What is the Expected Return on the Market?

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Outline

- $M \& \mathbb{P}^* \rightarrow \mathbb{P}$
- Option-implied lower bound on the equity premium
- Option-implied upper and lower bounds on all moments
- What is the probability that the market goes up? What is the expected excess return \textit{conditional} on an up-move or on a down-move?
A lower bound on the equity premium

1 month horizon, annualized, in %
A lower bound on the equity premium

1 year horizon, in %
The equity premium

According to John Campbell’s Princeton Lecture in Finance
Assumptions

- No arbitrage
- Liquid market in index options
Whose expected return?

- Take the perspective of an investor who is *unconstrained*, who *holds the market* over the horizon of interest, and whose relative risk aversion (which need not be constant) is *at least one*.
- I have nothing to say about
  - Constrained investors
  - Irrational investors
  - Unconstrained rational investors who don’t hold the market
  - The connection between prices and cashflows or consumption
An identity

- Notation: \( \frac{1}{R_{f,T}} \mathbb{E}^* X_T = \mathbb{E} M_T X_T \)
- Notice that

\[
\text{var}^* R_T \frac{R_T}{R_{f,T}} = \frac{1}{R_{f,T}} \mathbb{E}^* R_T^2 - \frac{1}{R_{f,T}} (\mathbb{E}^* R_T)^2 \\
= \mathbb{E}(M_T R_T^2) - R_{f,T} \\
= \mathbb{E} R_T - R_{f,T} + \mathbb{E}(M_T R_T^2) - \mathbb{E} R_T \\
= \mathbb{E} R_T - R_{f,T} + \text{cov}(M_T R_T, R_T) \leq 0
\]

- This connects something we can measure to something interesting plus something we can control
A lower bound on the equity premium

Result (A bound on the equity premium, version 1)

If there is a one-period investor who holds the market from time 0 to time $T$, and whose risk aversion $\gamma(c) \equiv -cu''(c)/u'(c) \geq 1$, then

$$E [R_T - R_{f,T}] \geq \frac{\text{var}^* R_T}{R_{f,T}}$$

With log utility, $\gamma(x) \equiv 1$, this holds with equality

- Does not require power utility: $\gamma$ doesn’t have to be constant
- Does not require lognormality
A lower bound on the equity premium

Proof.

- The given assumption implies that the SDF is proportional to $u'(R_T)$, so we must show that $\text{cov}(R_T u'(R_T), R_T) \leq 0$.
- This holds because $R_T u'(R_T)$ is decreasing in $R_T$: its derivative is

$$u'(R_T) + R_T u''(R_T) = -u'(R_T) [\gamma(R_T) - 1] \leq 0$$
A lower bound on the equity premium

- Paper extends to the case of an intertemporal consumer-investor:

\[ V[W] = \max_{C, W_i} u(C) + \beta \mathbb{E} V \left( (W - C) \sum_i w_i R_T^{(i)} \right) \]

- The right coefficient of risk aversion is not \(-\frac{C u''(C)}{u'(C)}\) but \(-\frac{W V''(W)}{V'(W)}\)

- The envelope condition, \(u'(C) = V'(W)\), implies that

\[ -\frac{W V''(W)}{V'(W)} = -\frac{C u''(C)}{u'(C)} \times \frac{\partial \log C}{\partial \log W} \]

\[ \text{must be} \geq 1 \]
A lower bound on the equity premium

- If your preferred model is Bansal-Yaron or Campbell-Cochrane, the above does not apply: it requires separable utility.
- But we can deal with these cases too: under conditional lognormality,

\[ \mathbb{E} [R_T - R_{f, T}] \geq \frac{\text{var}^* R_T}{R_{f, T}} \]

if and only if the conditional Sharpe ratio is greater than the conditional volatility of the market—which holds in the data.

- The inequality also holds with Epstein-Zin preferences whenever \( \gamma \geq 1 \) and EIS is sufficiently close to 1 (no need for lognormality here).
A lower bound on the equity premium

\[
\frac{\text{var}^* R_T}{R_{f,T}} \leq \mathbb{E} R_T - R_{f,T} \leq R_{f,T} \cdot \sigma(M) \cdot \sigma(R_T)
\]

- **Left-hand inequality** is the new result
  - Good: relates unobservable equity premium to an observable quantity
  - Bad: requires an economic assumption

- **Right-hand inequality** is the Hansen-Jagannathan bound
  - Good: no assumptions
  - Bad: neither side is directly observable
A lower bound on the equity premium

- Merton (1980) estimated equity premium from

  \[ \text{instantaneous risk premium} = \gamma \sigma^2 \]

- Assumes power utility and the market’s price follows a diffusion
- No distinction between risk-neutral and real-world variance in a diffusion-based model (Girsanov’s theorem)
- Beyond diffusions, the appropriate generalization relates the risk premium to the *risk-neutral* variance
Measuring risk-neutral variance

- The return on the market is $R_T \equiv (P_T + D_0)/P_0$
  - I assume that dividends are paid at time $T$ but known at time 0
- The forward price of the market to time $T$ is known at time 0:
  \[ F_{0,T} = \mathbb{E}^* P_T = P_0 R_{f,T} - D_0 \]
- The forward price can be calculated directly from option prices as the unique strike at which call and put prices are equal
- Implied dividend yield is $D_0/P_0 = R_{f,T} - F_{0,T}/P_0$
Implied forward-looking dividend yield on S&P 500 index

Annual horizon, 20-day moving average
Measuring risk-neutral variance

**Result**

*The risk-neutral variance can be computed from call and put prices:*

\[
\text{var}^* R_T = \frac{2R_{f,T}}{P_0^2} \left( \int_{0}^{F_{0,T}} \text{put}_{0,T}(K) \, dK + \int_{F_{0,T}}^{\infty} \text{call}_{0,T}(K) \, dK \right)
\]

- Note that for any \( x \), we have
  \[
x^2 = 2 \int_{0}^{\infty} \max\{0, x - K\} \, dK
  \]

- Setting \( x = P_T \) and taking risk-neutral expectations,
  \[
  \mathbb{E}^* P_T^2 = 2 \int_{0}^{\infty} \mathbb{E}^* \max\{0, P_T - K\} \, dK = 2R_{f,T} \int_{0}^{\infty} \text{call}_{0,T}(K) \, dK
  \]
VIX and SVIX

\[ \text{VIX}^2 = \frac{2R_{f,T}}{T} \left( \int_0^{F_0,T} \frac{1}{K^2} \text{put}_T(K) \, dK + \int_{F_0,T}^\infty \frac{1}{K^2} \text{call}_T(K) \, dK \right) \]

\[ \text{SVIX}^2 = \frac{2R_{f,T}}{T} \left( \int_0^{F_0,T} \frac{1}{P_0^2} \text{put}_T(K) \, dK + \int_{F_0,T}^\infty \frac{1}{P_0^2} \text{call}_T(K) \, dK \right) \]

- These are definitions, not statements about pricing
- What VIX does not measure: \( \text{VIX}^2 \neq \frac{1}{T} \mathbb{E}^* \left[ \int_0^T \sigma^2_t \, dt \right] \)
- Both measure variability of the simple return \( R_T = \frac{S_T}{S_0} \)

Entropy: \( \text{VIX}^2 = \frac{2}{T} (\log \mathbb{E}^* R_T - \mathbb{E}^* \log R_T) \)

Variance: \( \text{SVIX}^2 = \frac{1}{T} \text{var}^* R_T \)

- Entropy is sensitive to left tails, variance is sensitive to right tails
Figure: VIX (dotted) and SVIX (solid). Jan 4, 1996–Jan 31, 2012
Figure shows 10-day moving average. $T = 1$ month
Figure: VIX minus SVIX. Jan 4, 1996–Jan 31, 2012
Figure shows 10-day moving average. $T = 1$ month
Conditionally lognormal models can’t match the data

- If returns and the SDF are conditionally lognormal with return volatility $\sigma_R$ then we can calculate VIX and SVIX in closed form:

  $$SVIX^2 = \frac{1}{T} e^{2rT} \left( e^{\sigma_R^2 T} - 1 \right)$$

  $$VIX^2 = \sigma_R^2$$

- VIX would be lower than SVIX

- No conditionally lognormal model can match the data
A lower bound on the equity premium

Lower bound on 1mo equity premium, annualized, 10-day moving average
A lower bound on the equity premium

Lower bound on 3mo equity premium, annualized, 10-day moving average
A lower bound on the equity premium

Lower bound on 1yr equity premium, annualized, 10-day moving average
A lower bound on the equity premium

<table>
<thead>
<tr>
<th>horizon</th>
<th>mean</th>
<th>s.d.</th>
<th>min</th>
<th>1%</th>
<th>10%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>90%</th>
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<td>5.48</td>
<td>7.02</td>
<td>13.7</td>
<td>21.2</td>
</tr>
</tbody>
</table>

**Table:** Mean, standard deviation, and quantiles of EP bound (in %)
A lower bound on the equity premium

- Equity premium was very high at times of stress
- Also high from 1998–2000
  - Forecasts based on market valuation ratios incorrectly predicted a low or even negative equity premium during this period
  - By construction, the var\(^*\) lower bound can never be less than zero
- No out-of-sample issues (Ang-Bekaert, Goyal-Welch) because no parameter estimation is required
A lower bound on the equity premium

- Suppose you’re skeptical. What *trade* should you do?
- Short the portfolio of options, i.e. short an at-the-money-forward straddle and (equally weighted) out-of-the-money calls and puts
- An investor who puts on such a trade ends up short the market if the market rallies and long the market if the market sells off
- Contrarian position: investor provides liquidity to the market
- Investors demanded extraordinarily high risk premia for providing liquidity at the height of the credit crisis
How *not* to think about what’s going on

- If the world were lognormal you could think of what we’re doing as
  - Exploiting a relationship between risk premia and $\mathbb{P}$-variance
  - $\mathbb{Q}$-variance (SVIX) equals $\mathbb{P}$-variance
- In the real, non-lognormal, world, this is the wrong intuition
  - Risk premia sensitive to higher moments as well as $\mathbb{P}$-variance
  - $\mathbb{Q}$-variance (SVIX) not equal to $\mathbb{P}$-variance
Robustness

- Can’t trade deep-OTM options
- Even near-the-money, can’t trade *all* strikes
  - Both these effects make the bound *conservative*: it would be *even higher* if we had perfect data
- If the investor holds bonds, or earns labor income, we get the same result under a slightly stronger assumption
  - If \( L'(R_T) \geq 0 \) and \( L(R_T) \leq \kappa W_T \) then we need risk aversion at least \( 1 + \kappa \)
  - If the agent has at least as much wealth in the market as labor (or bond) income then we need risk aversion at least 2
A lower bound on the equity premium

Rescaled 6mo equity premium bound and S&P 500
What if the bound were tight?

- \( REP_{t \rightarrow t+T} \approx \alpha + \beta \cdot EPB_t + \varepsilon_{t+T} \)
- OLS with Hansen-Hodrick standard errors to account for heteroskedasticity and overlapping observations
- Null hypothesis: bound holds with equality, \( \alpha = 0, \beta = 1 \)

<table>
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<th>horizon</th>
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<th>s.e.</th>
<th>( \hat{\beta} )</th>
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<td>[0.09]</td>
<td>1.54</td>
<td>[1.29]</td>
<td>3.6%</td>
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</table>
Generalizing the bound

Modify the previous assumptions as follows:

1. There is a one-period investor as before, except that relative risk aversion $\gamma(x) \equiv -xu''(x)/u'(x)$ satisfies $\gamma(x) \in [\underline{\gamma}, \overline{\gamma}]$, or

2. There is an intertemporal investor as before, except that relative risk aversion $\Gamma(x) \equiv -xJ''[x]/J'[x]$ satisfies $\Gamma(x) \in [\underline{\gamma}, \overline{\gamma}]$, or

3. There is an Epstein-Zin (1989) investor as before, except that risk aversion $\gamma \in [\underline{\gamma}, \overline{\gamma}]$, or

4. The SDF and market return are conditionally jointly lognormal, and the ratio of the market’s conditional Sharpe ratio to its conditional volatility lies between $\underline{\gamma}$ and $\overline{\gamma}$.
Generalizing the bound

Result

Under any one of these assumptions,

\[
\frac{\mathbb{E}^* \left( R_T^{\theta + \gamma} \right)}{\mathbb{E}^* \left( R_T^\gamma \right)} \leq \mathbb{E} \left( R_T^\theta \right) \leq \frac{\mathbb{E}^* \left( R_T^{\theta + \gamma} \right)}{\mathbb{E}^* \left( R_T^\gamma \right)}
\]

for any \( \theta \geq 0 \)

and

\[
\frac{\mathbb{E}^* \left( R_T^{\theta + \gamma} \right)}{\mathbb{E}^* \left( R_T^\gamma \right)} \leq \mathbb{E} \left( R_T^\theta \right) \leq \frac{\mathbb{E}^* \left( R_T^{\theta + \gamma} \right)}{\mathbb{E}^* \left( R_T^\gamma \right)}
\]

for any \( \theta \leq 0 \).

If one of the bounds on relative risk aversion holds with equality, then the corresponding inequality holds with equality.
Generalizing the bound

Result

For any \( \theta \), we have

\[
\mathbb{E}^* R_T^\theta = R_{f,T}^\theta + R_{f,T} \left\{ \int_0^{F_0,T} \frac{\theta(\theta - 1)}{P_0^\theta} (P_0 R_{f,T} - F_{0,T} + K)^{\theta-2} \text{put}_T(K) \, dK 
+ \int_{F_0,T}^{\infty} \frac{\theta(\theta - 1)}{P_0^\theta} (P_0 R_{f,T} - F_{0,T} + K)^{\theta-2} \text{call}_T(K) \, dK \right\}
\]

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Bounds on the equity premium

Bounds on 1mo equity premium, annualized, 10-day moving average, $\gamma = 1, 2, 4$
Bounds on the equity premium

Bounds on 3mo equity premium, annualized, 10-day moving average, $\gamma = 1, 2, 4$

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Bounds on the equity premium

Bounds on 1yr equity premium, annualized, 10-day moving average, \(\gamma = 1, 2, 4\)
### Bounds on the equity premium

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**Table:** Mean, standard deviation, and quantiles of equity premium bounds at the 1-year horizon (measured in %) for different levels of risk aversion.
Implied real-world volatility, $\sqrt{\text{var} \, R_T}$

Forward-looking 1mo volatility, annualized, 10-day moving average, $\gamma = 1, 2, 4$
Implied real-world volatility, $\sqrt{\text{var}\ R_T}$

Forward-looking 3mo volatility, annualized, 10-day moving average, $\gamma = 1, 2, 4$
Implied real-world volatility, $\sqrt{\text{var} R_T}$

Forward-looking 1yr volatility, annualized, 10-day moving average, $\gamma = 1, 2, 4$
\[ \sqrt{\text{var} R_T}, \sqrt{\text{var}^* R_T}, \text{ and realized volatility} \]

1mo realized vol, annualized, 10-day moving average
$\sqrt{\text{var}} R_T$, $\sqrt{\text{var}^*} R_T$, and realized volatility

1mo vol* (black) and vol (red), annualized, 10-day moving average, $\gamma = 1$
\[ \sqrt{\text{var} R_T}, \sqrt{\text{var}^* R_T}, \text{and realized volatility} \]

1yr vol* (black) and vol (red), 10-day moving average, \( \gamma = 1 \)
Forecasting volatility

\[ R^2 = 50\% : \text{RVAR}_t = 0.01 + 0.71 \text{RVAR}_{t-1} \]

\[ R^2 = 57.8\% : \text{RVAR}_t = -0.02 + 1.21 \text{var}^* R_t \]

\[ R^2 = 58.1\% : \text{RVAR}_t = -0.02 + 1.40 \text{var} R_t \]

\[ R^2 = 58.1\% : \text{RVAR}_t = -0.02 + 1.50 \text{var} R_t + -0.09 \text{var}^* R_t \]

\[ R^2 = 58.3\% : \text{RVAR}_t = -0.01 + 1.01 \text{var}^* R_t + 0.14 \text{RVAR}_{t-1} \]

\[ R^2 = 58.7\% : \text{RVAR}_t = -0.02 + 1.13 \text{var} R_t + 0.16 \text{RVAR}_{t-1} \]

\[ R^2 = 58.9\% : \text{RVAR}_t = -0.02 + 2.64 \text{var} R_t + \\
\quad + 3.89 \text{var}^* R_t + 0.21 \text{RVAR}_{t-1} \]
Forecasting volatility

- Both real-world and risk-neutral forward-looking variance easily outperform lagged realized variance.
- If anything, real-world forward-looking variance performs slightly better than risk-neutral variance (as one would hope) but because the two are so highly correlated, it is hard to make any clear inference.
- Lagged realized variance contributes almost no incremental explanatory power once real-world or risk-neutral forward-looking variances are included.
- Same conclusions hold if forward-looking real-world variance is calculated for any $\gamma$ between 1 and 2.
- In view of equity premium results, it seems reasonable to entertain values of $\gamma$ in a range between about 1 and 1.5.
Forward-looking Sharpe ratio

1yr (black) and 1mo (red) horizon. $\gamma = 1$
Forward-looking Sharpe ratio

1yr (black) and 1mo (red) horizon. $\gamma = 1.25$
Forward-looking Sharpe ratio

1yr (black) and 1mo (red) horizon. $\gamma = 1.5$
Up-SVIX and down-SVIX

\[ \text{var}^* R_T = \frac{2R_{f,T}}{P_0^2} \int_0^{F_{0,T}} \text{put}_T(K) \, dK + \int_{F_{0,T}}^\infty \text{call}_T(K) \, dK \]

- “down-variance”
- “up-variance”

Natural to think about what the calls and puts tell us separately.

Can we give a nice interpretation to down- and up-variance?

Yes, if we think from the perspective of a log investor.
Up-SVIX and down-SVIX

\[ P(R_T > R_{f,T}) = -R_{f,T} \text{call}'(F_{0,T}) + \frac{\text{call}(F_{0,T})}{P_0} \]

\[ \mathbb{E}[(R_T - R_{f,T}) \mathbf{1}\{R_T > R_{f,T}\}] = \frac{R_{f,T}}{P_0} \text{call}(F_{0,T}) + \frac{\text{up-variance}}{R_{f,T}} \]

\[ \mathbb{E}[(R_T - R_{f,T}) \mathbf{1}\{R_T < R_{f,T}\}] = -\frac{R_{f,T}}{P_0} \text{call}(F_{0,T}) + \frac{\text{down-variance}}{R_{f,T}} \]

- These are real-world probabilities, \( P \) not \( P^* \)
- Rule of thumb: \( \mathbb{E}[R_T - R_{f,T} \mid R_T > R_{f,T}] \approx 2 \times \text{up-variance} \)
- \( \mathbb{E}[R_T - R_{f,T} \mid R_T < R_{f,T}] \approx 2 \times \text{down-variance} \)
Probability of an up-move, $\mathbb{P}(R_T > R_{f,T})$

$T = 1$ mo
Probability of an up-move, $\mathbb{P}(R_T > R_{f,T})$

$T = 1 \text{ yr}$
Conditional expected return \( \mathbb{E}(R_T | R_T \geq R_{f,T}) \)

Black: up-move. Red: down-move (sign flipped). Not annualized. \( T = 1 \) mo
Conditional expected return $\mathbb{E}(R_T | R_T \geq R_{f,T})$

Black: up-move. Red: down-move (sign flipped). Not annualized. $T = 1$ yr
Conclusion

- The equity premium is \textit{volatile} and was \textit{high} during the crisis.
- Bad: Need a theoretical assumption: this is the equity premium perceived by an unconstrained index investor with risk aversion no less than one.
- Good: No need to assume that the world is stationary or ergodic, because there’s no need to replace $E$ with $\frac{1}{T} \sum$. 

\[ \sum \]