(b_2)]
\]
Model Outline: Commitment

- Period one auction revenue given by

\[
\frac{\hat{B}}{R} [1 - g(b_2)] \times [b_2 - b_1]
\]

Under our assumptions, concave in \( b_2 \)
Model Outline: Commitment

Revenue

\( b_1 \)

\( \bar{b} \)

New Debt: \( b_2 \)
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Under our assumptions, concave in \( b_2 \)

- Two solutions for any feasible (positive) revenue
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- Assume always on LHS: **Commitment Condition**

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g'(b_2)(b_2 - b_1) \leq 1 - g(b_2)
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Terminal period: **Contemporaneous commitment to debt issuance**
Characterizing the Solution: Lifetime BC

- Idea: Collapse flow BC into Lifetime BC
  - Derive **Lifetime-Laffer Curve** → Multiplicity
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- Re-write BC1 as a function of $b_2$

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b_1 = b_2 - \frac{Rd_1}{[1 - g(b_2)]\hat{B}}\]
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- Substitute into BC0

$$D = \frac{[1 - g(b_2)]\hat{B}}{R^2}[b_2 - b_0]$$
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- Where $D = d_0 + \frac{d_1}{R}$
Multiplicity via the Lifetime-Laffer Curve

- Let

\[ D^*(b_0) = \max_{b_2} \left[ \frac{1 - g(b_2)}{R^2} \right] \hat{B} [b_2 - b_0] \]
Multiplicity via the Lifetime-Laffer Curve

- Let

\[ D^*(b_0) = \max_{b_2} \frac{[1 - g(b_2)]}{R^2} [b_2 - b_0] \]

Proposition

Suppose that \( 0 < D < D^*(b_0) \). Then two solutions exist if and only if the sovereign's primary deficit stream is sufficiently front-loaded.

Call \( b_2 \) for each of these solutions \( b_L \) and \( b_H \)
Graphical Example
The Logic of Front-Loading

Formal **Front-Loading Condition**

\[
\frac{d_1}{R} \leq \frac{\hat{B}[1 - g(b_H(D))]^2}{R^2 g'(b_H(D))}
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The Logic of Front-Loading

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\frac{\partial RevPeak_1}{\partial b_1} > 0
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    \[
    \frac{\partial}{\partial b_1} \text{RevPeak}_1 > 0
    \]
  - Holds for \( b_H \rightarrow \) Holds for \( b_L \)
The Logic of Front-Loading
Policy 1: Austerity

- Subject of *much* debate in recent years. Can it work?
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**Proposition**

*If* $d_0 \leq 0$, *then at most one solution exists, and it is on the LHS of the Lifetime-Laffer Curve*
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*If* $d_0 \leq 0$, *then at most one solution exists, and it is on the LHS of the Lifetime-Laffer Curve*

- Immediate austerity (enough to induce buyback) works
- Dilution makes buyback easier $\rightarrow$ Kills self-fulfilling dynamics

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Proposition

*If* $0 < D < D^*(b_0)$ and $d_1 \leq 0$, *then two solutions exist.*

- Delayed austerity *guarantees* existence of two solutions
- Front-loads deficit stream
Policy 2: Liquidity Provision

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- Add central bank as deep-pocketed third party: Specifies \(< \hat{q}_0, \hat{q}_1 >\) at which it is willing to purchase requisite debt to fill primary deficits
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- Sovereign receives a choice
  - If multiple financing trajectories are available, he goes with the one with the lowest default probability
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Proposition

*The central bank can costlessly eliminate the high-debt solution by pledging to provide liquidity at $< q_{0,L}, q_{1,L} >$.  

Extension 1: $T$-Periods

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- Can still construct Lifetime-Laffer Curve with all same properties.

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Suppose that $0 < D < D^*(b_0)$. Then two solutions to the $T$-period model exist if and only if the sovereign’s primary deficit stream is sufficiently front-loaded.
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**Proposition**

*Suppose that $0 < D < D^*(b_0)$. Then two solutions to the $T$-period model exist if and only if the sovereign’s primary deficit stream is sufficiently front-loaded.*

- Calibrate $T$-period model to Irish data: 2008-2013
  - Explains 380 bp of 450 bp spread
  - Very little change in counterfactual $B/Y$
Extension 2: Deficit Response

- In $T$-period model, suppose that $d_t(b_T)$
  - Sovereign responds to expected debt build-up (or spreads, default prob, etc.)
  - Assume $d_t(\cdot)$ continuous, twice differentiable

Proposition

*Under a feasibility condition, a positive economy* $<< b_0, \{d_t(\cdot)\}^{T-1}_{t=0} >>$ *will have at least two distinct financing trajectories. Further, if each $d_t(\cdot)$ is increasing and convex and a front-loading condition holds, then exactly two solutions exist.*
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• Can augment model to include possibility of banking sector bailout
  • Much more action on $B/Y$
Extension 3: Uncertainty

- Introduce rollover risk in period 1
  - $N < \infty$ potential states with distribution $\{\pi_s\}_{s=1}^{N}$
  - Period 1 deficit is state-dependent: $d_1(s)$
  - Commitment condition, budget condition, and no-arbitrage condition hold in each state
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  - Period 1 deficit is state-dependent: $d_1(s)$
  - Commitment condition, budget condition, and no-arbitrage condition hold in each state
- Economy is described by $<< b_0, d_0, \{d_1(s)\}_{s=1}^{N} >>$ and the distribution across $s$
- Solution given by $<< b_1, \{b_2(s)\}_{s=1}^{N} >>$
Extension 3: Uncertainty

Proposition

Under a feasibility condition and a front-loading condition, a positive economy \(<\langle b_0, d_0, \{d_1(s)\}_{s=1}^N \rangle\rangle\) will have at least two distinct solutions. Further, if \(<\langle b_{L,1}, \{b_L(s)\}_{s=1}^N \rangle\rangle\) and \(<\langle b_{H,1}, \{b_H(s)\}_{s=1}^N \rangle\rangle\) are components of those two distinct solutions and wlog it must be the case that \(b_L(s) \leq b_H(s)\) for any \(s \in S\) and that only the lesser solution will be numerically stable.
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Generalizes deterministic existence result
Example with Uncertainty: $N = 25$
Conclusion

- Tractable, three period model in which multiple financing trajectories arise as a result of coordination failures with long-term debt
- Calibrate to Ireland $\rightarrow$ Substantial impact on spreads
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- Analyzed policy
  1. Liquidity provision by central bank effective
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- Analyzed policy
  1. Liquidity provision by central bank effective
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- Further work:
  - Infinite-horizon limiting case: Commitment in all periods (in progress)
  - Empirical identification
  - Application to other markets (commercial paper, municipal debt, etc.)