Comments on
“Aggregate Implications of Innovation Policy”
by Atkeson and Burstein

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models of growth and firm heterogeneity

• endogenous growth
  – Aghion and Howitt [1992]
  – Grossman and Helpman [1991]
  – Romer [1990]
  – …and many more

• adding richer firm heterogeneity and dynamics
  – Klette and Kortum [2004]
  – Lentz and Mortensen [2008]
  – Luttmer [2007, 2011]
  – …and some more

▶ AB nest “some of the canonical models”
▶ is the result ready for policy analysis?
notation

\[ A_t = Ae^{\alpha t} \]  \hspace{1cm} (1)

\[ H_t = He^{\eta t} \]  \hspace{1cm} (2)

\[ y_t[z] = z (k_t[z])^{1-\beta} (l_t[z])^\beta \]  \hspace{1cm} (3)

\[ Y_t = \left( \int (y_t[z])^{1-1/\varepsilon} \, dM_t[z] \right)^{1/(1-1/\varepsilon)} \]  \hspace{1cm} (4)

\[ M_t = \int dM_t[z] \]  \hspace{1cm} (5)

\[ Z_t^{\varepsilon-1} = \int z^{\varepsilon-1} \, dM_t[z] \]  \hspace{1cm} (6)

(an aggregate, not an average)
the simplest version of AB

\[ H_t = L_{Y,t} + L_{R,t}, \quad \lambda_t = DU(C_t/H_t) \]

\[ Y_t = Z_t K_t^{1-\beta} L_{Y,t}^{\beta}, \quad w_t = \left(1 - \frac{1}{\varepsilon}\right) \frac{\beta Y_t}{L_{Y,t}} \]

\[ N_t = A_t M_t \times L_{R,t}, \quad w_t = \frac{1}{1 - \tau} \left( A_t M_t \right) \times \frac{\Gamma^{\varepsilon-1} Q_t}{M_t} \tag{1} \]

\[ \frac{dK_t}{dt} = -\delta_K K_t + Y_t - C_t \]
\[ \frac{dM_t}{dt} = -\delta_Z M_t + N_t \tag{2} \]
\[ \frac{d\left[Z_t^{\varepsilon-1}\right]}{dt} = -\delta_Z Z_t^{\varepsilon-1} + N_t \times \frac{\left(\Gamma Z_t\right)^{\varepsilon-1}}{M_t} \tag{3} \]

\[ \frac{d[\lambda_t K_t]}{dt} = \rho \lambda_t K_t - \lambda_t \left( \left( 1 - \frac{1}{\varepsilon} \right) (1 - \beta) Y_t - (Y_t - C_t) \right) \]
\[ \frac{d[\lambda_t Q_t]}{dt} = \left( \rho + \frac{N_t}{M_t} \times \Gamma^{\varepsilon-1} \right) \lambda_t Q_t - \frac{\lambda_t Y_t}{\varepsilon} \tag{4} \]
the simplest version of AB

\[ H_t = L_{Y,t} + L_{R,t}, \quad \lambda_t = DU(C_t/H_t) \]
\[ Y_t = Z_t K_t^{1-\beta} L_{Y,t}^{\beta}, \quad w_t = \left(1 - \frac{1}{\varepsilon}\right) \frac{\beta Y_t}{L_{Y,t}} \]
\[ N_t = \frac{A_t M_t}{Z_t^{1-\phi}} \times L_{R,t}, \quad w_t = \frac{1}{1 - \tau} \frac{A_t M_t}{Z_t^{1-\phi}} \times \frac{\Gamma^{\varepsilon-1} Q_t}{M_t} \quad (1) \]

\[ \frac{dK_t}{dt} = -\delta_K K_t + Y_t - C_t \]
\[ \frac{dM_t}{dt} = -\delta_Z M_t + N_t \quad (2) \]
\[ \frac{d \left[ Z_t^{\varepsilon-1} \right]}{dt} = -\delta_Z Z_t^{\varepsilon-1} + N_t \times \frac{(\Gamma^{Z_t})^{\varepsilon-1}}{M_t} \quad (3) \]

\[ \frac{d[\lambda_t K_t]}{dt} = \rho \lambda_t K_t - \lambda_t \left( \left(1 - \frac{1}{\varepsilon}\right) (1 - \beta) Y_t - (Y_t - C_t) \right) \]
\[ \frac{d[\lambda_t Q_t]}{dt} = \left( \rho + \frac{N_t}{M_t} \times \Gamma^{\varepsilon-1} \right) \lambda_t Q_t - \frac{\lambda_t Y_t}{\varepsilon} \quad (4) \]
let’s lose the $M_t$

\[ H_t = L_{Y,t} + L_{R,t}, \quad \lambda_t = DU(C_t/H_t) \]
\[ Y_t = Z_t K_t^{1-\beta} L_{Y,t}^\beta, \quad w_t = \left( 1 - \frac{1}{\varepsilon} \right) \frac{\beta Y_t}{L_{Y,t}} \]
\[ (n_t) = \left( \frac{A_t H_t}{Z_t^{1-\phi}} \times \frac{L_{R,t}}{H_t} \right), \quad w_t = \frac{1}{1 - \tau} \left( \frac{A_t H_t}{Z_t^{1-\phi}} \times \frac{\Gamma^{\varepsilon-1} Q_t}{H_t} \right) \quad (1) \]

\[ \frac{dK_t}{dt} = -\delta K_t + Y_t - C_t \]

\[ \frac{d}{dt} \left[ Z_t^{\varepsilon-1} \right] = ( -\delta_Z + (n_t) \times \Gamma^{\varepsilon-1} ) \left[ Z_t^{\varepsilon-1} \right] \quad (2) \]

\[ \frac{d[\lambda_t K_t]}{dt} = \rho \lambda_t K_t - \lambda_t \left( \left( 1 - \frac{1}{\varepsilon} \right) (1 - \beta) Y_t - (Y_t - C_t) \right) \]
\[ \frac{d[\lambda_t Q_t]}{dt} = \left( \rho + (n_t) \times \Gamma^{\varepsilon-1} \right) \lambda_t Q_t - \frac{\lambda_t Y_t}{\varepsilon} \quad (3) \]
the $\beta = 1$ special case

\[ H_t = L_{Y,t} + L_{R,t}, \quad \lambda_t = DU(C_t/H_t) \]
\[ C_t = Z_tL_{Y,t}, \quad w_t = \left( 1 - \frac{1}{\varepsilon} \right) \frac{C_t}{L_{Y,t}} \]
\[ n_t = \left( \frac{A_tH_t}{Z_t^{1-\phi}} \times \frac{L_{R,t}}{H_t} \right), \quad w_t = \frac{1}{1 - \tau} \left( A_tH_t \right) \times \frac{\Gamma^{\varepsilon-1}Q_t}{H_t} \] (1)

\[ d\left[ Z_t^{\varepsilon-1} \right]/dt = (-\delta_Z + n_t \times \Gamma^{\varepsilon-1}) Z_t^{\varepsilon-1} \] (2)

\[ d[\lambda_tQ_t]/dt = (\rho + n_t \times \Gamma^{\varepsilon-1}) \lambda_tQ_t - \frac{\lambda_tC_t}{\varepsilon} \] (3)
sort-of-balanced growth

- science and population

\[ A_t = Ae^{\alpha t}, \quad H_t = He^{nt} \]

▷ constant number of entrepreneurs per capita

\[ n_t = \frac{A_t H_t}{Z_t^{1-\phi}} \times \frac{L_{R,t}}{H_t}, \quad \frac{L_{R,t}}{H_t} = l_R \] (1)

▷ constant “productivity” growth

\[ \frac{1}{Z_t} \frac{dZ_t}{dt} = n_t \times \frac{\Gamma^{\varepsilon-1} - \delta Z}{\varepsilon - 1} = \frac{\Gamma^{\varepsilon-1} - 1}{\varepsilon - 1} \times n_t + \frac{n_t - \delta Z}{\varepsilon - 1} \] (2)

▷ (1) and (2) imply

\[ \frac{A_t H_t}{Z_t^{1-\phi}} = \text{constant} \Rightarrow \frac{1}{Z_t} \frac{dZ_t}{dt} = \frac{\alpha + \eta}{1 - \phi} \]

\[ \Rightarrow n_t = \frac{1}{\Gamma^{\varepsilon-1}} \left( (\varepsilon - 1) \times \frac{\alpha + \eta}{1 - \phi} + \delta Z \right) \]
level effects of $\tau$

- steady state growth

\[
\frac{1}{Z_t} \frac{dZ_t}{dt} = \frac{\alpha + \eta}{1 - \phi}
\]

- levels determined by

\[
Z_t = \left( A_t H_t \times \frac{L_{R,t}}{n H_t} \right)^{1/(1-\phi)}
\]

\[
n = \frac{1}{\Gamma^{\varepsilon-1}} \left( (\varepsilon - 1) \times \frac{\alpha + \eta}{1 - \phi} + \delta_Z \right), \quad \frac{L_{R,t}}{H_t} = l_R = \ldots
\]

- subsidies direct allocation of labor across sectors via

\[
\left( 1 - \frac{1}{\varepsilon} \right) \frac{\beta Y_t}{L_{Y,t}} = w_t = \frac{1}{1 - \tau} \frac{A_t H_t}{Z_t^{1-\phi}} \times \frac{\Gamma^{\varepsilon-1} Q_t}{H_t}
\]

- Grossman-Helpman quality ladder model,

  - same instrument can be used to achieve efficient growth
what about those $M_t$?

- number of goods, establishments, firms,

\[
\frac{1}{M_t} \frac{dM_t}{dt} = n_t - \delta_Z
\]

\[
= \frac{1}{\Gamma \varepsilon - 1} \left( (\varepsilon - 1) \times \frac{\alpha + \eta}{1 - \phi} + \delta_Z \right) - \delta_Z
\]

\[
= \frac{1}{\Gamma \varepsilon - 1} \left( (\varepsilon - 1) \times \frac{DZ_t}{Z_t} + \delta_Z \right) - \delta_Z
\]

- could be anything...

- but, for firms and establishments in postwar US data,

\[
\frac{1}{M_t} \frac{dM_t}{dt} \approx \frac{1}{H_t} \frac{dH_t}{dt} = \eta
\]

- AB must choose some combination of $(\delta_Z, \varepsilon, \Gamma)$ for which this works

  - FFT$^\text{TM}$ = fragile fine tuning

  - any change in $\delta_Z$, $\Gamma$ or $(dZ_t/dt)/Z_t$ will cause (*) to fail
count data

central planning

with \(X_t = Z_t^{1-\phi}/(A_t H_t)\), the planner solves, in de-trended variables,

\[
\max_{C_t \geq 0} \int_0^\infty e^{-\rho_* t} U(C_t) dt
\]

subject to

\[
\begin{align*}
dK_t/dt &= -\delta_* K_t + Y_t - C_t, \\
dX_t/dt &= -\delta_X X_t + \gamma(1 - L_t),
\end{align*}
\]

where

\[
Y_t = X_t^{1/(1-\phi)} K_t^{1-\beta} L_t^\beta, \quad L_t \in [0, 1],
\]

and \((K_0, X_0)\) given.

- like Lucas [1988], but without growth...

- following \((K_0, X_0)\) at steady state, the solution is

\[
L_t = \frac{(\rho_* + \delta_X)\beta}{\delta_X 1-\phi + (\rho_* + \delta_X)\beta}, \quad \text{where} \quad \frac{\delta_X}{1-\phi} = \frac{\alpha + \eta}{1-\phi} + \frac{\delta_Z}{\varepsilon - 1}
\]

- if \(\varepsilon - 1 = 1 - \phi\) then \(\tau = 0\) is optimal

- the “mystery factor” provides just the right incentives
my hunch about what is important

▶ more heterogeneity, understanding slow convergence
  – see Luttmer [2011; 2012, 2018]

▶ Roy models, richer input-output structures

▶ different types of labor are used with different intensities in the various consumption and capital accumulation sectors

▶ tax policy that affects the level and composition of the capital stock can have important distributional consequences