Corporate Profits Tax and Innovative Investments

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Introduction

- Impact of changes in corporate profits taxes on dynamics of aggregate output?
- How much do changes in firms’ incentives to invest in innovation matter?

Barro-Furman (2018), CBO, Penn-Wharton Budget Model
  - R&D capital accumulation, parallel to physical capital

Atkeson-Burstein (2018) model of firm dynamics and innovation:
  - Expensing for tax purposes of investment in entry vs. innovation by incumbents, business stealing, markups, gap btw private & social returns due to spillovers
  - Extend results on aggregate implications of policy-induced changes in innovation
AB Model

Pros:

- Nest range of models that allow for innovation by incumbents and entrants
  - Jones (2002)

- Tractable aggregate transition dynamics and welfare


Cons:

- Does not nest models that need state variable for firm types
Ten-year horizon: consideration of impact of corporate profits taxes on firms’ incentives to innovate does not results in large absolute differences (relative to e.g. business cycle variation)

Long-run: potentially large impact (positive or negative) depending on

- intertemporal knowledge spillovers

- is there too much or too little entry in the initial equilibrium?
Talk Outline

- Simple model
  - only entrants invest
  - two sufficient statistics for long-run, transition dynamics
  - simple bound on impact elasticity

- Add investment by incumbent firms
  - reallocation of innovation between entrants and incumbents

- Calibration and policy experiment

- Results
Model outline

- Two final goods
  - output: consumption and physical investment
  - research good: input to innovation

- Output produced from differentiated intermediate goods
  - produced with physical capital and production labor

- Research good produced with research labor

- Entrants invest research good to acquire a differentiated good
  - create new intermediate goods or
  - improve upon existing goods (business stealing)

- Later: add innovation by incumbents
Intermediate Goods

\[ y_t(z) = \exp(z)k_t(z)^\alpha l_{pt}(z)^{1-\alpha} \]

Measure of products \( M_t(z) \), \( M_t = \sum_z M_t(z) \)

Aggregate Output

\[ Y_t = \left[ \sum_z y_t(z) \frac{\rho-1}{\rho} M_t(z) \right]^{\frac{\rho}{\rho-1}} = C_t + K_{t+1} - (1 - d_k)K_t \]

Assuming constant markups \( \mu > 1 \) and common factor prices

\[ Y_t = Z_t K_t^\alpha L_{pt}^{1-\alpha} \quad K_t = \sum_z k_t(z)M_t(z) \quad L_{pt} = \sum_z l_{pt}(z)M_t(z) \]

Aggregate productivity

\[ Z_t = \left[ \sum_z \exp(z)^{\rho-1} M_t(z) \right]^{1/(\rho-1)} \]
Inputs into Innovative Investment

Research good

\[ Y_{rt} = A_{rt}Z_t^{\phi-1}L_{rt} \]

Research labor \( L_{rt} = L_t - L_{pt} \).

Intertemporal knowledge spillovers \( \phi \leq 1 \)

Research TFP \( = A_{rt}Z_t^{\phi-1} \) \quad Jones (2002) and Bloom et. al. (2017)

Exogenous Scientific Progress \( A_{rt} \) grows at \( \bar{g}_{Ar} \), labor \( L_t \) grows at \( \bar{g}_L \)

\( \phi < 1 \) ideas get “harder to find”

Balanced growth path — \( Y_{rt} \) constant

\[ \bar{g}_Z = \frac{\bar{g}_L + \bar{g}_{Ar}}{1 - \phi} \]
Innovation technologies imply tractable equilibrium aggregation:

\[
\log Z_{t+1} - \log Z_t = G(x_{et})
\]

subject to

\[x_{et} = Y_{rt}\]

- Each entrant spends \( \frac{1}{M_t} \) units of the research good at \( t \) \( \implies x_{et}M_t \) new products
- Fraction \( \delta_e \) stolen, \( 1 - \delta_e \) new to society
- Externalities in productivity:
  \[
  \mathbb{E} \exp(z')^{\rho - 1} = \eta_e \frac{Z_t^{\rho - 1}}{M_t}
  \]

Evolution of aggregate productivity

\[
\log Z_{t+1} - \log Z_t = \frac{1}{\rho - 1} \log [(1 - \delta_{ct}) + \eta_e x_{et}]
\]

Product exit \( \delta_{ct} = \delta_0 + \delta_e x_{et} \)
Innovations technologies imply tractable equilibrium aggregation:

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- Externalities in productivity: \(E \exp(z')^{\rho-1} = \eta_e \frac{Z_t^{\rho-1}}{M_t}\)

Evolution of aggregate productivity

\[
\log Z_{t+1} - \log Z_t = \frac{1}{\rho - 1} \log [(1 - \delta_{ct}) + \eta_e x_{et}]
\]

Product exit \(\delta_{ct} = \delta_0 + \delta_e x_{et}\)
Firm chooses investment $K_{t+1}$ to maximize $\sum_{t=0}^{\infty} Q_t D_{kt}$

$$D_{kt} = (1 - \tau) R_{kt} K_t - (1 - \tau \lambda_k) (K_{t+1} - (1 - \delta_k) K_t)$$

$$\left( \frac{1 - \tau}{1 - \tau \lambda_k} \right) R_{kt+1} - \delta_k = R_{t+1} \quad \text{where} \quad R_{kt} = \frac{\alpha}{\mu} \frac{Y_t}{K_t}$$

Long run change in user cost of capital

$$\log \frac{\bar{K}_t'}{\bar{Y}_t'} - \log \frac{\bar{K}_t}{\bar{Y}_t} = \log \left( \frac{1 - \tau'}{1 - \lambda_k' \tau'} \right) - \log \left( \frac{1 - \tau}{1 - \lambda_k \tau} \right)$$
Value of an incumbent firm with productivity $z$, $V_t(z) = V_t \exp(z) \frac{1}{\rho-1}$

\[ V_t = (1 - \tau) \frac{\mu - 1}{\mu} Y_t + \frac{1}{1 + R_t} V_{t+1} (1 - \delta_{ct}) \frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}} \]

Entry condition:

\[ (1 - \tau \lambda_e) P_{rt} \frac{1}{M_t} = \frac{1}{1 + R_t} V_{t+1} \frac{\eta_e}{M_t} \frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}} \]

Innovation intensity $i_{rt} \equiv \frac{P_{rt} Y_{rt}}{Y_t}$, and labor allocation $L_{rt} = \frac{i_{rt}}{i_{rt} + (1 - \alpha)/\mu}$ (w.l.o.g $L_t = 1$)

Long-run change in user cost of intangible capital

\[ \log \bar{Z}_t' - \log \bar{Z}_t = \log \left( \frac{1 - \tau'}{1 - \lambda_e' \tau'} \right) - \log \left( \frac{1 - \tau}{1 - \lambda_e \tau} \right) \]

and given $Y_{rt} = A_{rt} Z_t^{\phi-1} L_{rt}$ constant between BGPs:

\[ \log \bar{Z}_t' - \log \bar{Z}_t = \frac{1}{1 - \phi} (\log \bar{L}_t' - \log \bar{L}_t) \]
Transition Dynamics

\[ \log Z_{t+1} - \log Z_t = G(Y_{rt}) \]

Impact Elasticity

\[ \log Z_{t+1} - \log Z_t - \bar{g}_Z \approx \Theta (\log Y_{rt} - \log \bar{Y}_r) \]

\[ \Theta \equiv G'(\bar{Y}_r) \bar{Y}_r \]

Combined with research good production function, \( Y_{rt} = A_{rt}Z_t^{\phi-1}L_{rt} \):

\[ \log Z'_{t+1} - \log \bar{Z}_{t+1} = \sum_{k=0}^{t} \Gamma_k (\log L_{rt} - \log \bar{L}_r) \]

\[ \Gamma_0 = \Theta \quad \Gamma_{k+1} = [1 - \Theta(1 - \phi)] \Gamma_k \]

\[ \log Y'_t - \log \bar{Y}_t = \frac{1}{1 - \alpha} (\log Z'_t - \log \bar{Z}_t) + (\log L'_{pt} - \log \bar{L}_p) - \frac{\alpha}{1 - \alpha} (\log R'_{kt} - \log \bar{R}_k) \]

Welfare
One-period reallocation of labor towards research

Aggregate Output, % deviation from BGP

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Impact Elasticity
Increase in aggregate productivity above trend
Dynamics: Full Intertemporal Knowledge Spillovers
Permanent increase in aggregate productivity above trend
Limited Intertemporal Knowledge Spillovers

Aggregate productivity reverts back to its initial path.
Bounding the Impact Elasticity $\Theta$

$$\Theta = \frac{1}{\rho - 1} \frac{\exp(\bar{g}Z)^{\rho-1} - \exp(G(0))^{\rho-1}}{\exp(\bar{g}Z)^{\rho-1}} \leq \bar{g}z - G(0)$$

Impact elasticity $\leq \frac{\bar{g}z - G(0)}{\bar{x}_e / \bar{Y}_r} = \frac{\text{contribution of entrants’ innovation to growth}}{\text{entrants’ share of innovative investment}}$
Product-level innovation technologies imply tractable equilibrium aggregation:

\[ \log Z_{t+1} - \log Z_t = G(x_{ct}, x_{mt}, x_{et}) \]

subject to

\[ x_{ct} + x_{mt} + x_{et} = Y_{rt} \]

Evolution of aggregate productivity

\[ Z_{t+1}^{\rho-1} = [(1 - \delta_{ct}) \zeta(x_{ct}) + \eta_m h(x_{mt}) + \eta_e x_{et}] \frac{Z_{t}^{\rho-1}}{M_t} M_t \]

Product exit

\[ \delta_{ct} = \delta_0 + \delta_m h(x_{mt}) + \delta_e x_{et} \]
Impact Elasticity

Start from

$$\log Z_{t+1} - \log Z_t = G(x_{ct}, x_{mt}, x_{et})$$

where

$$x_{ct} + x_{mt} + x_{et} = Y_{rt}$$

Differentiating

$$\log Z_{t+1} - \log Z_t - \tilde{g}_Z \approx \Theta (\log Y_{rt} - \log \bar{Y}_r)$$

$$\Theta = \frac{dx_c}{dY_r} \Theta_c + \frac{dx_m}{dY_r} \Theta_m + \frac{dx_e}{dY_r} \Theta_e$$

where$$\Theta_i = G_i(\bar{x}_c, \bar{x}_m, \bar{x}_e) \bar{Y}_r$$

and

$$\frac{dx_c}{dY_r} + \frac{dx_m}{dY_r} + \frac{dx_e}{dY_r} = 1$$

$$\Theta$$ is now policy dependent
Intermediate good firms and corporate profits tax

Value of incumbents:

\[ V_t = (1 - \tau) \frac{\mu - 1}{\mu} Y_t - (1 - \tau \lambda_c) P_{rt} (x_{ct} + x_{mt}) \]

\[ + \frac{1}{1 + R_t} V_{t+1} \left[ (1 - \delta_{ct}) \zeta (x_{ct}) + \eta_m h(x_{mt}) \right] \frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}} \]

Entry condition:

\[ (1 - \tau \lambda_e) P_{rt} \frac{1}{M_t} = \frac{1}{1 + R_t} \frac{\eta_e}{M_t} \frac{Z_t^{\rho-1}}{Z_{t+1}^{\rho-1}} \]

Interior equilibrium investment allocation, given \( Y_{rt} \)

\[ \frac{(1 - \tau \lambda_c)}{(1 - \tau \lambda_e)} \eta_e = \eta_m h' (x_{mt}) = (1 - \delta_{ct}) \zeta' (x_{ct}) \]

\[ Y_{rt} = x_{ct} + x_{mt} + x_{et} \]
Equal expensing for incumbents and entrants: $\lambda_e = \lambda_c$

Policy affects $Y_{rt}$ but not mapping between $Y_{rt}$ and $\log(Z_{t+1}) - \log(Z_t)$

Formulas for dynamics in simple model go through

If $\lambda_e = \lambda_c < 1$, $\downarrow \tau$ equivalent to $\uparrow$ uniform subsidy to all innovative investments $\Rightarrow \uparrow i_r$

- No business stealing: $\Theta = \Theta_e = \Theta_c = \Theta_m$

- Business stealing: $\Theta_e < \Theta_c$. $\uparrow x_e$ and $\downarrow x_c$, under regularity condition, $\Theta \leq \Theta_e$

$$\Theta_e \leq \frac{\tilde{g}_Z - G(\bar{x}_c, \bar{x}_m, 0)}{\bar{x}_e / \bar{Y}_r} = \text{contribution of entrants’ innovation to growth}$$

entrants’ share of innovative investment
Equal expensing for incumbents and entrants: $\lambda_e = \lambda_c$

Policy affects $Y_{rt}$ but not mapping between $Y_{rt}$ and $\log(Z_{t+1}) - \log(Z_t)$

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Different expensing for incumbents and entrants

Interior equilibrium investment allocation, given $Y_{rt}$

$\frac{(1 - \tau \lambda_c)}{(1 - \tau \lambda_e)} \eta_e = \eta_m h' (x_{mt}) = (1 - \delta_{ct}) \zeta' (x_{ct})$

$Y_{rt} = x_{ct} + x_{mt} + x_{et}$ \hspace{1cm} $\log Z_{t+1} - \log Z_t = G(x_{ct}, x_{mt}, x_{et})$

Policies affect mapping btw $Y_{rt}$ and $\log(Z_{t+1}) - \log(Z_t)$, but mapping time-invariant

Transition dynamics around new BGP allocation of investment

$\log Z'_{t+1} - \log \bar{Z}_{t+1} \approx \sum_{k=0}^{t} \Gamma'_k \left[ \log L'_{r-k} - \log \bar{L}_r - \left( \log \bar{Y}'_r - \log \bar{Y}_r \right) \right]$

$\Gamma'_0 = \Theta'$ and $\Gamma'_{k+1} = [1 - (1 - \phi)\Theta'] \Gamma'_k$

Long-run efficiency gains from reallocation of investments: $\log \bar{Y}'_r - \log \bar{Y}_r$

If $\lambda_c > \lambda_e$, then it is possible that $\Theta'_e > \Theta'_c$ and $\Theta' > \Theta'_e$, curvatures of $h$ and $\zeta$ matter
Measurement

\[ \Theta_e \leq (\bar{g}_Z - G(\bar{x}_c, \bar{x}_m, 0)) \frac{\bar{Y}_r}{\bar{x}_e} \]

1 Contribution of Entrants to Growth

- with no business stealing = \( \frac{1}{\rho - 1} \) employment share of entering firms
- estimates of \( \bar{g}_Z - G(\bar{x}_c, \bar{x}_m, 0) \) from GHK and Akcigit and Kerr (2016)

2 Share of entrants’ innovation expenditure in total innovation expenditure

- NIPA measures of innovation expenditures of incumbent firms
- impute investment in entry equal to value of entering firms

Intertemporal Knowledge Spillovers \( \phi \)

- \( \phi = 0.96 \) — \( \approx \) endogenous growth
- \( \phi = -1.6 \) — Fernald and Jones (2014)
Measurement

\[ \Theta_e \leq (\bar{g}_Z - G(\bar{x}_c, \bar{x}_m, 0)) \frac{\bar{Y}_r}{\bar{x}_e} \]

1 Contribution of Entrants to Growth

\[ \frac{1}{\rho - 1} \text{employment share of entering firms} = \frac{0.027}{3} = 0.009 \]

\[ \text{AK 2016} \quad \frac{\bar{g}_Z - G(\bar{x}_c, \bar{x}_m, 0)}{\bar{g}_Z} = 0.257 \quad \Rightarrow \quad \bar{g}_Z - G(\bar{x}_c, \bar{x}_m, 0) = 0.0037 \]

2 Share of entrants’ innovation expenditure in total innovation expenditure

- incumbent firms invest 6.1% of output in innovation
- imputed innovative investment of entering firms 3.1% of output

Intertemporal Knowledge Spillovers \( \phi \)

\[ \phi = 0.96 \approx \text{endogenous growth} \]

\[ \phi = -1.6 \quad \text{— Fernald and Jones (2014)} \]
Implied parameter values

1. Impact elasticity of entrants, $\Theta_e$
   - $\Theta_e = 0.030$ — no business stealing
   - $\Theta_e = 0.011$ — AK 2016

2. Intertemporal Knowledge Spillovers $\phi$
   - $\phi = 0.96$ — $\approx$ endogenous growth
   - $\phi = -1.6$ — Fernald and Jones (2014)

3. Ratio of growth rate to interest rate and physical capital share
   - $\tilde{\beta} = 0.986$
   - $\alpha = 0.25$
Policy Experiment

Unanticipated, permanent

Corporate profit taxes $\tau = 0.38$, $\tau' = 0.26$ (BF2018)

- $\lambda_k = 0.87$ ($K/Y = 2.12$), $\lambda'_k = 1$ ($\Delta \log K/Y = 0.08$, BF2018)
- expensing of innovative investments: $\lambda_c = 1$ and $\lambda_e = 0$

Three cases:

1. innovation unchanged

2. only entry responds

   $$\log \bar{i}'_r - \log \bar{i}_r = [\log (1 - \tau') - \log (1 - \tau)] \times \frac{\text{div}}{\text{prof}},$$

3. entry & incumbents respond

Physical investment / output up by 1.4 percentage points

Innovation intensity of the economy up by 0.8 (case 2) or 0.3 (case 3) p.p.
## Aggregate productivity

<table>
<thead>
<tr>
<th>Log change</th>
<th>10 years</th>
<th>Long run</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \phi = -1.6 )</td>
<td>( \phi = 0.96 )</td>
</tr>
<tr>
<td><strong>1. Exogenous productivity</strong></td>
<td></td>
<td></td>
</tr>
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<td></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>2. Endogenous entry</strong></td>
<td></td>
<td></td>
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<tr>
<td>No business stealing</td>
<td>0.024</td>
<td>0.020</td>
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<tr>
<td>With business stealing</td>
<td>0.009</td>
<td>0.007</td>
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<td><strong>3. Endogenous entry and incumbents</strong></td>
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<tr>
<td>No business stealing</td>
<td>0.030</td>
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<td>With business stealing</td>
<td>−0.017</td>
<td>−0.015</td>
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<tr>
<td>Log change</td>
<td>10 years</td>
<td>Long run</td>
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<td></td>
<td>$\phi = -1.6$</td>
<td>$\phi = 0.96$</td>
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<tr>
<td>1. Exogenous productivity</td>
<td>0.019</td>
<td>0.019</td>
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<tr>
<td>2. Endogenous entry</td>
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<tr>
<td>No business stealing</td>
<td>0.034</td>
<td>0.029</td>
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<td>With business stealing</td>
<td>0.016</td>
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<td>3. Endogenous entry and incumbents</td>
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<tr>
<td>No business stealing</td>
<td>0.047</td>
<td>0.040</td>
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<tr>
<td>With business stealing</td>
<td>0.002</td>
<td>0.002</td>
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Welfare (equivalent variation)

<table>
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<th>$\phi = 0.96$</th>
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</thead>
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<tr>
<td>1. Exogenous productivity</td>
<td>0.004</td>
<td>0.004</td>
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<tr>
<td>2. Endogenous entry</td>
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<tr>
<td>No business stealing</td>
<td>0.03</td>
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<td>With business stealing</td>
<td>0.02</td>
<td>0.07</td>
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<td>3. Endogenous entry and incumbents</td>
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<tr>
<td>No business stealing</td>
<td>0.04</td>
<td>0.22</td>
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<tr>
<td>With business stealing</td>
<td>$-0.04$</td>
<td>$-0.11$</td>
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</tbody>
</table>
Endogenous entry
Endogenous entry and incumbents

Innovation Intensity (low \( \phi \))

- no business stealing
- business stealing
- fixed productivity

Innovation Intensity (high \( \phi \))

- no business stealing
- business stealing
- fixed productivity

Capital investment/GDP (low \( \phi \))

Capital investment/GDP (high \( \phi \))
Endogenous entry

**Productivity (low $\phi$)**
- no business stealing
- business stealing
- fixed productivity

**Productivity (high $\phi$)**
- no business stealing
- business stealing
- fixed productivity

**Output (low $\phi$)**

**Output (high $\phi$)**
Endogenous entry and incumbents

**Productivity (low $\phi$)**
- no business stealing
- business stealing
- fixed productivity

**Productivity (high $\phi$)**
- no business stealing
- business stealing
- fixed productivity

**Output (low $\phi$)**

**Output (high $\phi$)**
Endogenous entry and incumbents

Output (no business stealing)

Output (with business stealing)

Productivity (no business stealing)

Productivity (with business stealing)
How do consideration of the impact of corporate profits taxes on firms’ incentives to invest in innovation shape the model’s predicted response of aggregate output?

Ten-year horizon: response not very different (in absolute terms) compared to e.g. business cycle variation

Long-run: can make a big difference, depending on:

- intertemporal knowledge spillovers
- is there too much or too little entry in the initial equilibrium (due to business stealing and ex-ante differential expensing)?
Investments by incumbents and productivity growth

Product-level technologies:

1. New products by entrants, $x_{et}M_t$, fraction $\delta_e$ stolen, $\mathbb{E} \exp(z')^{\rho - 1} = \eta_e \frac{Z_t^{\rho - 1}}{M_t}$

2. New products by incumbent firms, $h(x_{mt})M_t$, $\delta_m$ stolen, $\mathbb{E} \exp(z')^{\rho - 1} = \eta_m \frac{Z_t^{\rho - 1}}{M_t}$

3. Improvements of incumbents’ products, if continue, $\mathbb{E} \exp(z')^{\rho - 1} = \zeta(x_{ct}) \frac{Z_t^{\rho - 1}}{M_t}$

Total measure of products

$$M_{t+1} = [(1 - \delta_{ct}) + h(x_{mt}) + x_{et}] M_t$$

Evolution of aggregate productivity

$$Z_t^{\rho - 1} = [(1 - \delta_{ct}) \zeta(x_{ct}) + \eta_m h(x_{mt}) + \eta_e x_{et}] \frac{Z_t^{\rho - 1}}{M_t} M_t$$

Product exit

$$\delta_{ct} = \delta_0 + \delta_m h(x_{mt}) + \delta_e x_{et}$$
Suppose undistorted physical capital Euler

Consumption Equivalent Welfare

\[
\log \xi \approx (1 - \tilde{\beta}) \sum_{t=0}^{\infty} \tilde{\beta}^t \frac{\bar{Y}}{\bar{C}} \left[ (\log Z'_t - \log \tilde{Z}_t) + (1 - \alpha) (\log L'_{pt} - \log \tilde{L}_p) \right]
\]

\[
\tilde{\beta} = \frac{\exp(\bar{g}_Y)}{1 + R}
\]

Socially Optimal Research Intensity

\[
i_r^* = \frac{\tilde{\beta} \Theta}{1 - \tilde{\beta} [1 - (1 - \phi) \Theta]}
\]