Comments on
"Aggregate Implications of Innovation Policy" by Atkeson and Burstein

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models of growth and firm heterogeneity

- endogenous growth
  - Aghion and Howitt [1992]
  - Grossman and Helpman [1991]
  - Romer [1990]
  - ...and many more

- adding richer firm heterogeneity and dynamics
  - Klette and Kortum [2004]
  - Lentz and Mortensen [2008]
  - Luttmer [2007, 2011]
  - ...and some more

▶ AB nest "some of the canonical models"

▶ is the result ready for policy analysis?

notation

$$A_t = Ae^{\alpha t} \tag{1}$$

$$H_t = He^{\eta t} \tag{2}$$

$$y_t[z] = z\left(k_t[z]\right)^{1-\beta}\left(l_t[z]\right)^{\beta} \tag{3}$$

$$Y_t = \left(\int \left(y_t[z]\right)^{1-1/\varepsilon} \mathrm{d}M_t[z]\right)^{1/(1-1/\varepsilon)} \tag{4}$$

$$M_t = \int \mathrm{d}M_t[z] \tag{5}$$

(an aggregate,
not an average)

$$Z_t^{\varepsilon-1} = \int z^{\varepsilon-1}\mathrm{d}M_t[z] \tag{6}$$

the simplest version of AB

temporary equilibrium conditions

$$H_t = L_{Y,t} + L_{R,t}, \qquad \lambda_t = \mathrm{D}U(C_t/H_t)$$

$$Y_t = Z_t K_t^{1-\beta} L_{Y,t}^{\beta}, \qquad w_t = \left(1 - \frac{1}{\varepsilon}\right) \frac{\beta Y_t}{L_{Y,t}}$$

$$N_t = \frac{A_t M_t}{Z_t^{1-\phi}} \times L_{R,t}, \qquad w_t = \frac{1}{1-\tau} \frac{A_t M_t}{Z_t^{1-\phi}} \times \frac{\Gamma^{\varepsilon-1} Q_t}{M_t} \tag{1}$$

state dynamics

$$\mathrm{d}K_t/\mathrm{d}t = -\delta_{\mathrm{K}} K_t + Y_t - C_t$$

$$\mathrm{d}M_t/\mathrm{d}t = -\delta_{\mathrm{Z}} M_t + N_t \tag{2}$$

$$\mathrm{d}\left[Z_t^{\varepsilon-1}\right]/\mathrm{d}t = -\delta_{\mathrm{Z}} Z_t^{\varepsilon-1} + N_t \times \frac{(\Gamma Z_t)^{\varepsilon-1}}{M_t} \tag{3}$$

present values

$$\mathrm{d}\left[\lambda_t K_t\right]/\mathrm{d}t = \rho \lambda_t K_t - \lambda_t \left(\left(1 - \frac{1}{\varepsilon}\right)(1-\beta) Y_t - (Y_t - C_t)\right)$$

$$\mathrm{d}\left[\lambda_t Q_t\right]/\mathrm{d}t = \left(\rho + \frac{N_t}{M_t} \times \Gamma^{\varepsilon-1}\right) \lambda_t Q_t - \frac{\lambda_t Y_t}{\varepsilon} \tag{4}$$

the simplest version of AB

standing on the shoulders of giants

$$
\begin{aligned}
H_t &= L_{Y,t} + L_{R,t}, & \lambda_t &= \mathrm{D}U(C_t/H_t) \\
Y_t &= Z_t K_t^{1-\beta} L_{Y,t}^{\beta}, & w_t &= \left(1 - \frac{1}{\varepsilon}\right) \frac{\beta Y_t}{L_{Y,t}} \\
N_t &= \frac{A_t M_t}{Z_t^{1-\phi}} \times L_{R,t}, & w_t &= \frac{1}{1-\tau} \frac{A_t M_t}{Z_t^{1-\phi}} \times \frac{\Gamma^{\varepsilon-1} Q_t}{M_t}
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
\mathrm{d}K_t/\mathrm{d}t &= -\delta_{\mathrm{K}} K_t + Y_t - C_t \\
\mathrm{d}M_t/\mathrm{d}t &= -\delta_{\mathrm{Z}} M_t + N_t
\end{aligned} \tag{2}
$$

$$
\mathrm{d}\left[Z_t^{\varepsilon-1}\right]/\mathrm{d}t = -\delta_{\mathrm{Z}} Z_t^{\varepsilon-1} + N_t \times \frac{(\Gamma Z_t)^{\varepsilon-1}}{M_t} \tag{3}
$$

$$
\begin{aligned}
\mathrm{d}\left[\lambda_t K_t\right]/\mathrm{d}t &= \rho \lambda_t K_t - \lambda_t \left(\left(1 - \frac{1}{\varepsilon}\right) (1-\beta) Y_t - (Y_t - C_t) \right) \\
\mathrm{d}\left[\lambda_t Q_t\right]/\mathrm{d}t &= \left(\rho + \frac{N_t}{M_t} \times \Gamma^{\varepsilon-1}\right) \lambda_t Q_t - \frac{\lambda_t Y_t}{\varepsilon}
\end{aligned} \tag{4}
$$

let's lose the M_t

$$
\begin{aligned}
H_t &= L_{Y,t} + L_{R,t}, & \lambda_t &= \mathrm{D}U(C_t/H_t) \\
Y_t &= Z_t K_t^{1-\beta} L_{Y,t}^{\beta}, & w_t &= \left(1 - \frac{1}{\varepsilon}\right) \frac{\beta Y_t}{L_{Y,t}} \\
n_t &= \frac{A_t H_t}{Z_t^{1-\phi}} \times \frac{L_{R,t}}{H_t}, & w_t &= \frac{1}{1-\tau} \frac{A_t H_t}{Z_t^{1-\phi}} \times \frac{\Gamma^{\varepsilon-1} Q_t}{H_t}
\end{aligned} \tag{1}
$$

$$
\mathrm{d}K_t/\mathrm{d}t = -\delta_{\mathrm{K}} K_t + Y_t - C_t
$$

$$
\mathrm{d}\left[Z_t^{\varepsilon-1}\right]/\mathrm{d}t = \left(-\delta_{\mathrm{Z}} + n_t \times \Gamma^{\varepsilon-1}\right) Z_t^{\varepsilon-1} \tag{2}
$$

$$
\begin{aligned}
\mathrm{d}\left[\lambda_t K_t\right]/\mathrm{d}t &= \rho \lambda_t K_t - \lambda_t \left(\left(1 - \frac{1}{\varepsilon}\right)(1-\beta) Y_t - (Y_t - C_t) \right) \\
\mathrm{d}\left[\lambda_t Q_t\right]/\mathrm{d}t &= \left(\rho + n_t \times \Gamma^{\varepsilon-1}\right) \lambda_t Q_t - \frac{\lambda_t Y_t}{\varepsilon}
\end{aligned} \tag{3}
$$

the $\beta = 1$ special case

$$
\begin{aligned}
H_t &= L_{Y,t} + L_{R,t}, & \lambda_t &= \mathrm{D}U(C_t/H_t) \\
C_t &= Z_t L_{Y,t}, & w_t &= \left(1 - \frac{1}{\varepsilon}\right)\frac{C_t}{L_{Y,t}} \\
n_t &= \frac{A_t H_t}{Z_t^{1-\phi}} \times \frac{L_{R,t}}{H_t}, & w_t &= \frac{1}{1-\tau}\frac{A_t H_t}{Z_t^{1-\phi}} \times \frac{\Gamma^{\varepsilon-1} Q_t}{H_t}
\end{aligned} \tag{1}
$$

$$
\mathrm{d}\left[Z_t^{\varepsilon-1}\right]/\mathrm{d}t = \left(-\delta_{\mathrm{Z}} + n_t \times \Gamma^{\varepsilon-1}\right) Z_t^{\varepsilon-1} \tag{2}
$$

$$
\mathrm{d}\left[\lambda_t Q_t\right]/\mathrm{d}t = \left(\rho + n_t \times \Gamma^{\varepsilon-1}\right) \lambda_t Q_t - \frac{\lambda_t C_t}{\varepsilon} \tag{3}
$$

sort-of-balanced growth

- science and population

$$A_t = Ae^{\alpha t}, \quad H_t = He^{\eta t}$$

▷ constant number of entrepreneurs per capita

$$n_t = \frac{A_t H_t}{Z_t^{1-\phi}} \times \frac{L_{R,t}}{H_t}, \qquad \frac{L_{R,t}}{H_t} = l_R \tag{1}$$

▷ constant "productivity" growth

$$\frac{1}{Z_t}\frac{\mathrm{d}Z_t}{\mathrm{d}t} = \frac{n_t \times \Gamma^{\varepsilon-1} - \delta_Z}{\varepsilon - 1} = \underbrace{\frac{\Gamma^{\varepsilon-1} - 1}{\varepsilon - 1} \times n_t}_{\text{process}} + \underbrace{\frac{n_t - \delta_Z}{\varepsilon - 1}}_{\text{variety}} \tag{2}$$

▶ (1) and (2) imply

$$\frac{A_t H_t}{Z_t^{1-\phi}} = \texttt{constant} \Rightarrow \qquad \frac{1}{Z_t}\frac{\mathrm{d}Z_t}{\mathrm{d}t} = \frac{\alpha + \eta}{1 - \phi}$$

$$\Rightarrow \qquad n_t = \frac{1}{\Gamma^{\varepsilon-1}}\left((\varepsilon - 1) \times \frac{\alpha + \eta}{1 - \phi} + \delta_Z\right)$$

level effects of τ

- steady state growth

$$\frac{1}{Z_t}\frac{\mathrm{d}Z_t}{\mathrm{d}t} = \frac{\alpha+\eta}{1-\phi}$$

- *levels* determined by

$$Z_t = \left(A_t H_t \times \frac{L_{R,t}}{nH_t}\right)^{1/(1-\phi)}$$
$$n = \frac{1}{\Gamma^{\varepsilon-1}}\left((\varepsilon-1)\times\frac{\alpha+\eta}{1-\phi}+\delta_{\mathrm{Z}}\right), \quad \frac{L_{R,t}}{H_t} = l_R = \ldots$$

- subsidies direct allocation of labor across sectors via

$$\left(1-\frac{1}{\varepsilon}\right)\frac{\beta Y_t}{L_{Y,t}} = w_t = \frac{1}{1-\tau}\frac{A_t H_t}{Z_t^{1-\phi}} \times \frac{\Gamma^{\varepsilon-1}Q_t}{H_t}$$

- Grossman-Helpman quality ladder model,
  - same instrument can be used to achieve efficient *growth*

what about those M_t?

- number of goods, establishments, firms,

$$\begin{aligned}\frac{1}{M_t}\frac{\mathrm{d}M_t}{\mathrm{d}t} &= n_t - \delta_{\mathrm{Z}} \\ &= \frac{1}{\Gamma^{\varepsilon-1}}\left((\varepsilon-1)\times\frac{\alpha+\eta}{1-\phi}+\delta_{\mathrm{Z}}\right)-\delta_{\mathrm{Z}} \\ &= \frac{1}{\Gamma^{\varepsilon-1}}\left((\varepsilon-1)\times\frac{\mathrm{D}Z_t}{Z_t}+\delta_{\mathrm{Z}}\right)-\delta_{\mathrm{Z}}\end{aligned}$$

  – could be anything...

- but, for firms and establishments in postwar US data,

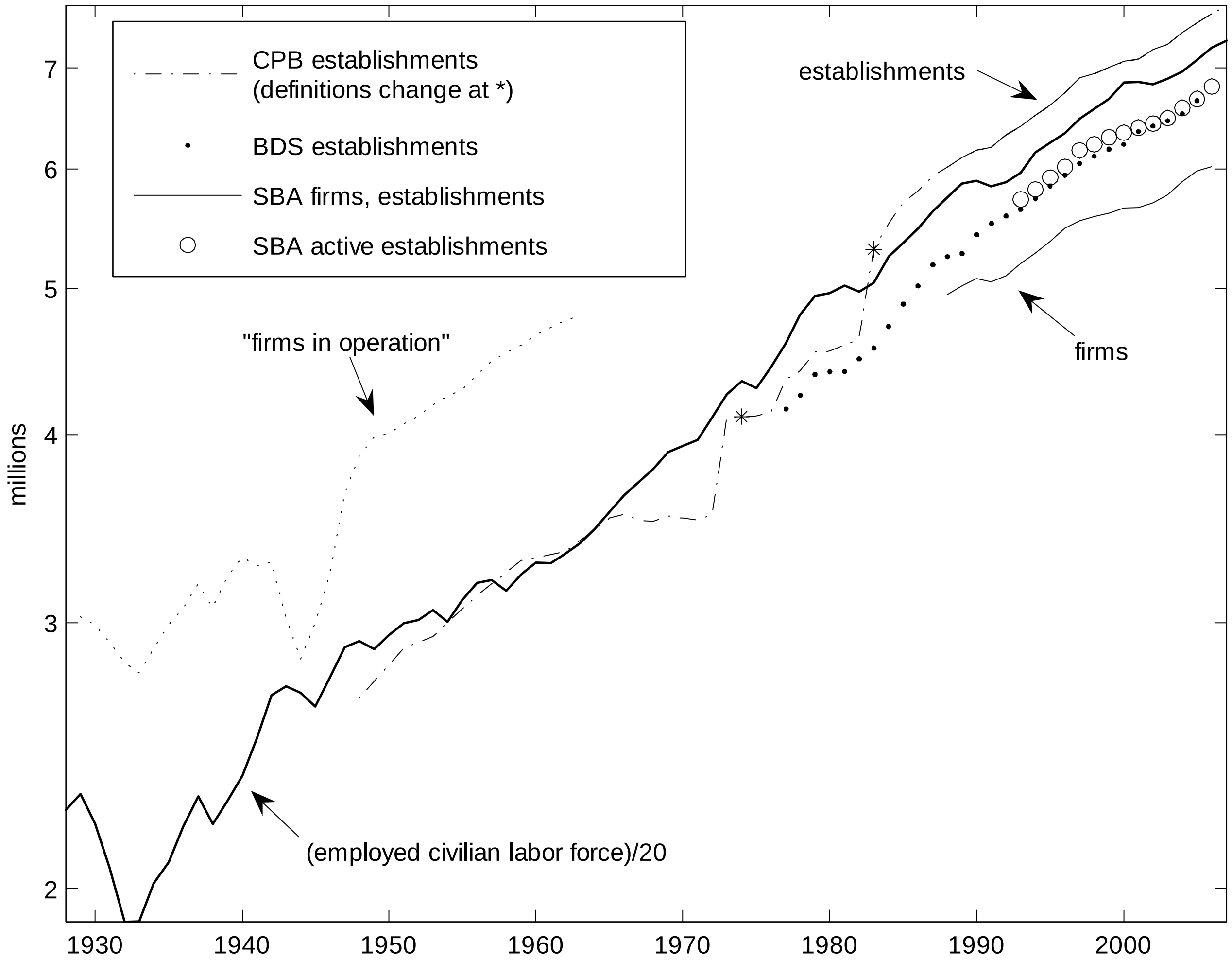
$$\frac{1}{M_t}\frac{\mathrm{d}M_t}{\mathrm{d}t} \approx \frac{1}{H_t}\frac{\mathrm{d}H_t}{\mathrm{d}t} = \eta \qquad (*)$$

- AB must choose some combination of $(\delta_{\mathrm{Z}}, \varepsilon, \Gamma)$ for which this works

  – $\mathrm{FFT}^{\mathrm{TM}}$ = fragile fine tuning

  – any change in δ_{Z}, Γ or $(\mathrm{d}Z_t/\mathrm{d}t)/Z_t$ will cause (*) to fail

count data



from Luttmer, *Annual Review of Economics, 2010.*

central planning

with $X_t = Z_t^{1-\phi}/(A_t H_t)$, the planner solves, in de-trended variables,

$$\max_{C_t \geq 0} \int_0^\infty e^{-\rho_* t} U(C_t) \mathrm{d}t$$

subject to

$$\begin{aligned} \mathrm{d}K_t/\mathrm{d}t &= -\delta_* K_t + Y_t - C_t, \\ \mathrm{d}X_t/\mathrm{d}t &= -\delta_\mathrm{X} X_t + \gamma(1 - L_t), \end{aligned}$$

where

$$Y_t = X_t^{1/(1-\phi)} K_t^{1-\beta} L_t^\beta, \qquad L_t \in [0, 1],$$

and (K_0, X_0) given.

- like Lucas [1988], but without growth...
- following (K_0, X_0) at steady state, the solution is

$$L_t = \frac{(\rho_* + \delta_\mathrm{X})\beta}{\frac{\delta_\mathrm{X}}{1-\phi} + (\rho_* + \delta_\mathrm{X})\beta}, \quad \text{where } \frac{\delta_\mathrm{X}}{1-\phi} = \frac{\alpha + \eta}{1-\phi} + \frac{\delta_\mathrm{Z}}{\varepsilon - 1}$$

  – if $\varepsilon - 1 = 1 - \phi$ then $\tau = 0$ is optimal
  – the "mystery factor" provides just the right incentives

my hunch about what is important

- ▶ more heterogeneity, understanding slow convergence
  - see Luttmer [2011; 2012, 2018]

- ▷ Roy models, richer input-output structures

- ▷ different types of labor are used with different intensities in the various consumption and capital accumulation sectors

- ▶ tax policy that affects the level and composition of the capital stock can have important distributional consequences