Comparative Valuation Dynamics in Models with Financial Frictions

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Motivation

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This prompts the following questions:

- What are the differences between those models?
- What are their similarities?
- How could we tell them apart if we had the “right” data?
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Research Goal: Compare/contrast implications of DSGE models with financial frictions
Research Objective

- **Research Goal**: Compare/contrast implications of DSGE models with financial frictions

- **Which Models?**
  - Continuous time with Brownian information structure
  - Non-trivial financial/intermediary sector
  - Financial/contractual frictions impeding the allocation of aggregate risk across economic agents
  - Non-linear behaviors
Research Objective

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- **Which Models?**
  - Continuous time with Brownian information structure
  - Non-trivial financial/intermediary sector
  - Financial/contractual frictions impeding the allocation of aggregate risk across economic agents
  - Non-linear behaviors

- **Which Comparisons?**
  - Macroeconomic quantity implications
  - Asset pricing implications
  - Macro- and micro-prudential policy
A “Nesting” (and Work-in-Progress) Model

"Experts":
- Time Preference: $\rho_e$
- Risk Aversion: $\gamma_e$
- IES: $\psi_e$
- Productivity: $a_e$

"Households":
- Time Preference: $\rho_h$
- Risk Aversion: $\gamma_h$
- IES: $\psi_h$
- Productivity: $a_h$

Technology:
- $\frac{dk_t}{k_t} = (g_t + \iota_t - \delta) dt + \sqrt{\kappa} \sigma_A \cdot dZ_t$
- $dg_t = -\lambda_g (g_t - g) dt + \sqrt{\kappa} \sigma_g \cdot dZ_t$
- $d \nu_t = -\lambda \nu (\nu_t - \nu) dt + \sqrt{\kappa} \sigma \nu \cdot dZ_t$

DSGE Models with Financial Frictions
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Nesting Financial Frictions’ Models

**Technology**

\[
\frac{dk_i}{k_i} = (g_i + \delta) dt + \sqrt{\nu_i} \sigma_A \cdot dZ_i
\]

\[
dg_i = -\lambda_g (g - g_i) dt + \sqrt{\nu_g} \sigma_g \cdot dZ_i
\]

\[
d\nu_i = -\lambda_i (\nu - \nu_i) dt + \sqrt{\nu_i} \sigma_i \cdot dZ_i
\]

**“Experts”**: 
- Time Preference: \( \rho_e \)
- Risk Aversion: \( \gamma_e \)
- IES: \( \psi_e = 1/\gamma_e \)
- Productivity: \( a_e \)

**“Households”**: 
- Time Preference: \( \rho_h = 1 \)
- Risk Aversion: \( \gamma_h = 1 \)
- IES: \( \psi_h = 1 \)
- Productivity: \( a_h = -\infty \)

**Basak & Cuoco 1998**

- Assets
  - Physical Capital \( q(t) k_e(t) \)
  - Risk Free Short Term Debt
  - Net Worth \( n_e(t) \)
- Liabilities
  - External Equity

- Assets
  - Physical Capital \( q(t) k_h(t) \)
  - Risk Free Short Term Bonds
  - Net Worth \( n_h(t) \)
- Liabilities
  - External Equity
  - Equities

**DSGE Models with Financial Frictions**

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Nesting Financial Frictions’ Models

“Experts”:
Time Preference: $\rho_e$
Risk Aversion: $\gamma_e = 1$
IES: $\psi_e = 1$
Productivity: $a_e$

Physical Capital $q(t)$ $k(t)$
Risk Free Short Term Debt
Net Worth $n(t)$
External Equity

“Households”:
Time Preference: $\rho_h$
Risk Aversion: $\gamma_h = 1$
IES: $\psi_h = 1$
Productivity: $a_h = -\infty$

Physical Capital $q(t)$ $k_h(t)$
Risk Free Short Term Bonds
Equities
Net Worth $n_h(t)$

Technology
$$\frac{dk_i}{k_i} = (g_t + \delta) dt + \sqrt{\nu_t} \sigma_A \cdot dZ_t$$
$$dg_i = \lambda_i(g_t - g) dt + \sqrt{\nu_t} \sigma_g \cdot dZ_t$$
$$d\nu_i = \lambda_i(\nu_t - \gamma) dt + \sqrt{\tau_t} \sigma_\nu \cdot dZ_t$$

He & Krishnamurthy 2013

DSGE Models with Financial Frictions
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Nesting Financial Frictions’ Models

Assets

Risk Free
Short Term
Debt

Physical Capital
q(t) ke(t)

Net Worth
n_e(t)

Liabilities

External Equity

Derivatives

"Experts":
Time Preference: \( \rho_e \)
Risk Aversion: \( \psi_e = 1 \)
IES: \( \gamma_e = 1 \)
Productivity: \( a_e \)

Technology

\[
\frac{dk_t}{k_t} = (\theta + i_t - \delta) dt + \sqrt{\nu_t} \sigma_A \cdot dZ_t
\]

\[
dg_t = -\lambda_g (g_t - g) dt + \sqrt{\nu_t} \sigma_g \cdot dZ_t
\]

\[
d\nu_t = -\lambda \nu_t (\nu_t - \nu) dt + \sqrt{\nu_t} \sigma \nu \cdot dZ_t
\]

"Households":
Time Preference: \( \rho_h \)
Risk Aversion: \( \psi_h = 1 \)
IES: \( \gamma_h = 1 \)
Productivity: \( a_h < a_e \)

BRUNNERMEIER & SANNIKOV 2014

DSGE Models with Financial Frictions
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Nesting Financial Frictions’ Models

**“Experts”:**
- Time Preference: \( \rho_e \)
- Risk Aversion: \( \gamma_e \)
- IES: \( \psi_e \neq 1/\gamma_e \)
- Productivity: \( \alpha_e \)

**Physical Capital:** \( q(t) k_e(t) \)

**Risk Free Short Term Debt:**

**Net Worth:** \( n_e(t) \)

**External Equity:**

**Assets**

**Liabilities**

**Technology**

\[
\frac{dk_i}{k_i} = \left( g_i + \iota_i - \delta \right) dt + \sqrt{\nu_i} \sigma_i dZ_i
\]

\[
dg_i = -\lambda_g (g_i - g) dt + \sqrt{\nu_i} \sigma_g dZ_i
\]

\[
dv_i = -\lambda_v (v_i - v) dt + \sqrt{\nu_i} \sigma_v dZ_i
\]

**“Households”:**
- Time Preference: \( \rho_h \)
- Risk Aversion: \( \gamma_h \)
- IES: \( \psi_h \neq 1/\gamma_h \)
- Productivity: \( \alpha_h = -\infty \)

**Physical Capital:** \( q(t) k_h(t) \)

**Risk Free Short Term Bonds**

**Net Worth:** \( n_h(t) \)

**Equities**

**Assets**

**Liabilities**

**Di Tella 2017**

DSGE Models with Financial Frictions

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Nesting Heterogenous Agents’ Models

Technology

\[
\frac{dk_t}{k_t} = (g_t + \delta) dt + \sqrt{\nu_t} \sigma_A \cdot dZ_t
\]

\[
g_t = \lambda_g (g_t - g) dt + \sqrt{\nu_t} \sigma_g \cdot dZ_t
\]

\[
\nu_t = -\lambda \nu (\nu_t - \nu) dt + \sqrt{\nu_t} \sigma \nu \cdot dZ_t
\]

Garleanu & Panageas 2015

Longstaff & Wang 2012

“Experts”:
- Time Preference: \( \rho_e \)
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- Productivity: \( a_e \)

“Households”:
- Time Preference: \( \rho_h \)
- Risk Aversion: \( \gamma_h \)
- IES: \( \psi_h \neq 1/\gamma_h \)
- Productivity: \( a_h = a_e \)
Nesting Long Run Risk Models

**Technology**

\[ \frac{dk_i}{k_i} = \left( g_t + \delta - \delta \right) dt + \sqrt{\nu_t} \sigma_A \cdot dZ_t \]

\[ dg_t = -\lambda_g \left( g_t - g \right) dt + \sqrt{\nu_t} \sigma_g \cdot dZ_t \]

\[ d\nu_t = -\lambda_\nu \left( \nu_t - \nu \right) dt + \sqrt{\nu_t} \sigma_\nu \cdot dZ_t \]

**“Experts”**: Time Preference: \( \rho_e \) Risk Aversion: \( \gamma_e > 1 \) IES: \( \psi_e > 1 \) Productivity: \( a_e \)

**“Households”**: Time Preference: \( \rho_h = \rho_e \) Risk Aversion: \( \gamma_h = \gamma_e \) IES: \( \psi_h = \psi_e \) Productivity: \( a_h = a_e \)

**Assets**

- Physical Capital: \( q(t) k_e(t) \)
- Derivatives

**Liabilities**

- Risk Free Debt
- Derivatives

**Bansal & Yaron 2004**

- Physical Capital: \( q(t) k_h(t) \)
- Risk Free Short Term Bonds
- Equities
- Derivatives

- Net Worth: \( n_e(t) \)
- Net Worth: \( n_h(t) \)
Agent “i” value function:

\[ V_{i,t} = \frac{(n_{i,t}\xi_i(X_t))^{1-\gamma_i}}{1 - \gamma_i} \]
Markovian Equilibrium

- Agent \( i \) value function:

\[
V_{i,t} = \frac{(n_{i,t} \xi_i(X_t))^{1-\gamma_i}}{1 - \gamma_i}
\]

- Look for an equilibrium where agent’s strategies are a function of the model’s state vector \( X_t \in \Omega \)
  - Exogenous states (exp. TFP growth \( g_t \), stochastic vol. \( \nu_t \))
  - Endogenous states (wealth distribution, here simply \( w_t := \frac{n_{e,t}}{N_t} \))
Markovian Equilibrium

- Agent “i” value function:

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- \( X_t \) follows dynamics:

\[ dX_t = \mu_X(X_t) \, dt + \sigma_X(X_t) \cdot dZ_t \]
Agent “i” value function:

\[ V_{i,t} = \frac{(n_{i,t} \xi_i(X_t))^{1-\gamma_i}}{1 - \gamma_i} \]

Look for an equilibrium where agent’s strategies are a function of the model’s state vector \( X_t \in \Omega \)

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Endogenous state space partition \( \Omega = \Omega_u \cup \Omega_c \cup \Omega_d \)
Solution Strategy and Numerical Challenges

- Complex fixed point problem in the space of functions:
  \[ \{\xi_i(X)\} \rightarrow (r(X), \pi(X), q(X), \mu_X(X), \sigma_X(X)) \rightarrow \{\xi_i(X)\} \]
Solution Strategy and Numerical Challenges

- Complex fixed point problem in the space of functions:
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- \( \{\xi_i\}_{i \in \{e,h\}} \) solve a set of second order non-linear elliptic PDEs
Solution Strategy and Numerical Challenges

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- Numerical considerations:
  - Finite difference scheme
  - Non-linear PDE that requires iteration scheme
  - Inversion of very large (sparse) matrices
  - Boundary conditions
  - No guarantee of convergence
Solution Strategy and Numerical Challenges

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  - No guarantee of convergence

- Implementation:
  - Core computations performed in C++ (allowing for HPC)
  - Shell in high-level languages (Matlab, Python) will be available
Traditional Diagnostics

- Stochastic discount factor(s)

\[
\frac{dS_{i,t}}{S_{i,t}} = -r(X_t)dt - \pi_i(X_t) \cdot dZ_t
\]
Traditional Diagnostics

- Stochastic discount factor(s)

\[
\frac{dS_{i,t}}{S_{i,t}} = -r(X_t) dt - \pi_i(X_t) \cdot dZ_t
\]

- Aggregate state dynamics

\[
dX_t = \mu_X(X_t) dt + \sigma_X(X_t) \cdot dZ_t
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Traditional Diagnostics

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dX_t = \mu_X(X_t)dt + \sigma_X(X_t) \cdot dZ_t
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- Stationary density \( f(X) \), solution of KF equation
Stationary Distribution

[Diagram showing two contour plots with the x-axis labeled 'omega' and the y-axis labeled 'g'.]
Shock elasticities as counterparts to impulse response functions
Shock elasticities as counterparts to impulse response functions

Consider an (exponential) martingale perturbation $H_{(0,s)}$

$$d \ln M_t = \mu_M(X_t)dt + \sigma_M(X_t) \cdot dZ_t$$

$$\epsilon_M(x, t) : = \lim_{s \to 0} \frac{1}{s} \mathbb{E} \left[ \frac{M_t}{M_0} H_{(0,s)} \big| X_0 = x \right]$$
Shock elasticities as counterparts to impulse response functions

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$$\epsilon_M(x, t) := \lim_{s \to 0} \frac{1}{s} \mathbb{E} \left[ \frac{M_t}{M_0} H_{(0,s)} | X_0 = x \right]$$

Applications for a cash-flow $C_t$ received at time $t$

- Shock exposure elasticity $\epsilon_C(x, t)$;
- Shock cost elasticity $\epsilon_{SC}(x, t)$;
- Shock price elasticity $\epsilon_C(x, t) - \epsilon_{SC}(x, t)$
Shock Exposure Elasticities

First Shock
- $g = -1sd; s = -1sd$
- $g = -1sd; s = 1sd$
- $g = 1sd; s = -1sd$
- $g = 1sd; s = 1sd$
- $g = mean; s = mean$

Second Shock

Third Shock

First Shock
- $\omega = -1sd; g = -1sd$
- $\omega = -1sd; g = 1sd$
- $\omega = 1sd; g = -1sd$
- $\omega = 1sd; g = 1sd$
- $\omega = mean; g = mean$

Second Shock

Third Shock
Shock Price Elasticities for “Experts”

DSGE Models with Financial Frictions
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Shock Price Elasticities for “Households”

DSGE Models with Financial Frictions
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Study interaction between different aggregate shocks and financial frictions
Conclusion / Next Steps

- Study interaction between different aggregate shocks and financial frictions
- Consider additional types of financial constraints
Conclusion / Next Steps

- Study interaction between different aggregate shocks and financial frictions
- Consider additional types of financial constraints
- Analyze link between heterogenous preference models, heterogenous belief models, financial frictions’ models