Perfect Competition in Markets with Adverse Selection

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Agenda

Adverse selection is considered a first-order problem in many markets, which are already heavily regulated in *complicated ways*:

- Mandates, community rating, risk adjustment, differential subsidies, regulation of contract characteristics.

All of these affect contract characteristics.

This is a challenge to the standard models (Akerlof / Eivan Finkelstein and Cullen, and Rothschild and Stiglitz).
This paper

Develops a price-taking model of adverse selection.

- Contract characteristics are endogenous.
- Consumers can be heterogeneous in more than one dimension.
- Equilibrium always exists.

Basic idea:

- Start from broad set of potential contracts.
- Use the same logic as price-taking models (Akerlof and Einav-Finkelstein and Cullen) to determine both prices and which contracts are traded.
Determining prices

\[ D(p) \]

\[ AC(q) \]

\[ p^* \]
Determining which contracts are traded
Coverage (Density)

No Mandate
Mandate

Coverage
Outline

1. Model
2. Competitive Equilibrium
3. Application: Equilibrium Effects of a Mandate
4. Inefficiency and Policy Interventions
Model

- **Consumers** $\theta \in \Theta$, distributed according to a probability distribution $\mu$.
- **Contracts** (or products) $x \in X$.
- Agent $\theta$ has **utility**
  \[ U(x, p, \theta) \]
  of buying $x$ at a price $p$, and the **cost** is
  \[ c(x, \theta) \geq 0 \]

Single, exogenous product: $X = \{0, 1\}$.

Quasilinear utility,

$$U = u(x, \theta) - p.$$

Single product is often not realistic.

No predictions on contract terms.

In particular, the model is silent about intensive margin regulations.

Yields useful predictions on pricing and efficiency.
Equilibria

- All that matters are willingness to pay and costs, \( u(1, \theta) \) and \( c(1, \theta) \).
- Can define demand \( D(P) \), and average cost \( AC(Q) \) curves.
- Equilibria are intersection of demand and average cost.
Toy example: Rothschild and Stiglitz *QJE* 1976

- All consumers have same wealth, same risk preferences, and may suffer a loss of the same size.
- Only two types, who differ in their probability of a loss, $\Theta = \{L, H\}$.
- Contracts specify % of loss covered, $X = [0, 1]$.
- Even in this setting, equilibria do not necessarily exist.
Interesting example: Einav, et al. *AER* 2013

- A model of health insurance.
- Higher dimensional heterogeneity of consumers:
  - Loss distributions.
  - Risk aversion.
  - Moral hazard parameters.
- Will calibrate this model to illustrate ideas, with set of contracts $X = [0, 1]$ being % of coverage.
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- A model of health insurance.
- Higher dimensional heterogeneity of consumers:
  - Loss distributions.
  - Risk aversion.
  - Moral hazard parameters.
- Will calibrate this model to illustrate ideas, with set of contracts $X = [0, 1]$ being % of coverage.
- Assuming CARA preferences,

\[
\begin{align*}
    u(x, \theta) &= x \cdot M_\theta + \frac{x^2}{2} \cdot H_\theta + \frac{1}{2}x(2 - x) \cdot S^2_\theta A_\theta, \\
    c(x, \theta) &= x \cdot M_\theta + x^2 \cdot H_\theta.
\end{align*}
\]
Assumptions

1. \( X \) and \( \Theta \) are compact subsets of Euclidean space.
2. \( U(x, p, \theta) = u(x, \theta) - p \), where \( u \) is Lipschitz in \( x \).
3. \( u \) and \( c \) are continuous.
Assumptions

Simpler assumptions for the talk:

1. $\mathcal{X}$ and $\Theta$ are compact subsets of Euclidean space.
2. $U(x, p, \theta) = u(x, \theta) - p$, where $u$ is Lipschitz in $x$.
3. $u$ and $c$ are continuous.
Prices and allocations

- A **price** is a measurable function $p$ over $X$, price of contract $x$ denoted $p(x)$.
- An **allocation** $\alpha$ is a measure over $\Theta \times X$ such that $\alpha|\Theta = \mu$.
- Given $(p, \alpha)$, **consumers are optimizing** if, for $(x, \theta)$ with probability 1 according to $\alpha$, for all $x' \in X$,

$$u(x, \theta) - p(x) \geq u(x', \theta) - p(x').$$
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Given $(p, \alpha)$, consumers are optimizing if, for $(x, \theta)$ with probability 1 according to $\alpha$, for all $x' \in X$,

$$u(x, \theta) - p(x) \geq u(x', \theta) - p(x').$$

Conditional moments are denoted as

$$E_x[c] = E[c(\tilde{x}, \tilde{\theta})|\alpha, \tilde{x} = x].$$
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Weak equilibrium

**Definition**

A price-allocation pair \((p, \alpha)\) is **weak equilibrium** if

1. Consumers optimize.
2. All contracts make 0 profits,

\[
p(x) = E_x[c]
\]

almost everywhere according to \(\alpha\).
Weak Equilibrium Example: Rothschild-Stiglitz

\( X = [0, 1] \) and \( \Theta = \{L, H\} \).
But there are *many* other weak equilibria

- $X = [0, 1]$ and $\Theta = \{L, H\}$. 
Definition: Perturbations

- A behavioral type $x$ is an agent who always demands contract $x$, $u(x, x) = \infty$, $u(x', x) = 0$ if $x' \neq x$, and $c(x, x) = 0$. 
**Definition: Perturbations**

- A behavioral type $x$ is an agent who always demands contract $x$, $u(x, x) = \infty$, $u(x', x) = 0$ if $x' \neq x$, and $c(x, x) = 0$.

- A perturbation $(\bar{X}, \eta)$ is an economy with a finite set of contracts $\bar{X} \subseteq X$, set of types $\Theta \cup \bar{X}$, and distribution of types $\mu + \eta$, where the support of $\eta$ is $\bar{X}$.

- We can define unrefined equilibria of perturbations because every perturbation is a particular case of the model.
Equilibrium of a Perturbation: Example
Equilibrium of a Perturbation: Example
A sequence of perturbations \((\bar{X}^n, \eta^n)_{n \in \mathbb{N}}\) converges to the original economy if

1. Every point in \(X\) is the limit of a sequence \((x^n)_{n \in \mathbb{N}}\) with each \(x^n \in \bar{X}^n\).

2. The mass of behavioral types \(\eta^n(\bar{X}^n)\) converges to 0.
Definition: Perturbations (continued)

Consider a sequence of perturbations \((\bar{X}^n, \eta^n)_{n \in \mathbb{N}}\) converging to the original economy. A sequence of weak equilibria \((p^n, \alpha^n)_{n \in \mathbb{N}}\) converges to \((p^*, \alpha^*)\) if

1. The allocations \(\alpha^n \in \Delta((\Theta \cup X) \times X)\) converge to \(\alpha^*\) weakly.

2. For every sequence \((x^n)_{n \in \mathbb{N}}\), with each \(x^n \in \bar{X}^n\) and limit \(x \in X\), we have that \(p^n(x^n)\) converges to \(p^*(x)\).
Equilibrium

Definition

\((p^*, \alpha^*)\) is a competitive equilibrium if there exists a sequence of perturbations converging to the original economy with a sequence of weak equilibria that converges to \((p^*, \alpha^*)\).
Equilibrium: Example
Equilibrium: Example

The diagram illustrates an equilibrium example with points L and H connected by lines $IC^H$ and $IC^L$. The $x$-axis and $y$-axis are labeled accordingly.
Equilibrium: Example

\[ p(x) \]

Graph showing the relation between \( p(x) \) and \( x \) with points labeled \( L \) and \( H \). The curves are labeled \( IC^H \) and \( IC^L \).
Existence

Theorem

A competitive equilibrium exists.
Proof Outline

- **Step 1:** Every perturbed economy has an equilibrium, by a standard fixed point argument.
- **Step 2:** Equilibrium prices in every perturbed economy are uniformly Lipschitz.
- **Step 3:** Every sequence of perturbations converging to the original economy has a convergent subsequence, and the limit is an equilibrium of the original economy.
Equilibrium properties

Proposition

1. Every equilibrium is a weak equilibrium.

2. Equilibrium prices are continuous and almost everywhere differentiable.

3. For every contract \( x \) with strictly positive equilibrium price there exists a consumer \( \theta \) who is indifferent between her current contract and \( x \). Moreover,

   \[
   c(x, \theta) \geq p(x).
   \]
The paper shows that the competitive model is a particular limiting case of Bertrand competition with differentiated products.
Strategic Foundations

- The paper shows that the competitive model is a particular limiting case of Bertrand competition with differentiated products.

- In particular, this means that competitive models (Rothschild and Stiglitz, Akerlof, Riley) are limiting cases of the differentiated-products models used in empirical IO (Starc, Veiga and Weyl).

- Key assumptions: Many, small firms, with a small degree of differentiation.
Bertrand Game (definitions)

- Fix a perturbation \((E, \bar{X}, \eta)\).
- \(n\) firms selling differentiated varieties of each product \(x\).
- Logit shares \(S(P, p, x, \theta)\) equal to
  \[
  e^{\sigma \cdot (u(x, \theta) - P)} \over \sum_{x' \neq x} n \cdot e^{\sigma \cdot (u(x', \theta) - p(x'))}.
  \]
- Profits
  \[
  \Pi(P, p, x) = \int_{\theta} S(P, p, x, \theta) \cdot (P - c(x, \theta)) \, d(\mu + \eta)
  \]
  if firms produce less than scale \(\bar{q}\) or \(-\infty\) if the firm produces more.
Proposition

There exists a constant $K$ such that, if

$$\frac{1}{n} < \overline{q} < K,$$

then an equilibrium exists. Moreover, profits per unit sold are lower than $\frac{2}{\sigma}$.

- Can show that with fixed small scale and large number of firms, as elasticities go to infinity equilibria converge to the perfectly competitive outcome.
- Bottom line: perfect competition is the limit of a Bertrand game with many, small, and undifferentiated firms.

In one-dimensional case, coincides with some standard notions from the signaling and screening literatures:
- Rothschild and Stiglitz, when their equilibrium exists.
- Riley *Ecma* (1979) reactive equilibrium.

But differs from notions that allow firms to cross-subsidize contracts:
- Netzer and Scheuer *IER* (2014).

Veiga and Weyl (2014)
- Complementary to our work, many similar comparative statics.
- Key differences are imperfect competition and product variety (tomato sauce).
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Calibration: Einav et al. health insurance model

\[ u(x, \theta) = x \cdot M_\theta + \frac{x^2}{2} \cdot H_\theta + \frac{1}{2}x(2 - x) \cdot S^2_\theta A_\theta, \text{ and} \]
\[ c(x, \theta) = x \cdot M_\theta + x^2 \cdot H_\theta. \]

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Log covariance

\[ \sigma^2_{\log H} = 0.28 \]

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<td>\sigma^2_{\log H} = 0.28</td>
<td>-0.03</td>
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<tr>
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Equilibrium prices and adverse selection

![Graph showing equilibrium prices and average loss parameter. The graph plots contract ($) on the x-axis and price ($) on the y-axis. The black line represents the equilibrium prices, and the red dots represent the average loss parameter.]
Equilibrium demand profile

Risk Aversion, $A_\theta$

Average Loss, $M_\theta$

$1,000$ $10,000$ $100,000$

$10^{-6}$ $10^{-5}$ $10^{-4}$

$0.2$ $0.4$ $0.6$ $0.8$ $1$
Simulating a Mandate

The graph illustrates the relationship between the contract value and the corresponding prices and losses under different conditions:

- **No Mandate**
  - Prices
  - Losses

- **Mandate**
  - Prices
  - Losses

The graph shows two curves:

1. **No Mandate – Prices**
2. **Mandate – Prices**
3. **No Mandate – Losses**
4. **Mandate – Losses**

The x-axis represents the contract value, while the y-axis represents the corresponding prices and losses in dollars.
85% of consumers purchase minimum coverage, up from 80% before the mandate.
Consider an economy with mandated coverage \([m + dm, 1]\), and equilibria \((p_{dm}, \alpha_{dm})\).

We will derive comparative statics with respect to \(dm\).

Denote \((p_0, \alpha_0)\) as \((p, \alpha)\).

See the paper for necessary regularity conditions.

Define the intensive margin selection coefficient as

\[
S_I(x) = \partial_x E_x[c] - E_x[mc].
\]
Theoretical Results

Proposition

The change in the prices of minimum coverage is

$$\lim_{x \to m} \partial_{dm} p_{dm}(x)|_{dm=0} = -S_I(m) + \xi,$$

where the error term $\xi$ is small if $g(m)/G(m)$ is small.

Proposition

Whenever

$$S_I(m) \neq 0,$$

there are consumers who change their decisions beyond the direct effects of the mandate.
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Informal Example

- Will cover main ideas behind optimal regulation in a simple informal example.
- Let $X = [0, 1]$.
  - Assume that consumers only adjust in the “intensive margin” when prices change.
- A benevolent government can regulate menus and prices, but has the same information as the firms.
- Kaldor-Hicks efficiency, no excess burden of public funds.
Equilibrium Inefficiency

- Starting point for regulation: equilibria are inefficient.
- Definitions:
  - Marginal cost
    \[ mc(x, \theta) = \partial_x c(x, \theta). \]
  - Marginal utility
    \[ mu(x, \theta) = \partial_x u(x, \theta). \]
Equilibrium is Inefficient

Private optimum where marginal utility equals $p'$. 

\[
x_{eq} = p' \quad \mu_{\theta}
\]
Equilibrium is Inefficient

But social optimum where marginal utility equals marginal cost.
Equilibrium is Inefficient

Two distortions: $E_x[mc] \neq p'$ and $mc_\theta \neq E_x[mc]$.
Equilibrium is Inefficient

Sources: adverse / advantageous selection and multidimensional heterogeneity.
Optimal Regulation

- Effectively, any regulation can be implemented setting \( p(x) \).
- It is insightful to write a formula for the per unit subsidy the government must give firms,

\[
p(x) + t(x) = E_x[c].
\]

- Will now find necessary conditions for optimum by perturbing a price schedule. This is an old trick in optimal tax theory that has seen a revival since the 2000s.
- Increase \( p' \) by \( dp' \) in the interval \( x + dx \).
Perturbation
Perturbation

The diagram illustrates the perturbation of a function $p(x)$, showing the original function $p(x)$ and an approximate function $\tilde{p}(x)$, with a shaded region indicating the difference between the two.
Perturbation (continued)

- Denote intensive margin elasticity as $\epsilon(x, \theta)$.
- In an optimal price schedule, it must be that

$$E_x[\epsilon \cdot (mu - mc)] = 0.$$
Denote intensive margin elasticity as $\epsilon(x, \theta)$.
In an optimal price schedule, it must be that

$$\mathbb{E}_x[\epsilon \cdot (mu - mc)] = 0.$$ 

We have

$$0 = \mathbb{E}_x[p' - mc] \cdot \mathbb{E}_x[\epsilon] + \text{Cov}_x[-mc, \epsilon]$$
$$= (S_l - t') \cdot \mathbb{E}_x[\epsilon] - \text{Cov}_x[mc, \epsilon].$$
Consequences

Optimal regulation is a modified risk adjustment formula: risk adjustment plus covariance term:

\[ t'(x) = S_l(x) - \frac{\text{Cov}_x[\epsilon, mc]}{E_x[\epsilon]} \]
Regulation Example

- No Mandate – Prices
- No Mandate – Losses
- Mandate – Prices
- Mandate – Losses
Regulation Example
Consequences

- The mandate raises welfare by $127 per consumer.
- Optimal regulation increases it by $279.
- A simple policy like the mandate can increase efficiency.
- But also has important unintended consequences: with adverse selection mandates subsidize low-quality coverage.
- Optimal policy also addresses selection in the intensive margin.
Conclusion

- Key idea is to apply the supply and demand approach from the one-contract model to a more general case.
- Gives a simple model to explain what contracts are traded, and effects of policy.
- Standard policies have important unintended consequences, and regulation should also address selection on the intensive margin.

Thank You!