Credible Mechanism Design

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Three Classic Auctions
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Ascending auction: strategy-proof, not sealed-bid

100 BC, Roman Empire (evidence is circumstantial)
1700 AD, England (art, ships, chattels)
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First-price auction: sealed-bid, not strategy-proof
Widespread since 20th century.
(procurement, real estate, treasury bonds)
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Second-price auction: strategy-proof and sealed-bid

Invented by Vickrey (1961).
Are the other auctions obsolete?
The Standard Approach

Incentive-compatibility for the bidders, but not for the auctioneer.

[The auctioneer] binds himself in such a way that all the bidders know that he cannot change his procedures after observing the bids, even though it might be in his interest ex post to renege. In other words, the organizer of the auction moves as the Stackelberg leader or first mover.

McAfee & McMillan 1987
The Standard Approach: Example

Spectrum auction

We will compute the least-cost feasible set of TV stations to buy. (Since the problem is NP-hard, we will not always succeed.) Tell us your reservation value for remaining on the air; we will tell you whether we’re buying you out, and how much we’re paying you. You should tell us the truth. Trust us, we are the American government!
Bending the rules

In a second-price auction:

1. Receive sealed bids $b_1 > b_2$. 
Bending the rules

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1. Receive sealed bids \( b_1 > b_2 \).
2. Pretend (to bidder 1) that \( \hat{b}_2 = b_1 - \epsilon \).
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3. Neither bidder notices.
4. Strict profit.

Vulnerable to shill bids! (Vickrey 1961)
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**In a first-price auction:**

1. Receive bids $b_1 > b_2$.
2. Invert bid function $b_1^{-1}(b_1) = v_1$.
3. Make TIOLI offer (to bidder 1) of $v_1 - \epsilon$. 
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4. Bidder 1 brings a lawsuit and wins.
A framework for **partial** commitment

Credible mechanism design.
Mediator can make any deviation that a single agent cannot detect. **credible** $\equiv$ incentive-compatible for Mediator to play ‘by the book’.
A framework for **partial** commitment

Credible mechanism design.
Mediator can make any deviation that a single agent cannot detect.

_credible_ ≡ incentive-compatible for Mediator to play ‘by the book’.

Includes shill bidding as special case.
General multi-step interactions.
Builds on bilateral commitments (Li 2016).
A framework for partial commitment

Credible mechanism design.
Mediator can make any deviation that a single agent cannot detect. **credible** ≡ incentive-compatible for Mediator to play ‘by the book’.

An application to optimal auctions

Invoke the machinery of extensive forms.
Characterize credible optimal auctions.
optimal, *ex post* IR auctions

regular i.i.d. values, ‘continuous’ type spaces, auctioneer wants profit
optimal, ex post IR auctions

regular i.i.d. values, ‘continuous’ type spaces,
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optimal, \textit{ex post} IR auctions

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implied by the Green-Laffont-Holmström Theorem.
optimal, *ex post* IR auctions

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sealed-bid $\cap$ credible?
optimal, *ex post* IR auctions

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Corollary: sealed-bid $\cap$ strategy-proof $\cap$ credible $= \emptyset$
optimal, *ex post* IR auctions

regular i.i.d. values, ‘continuous’ type spaces, auctioneer wants profit
optimal, *ex post* IR auctions

regular i.i.d. values, ‘continuous’ type spaces, auctioneer wants profit

ascending auction ∈ strategy-proof ∩ credible
Related literature (incomplete)

Commit to today’s mechanism, not tomorrow’s mechanism
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Not committing to outcome - seller can ‘re-open negotiations’

“You thought I was running a first-price auction, but now I’ve torn up the rule-book.”
Related literature (incomplete)

Commit to today’s mechanism, not tomorrow’s mechanism


Not committing to outcome - seller can ‘re-open negotiations’


What’s different today:

1. One-shot interaction.
2. No need for equilibrium refinements - ‘plain old’ BNE.
3. Commitment is linked to what agents can observe.
The Framework

1. A set of agents $N$
2. Finite type spaces $(\Theta_i)_{i \in N}$
3. Joint distribution $H : \Theta_N \rightarrow (0, 1]$ (full support)
4. Outcomes $X$
5. Utility $u_i : X \times \Theta_N \rightarrow \mathbb{R}$
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7. Mediator utility $u_0 : X \times \Theta_{\mathcal{N}} \rightarrow \mathbb{R}$
   - e.g. auction revenue, social surplus, hospitals’ welfare.
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   - e.g. auction revenue, social surplus, hospitals’ welfare.
8. For each agent, a partition $\mathcal{X}_i$ of $X$.
   - e.g. I observe my payment, but not your payment.
Implementation via extensive forms

$G$ denotes an extensive game form with consequences in $X$.

1. Finitely many histories.
2. No chance moves.
3. Perfect recall.
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\( \mathcal{G} \) denotes an **extensive game form with consequences in** \( \mathcal{X} \).

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\[ S_i : \text{infosets} \times \Theta_i \rightarrow \text{actions} \]
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\( S_i : \text{infosets} \times \Theta_i \rightarrow \text{actions} \)

\((G, S_N)\) is Bayesian Incentive Compatible (BIC) if

\[ \forall i : S_i \in \arg\max \mathbb{E} \theta_N [u_i^G (S_i', S_N \setminus i, \theta_N)] \]
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\((\mathcal{G}, S_N)\) is **Bayesian Incentive Compatible** (BIC) if

\[ \forall i : S_i \in \arg\max_{S_i'} \mathbb{E}_{\theta_N}[u_i^G(S_i', S_{N\setminus i}, \theta_N)] \]

Additionally assume (WLOG):

4. For every history \( h \), there exists \( \theta_N \) such that \( h \) is on the path-of-play.
5. Every infoset has at least two actions.
1 can observe: \{a, b\}\{c\}
2 can observe: \{a\}\{b\}\{c\}
Query 1 for his action.

1 can observe: \(\{a, b\}\{c\}\)

2 can observe: \(\{a\}\{b\}\{c\}\)
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1 can observe: \{a, b\}\{c\}
2 can observe: \{a\}\{b\}\{c\}
Query 2 for his action.

1 can observe: \(\{a, b\}\{c\}\)

2 can observe: \(\{a\}\{b\}\{c\}\)
Query 2 for his action.

1 can observe: $\{a, b\}\{c\}$
2 can observe: $\{a\}\{b\}\{c\}$
The book says: Outcome $c$ results.

1 can observe: $\{a, b\}\{c\}$
2 can observe: $\{a\}\{b\}\{c\}$
Deviate. Choose $b$ instead of $c$.

1 can observe: \{a, b\}\{c\}

2 can observe: \{a\}\{b\}\{c\}
Innocent explanation for 1.

1 can observe: \{a, b\}\{c\}
2 can observe: \{a\}\{b\}\{c\}
Innocent explanation for 2.

1 can observe: \{a, b\}\{c\}

2 can observe: \{a\}\{b\}\{c\}
Safe for the mediator to deviate!

1 can observe: \{a, b\}\{c\}
2 can observe: \{a\}\{b\}\{c\}
The Substantive Assumption
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Sequential private communication

Mediator makes ‘telephone calls’ to individual agents.
Can misrepresent to $i$ what $j$ has done.
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Sequential private communication

Mediator makes ‘telephone calls’ to individual agents. Can misrepresent to \( i \) what \( j \) has done.

Rules out a trivial ‘revelation mechanism’: theards in the town square.

Examples include: Art auctions, ad auctions, school choice, NRMP, etc.
Full commitment isn’t easy!

Hire a neutral third-party

Moral hazard.
Linear incentives in practice and in theory (Carroll 2015).

Reputation
Depends on detection rate and discount factor.

One-off high-stakes auctions: Spectrum, oil.

Reveal bids publicly and simultaneously
Bidders desire privacy and anonymity.

Law may forbid full disclosure (2017 FCC Incentive Auction).

Cryptography - secure multiparty computation
Large setup costs.
Bidders must have crypto expertise, or must ‘trust the code’.
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A Partial Commitment Game

1. Mediator chooses some $i \in N$, sends message $m$, set of acceptable replies $R$.
2. $i$ privately observes $(m, R)$, chooses $r \in R$.
3. Mediator privately observes $r$.
4. Mediator can:
   4.1 Either: Go to step 1.
   4.2 Or: Choose an outcome and end the game.

$\Psi_0$ is the set of all Mediator pure strategies.
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An isomorphism

For any \( G \), can define \( S_0 \) that is ‘equivalent’ for the agents.
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**Full commitment:** To ‘run’ $G$, Mediator commits to $S_0^G \in \Psi_0$. 
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How the Mediator can deviate

Consider protocol \((G, S_N)\), and \(S_0^G\) that ‘runs’ \(G\).
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\(o_i\) observation for \(i =\)

communication sequence \((m_i^t, R_i^t, r_i^t)_{t=1}^T\)

& cell of outcome partition \(\mathcal{X}_i\)
How the Mediator can deviate

Consider protocol \((G, S_N)\), and \(S_0^G\) that ‘runs’ \(G\).

\(o_i\) observation for \(i = \) communication sequence \((m^t_i, R^t_i, r^t_i)^T_{t=1}\) & cell of outcome partition \(X_i\)

\[
S_i: \text{messages} \times \Theta_i \rightarrow \text{replies}
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\text{infosets} \quad \text{actions}
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observation resulting from \(S_0, S_N, \theta_N\) denoted \(o_i(S_0, S_N, \theta_N)\)
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observation resulting from \(S_0, S_N, \theta_N\) denoted \(o_i(S_0, S_N, \theta_N)\)

\(o'_i\) has an **innocent explanation** if:

\[ \exists \theta'_{-i} : o'_i = o_i(S_0^G, S_N, (\theta_i, \theta'_{-i})). \]
How the Mediator can deviate

Consider protocol \((G, S_N)\), and \(S_0^G\) that ‘runs’ \(G\).

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observation resulting from \(S_0, S_N, \theta_N\) denoted \(o_i(S_0, S_N, \theta_N)\)

\(o_i'\) has an innocent explanation if:

\(\exists \theta'_{-i} : o_i' = o_i(S_0^G, S_N, (\theta_i, \theta'_{-i}))\).

\(S_0\) is safe if \(\forall i : \forall \theta_N : o_i(S_0, S_N, \theta_N)\) has an innocent explanation.
Mediator’s deviation

1 can observe: \{a, b\} \{c\}
2 can observe: \{a\} \{b\} \{c\}
Innocent explanation for 1’s observation

1 can observe: \( \{a, b\}\{c\} \)
2 can observe: \( \{a\}\{b\}\{c\} \)
Innocent explanation for 2’s observation

1 can observe: \(\{a, b\} \{c\}\)
2 can observe: \(\{a\} \{b\} \{c\}\)
The Mediator can deviate ‘before the end’.

1 observes: \{a\}{b, c}
2 & 3 observe: \{a\}{b}{c}
The Mediator can deviate ‘before the end’.

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1 observes: \{a\}\{b, c\}
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Innocent explanation for 1.
The Mediator can deviate ‘before the end’.

1 observes: \{a\}\{b, c\}

2 & 3 observe: \{a\}\{b\}\{c\}

Innocent explanation for 2 and 3.
Defining “Credible”

**Definition**

\((G, S_N)\) is **credible** if:

\[
S_0^G \in \arg\max_{S_0 | S_0 \text{ is safe}} \mathbb{E}_{\theta_N}[u_0(S_0, S_N, \theta_N)]
\]

mediator’s expected utility
Defining “Credible”

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\(\text{mediator’s expected utility}\)
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mediator’s expected utility
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Definition

$$(G, S_N)$$ is credible if:

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mediator's expected utility
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\]

\((G, S_N)\) is BIC and credible. \(\leftrightarrow\) \((S_0^G, S_N)\) is a BNE of the partial commitment game.


Credible Optimal Auctions

Following Myerson (1981)

1. One object.
2. $N$ bidders.
3. *Ex post* IR, so only winner pays.
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5. Private values $u_i(y, t, \theta_i) = 1_{i=y}[\theta_i - t]$
6. Auctioneer wants revenue $u_0(y, t) = 1_{i \in N}t$
Credible Optimal Auctions

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6. Auctioneer wants revenue \( u_0(y, t) = 1_{i \in N} t \)
7. \( i \) observes whether or not he gets the object, and how much he pays.
We must bridge a gap.

Extensive games

Transparent.
Reasonably general.
**Move in discrete steps.**
We must bridge a gap.

Extensive games
- Transparent.
- Reasonably general.
- Move in discrete steps.

Optimal Auctions
- Virtual values characterization.
- Clean, beautiful.
- Requires continuum.
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Infinite actions complicate beliefs. (Myerson & Reny 2016)
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Discrete type spaces break revenue equivalence.
Distributions

Types are on a grid: \( \Theta_i = \Theta_j = \{ \theta^1, \theta^2, \ldots, \theta^K \} \subset \mathbb{R} \)

\( \theta^k - \theta^{k-1} = \epsilon \)

i.i.d. probability mass function \( p : \Theta_i \rightarrow (0, 1] \)
Distributions

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i.i.d. probability mass function \( p : \Theta_i \rightarrow (0, 1] \)

pseudo-pdf \( f(\theta^k) \equiv \frac{p(\theta^k)}{\epsilon} \)

cdf \( F(\theta^k) \equiv \sum_{j=1}^{k} p(\theta^j) \)
Distributions

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i.i.d. probability mass function \( p : \Theta_i \rightarrow (0, 1] \)

pseudo-pdf \( f(\theta^k) \equiv \frac{p(\theta^k)}{\epsilon} \)

cdf \( F(\theta^k) \equiv \sum_{j=1}^{k} p(\theta^j) \)

virtual value \( \eta(\theta_i) \equiv \theta_i - \frac{1 - F(\theta_i)}{f(\theta_i)} \)

Assume \( p \) is regular, i.e. \( \eta(\cdot) \) is strictly increasing.
Objective

Choose \((G, S_N)\) to maximize revenue subject to BIC and interim individual rationality (IR).
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**continuous (Myerson 1981)**

\((G, S_N)\) is optimal
\[\iff\]
\((G, S_N)\) is BIC
IR of lowest type binds
allocation maxes virtual value
Objective

Choose \((G, S_N)\) to maximize revenue subject to BIC and interim individual rationality (IR).

<table>
<thead>
<tr>
<th>discrete</th>
<th>continuous (Myerson 1981)</th>
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<tbody>
<tr>
<td>((G, S_N)) is optimal</td>
<td>((G, S_N)) is optimal</td>
</tr>
<tr>
<td>(\uparrow)</td>
<td>(\uparrow)</td>
</tr>
<tr>
<td>((G, S_N)) is BIC</td>
<td>((G, S_N)) is BIC</td>
</tr>
<tr>
<td>IR of lowest type binds</td>
<td>IR of lowest type binds</td>
</tr>
<tr>
<td>allocation maxes virtual value</td>
<td>allocation maxes virtual value</td>
</tr>
<tr>
<td>IC binds ‘locally downward’</td>
<td></td>
</tr>
</tbody>
</table>
Objective

Choose \((G, S_N)\) to maximize revenue subject to BIC and interim individual rationality (IR).

**discrete**

\((G, S_N)\) is optimal
\[\iff\]
\((G, S_N)\) is BIC
IR of lowest type binds
allocation maxes virtual value
IC binds ‘locally downward’

**continuous (Myerson 1981)**

\((G, S_N)\) is optimal
\[\iff\]
\((G, S_N)\) is BIC
IR of lowest type binds
allocation maxes virtual value

Each \(\theta_i\) must be exactly indifferent between ‘truth-telling’ and playing as though his type is \(\theta_i - \epsilon\).
Objective

Choose \((G, S_N)\) to maximize revenue subject to BIC and interim individual rationality (IR).

**discrete**

\((G, S_N)\) is optimal

\(\Updownarrow\)

\((G, S_N)\) is BIC

IR of lowest type binds

allocation maxes virtual value

IC binds ‘locally downward’

**continuous** *(Myerson 1981)*

\((G, S_N)\) is optimal

\(\Updownarrow\)

\((G, S_N)\) is BIC

IR of lowest type binds

allocation maxes virtual value

Slack in local ICs breaks revenue equivalence.

Discrete \(\rightarrow\) \(\neg\exists\) optimal second-price auction
Objective

Choose \((G, S_N)\) to maximize revenue subject to BIC and interim individual rationality (IR).

**discrete**

\((G, S_N)\) is \(d\)-optimal

\(\uparrow\)

\((G, S_N)\) is BIC

IR of lowest type binds
allocation maxes virtual value
IC binds ‘locally downward’

**continuous (Myerson 1981)**

\((G, S_N)\) is optimal

\(\uparrow\)

\((G, S_N)\) is BIC

IR of lowest type binds
allocation maxes virtual value

\(d\)(iscrete analog to continuous)-optimality.

allows ICs to be a *little* slack. (‘upward’ ICs bound slackness.)
Objective

Choose \((G, S_N)\) to maximize revenue subject to BIC and interim individual rationality (IR).

**discrete**

\((G, S_N)\) is \(d\)-optimal

\((G, S_N)\) is BIC

IR of lowest type binds allocation maxes virtual value

IC binds ‘locally downward’

**continuous (Myerson 1981)**

\((G, S_N)\) is optimal

\((G, S_N)\) is BIC

IR of lowest type binds allocation maxes virtual value

d-optimality is a necessary condition for optimality that is sufficient ‘in the limit’.
Assumption

\((G, S_N)\) breaks ties in order. \textit{That is, there exists a strict total order} \(\triangleright\) \textit{on} \(N\) \textit{such that if} \(\theta_i = \theta_j\) \textit{and} \(i \triangleright j\), then \(j\) \textit{does not win the object.}
Assumption

\((G, S_N)\) breaks ties in order. That is, there exists a strict total order \(\triangleright\) on \(N\) such that if \(\theta_i = \theta_j\) and \(i \triangleright j\), then \(j\) does not win the object.

Definition

\((G, S_N)\) is sealed-bid if every agent has exactly one infoset and is always called to play.
**Assumption**

\((G, S_N)\) breaks ties in order. That is, there exists a strict total order \(\triangleright\) on \(N\) such that if \(\theta_i = \theta_j\) and \(i \triangleright j\), then \(j\) does not win the object.

**Definition**

\((G, S_N)\) is sealed-bid if every agent has exactly one infoset and is always called to play.

**Proposition**

If \((G, S_N)\) is a second-price auction (with ‘truth-telling’ \(S_N\) and an optimal reserve), then \((G, S_N)\) strategy-proof, sealed-bid, and \(d\)-optimal.
Assumption

\((G, S_N)\) breaks ties in order. That is, there exists a strict total order \(\succ\) on \(N\) such that if \(\theta_i = \theta_j\) and \(i \succ j\), then \(j\) does not win the object.

Definition

\((G, S_N)\) is sealed-bid if every agent has exactly one infoset and is always called to play.

Proposition

If \((G, S_N)\) is a second-price auction (with ‘truth-telling’ \(S_N\) and an optimal reserve), then \((G, S_N)\) strategy-proof, sealed-bid, and \(d\)-optimal.

In the limit (as \(\epsilon \to 0\)), the second-price auction is the unique strategy-proof, sealed-bid, \(d\)-optimal mechanism.  
(Green-Laffont-Holmström)
Definition

\((G, S_N)\) is an **almost first-price auction** if it is sealed-bid, each action is associated with a bid \(\in \mathbb{R}\), and:

1. **The object is sold iff some agent bids above the reserve.**
2. **If i wins the object, then i pays his bid, and:**
   2.1 Either: i has the highest bid.
Definition

\((G, S_N)\) is an almost first-price auction if it is sealed-bid, each action is associated with a bid \(\in \mathbb{R}\), and:

1. The object is sold iff some agent bids above the reserve.

2. If \(i\) wins the object, then \(i\) pays his bid, and:
   2.1 Either: \(i\) has the highest bid.
   2.2 Or: \(\theta_i = \theta^K\), \(i\) wins all ties, and \(i\) has bid at least as much as any \(j\) does when \(\theta_j = \theta^K - \epsilon\).

Allows \(i\) to win with a slight under-bid when his type is \(\theta^K\) and someone else’s type is also \(\theta^K\) (highest possible type). Vanishes as \(\epsilon \to 0\).
Definition

\((G, S_N)\) is an almost first-price auction if it is sealed-bid, each action is associated with a bid \(b \in \mathbb{R}\), and:

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Theorem

Assume \((G, S_N)\) is d-optimal. \((G, S_N)\) is sealed-bid and credible if and only if \((G, S_N)\) is an almost first-price auction.
Proof Outline

sealed-bid and credible $\rightarrow$ almost first-price auction
**Proof Outline**

sealed-bid and credible → almost first-price auction

Suppose after $i$ plays $a$, there are two prices that $i$ might pay. Only $i$ observes $i$’s transfer. Whenever $S^G_0$ would charge the lower price, deviate to charge the higher price. → not credible.

Unique winning price for each action $b_i$: actions; $i \rightarrow \mathbb{R}$

($= 0$ for bids below reserve)
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Suppose $\theta_i < \theta^K$ or $i \triangleleft \max j$. If some agent has bid strictly more than $i$, should instead sell to that agent.

(Innocent explanation: “Someone else had a higher type”.)
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Suppose $\theta_i = \theta^K$ and $i = \max j$.

$$b_i(\theta^K) \geq b_i(\theta^K - \epsilon) \geq b_j(\theta^K - \epsilon)$$

by BIC
sealed-bid and credible → almost first-price auction

Suppose after $i$ plays $a$, there are two prices that $i$ might pay. Only $i$ observes $i$’s transfer. Whenever $S_0^G$ would charge the lower price, deviate to charge the higher price. → not credible.

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by credible
The story so far

regular i.i.d. values, ‘in the limit’
a technical assumption

Definition

\[(G, S_N) \text{ is strategy-proof if } \forall i : \forall S'_{N \setminus i} : S_i \text{ best responds to } S'_{N \setminus i}.\]
Definition

\((G, S_N)\) is **strategy-proof** if \(\forall i : \forall S'_{N \setminus i} : S_i\) best responds to \(S'_{N \setminus i}\).

\((G, S_N)\) has **threshold pricing** if the winning bidder \(i\) at type profile \((\theta_i, \theta_{N \setminus i})\) pays \(\inf\{\theta'_i \in \Theta_i\ such\ that\ i\ wins\ at\ (\theta'_i, \theta_{N \setminus i})\}\).
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Continuous case: strategy-proof \(\rightarrow\) threshold pricing

(Green-Laffont-Holmström)

Discrete case: Prices can wiggle in messy ways.
Definition

\((G, S_N)\) is **strategy-proof** if \(\forall i: \forall S'_{N\setminus i}: S_i \text{ best responds to } S'_{N\setminus i}.\)

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Continuous case: strategy-proof \(\rightarrow\) threshold pricing
(Green-Laffont-Holmström)

Discrete case: Prices can wiggle in messy ways.

Definition

\((G, S_N)\) is **d-strategy-proof** if it is strategy-proof and has threshold pricing.

A sufficient condition for strategy-proofness.
Necessary ‘in the limit’.
Types of $i$ consistent with history $h$ 

$$\Theta^h_i = \{ \theta_i \mid \forall l_i \prec h : i \text{ played } S_i(l_i, \theta_i) \}$$
Types of $i$ consistent with history $h$

\[ \Theta^h_i = \{ \theta_i \mid \forall l_i \prec h : i \text{ played } S_i(l_i, \theta_i) \} \]
Types of $i$ consistent with history $h$

$$\Theta_i^h = \{ \theta_i \mid \forall l_i < h : i \text{ played } S_i(l_i, \theta_i) \}$$
Types of \( i \) consistent with history \( h \)

\[
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Consider an ascending auction:

![Diagram with two sets $\Theta_1^h$ and $\Theta_2^h$]
Types of $i$ consistent with history $h$

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Consider an ascending auction:

[Diagram showing two branches of nodes connected at the bottom with a horizontal line labeled "optimal reserve"]
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\[ \Theta_1^h \quad \text{optimal reserve} \quad \Theta_2^h \]
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[Diagram showing two sets $\Theta_1^h$ and $\Theta_2^h$ with a horizontal line indicating the optimal reserve]
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Consider an ascending auction:

[Diagram showing two vertical lines with red and blue dots labeled $\Theta_1^h$ and $\Theta_2^h$ with an arrow labeled "optimal reserve" between them.]
Types of $i$ consistent with history $h$

$$\Theta_i^h = \{\theta_i \mid \forall l_i < h : i \text{ played } S_i(l_i, \theta_i)\}$$

Consider an ascending auction:
Types of $i$ consistent with history $h$

$$\Theta^h_i = \{\theta_i \mid \forall l_i \prec h : \text{i played } S_i(l_i, \theta_i)\}$$

Consider an ascending auction:

\[ \Theta^h_1 \]
\[
\Theta^h_2
\]

optimal reserve
Types of $i$ consistent with history $h$

$$\Theta^h_i = \{\theta_i \mid \forall I_i \prec h: i \text{ played } S_i(I_i, \theta_i)\}$$

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Consider an ascending auction:

[Diagram showing two sets $\Theta_1^h$ and $\Theta_2^h$ with points representing possible strategies, connected by lines to indicate the optimal reserve.]

optimal reserve
Types of $i$ consistent with history $h$

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Consider an ascending auction:

- $\Theta^h_1$
- $\Theta^h_2$

2 has quit.

optimal reserve
Types of $i$ consistent with history $h$

$$\Theta_i^h = \{\theta_i | \forall l_i \prec h : i \text{ played } S_i(l_i, \theta_i)\}$$

Consider an ascending auction:

1's payment

2 has quit.

optimal reserve
Feasible bids: $\theta_i \in \Theta_i$.
The **high bidder** has placed the highest bid so far that is (weakly) above the reserve. (break ties with $\triangleright$)

### Definition

$(G, S_N)$ is a **clean ascending auction** if:

1. At each history, some bidder chooses to:
   1.1 **EITHER** raise his bid to $b_i$ (where $b_i$ is no more than is necessary to become the high bidder)
   1.2 **OR** quit.

2. **If only the high bidder remains**, he wins and pays his bid.
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**Definition**

$(G, S_N)$ is a **clean ascending auction** if:

1. At each history, some bidder chooses to:
   1.1 Either raise his bid to $b_i$ (where $b_i$ is no more than is necessary to become the high bidder)
   1.2 OR quit.

2. If only the high bidder remains, he wins and pays his bid.

3. (reserve) If no bidder remains, then no bidder wins.

4. (ceiling) If no bidder can beat the high bid, then the high bidder wins and pays his bid.
Feasible bids: $\theta_i \in \Theta_i$
The high bidder has placed the highest bid so far that is (weakly) above the reserve. (break ties with $\triangleright$)

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3. *(reserve)* If no bidder remains, then no bidder wins.

4. *(ceiling)* If no bidder can beat the high bid, then the high bidder wins and pays his bid.

5. $S_i(\theta_i)$ specifies:
   5.1 If (conditional on current infoset) you could win at a price $\leq \theta_i$, keep bidding.
   5.2 If the required bid is $b_i > \theta_i$, quit.
Definition

An **ascending auction** is as a clean ascending auction, but additionally can have:

1. *Multiple quitting actions.*
Definition

An ascending auction is as a clean ascending auction, but additionally can have:

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2. ‘Dawdling’ infosets where a bidder chooses to continue (without raising his bid) or quit.
Definition

An **ascending auction** is as a clean ascending auction, but additionally can have:

1. **Multiple quitting actions.**
2. ‘Dawdling’ infosets where a bidder chooses to continue (without raising his bid) or quit.
3. ‘Chattering’ infosets at which a bidder who is already sure to win (and knows the price at which he will win) gives more information about his type.
4. ‘Chattering’ infosets at which a bidder who has already quit gives more information about his type.
Ascending auctions thread the needle

Theorem

Assume \((G, S_N)\) is \(d\)-optimal. If \((G, S_N)\) is an ascending auction, then \((G, S_N)\) is credible and \(d\)-strategy-proof.
Ascending auctions thread the needle

Theorem

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Theorem

Assume \((G, S_N)\) is \(d\)-optimal and \(|N| = 2\). If \((G, S_N)\) is credible and \(d\)-strategy-proof, then \((G, S_N)\) is an ascending auction.
Theorem

Assume \((G, S_N)\) is \(d\)-optimal. If \((G, S_N)\) is an ascending auction, then \((G, S_N)\) is credible and \(d\)-strategy-proof.

Theorem

Assume \((G, S_N)\) is \(d\)-optimal and \(|N| = 2\). If \((G, S_N)\) is credible and \(d\)-strategy-proof, then \((G, S_N)\) is an ascending auction.

We think this is true for \(|N| > 2\); proof under construction.
Is the ascending auction credible?
Is the ascending auction credible?

\[ \Theta_1^{\text{h}} \]

\[ \Theta_2^{\text{h}} \]

\[ \theta_1' \]

why not raise 1's price by \( \varepsilon \)?

2 has quit.

optimal reserve
Under New York City regulations auctioneers can fabricate bids up to an item's reserve price. Because a reserve price is secret and not listed in the catalog, bidders have no way of knowing which offers are real.

NYT, April 24, 2000
Q: Why not raise 1’s price to $\theta_1' + \epsilon$, even after bidder 2 has quit?
A: 1’s virtual value is positive.
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‘The book’ requires that 1 pay $\theta_1'$.

$$\underbrace{-\epsilon f(\theta_1') \theta_1'}_{\text{expected loss from 1 quitting}}$$
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$$-\epsilon f(\theta_1') \theta_1'$$

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A: 1’s virtual value is positive.

‘The book’ requires that 1 pay $\theta_1'$. 

$$\underbrace{-\epsilon f(\theta_1')\theta_1'}_{\text{expected loss from 1 quitting}} + \underbrace{(1 - F(\theta_1'))\epsilon}_{\text{expected gain from raising price}}$$
Q: Why not raise 1’s price to $\theta'_1 + \epsilon$, even after bidder 2 has quit?
A: 1’s virtual value is positive.

‘The book’ requires that 1 pay $\theta'_1$. 

\[-\epsilon f(\theta'_1)\theta'_1 + (1 - F(\theta'_1))\epsilon\]

\[\text{expected loss from 1 quitting} \quad \text{expected gain from raising price} \]
Q: Why not raise 1’s price to $\theta_1' + \epsilon$, even after bidder 2 has quit?  
A: 1’s virtual value is positive.

‘The book’ requires that 1 pay $\theta_1'$. 

\[-\epsilon f(\theta_1')\theta_1' + (1 - F(\theta_1'))\epsilon\]

expected loss from 1 quitting       expected gain from raising price

divide through by $\epsilon f(\theta_1')$

\[-[\theta_1' - \frac{1 - F(\theta_1')}{f(\theta_1')}]] < 0\]

virtual value
Why is only the ascending auction credible and d-SP?

A key feature of ascending auctions:
All the types who might still win pool on the same action.
Why is only the ascending auction credible and d-SP?

A key feature of ascending auctions:
All the types who might still win pool on the same action.

Suppose \((G, S_N)\) d-optimal, d-SP. not pooling \(\rightarrow\) not credible.
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Suppose \((G, S_N)\) d-optimal, d-SP. **not pooling → not credible.**
Why is only the ascending auction credible and d-SP?

A key feature of ascending auctions: All the types who might still win pool on the same action.

Suppose \((G, S_N)\) d-optimal, d-SP: not pooling → not credible.

pause bidder 2.
pretend (to 1) that 2 has this type.
Why is only the ascending auction credible and d-SP?

A key feature of ascending auctions:
All the types who might still win **pool on the same action**.

Suppose \((G, S_N)\) d-optimal, d-SP. **not pooling \(\rightarrow\) not credible.**
Why is only the ascending auction credible and d-SP?

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All the types who might still win pool on the same action.

Suppose \((G, S_N)\) d-optimal, d-SP. 
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(if 1 doesn’t quit, 
pause 1. pretend (to 2) 
that 1 has this type.)
Why is only the ascending auction credible and d-SP?

A key feature of ascending auctions:
All the types who might still win pool on the same action.

Suppose \((G, S_N)\) d-optimal, d-SP. not pooling \(\rightarrow\) not credible.

If 1 doesn’t quit, pause 1. pretend (to 2) that 1 has this type.

\(\Theta_1^h\)

\(\Theta_2^h\)

2 might reveal his type is low.

optimal reserve
Why is only the ascending auction credible and d-SP?

A key feature of ascending auctions:
All the types who might still win pool on the same action.

Suppose \((G, S_N)\) d-optimal, d-SP. not pooling \(\rightarrow\) not credible.

If 1 doesn’t quit, pause 1. pretend (to 2) that 1 has this type.

\(\Theta^h_1\)

Pretend to 1 that it’s higher.

\(\Theta^h_2\)

2 might reveal his type is low.
Why is only the ascending auction credible and d-SP?

A key feature of ascending auctions:
All the types who might still win pool on the same action.

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All the types who might still win pool on the same action.

Suppose \((G, S_N)\) d-optimal, d-SP. not pooling → not credible.

if 1 doesn’t quit, pause 1. pretend (to 2) that 1 has this type.

\[
\Theta_1^h
\]

optimal reserve

2 might reveal that his type is high

\[
\Theta_2^h
\]
Why is only the ascending auction credible and d-SP?

A key feature of ascending auctions:
All the types who might still win pool on the same action.

Suppose \((G, S_N)\) d-optimal, d-SP. not pooling \(\rightarrow\) not credible.
Returning to where we began

Most auctions in human history are (variations of) 1P, 2P, or AC.
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Second-price auction: sealed-bid **and** SP

Invented by Vickrey (1961).
Are the other auctions obsolete?
Returning to where we began

Most auctions in human history are (variations of) 1P, 2P, or AC.

Second-price auction: sealed-bid and SP
Invented by Vickrey (1961).
Are the other auctions obsolete?

1P and AC are not obsolete. They are not historical accidents.
An Auction Trilemma

Pick any two of three.
Bidders seldom display types on placards.

In the English system bids are ... usually transmitted by signal. Such signals may be in the form of a wink, a nod, scratching an ear, lifting a pencil, tugging at the coat of the auctioneer or even staring into the auctioneer’s eyes – all of them perfectly legal.

Cassady 1967
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Cassady 1967

Public communication affects aftermarkets and thus incentives. Ausubel & Cramton 2004, Carroll & Segal 2016, Dworczak 2017. (Outside the model today.)