Optimal Taxes on Capital in the OLG Model with Uninsurable Idiosyncratic Income Risk

Dirk Krueger    Alexander Ludwig

University of Pennsylvania, CEPR, CFS, NBER and Netspar
SAFE, Goethe University Frankfurt

Taxation and Fiscal Policy Conference
University of Chicago, May 2018
Motivation

• Partial results in general equilibrium models with idiosyncratic risk (starting from Aiyagari, 1995): $\tau_k > 0$?
• Overaccumulation of Capital? Davila et al. (2012):
  • characterize constrained efficient allocation: how much should private households save, given incomplete markets?
  • Key: Impact of precautionary savings on GE factor prices
  • This GE feedback effect not specifically addressed in Ramsey optimal taxation literature
  • Main focus of Davila et al. (2012) is not on implementation through tax policy
• This paper: Impact of idiosyncratic income risk on optimal $\tau_k$ in OLG economy.
Overview

• The Environment
  • Diamond (1965) style two period OLG model with neoclassical production in general equilibrium
  • Uninsurable idiosyncratic labor income risk in second period of life

• Government and Fiscal Policy:
  • Government has social welfare function with arbitrary social welfare weights on generations
  • Ramsey equilibrium. Fiscal policy tools: time-varying taxes on capital & lump-sum transfers

• Contributions
  • Analytical characterization of Ramsey tax transition
  • Clarification of feedback from precautionary savings on general equilibrium prices
  • Optimal Ramsey policy corrects pecuniary externality
Outline

1. Intro
2. Model
3. Analysis
4. Conclusion
The Model: Overview

• Extension of 2-period Diamond (1965) textbook OLG model

• Second period:
  
  • Positive labor endowment
  
  • Idiosyncratic productivity shock
  
  • Incomplete markets ⇒ Ex-post heterogeneity and imperfect insurance
The Model: Endowments

- Unit mass of households in each generation
- Two periods of work with exogenous labor supply
- Time endowment of 1 in each period
- Labor productivity of cohort born in period $t$:
  - young: $1 - \kappa$
  - old: $\kappa \eta_{t+1}$, ($\kappa = 0$: textbook model)
- $\eta_{t+1}$: positive values, integrating to $\int \eta_{t+1} d\Psi = 1$
- Aggregate labor input

\[ L_t = 1 - \kappa + \kappa \int \eta_{t+1} d\Psi = 1 \]
The Model: Preferences

- Household of generation $t \geq 0$:
  \[ V_t = u(c_t^y) + \beta \int u(c_{t+1}^o(\eta_{t+1}))d\Psi \]

- Initial old generation born at $t = -1$
  \[ V_{-1} = \int u(c_0^o(\eta_0))d\Psi \]
The Model: Technology

- Aggregate production function:
  \[ F(K_t, L_t) = K_t^\alpha (L_t)^{1-\alpha} \]

- Full depreciation of capital.

- Aggregate resource constraint
  \[ C_t + K_{t+1} = K_t^\alpha (L_t)^{1-\alpha} \]

- Define \( k_t = \frac{K_t}{L_t} = K_t \).
The Model: Government

- Social welfare function

\[
SWF = \begin{cases} 
\sum_{t=-1}^{\infty} \omega_t V_t & \text{for } \sum_{t=-1}^{\infty} \omega_t < \infty \\
\lim_{T \to \infty} \sum_{t=-1}^{T} \frac{1}{T} V_t & \text{for } \omega_t = 1 \forall t
\end{cases}
\]

where \( \omega_t \) is the Pareto weight on generation born at time \( t \).

- Instruments of the Ramsey government:
  - Proportional tax rate on capital \( \tau_t \)
  - Lump-sum transfer \( T_t \)
  - No access to \( \eta_t \)-contingent transfers
The Model: Household Budget Constraints

- Budget constraints:

\[ c_t^y + a_{t+1} = (1 - \kappa)w_t \]

\[ c_{t+1}^o = a_{t+1}R_{t+1}(1 - \tau_{t+1}) + \kappa \eta_{t+1}w_{t+1} + T_{t+1} \]

- Note: one-to-one mapping from capital taxes to capital income taxes:

\[ R_t(1 - \tau_t) = 1 + r_t(1 - \tau_t^k) \]

\[ \Leftrightarrow \quad \tau_t^k = \frac{R_t}{R_t - 1} \tau_t \]
Competitive Equilibrium for Given Fiscal Policy

Definition

Given initial condition \( a_0 = k_0 \) and sequence of taxes \( \tau = \{\tau_t\}_{t=0}^{\infty} \), a CE is allocation \( \{c^y_t, c^o_t(\eta_t), a_{t+1}, k_{t+1}\}_{t=0}^{\infty} \), prices \( \{R_t, w_t\}_{t=0}^{\infty} \), transfers \( \{T_t\}_{t=0}^{\infty} \) s.t.

1. Given prices \( \{R_t, w_t\}_{t=0}^{\infty} \) and policies \( \{\tau_t, T_t\}_{t=0}^{\infty} \), for each \( t \geq 0 \), \((c^y_t, c^o_{t+1}(\eta_{t+1}), a_{t+1})\) solves the household problem.

2. Factor prices satisfy:

\[
    R_t = \alpha k_t^{\alpha-1} \\
    w_t = (1 - \alpha)k_t^\alpha
\]

3. For each \( t \), government budget constraint is satisfied and markets clear

\[
    T_t = \tau_t R_t k_t \\
    a_{t+1} = k_{t+1} \\
    c^y_t + \int c^o_t(\eta_t)d\Psi + k_{t+1} = k_t^\alpha
\]
Analysis: Law of Motion of Capital Stock

• Define the saving rate as

\[ s_t = \frac{a_{t+1}}{(1 - \kappa)w_t} = \frac{k_{t+1}}{(1 - \kappa)(1 - \alpha)k_t^\alpha} \]

• For given \( k_0 > 0 \), law of motion of capital \( k_t \) in economy given by:

\[ k_{t+1} = s_t(1 - \kappa)(1 - \alpha)k_t^\alpha \]

• Next:
  • Determine \( s_t \) chosen in competitive equilibrium, given fiscal policy
  • Show: gov’t can implement any CE \( s_t \in (0, 1) \) by choice of \( \tau_{t+1} \).
  • Characterize optimal \( \{s_t\} \) chosen by Ramsey government.
  • Preferences:
    1. Most of talk: log utility: \( u(c) = \ln(c) \)
    2. Later: General Epstein-Zin-Weil preferences (which nests CRRA)
Analysis: Savings Rate in CE

- Household Euler equation in any period $t$

$$1 = \beta R_{t+1}(1 - \tau_{t+1})E_t \left[ \frac{c^y_{t+1}(\eta_{t+1})}{c^y_t} \right]^{-1}$$

- Exploiting hh budget constraints, firm’s optimality conditions, equilibrium dynamics of $k_t$, can find CE saving rate in GE:

$$s_t(\tau_{t+1}) = \frac{1}{1 + [\beta(1 - \tau_{t+1})\Gamma(\alpha, \kappa, \sigma = 1; \Psi)]^{-1}} \in (0, 1),$$

- Effect of income risk completely summarized by

$$\Gamma(\cdot; \Psi) = \int (\alpha + \kappa(1 - \alpha)\eta_{t+1})^{-1} d\Psi(\eta_{t+1}) > 1$$

which is strictly increasing in income risk.
Interpretation of CE Saving Rate for Log Utility

• Observations:
  
  • $s_t$ strictly decreasing in $\tau_{t+1}$
  
  • Government can implement any $s_t \in (0, 1)$ by choice of $\tau_{t+1}$
  
  • Mean preserving spread in $\eta$ ($MPS(\eta)$) increases $s_t$ by increasing $\Gamma(\cdot)$
  
  • $s_t$ is independent of $k_t$
Analysis: Ramsey Problem

- Primal approach to optimal taxation: government chooses \( \{s_t\} \)
- Recall:
  - Instruments: \( \{\tau_{t+1}\} \) (resp. \( \{\tau^k_{t+1}\} \)) and \( \{T_{t+1}\} \)
  - Social welfare function: \( SWF = \sum_{t=-1}^{\infty} \omega_t V_t \)
  - Maximize by choice of \( \{s_t\} \)
- Note that Ramsey tax policy is time-consistent:
  - For given \( k_t \) implied by past household decisions, government cannot alter lifetime utility of generation \( t \) through changing \( \tau_t \).
  - Since tax revenues from current old are rebated to this generation, remaining lifetime utility of the old is unaffected by the tax \( \tau_t \).
  - Thus government has no incentive to deviate in period \( t \) from period zero tax plan \( \{\tau_t\} \).
Analysis: Recursive Formulation of Ramsey Problem

- Recursive formulation: convenient for interpretation
- Government discount factor: \( \frac{\omega_{t+1}}{\omega_t} = \theta \in [0, 1] \).
- Objective function in Ramsey problem:

\[
W(k) = \max_{s \in [(0,1)]} \ln((1-s)(1-\kappa)(1-\alpha)k^\alpha) + \beta \int \ln (\kappa \eta w(s) + R(s) s(1-\kappa)(1-\alpha)k^\alpha) d\Psi(\eta) + \theta W(k'(s))
\]

\[
k'(s) = s(1-\kappa)(1-\alpha)k^\alpha
\]

\[
R(s) = \alpha [k'(s)]^{\alpha-1}
\]

\[
w(s) = (1-\alpha) [k'(s)]^\alpha
\]
Analysis: Determination of Optimal Saving Rate

- FOC of Ramsey government:

\[ PE(s) + CG(s) + FG(s) = 0 \]

- \( PE(s) <> 0 \): Partial equilibrium effect of altering saving rate, where \( PE(s^{CE}) = 0 \)

- \( CG(s) < 0 \): GE feedback on current generations, \( CG(s) \) increasing in risk if \( \kappa > 0 \)

- \( FG(s) \geq 0 \): GE feedback on future generations, \( FG(s) = 0 \) for \( \theta = 0 \)
Analysis: Decomposition of Optimal Savings Rate Determination

Decomposition into Three Effects

PE, CG, FG

Decomposition into Three Effects

PE
CG
FG
Total

s^CE
s^*

s

0.2
0.4
0.6
0.8

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Analysis: Ramsey Saving Rate for Log Utility

- Optimal savings rate:

\[ s(k) = s^* = \frac{\alpha(\beta + \theta)}{1 + \alpha \beta} \]

independent of \( k \), independent of \( \eta \) risk. Thus implied sequence of saving rates \( \{s_t\} \) constant over time.

- Precautionary savings & wage risk interactions cancel (future generations effect independent of risk):

\[
PE(s) = -\frac{1}{1 - s} + \frac{\alpha \beta}{s} \Gamma(\alpha, \kappa, \sigma = 1; \Psi),
\]
\[
CG(s) = \frac{\alpha \beta}{s} - \frac{\alpha \beta}{s} \Gamma(\alpha, \kappa, \sigma = 1; \Psi)
\]
Analysis: Implementation through Capital Taxes (Log Utility)

• Implementing capital taxes:

\[ \tau = 1 - \frac{(\theta + \beta)}{(1 - \alpha \theta) \beta \Gamma(\alpha, \kappa, \sigma = 1; \Psi)} \]

• \( MPS(\eta) \) increases optimal capital taxes by increasing \( \Gamma(\cdot, \Psi) \)

• Reason:
  
  • \( MPS(\eta) \) increases private precautionary savings in CE.

  • Ramsey government offsets this to implement socially optimal (Ramsey) saving rate which is independent of income risk.
Ramsey Optima with Log-Utility: Steady State ($\theta = 1$)

- Define:
  - $s_0$: Steady state saving rate in CE without income risk, with $\tau = 0$
  - $s_0(\eta)$: Steady state saving rate in CE with income risk, with $\tau = 0$
  - $s^{GR}$: Golden rule saving rate, maximizing steady state consumption.
  - $s^*$: Optimal Ramsey saving rate (in steady state, $\theta = 1$)

- Assume that $\beta < \left[ (1 - \alpha)\bar{\Gamma} - 1 \right]^{-1}$, hence $s_0 < s^{GR}$

- Three cases:
  1. High risk: Then $s^{GR} < s_0(\eta)$, $s^* < s_0(\eta)$ and $\tau^* > 0$
  2. Intermediate risk: Then $s^* < s_0(\eta) < s^{GR}$ and $\tau^* > 0$
  3. Low risk: Then $s_0(\eta) < s^{GR}, s_0(\eta) < s^*$ and $\tau^* < 0$
Analysis: Pareto Improving Tax Transition

- Intermediate risk: \( s^* < s_0(\eta) < s^{GR} \) and \( \tau^* > 0 \)

- Implementing \( s_t = s^* \) for all \( t \) by setting \( \tau_t = \tau^* \) for all \( t \) induces Pareto improving transition

- Intuition:
  - Capital crowding most severe in the long run
  - Show that welfare consequences most favorable for initial generations, least favorable in the long run
  - But: \( s^* \) maximizes steady state utility

- Nota bene: argument not restricted to log utility and applies as long as \( s^* < s_0(\eta) < s^{GR} \).
Analysis: Epstein-Zin-Weil Preferences

- $\sigma > 0$: risk aversion (RA), $\rho \geq 0$: inter-temporal elasticity of substitution (IES), $ce(c^o; \sigma, \eta\text{-risk})$: certainty equivalent of utility from old-age consumption.

- $\sigma = \frac{1}{\rho}$: CRRA preferences (power utility).

- $\sigma = \rho = 1$: log utility analyzed thus far.

- Preferences:

$$V_t = \left( \frac{(c^y_t)^{1 - \frac{1}{\rho}} - 1}{1 - \frac{1}{\rho}} \right) + \frac{\beta \{ ce(c^o_{t+1}; \sigma, \Psi) \}^{1 - \frac{1}{\rho}} - 1}{1 - \frac{1}{\rho}}$$

$$= V_t(c^y_t, ce(c^o_t, \sigma, \eta\text{-risk}), \rho, \beta)$$
Analysis: Results for Epstein-Zin-Weil Preferences

- Assume $\rho = 1, \sigma \neq 1$. Results for optimal Ramsey allocation along transition path as before:
  - $s^*$ time-invariant, independent of risk
  - $\tau^*$ time-invariant, increasing in risk.
  - Pareto improving tax transition possible.

- Assume $\rho \neq 1$: Analytical results only for steady state ($\theta = 1$):
  - $s^*$ increases (decreases) in $\eta$-risk and only if $\rho < 1$ ($\rho > 1$)
  - $\tau^*$ increases in $\eta$-risk if (i) $\rho \leq 1$ or (ii) $\rho > 1$ and $\frac{1}{\rho} \geq \sigma$
  - $\tau^*$ may decrease in income risk if $\rho > 1$ and $\frac{1}{\rho} < \sigma \leq \infty$, but only if $s_{CE}$ decreases in income risk.
Analysis: Why Might $\tau^*$ Decrease in Income Risk?

- Recall that $s^*$ satisfies $PE(s^*) + CG(s^*) + FG(s^*) = 0$

- Households ignore price externalities $(CG(s), FG(s))$, might cut saving rate $s_{CE}$ too much in response to increase in income risk.

- Wages of future generations falls too much. Captured through $FG(s) > 0$.

- Thus Ramsey government may find it optimal to reduce $\tau^*$ when income risk increases.
Outline

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Conclusion

• Studied Optimal Ramsey capital (income) taxation in OLG economy with uninsurable idiosyncratic income risk

• Feedback from precautionary savings to equilibrium factor prices

• Fully analytically tractable with IES=1
  • Closed form solution for Ramsey allocation along transition
  • Optimal saving rate independent of idiosyncratic risk
  • Optimal capital taxes increase with idiosyncratic risk

• Pareto improving transition in dynamically efficient economy possible

• EZW with $\rho \neq 1, \sigma \neq 1$: capital taxes may decrease in income risk
Outline

5  Numerical Analyses
Numerical Exploration

- Objective: Characterize Ramsey tax transition when analytical results not available (especially for $\rho > 1$)

- Parametrization: $\rho = 20$, $\sigma = 50$, log-normal $\eta$, $\sigma_\eta^2 \in \{0, \ldots, 2\}$

- Policy functions

- Transitions for $s_t, k_t, \tau_t^k, \Delta V_t$

- Recall: $\rho > 1$
  - Steady state Ramsey saving rate $s^*$ decreases in $\eta$-risk
  - $\tau^*$ may decrease in $\eta$-risk
Numerical Analyses: Policy Functions

(a) $s^*(k)$

(b) $k'^*(k)$
Numerical Analyses: Transition Summary

<table>
<thead>
<tr>
<th>$s_0(\eta)$</th>
<th>$s_\infty^*$</th>
<th>$\tau_k^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_\eta = 0$</td>
<td>0.41</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\eta = 0.25$</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\eta = 1$</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\eta = 2$</td>
<td>0.27</td>
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### Numerical Analyses: Transition Summary (cont’d)

<table>
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<tr>
<th>$\sigma_{\eta}$</th>
<th>$s_0(\eta)$</th>
<th>$s^*_\infty$</th>
<th>$\tau^k_\infty$</th>
</tr>
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<td>0.38</td>
<td>0.41</td>
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<tr>
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<tr>
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<tr>
<td>2</td>
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</tbody>
</table>
Numerical Analyses: Capital Transition

(c) $s_t$

(d) $k_t$
Numerical Analyses: Tax Transition & Welfare

(e) $\tau_t^k$

(f) $\Delta \nu_t$

(capital income tax rate over time: optimal transition policy)

(change in value function over time: optimal transition policy)
Literature on Optimal Taxation with Idiosyncratic Risk

- Ramsey tax literature:
  - Both risks: Panousi and Reis (2017)