Financial Networks and Contagion

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Introduction

• Possible contagions make understanding network structure of financial interactions critical.

• Need tools to help evaluate contagion risk.

• Although large cascades are usually (today) off-path, it’s important to keep them off-path.

• Need to understand effects of integration and diversification.
There is a growing literature on financial networks:
Rochet, Tirole (1996); Allen, Gale (2000); Eisenberg, Noe (2001); Lorenza, Battiston, Schweitzer (2009); Babus (2009); Allen, Babus (2009); Gai, Kapadia (2009); Blume et al (2011ab); Demange (2011); Billio et al (2011); Diebold, Yilmaz (2011); Dette, Pauls, Rockmore (2011); Cabrales, Gottardi, Vega-Redondo (2011); Cohen-Cole, Petacchini, Zenou (2012); Acemoglu et al (2012); Gouriéroux, Héam, Monfort (2012); Acemoglu, Ozdaglar, Tahbaz-Salehi (2013);...
Our Contributions:

• Develop network-based measures of cascades.

• *Distinguish* the effects of diversification and integration.

• Highlight nonmonotonic effects of diversification and integration on contagions.

• Offer a simple illustration of how the model can be used empirically.
Outline

• Model

• Cascades: Diversification and Integration

• Endogenous Values and Moral Hazard

• An Illustration with European Debt Data
Basics of the Model:

• Organizations (countries, banks, firms etc.) have claims on:
  • fundamental assets,
  • other organizations.

• When an organization’s value falls below a critical level, the values of others’ claims on it drop – **discontinuously**:
  • e.g., Greek tax receipts not enough to pay debt; creditors take >50% loss on value of their claims.

• Drop in value of one organization leads to drop in values of others they have financial arrangements with – cascades.
Model

- \{1, \ldots, n\}: Organizations (countries, firms, banks...)
- \{1, \ldots, m\}: Assets (primitive investments)
- \( p_k \): price of asset \( k \)
- \( D_{ik} \): holdings of asset \( k \) by organization \( i \)
Cross Holdings:

- $C_{ij}$: cross holdings: fraction of org $j$ owned by org $i$

- $C_{ii} = 0$: (don’t own yourself)

- $\hat{C}_{ii} = 1 - \sum_j C_{ji}$: fraction of org $i$ privately held
Approximate Debt by Shares:

\[ \text{Debt:} \quad \min[ C_{ij} V_j, X ] \]

Relevant range for contagions

\[ \text{Share/Debt Value} \]
Value of an Organization

book value:

\[ V_i = \sum_k D_{ik} p_k + \sum_j C_{ij} V_j \]

direct asset holdings

cross-holdings
Value of an Organization

\[ V_i = \sum_k D_{ik} \rho_k + \sum_j C_{ij} V_j \]

\[ V = D \rho + CV \]

\[ V = (I - C)^{-1} D \rho \]

Leontief calculation of book value
Value of an Organization

Book value:

\[ V = (I - C)^{-1} D p \]

Market value – value to final (private) investors.

\[ v_i = \hat{C}_{ii} V_i \]

\[ v = \hat{C} (I - C)^{-1} D p \]

\[ v = A D p \]

(cf. Brioschi et al. 89, Fedenia et al. 96)

\[ A_{ij} : \]

fraction of the returns owned by org \( j \) that ultimately accrue to private shareholders of \( i \)
Example

• Two organizations: \( n = 2 \)

• Each owns half of the other:

\[
\mathbf{C} = \begin{bmatrix}
0 & 0.5 \\
0.5 & 0 \\
\end{bmatrix}
\]

• Implied holdings by private investors:

\[
\hat{\mathbf{C}} = \begin{bmatrix}
0.5 & 0 \\
0 & 0.5 \\
\end{bmatrix}
\]
Example

- Two organizations: \( n = 2 \)

- Each owns half of the other:

\[
C = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix}
\]

- Final investors’ claims on assets:

\[
\hat{C} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \quad A = \hat{C} (I - C)^{-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}
\]
Example

\[ C_{21} = 0.5 \]

\[ \hat{C}_{11} = 0.5 \]

\[ \hat{C}_{22} = 0.5 \]

\[ C_{12} = 0.5 \]
Example

What happens to $1 of investment income to 1?
What happens to $1 of investment income to 1?
Example

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What happens to $1 of investment income to 1?
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Example

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What happens to $1 of investment income to 1?

\[ \hat{C} (I - C)^{-1} \]
If an organization’s value drops below some threshold $v_i$, its value falls by $\beta_i$.

$$b_i(v_i) = \begin{cases} \beta_i & \text{if } v_i < v_i \\ 0 & \text{otherwise.} \end{cases}$$

**with bankruptcies:**

$$v = A \left[ Dp - b(v) \right]$$
Value Drop

Value of $i$'s claim on $j$

$A_{ij} [Dp - b(v)]_j$

Discontinuous drop

Value of $j$'s asset ($p_j$)
Equilibria

• The equilibria form a complete lattice.

• We focus on the unique “best-case” equilibrium where the fewest organizations fail.

• Easy algorithm to find it:
  • Identify organizations that fail even if no others do (called a first failure).
  • Identify those that fail due to the failures identified above.
  • Iterate.
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• Endogenous Values and Moral Hazard

• An Illustration with European Debt Data
Three Necessary Components of a Cascade

• **A first failure**: some organization must fail.

• **Contagion**: some other organization(s) must be sufficiently exposed to the org. to fail too.

• **Wider propagation**: for a cascade to continue, the network must have sufficiently large components.
What Affects Cascades:

• **Diversification:** How many other organizations does a typical organization cross hold?

• **Integration:** How much of a typical organization is cross-held?
What Affects Cascades:

- Look at some simulations on random graphs
- Some analytic results too
Simulation Setup

- \( n = 100 \) organizations

- Simple random network \( G \):
  - \( \Pr(G_{ij} = 1) = \frac{d}{n - 1} \)
  - \( d = \) expected number of other organizations that an organization holds (\( d = \text{level of diversification} \))

- Fraction \( c \) of an org. is evenly split among those holding it; \( 1 - c \) held privately (\( c = \text{level of integration} \))

- So:
  \[
  C_{ij} = \frac{c \ G_{ij}}{d_j} \quad \hat{C}_{ii} = 1 - c
  \]
The Exercise

• One asset per organization (their investments).
  • Each starts at value $p_i = 1$;
  • Hence, values $v = A \ p = A \ 1$

• Pick one asset to devalue to 0.

• Threshold is $v_i = \theta v_i$ for all $i$; bankruptcy means lose all remaining value.

• Look at resulting cascade.
Diversification Preview: Dangerous Middle Levels

• Low diversification:
  • fragmented network, no widespread contagion

• Medium diversification
  • Connected network, contagion is possible
  • Exposure to only a few others makes it easy to spread

• High diversification
  • Little exposure to any single other organization
  • Failures do not spread
Diversification and Contagion: 93% threshold, c=.5

Percent of Orgs that Fail

Degree: Expected # of cross-holdings

θ = .93
Diversification and Contagion: Various Thresholds

Percent of Orgs that Fail

Degree: Expected # of cross-holdings
Diversification:
Dangerous Middle Levels

\[ d \text{ low} \quad d \text{ middle} \quad d \text{ high} \]
Diversification

• Nonmonotonicity is due to competing forces:
  
  • diversification increases component size;
  
  • diversification decreases contagion from one organization to its neighbors.
Integration

• Low integration: little exposure to others, failures don’t trigger others

• Middle integration: exposure to others substantial enough to trigger contagion

• High integration: difficult to get a first failure – failure of own assets does not trigger failure
Integration: .93 Threshold

Percent of Orgs that Fail

Degree: Expected # of cross holdings
Integration: .93 Threshold

Percent of Orgs that Fail

Degree: Expected # of cross-holdings
High Integration - First Failures, threshold .8

Frequency of first failures

Degree: Expected # of cross-holdings
Integration

- Increases exposure to others
- Decreases exposure to own idiosyncratic risks
- *Increases contagion, but can decrease first failures*
Proposition: Nonmonotonic Diversification and Integration

Network where each org. has:

• expected degree $d$ (in- and out $[d]$ or $[d]$);
• threshold $v$;
• integration $c$;
• initial asset values 1;
• $v_{\text{max}}, v_{\text{min}}$ be highest, lowest realized initial values.
Proposition: Nonmonotonic Diversification and Integration

If integration is very low or high \[ c(1 - c) < v_{\text{min}} - v \] there is no limit contagion.

Middle integration levels, diversification matters:

- low degree \[ d < 1 \], no limit contagion.
- medium degree \[ 1 \leq d < \left[ \frac{c(1-c)}{v_{\text{max}} - v} \right] \] get limit contagion.
- high degree \[ \left[ \frac{c}{v_{\text{min}} - v} \right] < d \], no limit contagion.
Summary on Cascades: Integration and Diversification:

• **A first failure:** some organization needs to fail
  
  integration decreases
  
  own-asset dependence

• **Initial Contagion:** some neighbors need to be sufficiently exposed to fail too
  
  integration increases
  
  diversification decreases
  
  exposure of neighbors
  
  exposure of specific neighbors

• **Interconnection:** to continue to cascade widely, the network must have sufficiently large components
  
  diversification increases
  
  component size
Other Random Graphs

Core - Periphery

- 10 Core organizations; 90 Periphery/Regional organizations
- Periphery orgs are randomly connected to 1 core org.
- Core orgs are connected to all other core orgs.
- Size of organizations: $p_{\text{Periphery}} = 1$; $p_{\text{Core}} = 8$
- $C_{CP} = C_{PC}$ fraction of each organization cross held by other types of orgs
- $C_{CC}$ cross holdings of core orgs by other core orgs.
- So, $\hat{C}_{CC} = 1 - C_{PC} - C_{CC}$; and $\hat{C}_{PP} = 1 - C_{CP}$
Fedwire Interbank payments, nodes accounting for 75% of total, Soromaki et al (2007), 25 nodes are completely connected.
Core Failure; .98 Threshold

Percent of Orgs that Fail

Core dependence on core ($0.2 \leq C_{CC} \leq 0.38$)

$C_{PC} = 1/20$
Core Failure; .98 Threshold

Percent of Orgs that Fail

Core dependence on core ($0.2 \leq C_{CC} \leq 0.38$)
Core Periphery Model

Given an initial core failure:

• Directly dependent periphery orgs fail unless their interdependence with core orgs is very low.

• If the contagion spreads to the other core orgs, then the whole system collapses. This is more likely when core orgs are more dependent on each other.
Periphery Failure; .98 Threshold

Percent of Orgs that Fail

Core dependence on core \((0.1 \leq C_{CC} \leq 0.38)\)

\[ C_{CP} = 1/2 \]
Periphery Failure; .98 Threshold

Percent of Orgs that Fail

Core dependence on core (0.1 ≤ C_{CC} ≤ 0.38)

C_{CP} = .6

.45

.5

.55

.35

.4

.3
Core-Periphery Model

Given a initial periphery failure:

- Only connection is to a core org and contagion requires very high dependency of the core org to the periphery.
- As core dependence on other core orgs begins to increase, the contagion spreads to other core orgs.
- As integration of core orgs further increases, the periphery failing is spread too evenly for a core failure

(The integration non-monotonicity)
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Endogenous Values:

Let $p_1 = p_2 = 10$ (D=I) and so $v_1 = v_2 = 10$

$C = \begin{bmatrix} 0 & .5 \\ .5 & 0 \end{bmatrix}$  

$A = \mathring{C} (I - C)^{-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$

- What if $v_1 = 8$ and $v_2 = 11$ and $b_2 = 6$ ?

- Without any intervention, 2 loses 6
Bailout:

\[ C = \begin{bmatrix} 0 & .5 \\ .5 & 0 \end{bmatrix} \quad A = \hat{C} (I-C)^{-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} \]

- Without any intervention, incur a cost 6 and so the values become: \( v_1 = 8 \) and \( v_2 = 6 \)
  
  (2 bears 2/3 of the cost and 1 bears 1/3)
Bailout:

\[
C = \begin{bmatrix}
0 & 0.5 \\
0.5 & 0
\end{bmatrix} \quad A = \hat{C} (I-C)^{-1} = \begin{bmatrix}
2/3 & 1/3 \\
1/3 & 2/3
\end{bmatrix}
\]

• Without any intervention, incur a cost 6 and so the values become: \(v_1=8\) and \(v_2=6\)
  
  \((2\text{ bears } 2/3\text{ of the cost and }1\text{ bears }1/3)\)

• If instead 1 gave a $ to 2, then values are
  \(v_1=9\) and \(v_2=11\)
Moral Hazard

- Suppose that 2 can choose:
  \[ b_2 \] either 2 or 6
  \[ v_2 \] either 10 or 11

Only gets a transfer if \( b_2 = 6 \) and \( v_2 = 11 \)

Best off by choosing highest bankruptcy cost and threshold
Outline

• Model

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• Endogenous Values and Moral Hazard (skip)

• An Illustration with European Debt Data
• Consider 6 key countries in Europe that have substantial cross-holdings of each other’s debt.

• Treat them as an isolated system (illustrating exercise, not for policy...).

• See what happens if values fall (contraction) and debt is devalued.
<table>
<thead>
<tr>
<th></th>
<th>France</th>
<th>Germany</th>
<th>Greece</th>
<th>Italy</th>
<th>Portugal</th>
<th>Spain</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>0</td>
<td>174862</td>
<td>1960</td>
<td>40311</td>
<td>6679</td>
<td>27015</td>
</tr>
<tr>
<td>Germany</td>
<td>198304</td>
<td>0</td>
<td>2663</td>
<td>227813</td>
<td>2271</td>
<td>54178</td>
</tr>
<tr>
<td>Greece</td>
<td>39458</td>
<td>32977</td>
<td>0</td>
<td>2302</td>
<td>8077</td>
<td>1001</td>
</tr>
<tr>
<td>Italy</td>
<td>329550</td>
<td>133954</td>
<td>444</td>
<td>0</td>
<td>2108</td>
<td>29938</td>
</tr>
<tr>
<td>Portugal</td>
<td>21817</td>
<td>30208</td>
<td>51</td>
<td>3188</td>
<td>0</td>
<td>78005</td>
</tr>
<tr>
<td>Spain</td>
<td>115162</td>
<td>146096</td>
<td>292</td>
<td>26939</td>
<td>21620</td>
<td>0</td>
</tr>
</tbody>
</table>
### Derived Exposures

\[ A = \begin{pmatrix}
(\text{France}) & (\text{Germany}) & (\text{Greece}) & (\text{Italy}) & (\text{Portugal}) & (\text{Spain}) \\
(\text{France}) & 0.71 & 0.13 & 0.13 & 0.17 & 0.07 & 0.11 \\
(\text{Germany}) & 0.18 & 0.72 & 0.12 & 0.11 & 0.09 & 0.14 \\
(\text{Greece}) & 0.00 & 0.00 & 0.67 & 0.00 & 0.00 & 0.00 \\
(\text{Italy}) & 0.07 & 0.12 & 0.03 & 0.70 & 0.03 & 0.05 \\
(\text{Portugal}) & 0.01 & 0.00 & 0.02 & 0.00 & 0.67 & 0.02 \\
(\text{Spain}) & 0.03 & 0.03 & 0.02 & 0.02 & 0.14 & 0.68 \\
\end{pmatrix} \]
Cascades

- Set $v_i$ to be a fraction $\theta$ of 2008 GDP.

- Look at 2011 GDP as the asset $p_i$.

- Calculate $v_i$.

- Calculate cascades in best equilibrium.

- Set $\beta_i = \frac{v_i}{2}$ (could rescale everything to debt levels – here based on GDP levels)
## Normalized GDPs

<table>
<thead>
<tr>
<th>Country</th>
<th>2008</th>
<th>2011</th>
<th>Drop %</th>
</tr>
</thead>
<tbody>
<tr>
<td>France</td>
<td>11.99</td>
<td>11.62</td>
<td>3</td>
</tr>
<tr>
<td>Germany</td>
<td>15.28</td>
<td>14.88</td>
<td>3</td>
</tr>
<tr>
<td>Greece</td>
<td>1.47</td>
<td>1.27</td>
<td>14</td>
</tr>
<tr>
<td>Italy</td>
<td>9.65</td>
<td>9.20</td>
<td>5</td>
</tr>
<tr>
<td>Portugal</td>
<td>1.06</td>
<td>1.00</td>
<td>6</td>
</tr>
<tr>
<td>Spain</td>
<td>6.70</td>
<td>6.25</td>
<td>7</td>
</tr>
<tr>
<td>( \theta ) fraction</td>
<td>.90</td>
<td>.93</td>
<td>.935</td>
</tr>
<tr>
<td>-------------------------</td>
<td>-----</td>
<td>-----</td>
<td>------</td>
</tr>
<tr>
<td>First Failure</td>
<td>Greece</td>
<td>Greece</td>
<td>Greece</td>
</tr>
<tr>
<td>Second Failure</td>
<td>Portugal</td>
<td>Spain</td>
<td></td>
</tr>
<tr>
<td>Third Failure</td>
<td>Spain</td>
<td>France</td>
<td></td>
</tr>
<tr>
<td>Fourth Failure</td>
<td>France Germany</td>
<td>Germany Italy</td>
<td></td>
</tr>
<tr>
<td>Fifth Failure</td>
<td>Italy</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
European Debt

- Portugal fragile: little exposure, but close to threshold
- Portugal triggers Spain, triggers France, Germany
- Italy is last to *cascade*: held by others, but much less exposed to Spain than France, Germany (but exposed to Fr, G)
Conclusions

- Values need to be derived from cross holdings carefully

- Diversification and Integration both face (different) competing effects, nonmonotonicities

- Model can serve as a foundation for studying bailouts and incentives...

- Can be taken to data...
Thank you!