The Global Diffusion of Ideas

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By the problem of economic development I mean simply the problem of accounting for the observed pattern, across countries and across time, in levels and rates of growth of per capita income. This may seem too narrow a definition, and perhaps it is, but thinking about income patterns will necessarily involve us in thinking about many other aspects of societies too. so I would suggest that we withhold judgment on the scope of this definition until we have a clearer idea of where it leads us.

Lucas (1988), "On the Mechanics..."

On the mechanics of economic development
RE **Lucas** - Journal of monetary economics, 1988 - Elsevier
Abstract This paper considers the prospects for constructing a neoclassical theory of growth and international trade that is consistent with some of the main features of economic development. Three models are considered and compared to evidence: a model ...
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Econometric policy evaluation: A critique
RE **Lucas** - Carnegie-Rochester conference series on public policy, 1976 - North-Holland
Cited by 7188 Related articles All 24 versions Cite Save

Asset prices in an exchange economy
RE **Lucas** Jr - Econometrica: Journal of the Econometric Society, 1978 - JSTOR
This paper is a theoretical examination of the stochastic behavior of equilibrium asset prices in a one-good, pure exchange economy with identical consumers. A general method of constructing equilibrium prices is developed and applied to a series of examples.
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Expectations and the Neutrality of Money
RE **Lucas** - Journal of economic theory, 1972 - Elsevier
This paper provides a simple example of an economy in which equilibrium prices and quantities exhibit what may be the central feature of the modern business cycle: a systematic relation between the rate of change in nominal prices and the level of real output. The ...
Cited by 5118 Related articles All 21 versions Cite Save

Some international evidence on output-inflation tradeoffs
RE **Lucas** - The American Economic Review, 1973 - JSTOR
This paper reports the results of an empirical study of real output-inflation tradeoffs, based on annual time-series from eighteen countries over the years 1951-67. These data are examined from the point of view of the hypothesis that average real output levels are ...
Cited by 3176 Related articles All 13 versions Cite Save More

Optimal fiscal and monetary policy in an economy without capital
RE **Lucas**, NL Stokey - Journal of monetary Economics, 1983 - Elsevier
Abstract This paper is concerned with the structure and time-consistency of optimal fiscal and monetary policy in an economy without capital. In a dynamic context, optimal taxation means distributing tax distortions over time in a welfare-maximizing way. For a barter ...
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"But we know from ordinary experience that there are group interactions that are central to individual productivity and that involve groups larger than the immediate family and smaller than the human race as a whole. Most of what we know we learn from other people. We pay tuition to a few of these teachers, either directly or indirectly by accepting lower pay so we can hang around them, but most of it we get for free, and often in ways that are mutual - without a distinction between student and teacher."

Lucas (1988), "On the Mechanics...", Section 6

The Global Diffusion of Ideas

- Long held belief that openness affects the diffusion of technologies/ideas

  "The progress of a society is all the more rapid in proportion as it is more completely subjected to external influences." Pirenne (1936)

- Empirical debate
  - Sachs & Warner (95), Coe & Helpman (95), Frankel & Romer (99), Rodriguez & Rodrik (00), Keller (09), Feyrer (09a,b), Pascali (2014)

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- Growth Miracles: Openness and protracted periods of growth

- But standard mechanisms imply relatively small effects
  - e.g., Connolly & Yi (14), Atkeson & Burstein (10)

The Global Diffusion of Ideas

- Provide explicit model of diffusion process based on local interactions
  - Kortum (1997), Eaton & Kortum (1999), Alvarez, Buera, & Lucas (2008), Lucas (2009), Lucas & Moll (2014), Perla & Tonetti (2014) Luttmer (2012, 2014), Jovanovic & Rob (1989)
- How does openness shape ideas to which individuals are exposed?
  - Alvarez, Buera, & Lucas (2014), Perla, Tonetti & Waugh (2014), Sampson (2014), Monge-Naranjo (2012)
- Combine new ideas with insights from others $\Rightarrow$ "general" Frechet limit
  - related to model of random networks in Oberfield (2013)
- Interface with static models of trade
  - Eaton & Kortum (2002), Bernard, Eaton, Jensen, & Kortum (2003), Alvarez & Lucas (2007)
- Country's "stock of knowledge" depends only on trade shares, trade partners' knowledge

  $\Rightarrow$ Use observed trade flows to quantify role of trade in facilitating idea flows

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The Global Diffusion of Ideas

- Which interactions facilitate exchange of ideas? Does it matter?
  - Learning from Sellers, Learning from Producers
  - Determines how gains spill over across countries, Speed of convergence

- Static and Dynamic Gains from Trade?
  - (Potentially) Large dynamic gains, protracted transition after openness
  - Learning from Sellers: Dynamic gains especially large close to autarky
  - Learning from Producers: Dynamic gains amplify static gains

- Nest workhorse models of innovation, diffusion
  - Learning from Sellers: at either extreme, small dynamic gains
  - Intermediate case: dynamic gains more important

- Quantitative exploration: Rich enough to take to cross-country data
  - Accounting for cross-sectional TFP-trade relationship...
  - Accounting for changes in TFP, growth miracles...

The Global Diffusion of Ideas

- Two potential questions
  1. What is role of cross-country idea flows in growth?
     - ⋆ Many channels, e.g., trade, FDI,...
  2. What is the role of trade in facilitating idea flows?
     - ⋆ "Productivity spillover" from trade
- Today's focus second question
- For most of talk: research effort is exogenous
  - Endogenize research intensity at end
  - Generalize result from Eaton & Kortum (1999)
    - ⋆ Across BGPs, research intensity does not depend on trade costs

Learning from an Arbitrary Source

Innovation and Diffusion

- Continuum of goods $s \in [0, 1]$
  - For each good m managers (m is large)
  - Bertrand Competition

- Manager with productivity q
  - Ideas arrive stochastically at rate $\tilde{\alpha}_t$
  - New idea has productivity zq'^{β}
    - ⋆ Insight from someone with productivity $q' \sim \tilde{G}_t(q')$
    - ⋆ Original component $z \sim H(z)$
  - Adopts if $zq'^{\beta} > q$

- β measures strength of diffusion
  - Pure innovation: $\beta = 0$ (Kortum, 1997)
  - Pure diffusion: $\beta = 1$, H degenerate (ABL, 2008, 2014; with Poisson arrivals)

Productivity Distribution

- Frontier of knowledge $\tilde{F}_t(q)$
  - m managers for each good
  - Probability best manager's productivity is $\leq q$

$$\begin{aligned} \frac{d}{dt}\log \tilde{F}_t(q) &= -m\alpha_t \Pr(zq'^{\beta} > q) \\ &= -m\alpha_t \int_0^{\infty} \left[1 - \tilde{G}_t\left(\left(q/z\right)^{1/\beta}\right)\right] dH(z) \end{aligned}$$

Frechet Limit

Assumptions

- Distr. of original component of ideas has Pareto tail: $\lim_{z\to\infty} \frac{1-H(z)}{z^{-\theta}} = 1$
- For now: $\tilde{G}_t$ has sufficiently thin right tail: $\lim_{q\to\infty} q^{\beta\theta}[1-\tilde{G}_t(q)] = 0$
  - Later: initial frontier $F_0(q)$ has sufficiently thin tail, (e.g., bounded)
- $\beta < 1$
- $m\tilde{\alpha}_t = \alpha e^{\gamma t}$

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Convenient to study productivity scaled by an exponential growth factor

$$F_t(q) = \tilde{F}_t\left(e^{\nu t} q\right) \qquad G_t(q) = \tilde{G}_t\left(e^{\nu t} q\right)$$

Proposition

As $t \to \infty$, $F_t(q) = e^{-\lambda_t q^{-\theta}}$, $\dot{\lambda}_t = \alpha \int_0^\infty x^{\beta\theta} dG_t(x) - \nu\theta\lambda_t$

- λ_t: stock of knowledge , $\nu = \frac{\gamma}{(1-\beta)\theta}$: growth rate of productivity

Simple Example

- Individuals learn from managers at frontier

$$G_t(q) = F_t(q)$$

- Then stock of knowledge evolves as

$$\dot{\lambda}_t = \Gamma(1-\beta)\alpha\lambda_t^{\beta} - \nu\theta\lambda_t$$

- The detrended stock of knowledge in a balance growth path

$$\lambda \propto \alpha^{\frac{1}{1-\beta}}$$

- Compounding: New ideas lead to even better insights

Trade

World Economy (BEJK, 2003)

- n countries, defined by
  - Labor, L_i
  - Stock of knowledge, λ_i
  - Iceberg trade costs, κ_{ij}

- Household in i has Dixit-Stiglitz preferences $C_i = \left[\int_0^1 c_i(s)^{\frac{\varepsilon-1}{\varepsilon}} ds\right]^{\frac{\varepsilon}{\varepsilon-1}}$

- Production is linear, uses only labor

- For manager in j, unit cost of providing good to country i is

$$\frac{w_j \kappa_{ij}}{q}$$

- Bertrand Competition:

$$p_i(s) = \min\left\{\frac{\varepsilon}{\varepsilon-1} \begin{matrix}\text{lowest} \\ \text{unit cost}\end{matrix}, \begin{matrix}\text{second lowest} \\ \text{unit cost}\end{matrix}\right\}$$

Static Trade Equilibrium

- Price index

$$P_i^{-\theta} \propto \sum_j \lambda_j (w_j \kappa_{ij})^{-\theta}$$

- Trade Shares

$$\pi_{ij} = \frac{\lambda_j (w_j \kappa_{ij})^{-\theta}}{\sum_k \lambda_k (w_k \kappa_{ik})^{-\theta}}$$

- Labor market clearing (under balanced trade)

$$w_i L_i = \sum_j \pi_{ji} w_j L_j$$

The Global Diffusion of Ideas

Source distributions

1. Learn from Sellers (Alvarez-Buera-Lucas)
   - in proportion to expenditure on good

$$G_i^S(q) \equiv \sum_j \int_{s \in S_{ij} | q_j(s) < q} \frac{p_i(s) c_i(s)}{P_i C_i} ds$$

2. Learn from Producers (Perla-Tonetti-Waugh, Sampson)
   - Equal exposure to active domestic producers (uniformly)

$$G_i^P(q) \equiv \sum_j \int_{s \in S_{ji} | q_i(s) \leq q} ds$$

S_{ij} be set of goods for which j is lowest-cost provider for i

The Global Diffusion of ideas

1. Learn from Sellers (Alvarez-Buera-Lucas)

2. Learn from Producers (Perla-Tonetti-Waugh, Sampson)

The Global Diffusion of ideas

1. Learn from Sellers (Alvarez-Buera-Lucas)

$$\dot{\lambda}_i \propto \qquad \alpha_i \sum_j \pi_{ij} \left(\frac{\lambda_j}{\pi_{ij}} \right)^{\beta}$$

- Expenditure-weighted average
- Selection: hold fixed λ_j, lower π_{ij} $\quad \Rightarrow \quad$ import goods with higher q

2. Learn from Producers (Perla-Tonetti-Waugh, Sampson)

The Global Diffusion of ideas

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2. Learn from Producers (Perla-Tonetti-Waugh, Sampson)

$$\dot{\lambda}_i \propto \qquad \alpha_i \left(\frac{\lambda_i}{\pi_{ii}} \right)^{\beta}$$

   - Impact of trade: Selection
     - High productivity producers likely to expand
     - Low productivity producers likely to drop out

The Global Diffusion of ideas

1. Learn from Sellers (Alvarez-Buera-Lucas)

$$\dot{\lambda}_i \propto \qquad \alpha_i \sum_j \pi_{ij} \left(\frac{\lambda_j}{\pi_{ij}} \right)^{\beta}$$

- To maximize growth:

$$\frac{\lambda_j}{\lambda_{j'}} = \frac{\pi_{ij}}{\pi_{ij'}}$$

2. Learn from Producers (Perla-Tonetti-Waugh, Sampson)

$$\dot{\lambda}_i \propto \qquad \alpha_i \left(\frac{\lambda_i}{\pi_{ii}} \right)^{\beta}$$

- Growth is maximized with the highest trade exposure (lowest π_{ii})

The Global Diffusion of ideas

1. Learn from Sellers (Alvarez-Buera-Lucas)

$$\dot{\lambda}_i \propto \qquad \alpha_i \sum_j \pi_{ij} \left(\frac{\lambda_j}{\pi_{ij}} \right)^\beta$$

- To maximize growth:

$$\frac{\lambda_j}{\lambda_{j'}} = \frac{\pi_{ij}}{\pi_{ij'}} \left(= \frac{\lambda_j \left(w_j \kappa_{ij}\right)^{-\theta}}{\lambda_{j'} \left(w_{j'} \kappa_{ij'}\right)^{-\theta}} \right)$$

2. Learn from Producers (Perla-Tonetti-Waugh, Sampson)

$$\dot{\lambda}_i \propto \qquad \alpha_i \left(\frac{\lambda_i}{\pi_{ii}} \right)^\beta$$

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Gains from Trade

Gains from Trade

Real income is

$$y_i \propto \frac{w_i}{P_i} \propto \left(\frac{\lambda_i}{\pi_{ii}}\right)^{1/\theta}$$

- Static gains from trade: hold λ fixed
- Dynamic gains from trade: operate through idea flows

Gains from Trade

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- Static gains from trade: hold λ fixed
- Dynamic gains from trade: operate through idea flows
- Symmetric world (independent of the specification of learning)

$$\frac{y^{FT}}{y^{AUT}} = \underbrace{n^{\frac{1}{\theta}}}_{static} \quad \underbrace{n^{\frac{\beta}{(1-\beta)\theta}}}_{dynamic} = n^{\frac{1}{\theta}\frac{1}{1-\beta}}$$

Gains from Trade

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Gains from Trade

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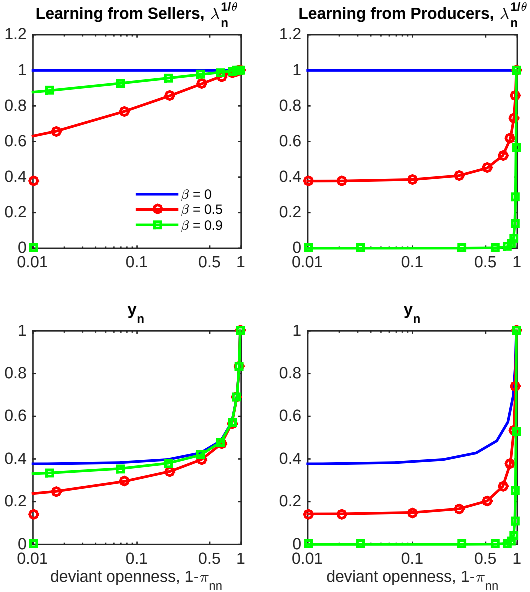
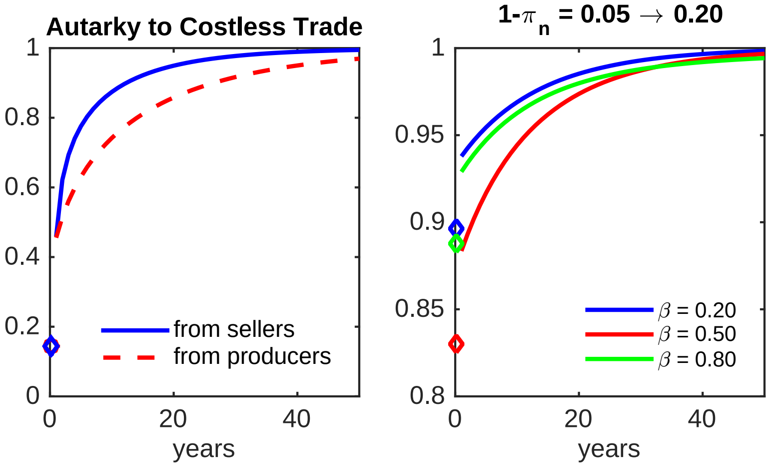
- Static gains from trade: hold λ fixed
- Dynamic gains from trade: operate through idea flows
- What is the fate of a single country that is isolated?
  - Consider world with n symmetric countries
  - Trade among $n-1$ countries is costless
  - Trade to and from "deviant" economy incurs iceberg cost κ_n

Long-Run Gains from Trade: Single Deviant



Trade Liberalization



$\beta = 0.5,\ \theta = 4,$ TFP Growth rate on BGP $= 0.01$

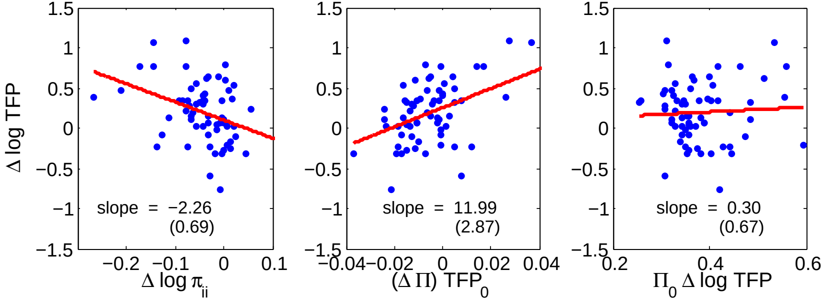
Gains from Trade: Takeaways

- Learning from sellers
  - Static gains relevant when economy relatively open
    Dynamic gains relevant when economy relatively closed
  - For moderately open economy: Dynamic gains from reducing trade barriers
    - Small if $\beta \approx 0$ or $\beta \approx 1$
    - Larger when β in intermediate range
- Learning from producers (uniformly)
  - Simple amplification of static gains
  - Slower transitional dynamics

Quantitative Exploration

Quantitative Exploration

- Questions:
  - Can openness account for a significant part of evolution of TFP?
  - Can model account for the cross-section relationship between TFP and trade?



Quantitative Exploration

- Generalized trade model: intermediate inputs, capital, non-traded goods (Alvarez & Lucas, 2007)
- Let L_{it} be equipped labor ($= K_{it}^{1/3} (emp_{it} \cdot h_{it})^{2/3}$, from the PWT)

Parameter	Value
Trade Elasticity, θ	4
Share of Non-Traded Goods	0.5
Intermediate Good Share of Cost	0.5

- Diffusion parameter, β:
  - Agnostic: Explore the effects for various β (recalibrating γ)
  - Heroic: $d \log TFP = 0.008 = \frac{\gamma}{\theta} \frac{1}{1-\beta} \Rightarrow \beta \approx 0.7$ ($\gamma =$ pop. growth)
- Focus on the case of learning from sellers to a country

Calibration

Two sets of residuals to exactly match trade and TFP

- Calibrate the evolution of trade costs, κ_{ijt}, to match bilateral trade flows
- Arrival rate of ideas, α_{it}: residual TFP level/growth

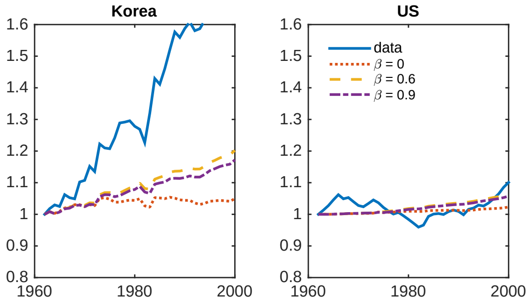
Two counterfactuals to measure contribution of trade

$$\ln \frac{TFP_i(\alpha_t,\kappa_t)}{TFP_i(\alpha_0,\kappa_0)} = \underbrace{\ln \frac{TFP_i(\alpha_0,\kappa_t)}{TFP_i(\alpha_0,\kappa_0)}}_{\text{cont. from trade}} + \underbrace{\ln \frac{TFP_i(\alpha_t,\kappa_t)}{TFP_i(\alpha_0,\kappa_t)}}_{\text{cont. from arrival rates}}$$

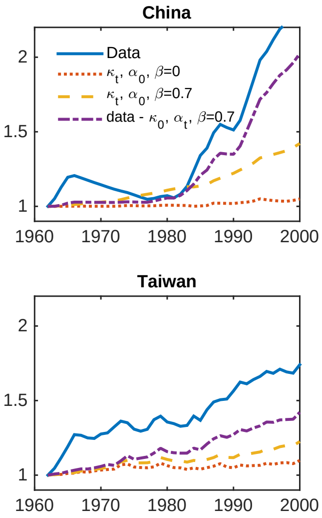
$$\ln \frac{TFP_i(\alpha_t,\kappa_t)}{TFP_i(\alpha_0,\kappa_0)} = \underbrace{\ln \frac{TFP_i(\alpha_t,\kappa_0)}{TFP_i(\alpha_0,\kappa_0)}}_{\text{cont. from arrival rates}} + \underbrace{\ln \frac{TFP_i(\alpha_t,\kappa_t)}{TFP_i(\alpha_t,\kappa_0)}}_{\text{cont. from trade}}$$

Development Dynamics, South Korea (vs. US)

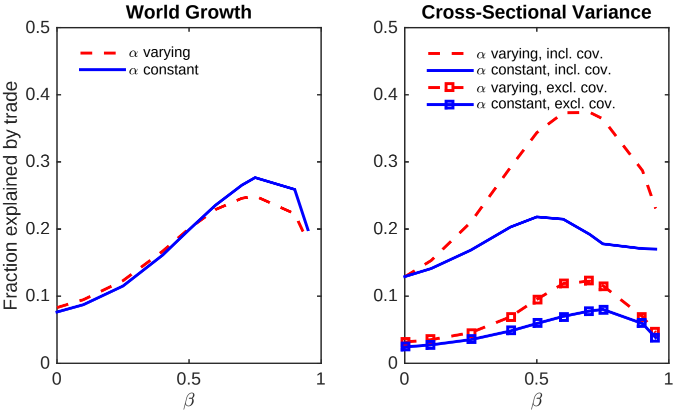
Hold research intensities (α_0) fixed at 1962 levels



Development Dynamics, Growth Miracles



Contribution of Trade: Transitions



Distribution of TFP in Cross-section

Two counterfactuals to measure contribution of trade

$$\ln \frac{TFP_i(\alpha_i,\kappa_{ij})}{TFP_i(\bar{\alpha},\bar{\kappa})} = \underbrace{\ln \frac{TFP_i(\bar{\alpha},\kappa_{ij})}{TFP_i(\bar{\alpha},\bar{\kappa})}}_{\text{cont. from trade}} + \underbrace{\ln \frac{TFP_i(\alpha_i,\kappa_{ij})}{TFP_i(\bar{\alpha},\kappa_{ij})}}_{\text{cont. from arrival rates}}$$

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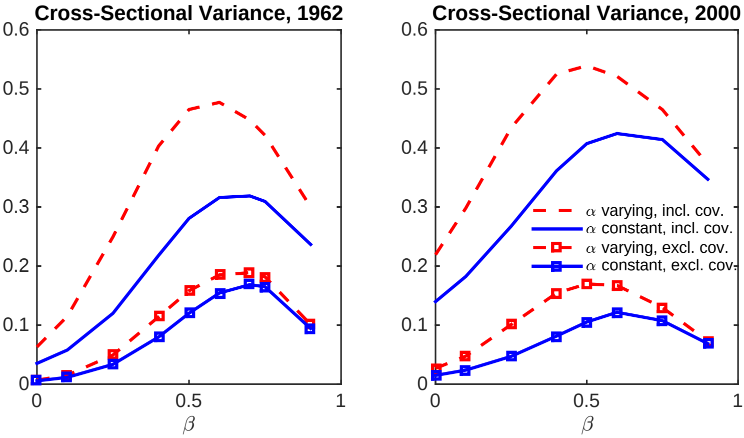
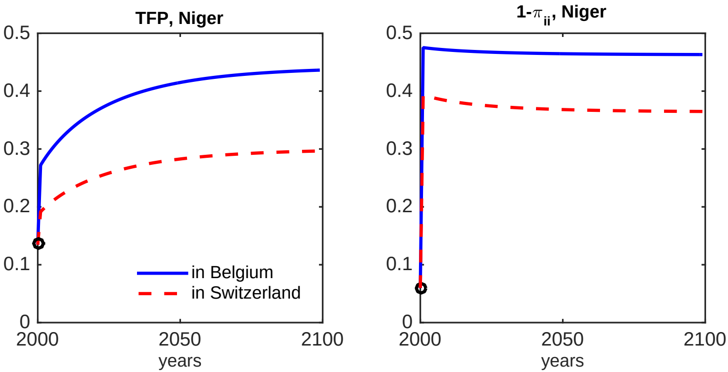
Distribution of TFP in Cross-Section



Effect of Moving Niger to ...



Incentives to Innovate

- Labor used for production or R&D

$$L_{it} = L_{it}^{Production} + L_{it}^{R\&D}$$

  - Arrival of ideas: $\alpha = \bar{\alpha} \times l$
  - Tax T_i on profit (Parente & Prescott, 1994)

- Result:
  - Across BGPs, $\frac{L_{it}^{R\&D}}{L_{it}}$ independent of trade barriers
    - Market size ↑, but competition ↑
    - Like Eaton & Kortum (2001), Atkeson & Burstein (2010)
  - But, openness $\Rightarrow$ same R&D effort leads to better insights
    - Related to Baldwin & Robert-Nicoud (2008)

Conclusions/Future Research

- Present tractable model that incorporates large class of diffusion mechanisms, based on local interactions
- Leading Example:
  - Large dynamics gains from trade, specially for intermediate values of β
  - able to account for the cross-sectional TFP-trade relationship
  - ... generate growth miracles with a significant role for trade
- Future research:
  - Infer value for β: aggregate TFP-trade dynamics, e.g., Feyrer (2009a,b), Hanson & Muendler (2013), Levchenko & Zhang(2014), Pascali (2014); micro evidence, e.g., Aitken & Harrison (1999), Javorcik (2004)
  - Multinational Production/FDI: Ramondo & Rodriguez-Clare (2014)

Frechet Limit

Proposition

Given assumptions, the frontier of knowledge evolves as:

$$\lim_{m\to\infty} \frac{d \ln F_t(q)}{dt} = -\alpha_t q^{-\theta} \int_0^\infty x^{\beta\theta} dG_t(x)$$

Define $\lambda_t = \int_{-\infty}^t \alpha_\tau \int_0^\infty x^{\beta\theta} dG_\tau(x)$

Corollary

Suppose that $\lim_{t\to\infty} \lambda_t = \infty$. Then $\lim_{t\to\infty} F_t(\lambda_t^{1/\theta} q) = e^{-q^{-\theta}}$.

Back

Learning from Producers

in proportion to employment

$$G_i(q) = \sum_{j=1}^{n} \int_0^q \underbrace{\frac{L_j w_j}{L_i w_i} \left(\frac{w_i \kappa_{ji}}{P_j} \right)^{1-\varepsilon} x^{\varepsilon - 1}}_{fraction\ of\ employment\ in\ x} \underbrace{\prod_{k \neq j} F_k \left(\frac{w_k \kappa_{ik}}{w_i \kappa_{ii}} x \right)}_{prob.\ j\ buys\ x\ from\ i} dF_i(x)$$

back

Learning from Producers

uniformly

$$G_i(q) = \sum_{j=1}^{n} \int_0^q \frac{1}{\pi_{ii}} \prod_{k \neq j} F_k\left(\frac{w_k \kappa_{jk}}{w_i \kappa_{ji}} x\right) dF_i(x)$$

The evolution of the stock of knowledge

$$\dot{\lambda}_i \propto \left(\frac{\lambda_i}{\pi_{ii}}\right)^{\beta}$$

back

Multivariate Pareto

$$H(z_1, ..., z_n) = \max \left\{ 1 - \left(\sum_j \left(\frac{z_i}{z_0} \right)^{-\frac{\Theta}{1-\rho}} \right)^{1-\rho}, 0 \right\}$$

- Each marginal is distribution is Pareto
- $\rho \in [0, 1]$ like a correlation

Back

Endogenous Growth Case, $\beta = 1$

Alvarez, Buera & Lucas (2013)

- Learning from sellers
- Trade only
- Evolution of the distribution of productivities

$$\frac{\partial \log(F_{it}(q))}{\partial t} = -\alpha \left[1 - \sum_{j=1}^{n} \int_0^q \prod_{k \neq j} F_{kt} \left(\frac{w_k \kappa_{ik}}{w_j \kappa_{ij}} x \right) dF_{jt}(x) \right]$$

Endogenous Growth Case, $\beta = 1$

Alvarez, Buera & Lucas (2013)

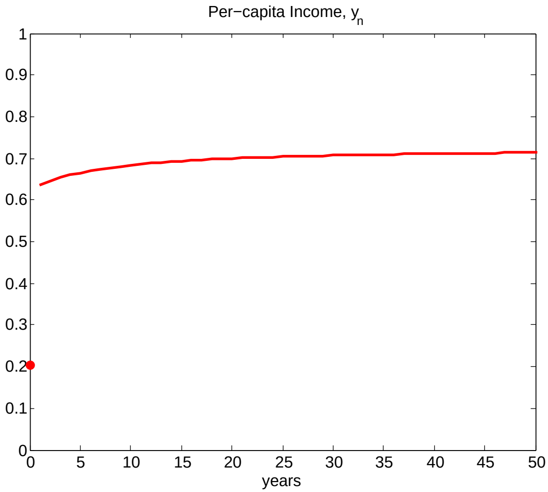
- Growth rate in a BGP, $\nu = n\alpha/\theta$

- Tails converge if $\kappa_{ij} < \infty$

$$\lim_{q\to\infty}\lim_{t\to\infty}\frac{1 - F_{it}\left(qe^{\nu t}\right)}{\lambda q^{-\theta}} = 1$$

- Distribution not Frechet (log-logistic if $\kappa_{ij} = w_i = 1$)

Single Deviant: Stock of Knowledge



Generalized Trade Model

- Technology requiring an intermediate aggregate and labor

$$y_i(\mathbf{q}) = \frac{1}{\eta^\eta \zeta^\zeta (1-\eta-\zeta)^{1-\eta-\zeta}} q_i x_i(\mathbf{q})^\eta k_i(\mathbf{q})^\zeta l_i(\mathbf{q})^{1-\eta-\zeta}$$

- Intermediate (investment) aggregate technology

$$X_i = \left[\int c_{xi}(\mathbf{q})^{1-1/\epsilon} dF_i(\mathbf{q})\right]^{\epsilon/(\epsilon-1)}$$

- Fraction μ of the goods are tradable, i.e.,

$$p_i^{1-\varepsilon} = (1-\mu)\int_0^\infty \left(\frac{p_i^\eta R_i^\zeta w_i^{1-\eta-\zeta}}{q}\right)^{1-\varepsilon} dF_j(q)$$
$$+ \mu \sum_{j=1}^{n} \int_0^\infty \left(\frac{p_j^\eta R_i^\zeta w_j^{1-\eta}\kappa_{ij}}{q}\right)^{1-\varepsilon} \prod_{k\neq j} F_k\left(\frac{p_k^\eta R_i^\zeta w_k^{1-\eta-\zeta}\kappa_{ik}}{p_j^\eta R_i^\zeta w_j^{1-\eta-\zeta}\kappa_{ij}} q\right) dF_j(q)$$

▸ Back

Speed of Convergence: Small Open Economy

For small open economy, speed of convergence is

- If agents learn from sellers

$$\gamma \left\{ 1 - \frac{\Omega_{ii}^{S} - \pi_{ii}}{1 + \theta\left(1 + \pi_{ii}\right)} + \frac{\beta}{1-\beta}\left(1 - \Omega_{ii}^{S}\right) \right\}$$

- If agents learn from producers

$$\gamma \left\{ 1 - \frac{\Omega_{ii}^{P} - \pi_{ii}}{1 + \theta\left(1 + \pi_{ii}\right)} + \frac{\beta}{1-\beta}\frac{\left(1 - \Omega_{ii}^{P}\right)\left(1 + \pi_{ii}\right)}{1 + \theta\left(1 + \pi_{ii}\right)} \right\}$$

where $\Omega_{ii}^{S} \equiv \frac{\pi_{ii}(\lambda_i/\pi_{ii})^{\beta}}{\sum_j \pi_{ij}(\lambda_j/\pi_{ij})^{\beta}}$ and $\Omega_{ii}^{P} \equiv \frac{r_{ii}(\lambda_i/\pi_{ii})^{\beta}}{\sum_j r_{ji}(\lambda_i/\pi_{ji})^{\beta}}$.

Back

Calibrating Trade Costs

Use trade data from Feenstra et al. (2005), GDP from PWT 8.0 and the equilibrium relations

$$\kappa_{ijt} = \kappa_{jit} = \left[\frac{1-\pi_{iit}}{\pi_{ijt}} \frac{1-\pi_{jjt}}{\pi_{jit}} \left(\frac{Z_{it}}{1-Z_{it}} \right) \left(\frac{1-Z_{jt}}{Z_{jt}} \right) \right]^{\frac{1}{2\theta}}$$

where Z_{it} solves

$$\pi_{iit} = \frac{(1-\mu) + \mu Z_{it}^{1-\frac{\varepsilon-1}{\theta}}}{(1-\mu) + \mu Z_{it}^{-\frac{\varepsilon-1}{\theta}}}.$$

Back

Distribution of TFP in 2000

- Calibration of homogeneous arrival rates α
  - $\alpha_i = \bar{\alpha} L_i^{\Upsilon}$, calibrate Υ to match $TFP \propto L^{0.15}$

Back