Empirical Analysis of Multi-Unit Auction Markets

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Why do empirical research on auctions?

1. Theory of auctions is well developed
2. Real-life auctions provide an environment where the data is generated under conditions that are close to those in economic models
3. Data available from economically and scientifically significant markets
Why do empirical research on multi-unit auctions?

1. Many economically significant markets are run using a multi-unit auction mechanism:
   - Treasury bill/bond auctions
   - Central bank monetary infusions
   - Deregulated electricity markets
   - Emissions markets

2. Revenue Equivalence does not hold for most real-world mechanisms!

3. Although equilibrium analysis is complicated, empirical work is possible using best-response conditions
   - However, counterfactual analyses are limited by our ability to compute equilibria – much more help is needed from theorists!!!
This talk

2. (Fast) Survey of literature on *multi*-unit auction empirics (2000- ).
3. Application: The 2007 Subprime Market Crisis Through the Lens of European Central Bank Auctions for Short-Term Funds (joint with Nuno Cassola and Jakub Kastl)
Single Unit Auctions (private values)

- **First-price sealed-bid auction**: you don’t see your opponents’ bids. Highest bid wins. Winner pays her bid, $b$. The winner’s profit is: $v - b$. Losers get nothing.

- **Second-price sealed-bid auction**: you don’t see your opponents’ bids. Highest bid wins. Winner pays the second highest bid in the auction. Therefore the winner’s profit is: $v$ minus the second highest bid. Losers get nothing.

- **Ascending-price drop-out auction**: Sort of like the game of chicken. Price starts ticking upwards from 0 until...wherever. You’re all paying the price until you decide to drop out! As soon as the second-to-last person drops out, the auction ends. Winner’s profit is $v$ minus the price at which the second-to-last guy dropped out.
Theoretical setup:

- $N$ bidders draw valuations, $v_i$, from a joint distribution $F(v_1, \ldots, v_N)$
- Simple setting: valuations are drawn independently from the same distribution $F(v)$.
- This is called the *symmetric independent private values* setting.
- Let’s take $F(v) = v$, i.e. the uniform distribution on $[0, 1]$. 
Second price auction:

- Claim: Regardless of what my competitors do, I can do no better than bidding my valuation.
- Reason 1: My payment does not depend on what I do.
- My payoff in every “contingency” is either
  - I win, and make $v_i - b^{2:N}$, where $b^{2:N}$ is the second highest bid.
  - I lose, and make zero.
  - My bid only affects whether I win or lose.
- If I bid my value, I do not regret winning or losing in any contingency.
- If I bid below my value, then I don’t win in some situations where I would want to win.
Ascending auction:

- Claim: Regardless of what my competitors do, I can do no better than dropping out at my valuation.
- Reason: My payment does not depend on what I do.
- If I drop out below my value, then I don’t win in some situations where I would be willing to win.
- If I drop out above my value, well, I make a loss.
- What is the expected revenue in ascending price auction with $N$ bidders, with uniform distributed valuations?
First price auction:

- Won’t bid valuation. Will want to “shade” your bid.
- How to optimally shade?

\[
\max_b (v_i - b) \Pr\{\text{win}|b\}
\]

How to calculate \( \Pr\{\text{win}|b\} \)?
Nash equilibrium:

- Suppose everybody plays the “strategy” $b(v)$, where $b(v)$ is an increasing function in $v$.
- Let’s look at the strategy, $b_i$, of bidder $i$, who has valuation $v_i$, assuming that all other bidders are using the strategy $b(v)$.

$$\max_{b_i} (v_i - b_i) \Pr\{b(v_j) \leq b_i, \forall j \neq i\}$$

$$\max_{b_i} (v_i - b_i) F^{N-1}(b^{-1}(b_i))$$

Take derivative w.r.t. $b_i$ and set to zero

$$F^{N-1}(b^{-1}(b_i)) =$$
\[(v_i - b_i)(N - 1)F^{N-2}(b^{-1}(b_i))f(b^{-1}(b_i)) \frac{1}{b'(b^{-1}(b_i))}\]

Now, \(v_i\) is not necessarily equal to \(b^{-1}(b_i)\), but now, let’s see what the above optimality condition tells us about the shape of \(b(v)\) if bidder \(i\) also plays \(b(v)\) (i.e. we’re looking at a symmetric equilibrium of the game). In this case \(b^{-1}(b_i) = v_i\), so

\[v_i - b(v_i) = \frac{1}{N - 1} \frac{F(v_i)b'(v_i)}{f(v_i)}\]

This is a first-order linear differential equation, with initial condition \(b(0) = 0\). Solving this, we get:

\[b(v_i) = v_i - \int_0^v F^{N-1}(\theta)d\theta = \int_0^{v_i} F^{N-1}(\theta)d\theta\]
Econometrics of Single Unit Auctions

- (Sub)set of bids from a sequence of auctions
- Structural “parameter:” $F_v(v)$
- Cool thing about auctions: the “error” terms are given explicitly by model.
- Second-price auction: suppose you observe all bids
  - $v = b$. Done. $G_b(b) = F_v(b)$.
- Suppose you only observe winner’s payment.
- Assume symmetry: $G_{b2:N}(x) = (N - 1)F_v^{N-1}(x)(1 - F_v(x))$
- Solve for $F_v(x)$
- In general (Haile and Tamer (2003), Athey and Haile (2002)):
  - Given iid random variables drawn from $F(v)$, the distribution
of the ith order statistic, $F_{i:N}(v)$ is related to the parent distribution by (the monotonic function):

$$F_{i:N}(v) = \frac{N!}{(N - i)! (i - 1)!} \int_0^{F(v)} s^{i-1} (1-s)^{N-i} ds$$
Econometrics of First Price Auctions

- Seems not as easy since \( b(v) \) is a nonlinear function.
- One solution: parameterize \( F_V(v, \theta) \).
- For each bid \( b_i \), find the value, \( v_i = b^{-1}(b_i; \theta) \), that rationalizes this bid for a given value of \( \theta \). Thus, the likelihood of observing \( b_i \) is \( f_V(b^{-1}(b_i; \theta)) \), where the mapping \( b(v) \) is the solution of the ODE:

\[
b(v_i) = v_i - \int_0^{v_i} \frac{F^{N-1}(\theta)d\theta}{F^{N-1}(v_i)}
\]

One actually write the ODE in terms of the inverse bid function \( \phi(b(v)) = b^{-1}(b(v)) \), which makes life easier.
\[
\phi'(b) = \frac{1}{N-1} F(\phi(b)) \frac{1}{f(\phi(b))} \frac{1}{\phi(b) - b}
\]

- Estimate using maximum likelihood.
- How to do this? Given \( \theta \), solve for \( \phi(b|\theta) \).
- Using \( \phi \), associate each bid with a valuation.
- Likelihood of observed bids = likelihood of valuations (times Jacobian)
- Disadvantage 1: Have to solve the ODE for every \( \theta \) iteration.
- Disadvantage 2: How do we know whether parametric assumption is good?
- Disadvantage 3: Support of the distribution of values depends on support of bids. The convergence of estimator no longer \( \sqrt{N} \) (actually faster), thus can’t use MLE standard error
formula. Need parametric bootstrap to calculate standard errors. OR, use in a Bayesian MCMC estimation framework.
Guerre, Perrigne and Vuong (2000, EMA)

\[
max_b (v_i - b) \Pr \{ \text{win} | b \} \\
(v_i - b) = \frac{\Pr \{ \text{win} | b \}}{\partial \Pr \{ \text{win} | b \} / \partial b}
\]

In the first-price auction, the FOC becomes:

\[
v_i - b = \frac{1}{N - 1} \frac{G(b)}{g(b)}
\]

(First step) Estimate \( v_i \) by:

\[
\hat{v}_i - b = \frac{1}{N - 1} \frac{\hat{G}(b)}{\hat{g}(b)}
\]

and (second step) estimate \( F_V(v) \) by the empirical cdf of \( \hat{v} \).
Estimators (for data with $L$ auctions):

$$
\tilde{G}(b) = \frac{1}{NL} \sum_{l=1}^{L} \sum_{p=1}^{N} 1\{B_{pl} \leq b\}
$$

$$
\tilde{g}(b) = \frac{1}{NLh_g} \sum_{l=1}^{L} \sum_{p=1}^{N} K_g\left(\frac{b - B_{pl}}{h_g}\right)
$$

where $h_g$ is a bandwidth parameter and $K_g(\cdot)$ is a kernel with compact support. Issue: kernel estimator of density is biased at the boundaries of the support.
Cross-Auction Heterogeneity

- Data: bids from \( l = 1, \ldots, L \) auctions, with \( N_l \) bidders per auction. Additional auction level covariate vector \( X_l \), which has \( d \) dimensions.

\[
\tilde{G}(b, x, n) = \frac{1}{\sum_{l=1}^{L} \frac{1}{N_l} \sum_{p=1}^{N_l} 1\{B_{pl} \leq b\}K_G\left(\frac{x - X_l}{h_G}, \frac{n - N_l}{h_{GN}}\right)}
\]

\[
\tilde{g}(b, x, n) = \frac{1}{\sum_{l=1}^{L} \frac{1}{N_l} \sum_{p=1}^{N_l} K_g\left(\frac{b - B_{pl}}{h_g}, \frac{x - X_l}{h_G}, \frac{n - N_l}{h_{GN}}\right)}
\]
Multi-unit auction formats

(a) Discriminatory auction

(b) Uniform price auction
Bidders don’t know the price where market will clear
Equivalently, bidders don’t know the residual supply curve.
Wilson (1979) ”share auction” model

- Marginal valuation function for bidder $i$: $v_i(q, s_i)$
- Submit decreasing bid function: $y_i(p)$
- Total quantity: $Q$
- Residual supply function:

$$RS_i(p) = \sum_{j \neq i} y_j(p) + Q$$

- Market clearing price:

$$p^c: y_i(p^c) = RS_i(p^c)$$
Wilson model cont.

- Probability distribution of market clearing price:

\[ H_i(p, y_i(p)) = Pr\{y_i(p) \leq R_{S_i}(p)\} \]
\[ = Pr\{p^{c} \leq p\} \]

- Expected surplus of risk-neutral bidder in *private value* discriminatory auction:

\[
\max_{y_i(\cdot)} \int_{p^c}^{p} \left( \int_{0}^{y_i(p^c)} v(q) dq - y_i^{-1}(q) dq \right) dH_i(p^c, y_i(p^c))
\]

surplus conditional on \( p^c \)

- Under interdependent values, market clearing price realizations convey information about ex-post utility
Wilson Euler eqn’s

Optimality condition in *private value discriminatory* auction:

\[
v_i(y_i(p)) = \underbrace{p}_{\text{bid for } y_i(p) \text{ units}} + \underbrace{\frac{H_i(p, y_i(p))}{\partial H_i(p, y_i(p))}}_{\text{"shading" factor}} \underbrace{\frac{\partial H_i(p, y_i(p))}{\partial p}}_{\partial p}
\]

Optimality condition in *private value uniform price* auction:

\[
v_i(y_i(p)) = \underbrace{p}_{\text{bid for } y_i(p) \text{ units}} - y_i(p) - y_i(p) \underbrace{\frac{\partial H_i(p, y_i(p))}{\partial p}}_{\partial p}
\text{"shading" factor}
\]
Alternatives to Euler eqn’s

- In almost all real world auctions, bidders confined to make step function bids with finite number of steps, residing in a price-quantity grid
- Characterize finite deviations to derive “bounds” on underlying marginal valuation function
  - McAdams 2008 – tighter bounds utilizing cleverly chosen array of deviations
  - Kastl 2008 – discrete bids lead to bidding below marginal value in uniform price auction; contrary to Wilson model!
Positive results in the IPV environment:
- No uniqueness results so far

Interdependent values?
Structural estimation

- Assuming bidders are behaving optimally, we can recover marginal valuations by estimating $H(p, y_i(p))$

- Techniques depend on informational assumptions
  - Hortaçsu (2002): in symmetric IPV environment, $y_i(p)$ are iid; can simulate empirical analog of $H(p, y_i(p))$
  - Hortaçsu (2002): if there is a small class of asymmetric bidders (e.g. “large” vs. “small”), $y_i(p)$ iid within class
  - Fevrier, Preguet, Visser (2004): estimation in pure common value model (with strong testable implications)
Use Euler eqn’s to estimate marginal values
Counterfactual revenue comparisons (Hortaçsu (2002))

- Calculate the “best-case” revenue of the uniform price auction, by assuming bidders revealed true marginal valuations.
  - This has been challenged by Kastl (2008)

- This also bounds the revenue in the private value Vickrey auction, where bidders indeed reveal their demands truthfully.

- *Ex-post* comparison: Make the comparison using point estimates of marginal valuations.

- *Ex-ante* comparison: Recognize that marginal valuations are also random variables with a distribution.
Counterfactual revenue comparison

![Graph showing counterfactual revenue comparison with two lines representing aggregated upper envelope of estimated marginal valuations and aggregated bids.](image-url)
Do bidders satisfy optimality conditions?

- Need “outside” information on marginal valuations (or costs in procurement context) to test optimality
- In Hortaçsu and Puller (2008, RAND), we utilize data on electricity generators’ marginal costs to test optimality of bidding in uniform price auction
- Large bidders in close agreement with Wilson model
- Small bidders deviate in ways that are not easy to reconcile with profit maximizing behavior
- Data was from a newly deregulated market (Texas ERCOT) with many of the small bidders as incumbents from regulated era
- Small bidders’ behavior led to significant efficiency losses – larger than efficiency losses caused by “optimal” market power exercise by large bidders
Testing the private values assumption

- No known nonparametric identification results in the common value case
- How to test for null hypothesis of private values?
  - Haile-Hong-Shum (2005): variation in number of bidders.
Bidders 1, 2, and 3 submit a bid curve (demand) $b_i(q)$

The auctioneer shows the bid of bidder 2, $b_2(q)$ to bidder 1 and allows her to submit another bid if she wants to

Bidder 1 makes use of this information and submits a new optimal bid, given the original information and this additional piece of information

This change in the bid (due to updating of priors) will be different under common and private values, because with an important common valuation component she updates her belief about the uncertain common valuation part using the observed bid $b_2(q)$
Bid updating

Updating of Bids

- Original dealer bid
- Updated dealer bid
- Customer bid

Price (Bid) vs. Quantity Share demanded

- 9.8735 x 10^5
- 9.873
- 9.8725
- 9.872
- 9.8715
- 9.871
- 0 0.02 0.04 0.06 0.08 0.1 0.12 0.14 0.16 0.18
Suppose that as in the previous example we observe both original and updated bid of bidder 1.

Estimate marginal values corresponding to the original and updated bids using Hortaçsu (2002).

Test for equality of the marginal valuations (subject to a few caveats).

Fail to reject PV for 3-month T-bills. (Some) Tests reject PV for 12 month T-bills.
Does the dealer change her valuation?

<table>
<thead>
<tr>
<th>Price (Bid)</th>
<th>Quantity Share demanded</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.8775</td>
<td>0.02</td>
</tr>
<tr>
<td>9.877</td>
<td>0.04</td>
</tr>
<tr>
<td>9.876</td>
<td>0.06</td>
</tr>
<tr>
<td>9.875</td>
<td>0.08</td>
</tr>
<tr>
<td>9.874</td>
<td>0.10</td>
</tr>
<tr>
<td>9.873</td>
<td>0.12</td>
</tr>
<tr>
<td>9.872</td>
<td>0.14</td>
</tr>
<tr>
<td>9.871</td>
<td>0.16</td>
</tr>
<tr>
<td>9.870</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Estimation of marg. value: Bidder 1/50

- Original dealer bid
- Updated dealer bid
- Customer bid
- Estimated MV – original bid
- Estimated MV – updated bid
- 95% CI

![Graph showing the estimation of marginal value for Bidder 1/50]
Under PV, we can calculate value of customer information for dealers, since we know dealer’s private valuation; hence profit from auction.

The value of customer bid info important in many other contexts, including stock trading.
The 2007 “Subprime” Liquidity Crisis in Europe: A Micro Perspective

Ali Hortaçsu

July 8, 2014
2007 Subprime Crisis:

- August 9th, 2007: BNP Paribas announces the freeze of 3 huge investment funds exposed to U.S. subprime mortgages
- in the 2nd half of 2007: volume in term interbank markets diminishes: liquidity shifts to less than 1-week maturities in secured (repo) transactions, and to overnight lending in unsecured.
“Black Swan” in the Euro Money Market (Taylor and Williams (2009))

Difference between 1−w unsecured rate (EURIBOR) and 1−w repo rate (EUREPO)

- August 9, 2007 (week 32)
This talk

1. Understand the heterogeneity of the crisis
2. Which banks are exposed, which banks are not
3. Use “revealed preference” information revealed in the observed strategic actions of banks in the money market
EURO money markets

MRO - main refinancing ops (1 wk; every week)
Standing facility -- overnight loans/deposits (MBR +/- 100 b.p)
LTRO - long-term refinancing ops (3 mo; every month)
FTO - fine-tuning ops (no fixed maturity; every month)
Key “Reported” Interest rates:

- **EURIBOR**: unsecured loans, average of rates submitted by panel banks: “daily quotes of the rate that each panel bank believes one prime bank is quoting to another prime bank for interbank term deposits within the Euro zone”

- **EUREPO**: collateralized loans, average of rates submitted by panel banks: “Panel Banks are submitting the best bids in the market. However, Panel banks submitting the bids are expected, under normal circumstances, to transact at these levels.”

- **EONIA swap rate**: “the average rate at which a representative panel of prime banks provide daily quotes that each Panel Bank believes is the Mid Market rate of EONIA swap quotations between prime banks”
  
  An “EONIA swap” is an interest rate swap transaction, where one party agrees to receive/pay a fixed rate to another party, against paying/receiving a floating rate named EONIA (which is average of all actual overnight unsecured transactions).
Bidding in ECB MRO auctions

Bidding Behavior in auctions

- Bid – EONIA (Before Turmoil)
- Bid – EONIA (After Turmoil)
This paper:

- The turmoil marked a drastic change in bidding behavior in the primary market.
- Since virtually all bidders started bidding more aggressively, was there a significant change in the fundamentals? (such as increase in their willingness-to-pay for ECB funding?)
- Can we use the data to tell which bidders are likely hit substantially by the crisis?
- Do our measures of distress predict future problems?
Demand for ECB liquidity)

- Bank $i$ needs $R_i$ units of liquidity
- Has $L_i$ units of “liquid” collateral: Euro gov’t bonds, can borrow at secured rate $s_i$ against these
- Has $K_i - L_i$ units of “illiquid” collateral: e.g. asset-backed securities (ABS), retail mortgage backed securities (RMBS)
- Marg. cost of borrowing $q$ units against illiquid collateral $s_i + c_i(q)$.
- Funding needs beyond $K_i$ can be satisfied at interbank unsecured rate $u_i$. 
Demand for ECB Liquidity, $v_i(q)$
Model: Divisible Good Auction

- **A1**: (Independence) *Conditional on observables* prior to auction, $s_i$ is independent across bidders
- **A1’**: $s_i$ are iid within (small number of) classes of bidders
- **A2**: (Private values) Learning $s_i$ does not affect $v(q, s_i)$.
- **Equilibrium**: Bayesian Nash
- **Euler equation**:

\[
v(q_k, s_i) = b_k + \frac{\Pr(b_{k+1} \geq p^c)}{\Pr(b_k > p^c > b_{k+1})} (b_k - b_{k+1})\]

- Uncertainty about market clearing price creates a wedge between marg valuation and bid!
Bidders don’t know the price where market will clear
Equivalently, bidders don’t know the residual supply curve
Estimation in the symmetric iid case

- Recall:

\[ v(q_k, s_i) = b_k + \frac{\Pr(b_{k+1} \geq p^c)}{\Pr(b_k > p^c > b_{k+1})} (b_k - b_{k+1}) \]

- Therefore we need the distribution of the market clearing price.

- Obtain this distribution by simulation: drawing with replacement \((N - 1)\) bids from the observed bids, subtract from the supply and intersect thus obtained residual supply with a bidder’s bid to obtain one possible market clearing price.

- Many such simulation draws will result in a distribution of the market clearing price.
Resampling method
Data

- 50 auctions of 1-week loans run by the ECB
- Discriminatory auction format
- ECB requires bidders to post collateral (next slide) with their respective central banks against the loans (otherwise penalties enforced). Eligible collateral broader than interbank repo transactions.
- Sources of heterogeneity, (conditional) private values:
  - Reserve requirements, $R_i$
  - Interbank funding opportunities: $s_i$, $u_i$
  - Collateral positions: $K_i$, $L_i$
### Data: Before and After Turmoil Start

<table>
<thead>
<tr>
<th>Summary Statistics</th>
<th>Mean Before</th>
<th>Mean After</th>
<th>Std Dev Before</th>
<th>Std Dev After</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bidders (cca 700 identities)</td>
<td>348.6</td>
<td>328.1</td>
<td>20.88</td>
<td>34.37</td>
</tr>
<tr>
<td>Submitted steps</td>
<td>1.47</td>
<td>2.02</td>
<td>0.73</td>
<td>1.34</td>
</tr>
<tr>
<td>Price bid</td>
<td>3.80</td>
<td>4.13</td>
<td>0.20</td>
<td>0.06</td>
</tr>
<tr>
<td>Quantity bid</td>
<td>0.004</td>
<td>0.005</td>
<td>0.009</td>
<td>0.014</td>
</tr>
<tr>
<td>Issued Amount (billions Euros)</td>
<td>292.34</td>
<td>202.19</td>
<td>1.42</td>
<td>4.51</td>
</tr>
</tbody>
</table>
We estimate marginal values for each bidder

- Compare marginal values for liquidity in the primary market to the repo-rate in the secondary market (EUREPO) and to the unsecured rate (EURIBOR).
Results: Aggregate value before turmoil

Bidding Behavior in auction32

- Marg val
- Bid
- EONIA
- RepoRate
- Euribor
Results: Aggregate value before turmoil

- Values bounded for most part from above by the unsecured rate (the riskiest one) EURIBOR and from below by the “risk-free” rate - fully collateralized rate EUREPO (highest quality collateral used)
- “First step” of marg val schedules: clustered around EURIBOR, i.e. \( u_i \) are close to EURIBOR before the turmoil
- Market cleared at the repo-rate. Some bidders gain infra-marginal surplus by utilizing (relatively) illiquid collateral to borrow at secured rate from ECB, but there’s rationing.
Results: Aggregate value after turmoil

Bidding behavior (−) and marginal values (x) in auctions after turmoil (auction 45)

- Marginal value
- Bid
- EURIBOR
- EUREPO
- EONIA
Results: Aggregate value in Nov 2007

- Most bidders cannot borrow at EURIBOR. Unsecured rates faced by bidders 5-8 b.p. above EURIBOR. Some much higher!
Interest rates and market clearing

Auction

Yield

Interest rates

1-w repo rate (EUREPO)
1-w unsecured rate (EURIBOR)
Clearing rate
Turmoil
Results: Clearing above unsecured rate

- Why are there auctions in which the market cleared above the unsecured rate?
- Do we believe the EURIBOR?
28. From at least approximately August 2005 to at least approximately May 2008, Barclays Euro swaps traders communicated with swaps traders at other financial institutions that were members of the EURIBOR Contributor Panel about requesting favorable EURIBOR submissions from the EURIBOR submitters at their respective banks. At Barclays, this conduct was primarily undertaken by a Barclays Euro swaps trader, Trader-5, who left Barclays and joined another financial institution in approximately May 2007. While Trader-5 worked at Barclays, Trader-5 communicated with traders at several other Contributor Panel banks about obtaining favorable EURIBOR submissions, and requested favorable EURIBOR submissions from the Barclays EURIBOR submitter. After Trader-5 joined another financial institution, Trader-5 continued communicating with traders at Barclays about requesting favorable EURIBOR settings.

29. As an example, during at least approximately February and March 2007, in addition to contacting the Barclays EURIBOR submitter, Trader-5 communicated with traders at four other EURIBOR Contributor Panel banks and requested that their respective EURIBOR submitters submit low 3-month EURIBOR contributions on March 19, 2007, which was the Monday before the March IMM date. That trading date was particularly significant for Trader-5, who stated that s/he had accumulated financial positions that would benefit from a low 3-month EURIBOR. In one example of Trader-5’s contacts with other banks, on February 6, 2007, in an electronic message, a trader at another Contributor Panel bank asked Trader-5: “[I]n march do you still want a very low 3m cash fixing for imm?” Trader-5 replied, “yeah.” The other trader
APPENDIX A

STATEMENT OF FACTS

This Statement of Facts is incorporated by reference as part of the non-prosecution agreement, dated June 26, 2012, between the United States Department of Justice, Criminal Division, Fraud Section, and Barclays Bank PLC. The parties agree that the following information is true and accurate:

BACKGROUND

LIBOR and EURIBOR

1. Since its inception in approximately 1986, the London Interbank Offered Rate (“LIBOR”) has been a benchmark interest rate used in financial markets around the world. Futures, options, swaps, and other derivative financial instruments traded in the over-the-counter market and on exchanges worldwide are settled based on LIBOR. The Bank of International Settlements has estimated that in the second half of 2009, the notional amount of over-the-counter interest rate derivatives contracts was valued at approximately $450 trillion. In addition, mortgages, credit cards, student loans, and other consumer lending products often use LIBOR as a reference rate.

2. LIBOR is published under the auspices of the British Bankers’ Association (“BBA”), a trade association with over 200 member banks that addresses issues involving the United Kingdom banking and financial services industries. The BBA defines LIBOR as:

   The rate at which an individual Contributor Panel bank could borrow funds, were it to do so by asking for and then accepting inter-bank offers in reasonable market size, just prior to 11:00 [a.m.] London time.

This definition has been in place since approximately 1998.

3. LIBOR is calculated for ten currencies. The LIBOR for a given currency is the result
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LIBOR and EURIBOR

1. Since its inception in approximately 1986, the London Interbank Offered Rate ("LIBOR") has been a benchmark interest rate used in financial markets around the world. Futures, options, swaps, and other derivative financial instruments traded in the over-the-counter market and on exchanges worldwide are settled based on LIBOR. The Bank of International Settlements has estimated that in the second half of 2009, the notional amount of over-the-counter interest rate derivatives contracts was valued at approximately $450 trillion. In addition, mortgages, credit cards, student loans, and other consumer lending products often use LIBOR as a reference rate.

2. LIBOR is published under the auspices of the British Bankers’ Association ("BBA"), a trade association with over 200 member banks that addresses issues involving the United Kingdom banking and financial services industries. The BBA defines LIBOR as:

   The rate at which an individual Contributor Panel bank could borrow funds, were it to do so by asking for and then accepting inter-bank offers in reasonable market size, just prior to 11:00 [a.m.] London time.

This definition has been in place since approximately 1998.

3. LIBOR is calculated for ten currencies. The LIBOR for a given currency is the result
Classifying troubled bidders

- Can we use our methodology to identify distressed banks?
- With current data yes.

<table>
<thead>
<tr>
<th>Based on</th>
<th>Bids</th>
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</thead>
<tbody>
<tr>
<td>Values</td>
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</tr>
<tr>
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</tr>
<tr>
<td>No</td>
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</table>

Notice that analysis based solely on bids (ignoring the equilibrium adjustments) would overestimate the number of affected banks! (88% rather than 66% based on values)
Predicting problems?

- In the future: if ECB utilized our approach “regularly”, banks would likely adjust their bidding behavior.
- Are there perhaps some incentive-compatible policies based on the stressed/distressed classification so that the eq’m bidding relationship necessary for our method would not be disrupted?
- A placebo test (doing the same exercise as above, but only based on 32 auctions pre-turmoil, where the artificial break is assumed at auction 16.) shows that the results are not due to “chance”.

Some insights about interbank (secondary) market from our model

- The quantity-weighted marginal value is between $s_i$ and $u_i$:

$$v_i = \alpha_i s_i + (1 - \alpha_i) u_i$$

- $\alpha_i$ is (approximately) the ratio of ECB-acceptable collateral that is also acceptable in the secondary market.

- **BUT**: Bank-specific rates not observed, but it may be reasonable that $s_i$ is fairly constant across banks (even though some bank-specific risk of the “capture” of collateral might be priced).

- Thought experiment: use the published rates $u$ (EURIBOR), $s$ (EUREPO) together with estimated $v_i$ and back out the implied convex combination weights $\alpha_i$. 
Linking values in the primary market with bank-specific interest rates in the secondary market

- Estimates: Median $\alpha$ pre-turmoil: 0.29, post-turmoil: −0.02!
- Per auction: even pre-turmoil about 40% of banks have estimated values above EURIBOR!
- post turmoil this fraction increases to 58%!
Distribution of crisis across Euro-zone countries

![Graph showing the distribution of crisis across Euro-zone countries with alpha by country (participants only). The x-axis represents the countries, and the y-axis represents the alpha values. The graph distinguishes between pre- and post-turmoil periods.]
Private Information

- For a subsample of banks we obtained data which is likely to be correlated with their private information, $s_i$
- Daily observation of monthly reserve requirements (used on the day of the auction), credit default swaps (CDS), size
## Analysis of Marginal Values

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Analysis of Marginal Values

- EUREPO highly correlated with values
- deficiency has a positive impact after turmoil
- liquidity obtained in LTROs (3-m maturity loans) results in lower values for liquidity in MROs pre-turmoil, but there is funding complementarity post-turmoil
- RCDS positive after turmoil
Do the estimated WTP for ECB funding predict balance sheet troubles (reported end of 2007)?
### Linking marginal values in 2007 to performance measures

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</table>

The dependent variables in this table are bank performance ratios (ROA, ROE, CTI and NIM) reported year-end 2007. The independent variables are the pre- vs. post-crisis (Aug. 9, 2007) change in a bank’s quantity-weighted average price bids (Δ bids) and estimated marginal valuations (Δ marginal values). We also control for the year-end 2006 performance ratios. Standard errors are in parentheses, with (***), (**), and (*) indicating p<0.01, p<0.05, and p<0.1.
Do the estimated WTP for ECB funding predict bailouts?
## Logit of 1st Wave Bailouts

<table>
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<th>Bailout Aug 07-Sep 08</th>
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<td>Δ Willingness-to-Pay post-pre Aug 2007</td>
<td>6.72** (3.33)</td>
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<td>7.37* (3.92)</td>
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<td>-26.43 (16.74)</td>
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<td>Mean of Dependent variable</td>
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<td>pseudo-$R^2$</td>
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<tr>
<td>N</td>
<td>384 369 297</td>
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We also found negative correlation between the change in marginal values pre- and post- August 2007 and the change in the stock prices pre- and post- August 2007.

This suggests that marginal values perhaps contain private information that is yet to be priced in the market.
Conclusion

- We decomposed the impact of 2007 subprime crisis to an equilibrium/strategic effect and a demand/fundamentals based effect.
- For over $\frac{2}{3}$ of the bidders, the value for obtaining liquidity in the primary market increased substantially. (Bids significantly increase for 88% of bidders.)
- This suggests that for many banks the primary market became the only viable option for obtaining liquidity (some auctions even cleared above the unsecured rate!)
- Due to increased heterogeneity of values and the associated misallocation of liquidity (as the secondary market does not function properly to rectify it) the primary auctions play an important role to improve allocative efficiency.
- Use of recent microeconometric methods to estimate banks’ implied funding costs could be a useful tool to evaluate financial health of banks.