Illiquidity in Financial Networks

Maryam Farboodi

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Outline

Introduction

Model
Motivation

- Contagion and systemic risk
- A lot of focus on bank inter-connections after the crisis
- Too-interconnected-to-fail
- Interconnections:
  - Propagate a shock from a bank to many banks
  - Exposes banks to counterparty risk
- Reasonable to study bank inter-connections in a network setting
- Focus on Counterparty risk: negative spillover between banks
  - Can induce run on banks in certain states
This Paper!

- Focuses on network formation among banks
- Externalities that banks form the "wrong" network
- Classic externality: Each bank does not fully internalize the failure of others due to his failure
  - Equilibrium social surplus and on profit hurt
- What I want to address: Change in division of surplus
  - Each bank does not care to hit the social surplus as long as his own share increases
Network Literature

- How does a negative shock propagate through the network?
- Can the network structure amplify a shock?
- A lot of work on random networks
- Some recent work focusing on contagion given the network structure
- Very little done on strategic network formation among banks
  - Network on a ring
  - Even less when counter party risk is involved
What is Difficult? :D

- What is a link between two banks (financial institutions)?
- Superposition of at least three networks:
  - OTC derivatives
  - Bilateral lending
  - Common asset holding
- Complex interaction between the three, banks adjust on different margins
Evidence

- Data: bilateral exposures not really known
  - OTC network: approximately a core-periphery network
  - Interconnected core
  - Core banks ("dealers") low net exposure, high gross
  - peripheries net lenders

Figure: OTC bank exposures as a Core-Periphery model
Financial Network

- Directed Graph $G(N, E)$
- Nodes are banks $\{i, j\}$
- Edges are lending $e_{ij}$: $i$ has lent to $j$

Figure: Simple interbank lending network with 5 banks
Basic Idea

- $i$ and $j$ choose to enter a lending relationship (edge $e_{ij}$ added to the network) if

$$
\Phi_i(G + ij) > \Phi_i(G) \\
\Phi_j(G + ij) > \Phi_j(G)
$$

- Has nothing to do with

$$
\sum_k \Phi_k(G + ij) > \sum_k \Phi_k(G)
$$
Equilibrium Concept: Stability

- **Pairwise Stability**
  - No node prefer to unilaterally break a connection
  - No pair of nodes prefer to add a connection

- **Coalitional Stability**
  - No coalition of nodes want to deviate
Outline

Introduction

Model
Environment

- 4 dates $t = 0, 1, 2, 3$
- 4 banks
  - 2 with risky investment opportunity ($R_1, R_2$)
  - 2 without ($S_1, S_2$)
- No discounting
Bank Choice Set

- Bank with investment opportunity
  - From/to which banks borrow/lend (size fix)
  - How much liquidity to hold
  - Invest in long term $h(.)$
- Bank without investment opportunity
  - To which banks lend
  - Invest in $h(.)$
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Model

Timeline

$t=0$
- Network Formed
- Liquidity Choice Made
- Face Value of Debt Determined
- Investment Takes Place

$t=1$
- Risky Investment Type Realized
- Type Revealed to Investor and Hed Counterparty
- Short-term debt Recalled
- If Obligation not Met, Investment Liquidated for L

$t=2$
- Liquidity Shock Realized
- Investment Fails and Return Zero if (Potential) Liquidity Need Not Met

$t=3$
- If Investment has not Failed and has not been Liquidated, Final Return Realized
- If Debt Obligation not met, Borrower Fails and All Return Destroyed
Banks

- Each bank has charter value $V^i$, $i \in \{R, S\}$
- Each bank has raised one unit from uninformed passive outside investors
- Banks can lend to each other
- All borrowing of the form of 1-period debt
- Universal risk neutrality
Technology

- **Risky Asset**
  - Investment \( I \) at date 0
  - Can be of three types \( U, M, B \)
  - Liquidity shock \( \rho \in \{+\alpha I, 0, -\alpha I\} \) at date 2
    - iid across the two investments
  - Can be liquidated at date 1 for \( L \)
  - Return realized at date 3 if not liquidated and potential liquidity need met
    - \( G_k(R) \sim [0, \bar{R}], k \in \{U, M, B\} \)
  - If debt not met: bankruptcy and return is destroyed

- **Safe long term asset**: \( h(.) \) concave
  - Safe final return
  - No intermediate liquidity need
## Asset Random Type

<table>
<thead>
<tr>
<th>Type</th>
<th>Probability</th>
<th>Liquidity Shock ($\rho$)</th>
<th>Return Dist.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>$p$</td>
<td>$-\alpha l$</td>
<td>$G_B(\cdot)/0$</td>
</tr>
<tr>
<td>M</td>
<td>$p_1$</td>
<td>0</td>
<td>$G_M(\cdot)$</td>
</tr>
<tr>
<td>U</td>
<td>$1 - p - p_1$</td>
<td>$\alpha l$</td>
<td>$G_U(\cdot)$</td>
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Introduction

Model

Information

- Each bank knows own and counterparty liquidity need at date 1
- Can withdraw short term debt
Assumptions

- Each bank can borrow from at most two banks
- Each bank can not allocate more than half of it's resources to any use (lend/risky investment/safe long term investment/etc)
- Each bank can lend 0.5 to any other bank
- If invest $I$, can choose to hold $\theta \in \{\alpha I, 0\}$ liquidity
### Risky Bank $R_1$ Balance Sheet

<table>
<thead>
<tr>
<th>Liabilities</th>
<th>Assets $(= A)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$I$</td>
</tr>
<tr>
<td>$b_{S_1 R_1}$ $\in {0, \frac{1}{2}}$</td>
<td>$\theta \in {\alpha I, 0}$</td>
</tr>
<tr>
<td>$b_{S_2 R_1}$ $\in {0, \frac{1}{2}}$</td>
<td>$h(A - I - \theta)$</td>
</tr>
<tr>
<td>$b_{R_2 R_1}$ $\in {0, \frac{1}{2}}$</td>
<td>$b_{R_1 R_2} \in {0, \frac{1}{2}}$</td>
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Link Formation Process

- Each bank chooses a set of links to form (lend and borrow): \( \{ \{ e_i \}, \{ e_i \} \} \)
- Connection formed
- Borrower chooses the amount of liquidity to hold
- Lender chooses the face value of debt
  - The face value would be such that if accepted, in the induced equilibrium it maximizes the surplus which goes to lender
- Can not offer to borrow from more than two banks
Possible Outcomes

Figure: Possible Outcome Networks
Equilibrium Concept: Stability

- **Pairwise Stability**
  - No node prefer to unilaterally break a connection
  - No pair of nodes prefer to add a connection

- **Coalitional Stability**
  - No coalition of nodes want to deviate

- I will only consider symmetric equilibria for now
Risky bank can affect the realized distribution of return through inducing a run in certain states.

Run can be induced through:

- Run erosions some surplus, but also change the distribution of surplus.
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Division of Surplus

- Resources in short supply
- Lender must be payed maximum amount

\[
D^* = \arg \max_D D(1 - G(D))
\]

\[
S = \int_{D^*}^{\bar{R}} Rg(R) dR
\]

\[
L = D^*(1 - G(D^*))
\]

\[
B = \int_{D^*}^{\bar{R}} (R - D^*)g(R) dR
\]
Example: Division of Surplus

**Figure:** Division of Surplus among lender and borrower at the face value which maximizes lender share. Plot is made for return distribution family is $G^n(R) = \frac{R^n}{\hat{R}^n} \sim [0, \hat{R}]$. 
Risky bank dislikes the eventual division of surplus in $M$ but likes it in $U$

- Connect to the other risky and not holding any liquidity
- Chance of failure in $M$ due to counterparty failure
- Preventive run by safe lender at date 1
- Some surplus destructed, but the division of the rest tilted towards risky banks
Results

- I show that under some parameter values, risky banks over connect.
- Equilibrium network is such that:
  - Connections between risky banks create counterparty risk.
  - Counterparty risk destroys social value (too much run).
  - Risky banks get a bigger share of the realized social value at the expense of others.
  - So they have an incentive to form those connections in the first place.
- Adding even less informed outsiders amplifies the losses.