The Global Diffusion of Ideas

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The Global Diffusion of Ideas

- Long held belief that openness affects the diffusion of technologies/ideas

- Empirical debate
  - Sachs & Warner (95), Coe & Helpman (95), Frankel & Romer (99), Rodriguez & Rodrik (00), Keller (09), Feyrer (09a,b)

- Growth Miracles: Openness and protracted periods of growth

- But standard mechanisms imply relatively small effects
  - e.g., Connolly & Yi (14)
The Global Diffusion of Ideas

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- But standard mechanisms imply relatively small effects
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The Global Diffusion of Ideas

- Provide explicit model of diffusion process based on local interactions

- How does openness shape ideas to which individuals are exposed?

- Combine new ideas with insights from others ⇒ “general” Frechet limit
  - related to model of random networks in Oberfield (2013)

- Interface with static models of trade, multinational production (MP)

- Comparative advantage: Asymmetric countries, transition dynamics
The Global Diffusion of Ideas

- How does openness affect development? Potential for growth miracles?

- Which interactions facilitate exchange of ideas? Does it matter?

- Role of policy, international barriers in shaping interactions?

- Rich and tractable enough to take to cross-country data
The Global Diffusion of Ideas

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  - (Potentially) Large dynamic gains, protracted transition after openness
  - Especially for countries close to autarky

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The Global Diffusion of Ideas

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  - Speed of convergence

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  - Free-trade not necessarily best policy

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Buera & Oberfield (FRB Chicago, Princeton)
The Global Diffusion of Ideas

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Rich and tractable enough to take to cross-country data

- Can we explain growth miracles?
- Work in progress, some preliminary results today...
Roadmap

- Learning from an arbitrary source distribution, Frechet Limit

- Trade
  - Illustrate implications of alternative learning mechanisms
  - Static and dynamic gains from trade
  - Long-run and short-run (liberalization)

- Incentives for Innovation

- Quantitative exploration
  - South Korea: trade and development in the postwar period

- (probably not today) Trade and Multinational Production
Learning from an Arbitrary Source
Innovation and Diffusion

- Continuum of goods $s \in [0, 1]$
  - For each good $m$ managers ($m$ is large)
  - Bertrand Competition

- Manager with productivity $q$
  - Ideas arrive stochastically at rate $\alpha_t$
  - New idea has productivity $zq'^\beta$
    * Insight from someone with productivity $q' \sim \tilde{G}_t(q')$
    * Original component $z \sim H(z)$
  - Adopts if $zq'^\beta > q$

- $\beta$ measures strength of diffusion
  - Pure innovation: $\beta = 0$ (Kortum (1997))
  - Pure diffusion: $\beta = 1$, $H$ degenerate (ABL (2008, 2014), with Poisson arrivals)
Productivity Distribution

- Distribution of productivity among managers $M_t(q)$

- Frontier of knowledge $\tilde{F}_t(q) = M_t(q)^m$
Productivity Distribution

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- Frontier of knowledge $\tilde{F}_t(q) = M_t(q)^m$

- The distribution of productivities at time $t + \Delta$

$$M_{t+\Delta}(q) = M_t(q) \left[ (1 - \alpha_t \Delta) + \alpha_t \Delta \Pr(q^{l/\beta} \leq q) \right]$$

- no new idea
- new idea $\leq q$
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- Taking the limit as $\Delta \to 0$

  $$\frac{d}{dt} \log M_t(q) = -\alpha_t \Pr(zq'^\beta > q)$$
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- Taking the limit as $\Delta \rightarrow 0$

$$\frac{1}{m} \frac{d}{dt} \log \tilde{F}_t(q) = \frac{d}{dt} \log M_t(q) = -\alpha_t \Pr(zq'^\beta > q)$$

$$= -\alpha_t \int_0^\infty \left[ 1 - \tilde{G}_t \left( (q/z)^{1/\beta} \right) \right] dH(z)$$
Frechet Limit

Assumptions

- Distr. of original component of ideas has Pareto tail: \( \lim_{z \to \infty} \frac{1-H(z)}{z-\theta} = 1 \)

- For now: \( \tilde{G}_t \) has sufficiently thin right tail: \( \lim_{q \to \infty} q^{\beta \theta} [1 - \tilde{G}_t(q)] = 0 \)
  
  - Later: initial distribution \( M_0(q) \) has sufficiently thin tail

- \( \beta < 1 \)
Frechet Limit

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  - Later: initial distribution \( M_0(q) \) has sufficiently thin tail
- \( \beta < 1 \)

Convenient to study productivity scaled by number of managers

\[
F_t(q) = \tilde{F}_t \left( m \left( \frac{1}{1-\beta} \right) q \right) \quad G_t(q) = \tilde{G}_t \left( m \left( \frac{1}{1-\beta} \right) q \right)
\]

Proposition

As \( m \to \infty, t \to \infty \),

\[
F_t(q) = e^{-\lambda_t q^{-\theta}}, \quad \dot{\lambda}_t = \alpha_t \int_0^\infty x^{\beta \theta} dG_t(x)
\]

- \( \lambda_t \): stock of knowledge
Simple Example

- Individuals learn from managers at frontier
  \[ G_t(q) = F_t(q) \]

- Then stock of knowledge evolves as
  \[ \dot{\lambda}_t = \Gamma (1 - \beta) \alpha_t \lambda_t^\beta \]

- Long-run growth requires the arrival rate grows, \( \frac{\dot{\alpha}_t}{\alpha_t} = \gamma \)

- Implies growth in stock of knowledge at rate
  \[ \frac{\dot{\lambda}}{\lambda} = \frac{\gamma}{1 - \beta} \]

- Compounding: New ideas lead to even better insights
Trade
World Economy (BEJK, 2003)

- $n$ countries, defined by
  - Labor, $L_i$
  - Stock of knowledge, $\lambda_i$
  - Iceberg trade costs, $\kappa_{ij}$

- Household in $i$ has Dixit-Stiglitz preferences $C_i = \left[ \int_0^1 c_i(s) \frac{\varepsilon - 1}{\varepsilon} ds \right]^{\frac{\varepsilon}{\varepsilon - 1}}$

- Production is linear, uses only labor

- For manager in $j$, unit cost of providing good to country $i$ is
  $$\frac{w_j \kappa_{ij}}{q}$$

- Bertrand Competition:
  $$p_i(s) = \min \left\{ \frac{\varepsilon}{\varepsilon - 1} \text{ lowest unit cost}, \frac{\varepsilon}{\varepsilon - 1} \text{ second lowest unit cost} \right\}$$
Static Trade Equilibrium

- Price index

\[ P_i^{-\theta} \propto \sum_j \lambda_j (w_j \kappa_{ij})^{-\theta} \]

- Trade Shares

\[ \pi_{ij} = \frac{\lambda_j (w_j \kappa_{ij})^{-\theta}}{\sum_k \lambda_k (w_k \kappa_{ik})^{-\theta}} \]

- Labor market clearing (under balanced trade)

\[ w_i L_i = \sum_j \pi_{ji} w_j L_j \]
The Global Diffusion of Ideas
Diffusion of ideas

1. Learn from Sellers
   - Equally exposed to goods consumed (Alvarez-Buera-Lucas)
   - Learn in proportion to quantity consumed (or expenditure)

2. Learn from Producers
   - Equal exposure to active domestic producers (Perla-Tonetti-Waugh, Sampson)
   - Exposed in proportion to labor used (Monge-Naranjo)
Source distributions

- Let $S_{ij}$ be set of goods for which $j$ is lowest-cost provider for $i$

- Learning from sellers
  - in proportion to expenditure on good
    $$G^S_i(q) \equiv \sum_j \int_{s \in S_{ij} | q_j(s) < q} \frac{p_i(s)c_i(s)}{P_iC_i} ds$$

- Learning from producers
  - in proportion to labor used to produce good
    $$G^P_i(q) \equiv \sum_j \int_{s \in S_{ji} | q_i(s) \leq q} \frac{1}{L_i q_i(s)} \frac{\kappa_{ji}}{c_j(s)} ds$$
Learning From Sellers

\[
\lambda_i = \alpha_i \int_0^\infty q^{\beta \theta} dG_i(q) \propto \alpha_i \sum_j \pi_{ij} \left( \frac{\lambda_j}{\pi_{ij}} \right)^\beta
\]

- Expenditure-weighted average

- **Selection:** hold fixed \( \lambda_j \)
  - lower \( \pi_{ij} \) \( \Rightarrow \) import goods with higher \( q \)

- To maximize growth:

\[
\frac{\lambda_j}{\lambda_j'} = \frac{\pi_{ij}}{\pi_{ij'}}
\]
Learning From Sellers

\[ \dot{\lambda}_i = \alpha_i \int_0^\infty q^{\beta \theta} dG_i(q) \propto \alpha_i \sum_j \pi_{ij} \left( \frac{\lambda_j}{\pi_{ij}} \right)^\beta \]

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- To maximize growth:

\[
\frac{\lambda_j}{\lambda_{j'}} = \frac{\pi_{ij}}{\pi_{ij'}} \left( \frac{\lambda_j (w_j \kappa_{ij})^{-\theta}}{\lambda_{j'} (w_{j'} \kappa_{ij'})^{-\theta}} \right)
\]

- Import more from high wage countries
- Conflicts with maximizing current welfare
Learning from Producers

- Stock of knowledge

\[ \dot{\lambda}_i = \alpha_i \int_{0}^{\infty} q^{\beta \theta} dG_i(q) \propto \alpha_i \sum_j r_{ji} \left( \frac{\lambda_i}{\pi_{ji}} \right)^\beta \]

- Revenue-weighted average:

\[ r_{ji} = \frac{\pi_{ji} P_j C_j}{\sum_k \pi_{ki} P_k C_k} \] is \( i \)'s revenue share

- Impact of trade: Selection
  - High productivity producers likely to expand
  - Low productivity producers likely to drop out
GAINS FROM TRADE
Static and Dynamic Gains from Trade

Real income is

\[ y_i \propto \frac{w_i}{P_i} \propto \left( \frac{\lambda_i}{\pi_{ii}} \right)^{1/\theta} \]

- **Static** gains from trade: hold \( \lambda \) fixed
- **Dynamic** gains from trade: operate through idea flows
A Symmetric World

- Consider world with $n$ symmetric countries

- Long-run gains from trade

$$\frac{y^{FT}}{y^{AUT}} = \underbrace{n^{\frac{1}{\theta}}}_{\text{static}} \underbrace{n^{\frac{\beta}{(1-\beta)\theta}}}_{\text{dynamic}} = n^{\frac{1}{\theta} \frac{1}{1-\beta}}$$

- Dynamic gains from trade
  - Increase with $\beta$
  - Similar to input-output multiplier

**Note:** For special case of symmetric world, specifications of learning are identical
Long-Run Gains from Trade: Reduction in common $\kappa$

Stock of Ideas, $\lambda^{1/\theta}$

per-capita income

$\beta = 0.1$
$\beta = 0.5$
$\beta = 0.9$

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Long-Run Gains from Trade: Single Deviant

What is the fate of a single country that is isolated?

- Trade among $n - 1$ countries is costless

- Trade to and from “deviant” economy incurs iceberg cost $\kappa_n$
Long-Run Gains from Trade: Single Deviant

Learning from Sellers, $\lambda_n^{1/\theta}$

Learning from Producers, $\lambda_n^{1/\theta}$

$\beta = 0$
$\beta = 0.5$
$\beta = 0.9$

$y_n$

$y_n$

$\kappa_n$

$\kappa_n$

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Single Deviant: Arrival Rate

Learning from Sellers

\[ \left( \frac{\lambda_n}{\lambda_1(\kappa_n=1)} \right)^{1/\theta} \]

Learning from Producers

\[ \left( \frac{\lambda_n}{\lambda_1(\kappa_n=1)} \right)^{1/\theta} \]

Deviant openness, \( \frac{1}{\kappa_n} \)

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Long-Run Gains: Takeaways

- Static gains relevant when economy relatively open
  Dynamic gains relevant when economy relatively closed

- For moderately open economy, dynamic gains non-monotonic in $\beta$

- Learning from producers: open economy can get better insights if more isolated

- Low arrival rate compounded when
  - Learning from producers
  - Close to autarky
Speed of Convergence: Small Open Economy

For small open economy, (relatively) simple expressions for speed of convergence

Lessons:

▶ Faster with high $\beta$

▶ $\alpha$ plays no role

▶ Slower with learning from domestic producers
Trade Liberalization, $\kappa_n = 100 \rightarrow \kappa'_n = 1$

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$\beta = 0.5, \ \theta = 5, \ \text{TFP Growth rate on BGP} = 0.01, \ \kappa_1 = 1$
Trade Liberalization, $\kappa_n = 100 \rightarrow \kappa'_n = 1$

stock of ideas, $\lambda_n^{1/\theta}$

per–capita income, $y_n$

$\beta = 0.6$, $\theta = 5$, TFP Growth rate on BGP = 0.01, $\kappa_1 = 1$
Incentives to Innovate

\[ L_{jt} = L_{jt}^{Production} + L_{jt}^{R&D} \]

- Across BGPs, \( \frac{L_{jt}^{R&D}}{L_{jt}} \) independent of trade barriers
  - Market size ↑, but competition ↑

- But, openness ⇒ same R&D effort leads to better insights
Quantitative Exploration
Quantitative Exploration

- Generalized trade model: intermediate inputs, capital, non-traded goods

- Let $L_{it}$ be equipped labor \( (= K_{it}^{1/3} (pop_{it} \cdot h_{it})^{2/3} , \text{from the PWT}) \)

- Question: Can openness account for a significant part of the evolution of TFP of growth miracles?
Calibration

- Calibrate the evolution of trade costs, $\kappa_{ijt}$, to match bilateral trade flows

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>5</td>
</tr>
<tr>
<td>Share of Non-Traded Goods</td>
<td>0.5</td>
</tr>
<tr>
<td>Intermediate Good Share of Cost</td>
<td>0.5</td>
</tr>
<tr>
<td>Capital Share of VA</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>TFP Growth on BGP</td>
<td>1% per year</td>
</tr>
</tbody>
</table>

- $\alpha_{it}$, $\beta$?
  - Homogenous $\alpha$. How much does diffusion contribute for various $\beta$?
  - Heterogenous $\alpha_{it}$. Match TFP in 1960. Allow $\alpha_i$ to change?
Distribution of TFP, 1962

\[ \beta = 0 \]

\[ \beta = 0.25 \]

\[ \beta = 0.75 \]

\[ \beta = 0.9 \]
Development Dynamics, South Korea (vs. US)

Learning From Sellers
Korean TFP

US TFP

Learning From Producers
Korean TFP

US TFP

data
β = 0
β = 0.25
β = 0.75
β = 0.9

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Development Dynamics, South Korea (vs. US)

TFP, Korea

TFP, US

\[ \beta = 0.0, \kappa_t, \alpha_0 \]
\[ \beta = 0.9, \kappa_t, \alpha_0 \]
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Other Applications/Extensions

- Trade and Multinational Production
- Incentives for Innovation: endogenizing $\alpha$
Multinational Production (MP)

- Multinational Production (build on Ramondo & Rodriguez-Clare (2013))

- Manager associated with
  - Home country $i$
  - Profile of productivities, $\{q_1, ..., q_n\}$

- Iceberg MP costs $\delta_{i,j}$

- Trade equilibrium: Eaton-Kortum
Manager with \( \{q_1, ..., q_n\} \) draws insight from good with \( q' \).

Location-specific \( \{z_1, ..., z_n\} \), drawn from \( H(z_1, ..., z_n) \).

New Profile

\[
\left\{ \max\{q_1, z_1^{1-\beta} q'^{\beta}\}, ..., \max\{q_n, z_n^{1-\beta} q'^{\beta}\} \right\}
\]

\( \{z_1, ..., z_n\} \) drawn from multivariate Pareto, correlation \( \rho \).

\( F_{it}(q_1, ..., q_n) \) is multivariate Frechet

\[
F_{it} = e^{-\lambda_{it} \left( \sum_j q_j^{-\frac{\theta}{1-\rho}} \right)^{1-\rho}} \quad \text{and} \quad \dot{\lambda}_{it} = \alpha \int_0^\infty q^{\beta\theta} dG_{it}(q)
\]
Multinational Production

- Learning from Sellers & Producers

\[ \dot{\lambda}_i \propto \alpha \sum_j \sum_k \pi_{ijk} \left( \frac{\lambda_k}{\pi_{ijk}^{1-\rho} \left[ \sum_l \pi_{ilk} \right]^\rho} \right)^\beta \]

\[ \dot{\lambda}_i \propto \alpha \sum_j \sum_k r_{jik} \left( \frac{\lambda_k}{\pi_{jik}^{1-\rho} \left[ \sum_l \pi_{jlk} \right]^\rho} \right)^\beta \]

where \( r_{jik} = \frac{w_j \pi_{jik}}{w_i} \)

- Autarky vs Free Trade, Free MP

\[ \frac{y^{FT}}{y^{AUT}} = n^{\frac{2-\rho}{\theta}} \times n^{\frac{(2-\rho)\beta}{1-\beta}} \]

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Trade and FDI

Are trade and FDI complements or substitutes?

- Let $y(\kappa, \delta)$ be real income for symmetric world with
  - trade costs $\kappa$
  - FDI costs $\delta$

- Depends on $\rho$. Two polar cases:

$$\lim_{\rho \to 0} \frac{y(\kappa, \delta)}{y(1, 1)} = \left[ \left( \frac{1 + (n - 1)\kappa^{-\theta(1-\beta)}}{n} \right) \left( \frac{1 + (n - 1)\delta^{-\theta(1-\beta)}}{n} \right) \right]^{\frac{1}{\theta(1-\beta)}}$$

and

$$\lim_{\rho \to 1} \frac{y(\kappa, \delta)}{y(1, 1)} = \max \left\{ \left( \frac{1 + (n - 1)\kappa^{-\theta(1-\beta)}}{n} \right), \left( \frac{1 + (n - 1)\delta^{-\theta(1-\beta)}}{n} \right) \right\}^{\frac{1}{\theta(1-\beta)}}$$
Opening to Trade and/or MP, $\rho = 0.5$

$\lambda_n^{1/\theta}$

$\beta = 0.5$, $\frac{\alpha^S}{\alpha^S + \alpha^P} = 0.1$, $\rho = 0.5$, $\kappa = 100 \rightarrow 2.15$, $\delta = 100 \rightarrow 3$
Opening to Trade and/or MP, $\rho = 0.1$

Stock of Ideas, $\lambda_n^{1/\theta}$

per–capita income, $y_n$

$\beta = 0.5, \frac{\alpha^S}{\alpha^S + \alpha^P} = 0.1, \rho = 0.1, \kappa = 100 \rightarrow 2.15, \delta = 100 \rightarrow 3$
Ongoing/Future Work

- Full quantitative exploration
  - Calibrate $\beta$ to match the effect of trade on GDP estimated by Feyrer (09) (?)
  - Quantify the dynamic gains from trade/ impact of trade on growth miracles, e.g., Korea development experience
Frechet Limit

**Proposition**
Given assumptions, the frontier of knowledge evolves as:

\[
\lim_{m \to \infty} \frac{d \ln F_t(q)}{dt} = -\alpha_t q^{-\theta} \int_0^\infty x^{\beta \theta} dG_t(x)
\]

Define \( \lambda_t = \int_{-\infty}^t \alpha_{\tau} \int_0^\infty x^{\beta \theta} dG_{\tau}(x) \)

**Corollary**
Suppose that \( \lim_{t \to \infty} \lambda_t = \infty \). Then \( \lim_{t \to \infty} F_t(\lambda_t^{1/\theta} q) = e^{-q^{-\theta}} \).
Learning from Producers

in proportion to employment

\[ G_i(q) = \sum_{j=1}^{n} \int_{0}^{q} \frac{L_j w_j}{L_i w_i} \left( \frac{w_i \kappa_{ji}}{P_j} \right)^{1-\varepsilon} x^{\varepsilon-1} \prod_{k \neq j} F_k \left( \frac{w_k \kappa_{ik}}{w_i \kappa_{ii}} x \right) dF_i(x) \]

fraction of employment in \( x \)

prob. \( j \) buys \( x \) from \( i \)
Learning from Producers

uniformly

\[ G_i(q) = \sum_{j=1}^{n} \int_0^{q} \frac{1}{\pi_{ii}} \prod_{k \neq j} F_k \left( \frac{w_k \kappa_{jk}}{w_i \kappa_{ji}} x \right) dF_i(x) \]

The evolution of the stock of knowledge

\[ \dot{\lambda}_i \propto \left( \frac{\lambda_i}{\pi_{ii}} \right)^\beta \]
Multivariate Pareto

\[ H(z_1, ..., z_n) = \max \left\{ 1 - \left( \sum_j \left( \frac{z_i}{z_0} \right)^{-\frac{\theta}{1-\rho}} \right)^{1-\rho}, 0 \right\} \]

- Each marginal is distribution is Pareto
- \( \rho \in [0, 1] \) like a correlation
Learning from sellers

Trade only

Evolution of the distribution of productivities

\[
\frac{\partial \log(F_{it}(q))}{\partial t} = -\alpha \left[ 1 - \sum_{j=1}^{n} \int_{0}^{q} \prod_{k \neq j} F_{kt} \left( \frac{w_{k} \kappa_{ik}}{w_{j} \kappa_{ij}} x \right) \, dF_{jt}(x) \right]
\]
Endogenous Growth Case, $\beta = 1$
Alvarez, Buera & Lucas (2013)

- Growth rate in a BGP, $\nu = n\alpha/\theta$

- Tails converge if $\kappa_{ij} < \infty$

$$
\lim_{q \to \infty} \lim_{t \to \infty} \frac{1 - F_{it}(q e^{\nu t})}{\lambda q^{-\theta}} = 1
$$

- Distribution not Frechet (log-logistic if $\kappa_{ij} = w_i = 1$)
Single Deviant: Stock of Knowledge

Per-capita Income, $y_n$

years

0 5 10 15 20 25 30 35 40 45 50

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

years

0 5 10 15 20 25 30 35 40 45 50
Generalized Trade Model

- Technology requiring an intermediate aggregate and labor

\[ y_i(q) = \frac{1}{\eta \zeta \zeta (1 - \eta - \zeta)^{1-\eta-\zeta}} q_i x_i(q)^\eta k_i(q)^\zeta l_i(q)^{1-\eta-\zeta} \]

- Intermediate (investment) aggregate technology

\[ X_i = \left[ \int c_{xi}(q)^{1-1/\epsilon} dF_i(q) \right]^{\epsilon/(\epsilon-1)} \]

- Fraction \( \mu \) of the goods are tradable, i.e.,

\[ p_i^{1-\epsilon} = (1 - \mu) \int_0^{\infty} \left( \frac{p_i^\eta R_i^\zeta w_i^1 - \eta - \zeta}{q} \right)^{1-\epsilon} dF_j(q) \]

\[ + \mu \sum_{j=1}^{n} \int_0^{\infty} \left( \frac{p_j^\eta R_i^\zeta w_j^1 - \eta - \zeta \kappa_{ij}}{q} \right)^{1-\epsilon} \prod_{k \neq j} F_k \left( \frac{p_k^\eta R_i^\zeta w_j^1 - \eta - \zeta \kappa_{ik}}{p_j^\eta R_i^\zeta w_j^1 - \eta - \zeta \kappa_{ij}} q \right) dF_j(q) \]
Speed of Convergence: Small Open Economy

For small open economy, speed of convergence is

- If agents learn from sellers

\[ \gamma \left\{ 1 - \frac{\Omega_{ii}^S - \pi_{ii}}{1 + \theta (1 + \pi_{ii})} + \frac{\beta}{1 - \beta} \left( 1 - \Omega_{ii}^S \right) \right\} \]

- If agents learn from producers

\[ \gamma \left\{ 1 - \frac{\Omega_{ii}^P - \pi_{ii}}{1 + \theta (1 + \pi_{ii})} + \frac{\beta}{1 - \beta} \left( 1 - \Omega_{ii}^P \right) \left( 1 + \pi_{ii} \right) \right\} \]

where \( \Omega_{ii}^S \equiv \frac{\pi_{ii} (\lambda_{i_1}/\pi_{ii})^\beta}{\sum_j \pi_{ij} (\lambda_{j_1}/\pi_{ij})^\beta} \) and \( \Omega_{ii}^P \equiv \frac{r_{ii} (\lambda_{i_1}/\pi_{ii})^\beta}{\sum_j r_{ji} (\lambda_{i_1}/\pi_{ji})^\beta} \).