Social Insurance, Information Revelation and Lack of Commitment

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Conference in honor of Robert E. Lucas
How to design the optimal social insurance system?

- Modern public finance approach builds on work of Atkeson-Lucas, Green, Phelan-Townsend...
- Standard approach: benevolent social planner (government) with full commitment
- General feature: the more information the government knows about agents, the better outcomes it achieves

More information is always better!
How to design the optimal social insurance system?

- A major concern: what if government cannot commit to insurance scheme?
  - more information now increases incentive to re-optimize
- How to design of social insurance when policymaker lacks commitment?
This paper

- A version of Atkeson-Lucas (1992) set up
  - agents receive privately observed idiosyncratic shocks
  - transmit information about those shocks to the gov’t
  - benevolent gov’t provides consumption allocations
- Key friction: the government cannot commit
  - we study Pareto frontier of PBEs
Main results

- **Recursive formulation**: all past info can be summarized with continuation utility $v$
  - generalizes Atkeson-Lucas
  - much harder to obtain without commitment
- **Key object**: complementarity between $v$ and information
  - does knowing more about high $v$ agents saves more or less resources than knowing more about low $v$ agents?
- **Efficient information revelation**: agents with higher $v$ reveal more precise info about their shock
  - positive complementarity
- Insurance often rationed: provided with prob $< 1$
Plan

1. Simple 2 period model
2. Dynamic model with iid shocks
Simplified model

- Continuum of agents with preferences
  \[ \theta \frac{c_1^{1-\rho}}{1-\rho} + \frac{c_2^{1-\rho}}{1-\rho} \]

- \( \theta \) is a privately observed shock \( \theta \in \Theta \)
- 1 unit of per capita endowment in each period
- Each agent is a member of a group \( i = 1, \ldots, I \) for some \( I \geq 1 \)
  - group \( i \) has measure \( \psi_i \) of agents
Politics

• Insurance is provided by a benevolent Utilitarian politician who lacks commitment

• Game has 3 stages
  1. Politician makes promises how to allocate consumption
  2. Agents report information to the politician
  3. Politician can break promise and re-optimize at a cost $\gamma > 0$

• We focus on the Pareto frontier of the set of PBE
  • strategies that maximize weighted lifetime utility of each group
  • corresponds closely to infinitely repeated game
  • probabilistic voting: groups that care more about economic platform get higher weight in election politics
Politics formally

- Variable: $u = \frac{c^{1-\rho}}{1-\rho}$, $C(u) = \left[\left((1-\rho)u\right)\right]^{1/(1-\rho)}$
- Message space $M$

Game

1. Politician's promises $u_{i,t}^{pr} : M \rightarrow \mathbb{R}$
2. Agents report: $\sigma_i : \Theta \rightarrow \Delta(M)$
   - $\sigma^{in}$ is fully informative if can invert $m$ to $\theta$
   - $\sigma^{un}$ is uninformative if agents babble
3. Final allocations: $u_{i,t} : M \rightarrow \mathbb{R}$.
   - feasibility
   - cost paid if $u_{i,t} - u_{i,t}^{pr} \neq 0$ for a positive measure of agents
Solve model backward

- Best allocation for politician \textbf{after} agents play \{σ_i\}:
  \[
  \max_{\{u_i\}} \sum_{i=1}^{l} \psi_i \mathbb{E}_{σ_i} [θu_{i,1} + u_{i,2}]
  \]
  s.t. feasibility

- For given \( λ^w \) define
  \[
  \mathcal{W} (σ) \equiv \max_u \mathbb{E}_σ [θu - λ^w C (u)]
  \]
Pareto frontier of PBEs

**Proposition:** each point on the Pareto frontier $v = (v_1, ..., v_I)$ associated with multipliers $\hat{\beta}, \chi, \lambda^w$ such that optimal strategies solve

$$\min \sum_i \psi_i \{ \mathbb{E}_{\sigma_i} [C(u_{i,1}) + \hat{\beta} C(u_{i,2})] + \chi W(\sigma_i) \}$$

subject to

$$\mathbb{E}_{\sigma_i} [\theta u_{i,1} + u_{i,2}] \geq \mathbb{E}_{\sigma'} [\theta u_{i,1} + u_{i,2}] \text{ for all } i, \sigma'$$

$$\mathbb{E}_{\sigma_i} [\theta u_{i,1} + u_{i,2}] = v_i \text{ for all } i$$
Infinite cost of deviation

- Suppose cost of breaking promises is infinite $\implies \chi = 0$
- Generalized Atkeson-Lucas:

$$\kappa (\nu, \sigma) \equiv \min_{u_1, u_2, \sigma} \mathbb{E}_{\sigma_i} [C(u_1) + \hat{\beta} C(u_2)]$$

subject to

$$\mathbb{E}_{\sigma} [\theta u_1 + u_2] \geq \mathbb{E}_{\sigma'} [\theta u_1 + u_2] \text{ for all } \sigma'$$

$$\mathbb{E}_{\sigma} [\theta u_1 + u_2] = \nu$$

- Optimal reporting strategy

$$\min_{\sigma} \kappa (\nu, \sigma)$$
Order reporting strategies

- CES preferences: \( \kappa (v, \sigma) = d(\sigma) C(v) \)
- Define *informativeness* as \( \sigma'' \geq \sigma' \) iff \( d(\sigma'') \leq d(\sigma') \)
    - completes Blackwell's informativeness
- Complementarity of informativeness and utility:

\[
\sigma'' \geq \sigma' \implies \frac{d}{dv} \left[ \kappa (v, \sigma') - \kappa (v, \sigma'') \right] \geq 0
\]
How do costs depend on info?
Finite cost of deviation

- Suppose cost of breaking promises is not too high $\Rightarrow \chi > 0$
- Optimal reporting strategy
  \[ \min_{\sigma} \kappa (v, \sigma) + \chi W (\sigma) \]
- Trade off: better information decreases cost of insurance but increases incentives to deviate
  \[
  \kappa (v, \sigma^{in}) \leq \kappa (v, \sigma) \leq \kappa (v, \sigma^{un}) \\
  W (\sigma^{in}) \geq W (\sigma) \geq W (\sigma^{un})
  \]
Trade off

\[ \kappa(v, \sigma^{in}) + \chi W(\sigma^{in}) \]

\[ \kappa(v, \sigma) + \chi W(\sigma) \]

\[ \kappa(v, \sigma^{un}) + \chi W(\sigma^{un}) \]
Optimal info
Optimal info revelation

Theorem

If $v_i \geq v_j$ then $\sigma_i^* \geq \sigma_j^*$.  

Key force: complementarity between informativeness and promised utility

Same result holds if agents and gov’t can use a mediator for communication a-la Myerson
Optimal info without public randomization
Optimal info with public randomization
Rationing

- Conditioning strategies on sunspot improves welfare
- **Proposition**: Suppose $\rho = 1$ and strategies can depend on sunspot. Then agents play only $\sigma^{in}$ and $\sigma^{un}$

- Interpretation: rationing of insurance
  - gov't provides second-best insurance access to which is rationed
  - rationing is non-trivial as long as $Y$ is not too high
  - holds even with $I = 1$
Full Model
Model

- Preferences
  \[ \sum_{t=0}^{\infty} \beta^t \theta_t U(c_t) \]

- \( \theta \in \Theta \) is the taste shock
  - \( \Theta \) is finite
  - \( \theta \) is iid, \( \pi(\theta) \) is probability of \( \theta \)

- Total endowment in each period is 1
- Each individual belongs to a family \( v \) from distribution \( \psi \)
Timing of the game

- Agents observe their types $\theta_t$ and report $m_t \in M$
- Government allocates utility $u_t$ to agents as a function of history of reports, sunspots, and initial promise $v$
Equilibrium

- **Perfect Bayesian Equilibrium**: agents and government play their best responses after all histories, allocations are feasible, government’s beliefs satisfy Bayes rule.

- **Best PBE**: maximizes the sum of the utility of all the agents subject to delivering at least $v$ to agent $v$.

- **Worst PBE**: agents play $\sigma^{un}$.
Recursive formulation

- For simplicity: assume $\psi$ is an invariant distribution
  - pins down $\zeta, \hat{\beta}, \lambda^w$
- Recursive problem:
  
  $$
  \kappa (v, \sigma) = \min_{u, w} \mathbb{E}_{\sigma} \left[ \zeta C (u) - \theta u + \hat{\beta} k (w) \right]
  $$

  subject to

  $$
  \mathbb{E}_{\sigma} [\theta u + \beta w] \geq \mathbb{E}_{\sigma'} [\theta u + \beta w] \text{ for all } \sigma'
  $$

  $$
  \mathbb{E}_{\sigma} [\theta u + \beta w] = v
  $$

- Optimal amount of info revelation for given $v$
  
  $$
  k (v) = \min_{\sigma} \mathbb{E} [\kappa (v, \sigma) + \chi W (\sigma)]
  $$
Policy functions with two shocks

A: Probability of info revelation
B: Promised utility policy

Shaded area: rationing of insurance
Persistent shocks

- Markov shocks $\pi (\theta | \theta^-)$
- State variables: prior $p$ and vector of promises $v$

$$v_i = E_\sigma [\theta u + \beta w | \theta_i^-]$$

- Bellman equation

$$k (v, p) = \min_{u, w, \sigma} \sum p_i E_\sigma [\zeta C (u) - \theta u + \hat{\beta} k (w, p') | \theta_i^-] + \chi W (\sigma, p)$$

subject to IC, PK, and Bayes’ rule

- Direct generalization of Fernandes-Phelan
Conclusion

- Social insurance without commitment
  - recursive formulation
  - characterization

- Main insight: more information is revealed by agents who accumulated higher promised utility
  - had more "good" shocks in the past

- Endogenous lower bound on promised utility
  no provision of incentives for low $v$