A Dynamic Duverger’s Law

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Introduction

- Duverger’s ‘law’ and ‘hypothesis’
  - Are predictions of static electoral models.
  - Are corroborated empirically with cross-sectional data on the number of parties competing in elections.
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• However, while electoral rules rarely change within countries, observed party systems are not typically stable over time.
• Political environments change over time (voter preferences, important issues, politicians’ characteristics, etc.), so adjustments to party systems should be expected.
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  ▶ Are corroborated empirically with cross-sectional data on the number of parties competing in elections.

• However, while electoral rules rarely change within countries, observed party systems are not typically stable over time.

• Political environments change over time (voter preferences, important issues, politicians’ characteristics, etc.), so adjustments to party systems should be expected.

• **Question**: Do different electoral rules yield systematically different party system dynamics?
Introduction

This paper has two goals

1. We introduce a simple dynamic model of party formation and maintenance.
   - The model suggests that plurality rule leads to less *variability* in the number of parties over time than proportional representation.
   - However, the model makes no *static* prediction about the number of parties in a given electoral system at any point in time.

2. We provide empirical evidence in favour of this prediction in elections from a panel of 44 democracies since 1945.
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Introduction

- In the model, the political environment evolves, so that a currently minor party
  - Finds it costly to compete in current elections.
  - But it may receive more support from future voters.

- Key strategic decision by the supporters of a currently minor party
  - Maintain their party or disband it to re-form a party under better electoral circumstances.
  - An active party generates an option value.

- How this decision is affected by the electoral system
  - Plurality rule imposes higher static costs to currently minor parties (more incentive to exit).
  - Plurality rule imposes higher future barriers to entry to currently minor parties (more incentive to stay).

- Less entry by new parties, but also less exit by established parties, may be expected under plurality rule.
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Literature

- Duverger’s law as an electoral district-level prediction.

- ‘Macro’-level studies of Duverger’s law

- Little work on the comparative dynamics of electoral systems.
Model: Party Formation and Maintenance

- Elections are held at time $t = 1, 2, \ldots$.
- The party that wins the election at $t$ implements policy $x^t \in \{x_{-1}, x_0, x_1\}$, with $x_{-1} < x_0 < x_1$. 
- Parties of type $0$ are present in all elections.
- Parties of type $-1$ and $1$ are formed and maintained by two long-lived interest groups of type $-1$ and $1$ respectively.
- Note: the party system at any time $t$ can feature one, two or three parties.
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Model: Probabilistic Elections

Preference state $s^t$

$$\begin{align*}
Pr(s^t = s_0) &= q \\
Pr(s^t = s_1) &= \frac{1-q}{2}
\end{align*}$$
Model: Probabilistic Elections

<table>
<thead>
<tr>
<th>Preference state $s^t$</th>
<th>Policy support $p^t$</th>
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<tbody>
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$Pr(s^t = s_0) = q$

$Pr(s^t = s_1) = \frac{1-q}{2}$

$\overline{p} > p > \underline{p}$

$\overline{p} + p + \underline{p} = 1$
## Model: Probabilistic Elections

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$p > \bar{p}$
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Pr$(s^t = s_0) = q$

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**Equations:****

- $Pr(s^t = s_0) = q$
- $Pr(s^t = s_1) = \frac{1-q}{2}$
- $\bar{p} > p > \underline{p}$
- $\bar{p} + p + \underline{p} = 1$
Model: Probabilistic Elections

- We model electoral systems as a mapping

\[ P_j^t \rightarrow x^t = x_j \]

  - party \( j \)'s win probability
  - policy outcome \( x^t = x_j \)

- Plurality rule and proportional representation are represented by different ‘contest success functions’.
Model: Probabilistic Elections

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<tr>
<th>Party structure $\phi^t$</th>
<th>Probability of winning</th>
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<tr>
<td>$X_1$</td>
<td>$P_1^t$</td>
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<tr>
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<tr>
<td>$X_{-1}$</td>
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Proportional representation
Model: Probabilistic Elections

Party structure $\phi^t$

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<tr>
<td>$s_1$</td>
<td>$x_1$</td>
<td>$\bar{p} + \alpha$</td>
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<tr>
<td>$s_0$</td>
<td>$x_0$</td>
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<tr>
<td>$s_{-1}$</td>
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<td>$p - \alpha$</td>
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- Minority penalty $\alpha > 0$
- Plurality rule
## Model: Probabilistic Elections

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Minority penalty $\alpha > 0$  
Plurality rule
Model: Probabilistic Elections

<table>
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<th>Party structure $\phi^{t-1}$</th>
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<th>Probability of winning</th>
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<tr>
<td>$x_1$</td>
<td>$X_1$</td>
<td>$P_1^t - \beta$</td>
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<td>$P_0^t$</td>
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Entry penalty $\beta > 0$  
Plurality rule
**Model: Probabilistic Elections**

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Entry penalty $\beta > 0$  
Plurality rule
Model: Payoffs and Equilibrium

- Interest group $j$ is risk-neutral and has preferences
  \[ x_j \succ j x_0 \succ j x_{-j}. \]

- Supporting a party is costly for activist $j$
  - Forming a new party imposes cost $c_\bar{c} > 0$.
  - Maintaining an existing party imposes cost $c \in (0, \bar{c})$.

- Interest groups have common discount factor $\delta \in (0, 1)$.
Model: Payoffs and Equilibrium

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- Interest groups have common discount factor $\delta \in (0, 1)$.

- We focus on Markov perfect equilibria $\sigma = (\sigma_{-1}, \sigma_1)$, in which interest groups condition their actions at $t$ only on the payoff-relevant state $\left( s^t, \phi^{t-1} \right)$. 
Results for Proportional Representation

- Under proportional representation, we consider a strategy profile $\sigma^{PR}$ such that interest group $j$ supports a party (existing or new) if and only if $s^t \neq s_{-j}$. 
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- Under $\sigma^{PR}$
  - Two or three parties compete in elections, depending on the preference state.
  - The party formation and maintenance decisions of party $j$ are independent of party structures.
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Proposition 1

We identify conditions under which $\sigma^{PR}$ is the unique Markov perfect equilibrium under proportional representation.
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- The option value of a maintained party (indexed by $\bar{c} - c$) must be low enough to ensure that party $j$ is disbanded when $s^t = s_{-j}$. 
Model: Results for Plurality Rule

- Under plurality rule, we focus on strategy profiles in which
  - Interest group $j$ forms a new party if and only if $s^t = s_j$.
  - Interest group $j$ supports an existing party if $s^t \in \{s_j, s_0\}$.

- Does interest group $j$ support an existing party when $s^t = s_j - j$?
  - Yes under profile $\sigma_{PL}$ with maximal participation.
  - No under profile $\sigma_{PL}$ with minimal participation.

- Under $\sigma_{PL}$, three parties compete in all elections.
- Under $\sigma_{PL}$, two parties compete in all elections, although their identities change with the preference state.
- Under both $\sigma_{PL}$ and $\sigma_{PL}$, the participation decisions of party $j$ are persistent.
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- Does interest group $j$ support an existing party when $s^t = s_{-j}$?
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Model: Results for Plurality Rule

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  - Interest group $j$ forms a new party if and only if $s^t = s_j$.
  - Interest group $j$ supports an existing party if $s^t \in \{s_j, s_0\}$.
- Does interest group $j$ support an existing party when $s^t = s_{j-}$?
  - Yes under profile $\overline{\sigma}^{PL}$ with maximal participation.
  - No under profile $\underline{\sigma}^{PL}$ with minimal participation.
- Under $\overline{\sigma}^{PL}$, three parties compete in all elections.
- Under $\underline{\sigma}^{PL}$, two parties compete in all elections, although their identities change with the preference state.
- Under both $\overline{\sigma}^{PL}$ and $\underline{\sigma}^{PL}$, the participation decisions of party $j$ are persistent.
Proposition 2

We identify conditions under which there exist bounds $\alpha$ and $\bar{\alpha}$ such that $\sigma^{PL}$ is a Markov perfect equilibrium whenever $\alpha > \alpha$ and $\sigma^{PL}$ is a Markov perfect equilibrium whenever $\alpha < \bar{\alpha}$. 
Proposition 2

We identify conditions under which there exist bounds $\underline{\alpha}$ and $\overline{\alpha}$ such that $\underline{\sigma}^{PL}$ is a Markov perfect equilibrium whenever $\alpha > \underline{\alpha}$ and $\overline{\sigma}^{PL}$ is a Markov perfect equilibrium whenever $\alpha < \overline{\alpha}$.

- $\underline{\sigma}^{PL}$ is an equilibrium if minority penalty is sufficiently high for interest group $j$ to disband a party in state $(s_{-j}, \{j\})$.
- $\overline{\sigma}^{PL}$ is an equilibrium if minority penalty is sufficiently low for interest group $j$ to maintain a party in state $(s_{-j}, \{-1,1\})$. 
Model: Comparative Predictions

• The parameter region satisfying Proposition 2 is nested in that satisfying Proposition 1, and it can be shown by example to be nonempty.
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At any time $t$, there can be more or less parties under plurality rule than under proportional representation.

- No variability in the number of parties under $\sigma_{PL}$.
- There is less expected party exits under $\sigma_{PL}$ than under $\sigma_{PR}$, because there is three-party competition in $s_0$ under $\sigma_{PR}$.
Model: Comparative Predictions

- The parameter region satisfying Proposition 2 is nested in that satisfying Proposition 1, and it can be shown by example to be nonempty.
- At any time $t$, there can be more or less parties under plurality rule than under proportional representation.
- However, in both equilibria $\sigma^{PL}$ and $\sigma^{PL}$ under plurality rule, there is less expected change in the number of parties than in equilibrium $\sigma^{PR}$ under proportional representation.
  - No variability in the number of parties under $\sigma^{PL}$.
  - There is less expected party exits under $\sigma^{PL}$ than under $\sigma^{PR}$, because there is three-party competition in $s_0$ under $\sigma^{PR}$. 
Empirical Evidence: Data

• We aim to provide a robust measure of the correlation between the variability in the number of parties and electoral systems across democratic countries.

• This is a stylised fact which has yet to be established.

• We use data from Constituency-Level Elections (CLE), which contains (eligible) district-level electoral results for multiple elections from 44 democracies.
Empirical Evidence: Measuring Proportionality

- Few electoral systems correspond to pure plurality rule and pure proportional representation.
- Our results are robust to using three commonly used proxies for electoral systems
  - The *majoritarian dummy variable* of Persson and Tabellini (2005).
  - *Effective district magnitude*: the average number of legislators elected per electoral district (excluding compensatory seats).
  - The *disproportionality index* of Gallagher (1991): the sum of squared differences between parties’ vote and seat shares.
Empirical Evidence: Measuring Proportionality

Figure: Electoral Proportionality: Three Measures
Empirical Evidence: Measuring Party Entry and Exit

We say party \( j \) enters in district \( d \) in election \( e \) of country \( c \) if

\[
p_{jdc}(e-1) < 0.05 \quad \text{and} \quad p_{jdce} \geq 0.05,
\]

and we define \( \text{exit} \) in a district similarly (our results are robust to different entry thresholds).
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p_{jdc(e-1)} < 0.05 \text{ and } p_{jdc(e)} \geq 0.05,
\]

and we define exit in a district similarly (our results are robust to different entry thresholds).

• Parties may not be active in all districts, but the number of electoral districts varies across electoral systems.

• Let \( \sigma_{dce} \) denote the fraction of the total seats in the national legislature contributed by district \( d \).

• Let \( n_{dce} \) denote the number of entering parties in district \( d \) during election \( e \) in country \( c \). The Total Entries \( N_{ce} \) is

\[
N_{ce} = \sum_{d=1}^{D_{ce}} n_{dce} \cdot \sigma_{dce},
\]

and we define Total Exits similarly.
Empirical Evidence: Specification

- Our main results consist of regressing our measure of party entry and exit on the majoritarian dummy.
- Our model supports a negative relationship.
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- Our main results consist of regressing our measure of party entry and exit on the majoritarian dummy.
- Our model supports a negative relationship.
- All continuous variables are specified in logarithms. This minimizes concerns that more variability in the number of parties could be mechanically driven by differences in the number of parties across electoral systems.
- We also control directly for the number of parties.
- This concern is especially valid since, according to the static Duverger’s law, the number of parties decreases in the disproportionality of electoral systems.
## Empirical Evidence: Duverger’s Law

### Table: Static Tests of Duverger’s Law

<table>
<thead>
<tr>
<th>Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Majoritarian Dummy</td>
<td>-0.23**</td>
<td>-0.17*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.10)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effective District Magnitude</td>
<td></td>
<td></td>
<td></td>
<td>0.09***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.02)</td>
<td></td>
<td></td>
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<tr>
<td>Disproportionality Index</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-2.76***</td>
<td>-2.77***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.86)</td>
<td>(0.88)</td>
</tr>
<tr>
<td>Decade, Regional and District Number Controls</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.06</td>
<td>0.23</td>
<td>0.10</td>
<td>0.23</td>
<td>0.31</td>
<td>0.42</td>
</tr>
<tr>
<td>Number of Observations</td>
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<td>454</td>
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</tbody>
</table>

Notes: Heteroskedasticity robust standard errors clustered by country are presented in parentheses. *** - 1%, ** - 5% and * - 10% significance level.
## Empirical Evidence: Main Results

**Table: Dynamic Tests of Duverger’s Law: Total Movements**

<table>
<thead>
<tr>
<th>Variable</th>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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</thead>
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<td>Majoritarian Dummy</td>
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<td>-0.42***</td>
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<td></td>
<td>(0.11)</td>
<td>(0.13)</td>
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<tr>
<td>Average District Magnitude</td>
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<td>0.12***</td>
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<tr>
<td></td>
<td></td>
<td>(0.04)</td>
<td>(0.04)</td>
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<td></td>
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</tr>
<tr>
<td>Disproportionality Index</td>
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<td></td>
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<td>-0.93</td>
<td>-0.01</td>
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<td>(0.79)</td>
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<td>Decade, Regional, Number of Districts and Parties Controls</td>
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<td>N</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>$R^2$</td>
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<td>0.27</td>
<td>0.04</td>
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</table>

*Notes:* Heteroskedasticity robust standard errors clustered by country are presented in parentheses. *** - 1%, ** - 5% and * - 10% significance level.
Empirical Evidence: Robustness to Vote Thresholds

Figure: Coefficient estimates for majoritarian dummy and effective district magnitude at different vote thresholds, along with their 95% confidence intervals
Conclusion

- We present a novel dynamic reinterpretation of Duverger’s law.
- We construct a minimal dynamic model with a clean time series prediction and show that this prediction is consistent with available evidence.
Conclusion

• We present a novel dynamic reinterpretation of Duverger’s law.

• We construct a minimal dynamic model with a clean time series prediction and show that this prediction is consistent with available evidence.

• Our ‘macro’ focus on the number of national parties, while important, rules out causal analysis.
  ▶ Our paper points to the importance of studying the comparative intertemporal properties of electoral systems.
  ▶ These questions could be posed in ways that allows causal claims and tests.