Social Learning in Financial Markets

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Motivation

- Many financial crises are driven by a deviation between prices and fundamental values: Buying booms followed by panicked selloffs.
- Busse and Green (2002): Equity prices respond within one minute of the firm being mentioned on CNBC
Focus of This Talk

• **Main Question:** Is there a CNBC effect for social media?

• I develop an asset pricing model where investors form their beliefs by paying attention to both their peers beliefs and market prices.

• **Main Prediction:** Asset prices may be biased towards the beliefs of well connected and wealthy investors.
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valyrian
$AMRS: awesome info must read

$AMRS: Oh so good!!! Great resource for DD biofuelsdigest.com/digest/

Bullish

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WinningTrades4U
$XG: Pre-Market. Tomorrow. Can guarantee that #InmensePower Behind this

bad-boy. Pumpers set down with $2 PT, but it will fly Bullish

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Diablo4ever
$SB: Time will tell. They are looking for money like any other financial

institutions. In the end, this bank remains... Bullish

via StockTwits For Android

AlejandroMasari
$TSLA: bloomberg.com/features/2016...
We currently have a large position in APPLE. We believe the company to be extremely undervalued. Spoke to Tim Cook today. More to come.
Related Work

• Colla and Mele (2010) place investors in a circle network. Price informativeness and volume increase when network weights are positive.

• Ozsoylev and Walden (2011): Investors are in a large, sparse network. Volume increases with connectedness. No explicit dynamics.

• Bailey, Cao, Kuchler and Stroebel (2017): House price increases of geographically-distant friends affect expectations of own house price
The Model
Setup

• There are $H$ households and 2 states of the world.

• Time is indexed by $t \in \{0,1, \ldots, \tau\}$. The state of the world is revealed at some time $\tau \gg 0$.

• There are two Arrow-Debreu assets which pay off when the state is revealed.
Endowments and Beliefs

• Each household $h$ will have endogenous sequences of beliefs
  
  \[ \pi^h(t) = (\pi_1^h(t), \pi_2^h(t)) \]

  and endowments
  
  \[ e^h(t) = (e_1^h(t), e_2^h(t)) \].

• Initial beliefs and endowments $\pi(0), e(0)$ are exogenous.

• Both assets are in unit net supply: $\sum_h e_i^h(t) = 1$. 
Prices and Wealth

- At time $t$, asset $i$’s price is $q_i(t)$. Prices are normalized so that $q_1(t) + q_2(t) = 1$.

- Household $h$’s wealth share at $t$ is $w^h(t) = q_1(t)e_1^h(t) + q_2(t)e_2^h(t)$.

- Total wealth is $\sum_h w^h(t) = q_1(t)\sum_h e_1^h(t) + q_2(t)\sum_h e_2^h(t) = 1$. 

Timing

At each round $t \geq 0$:

• Each household $h$ starts with an endowment $e^h(t)$

• Each household updates its beliefs $\pi^h(t)$ based on $\pi(t - 1)$ and $q(t - 1)$ (if $t = 0$ then beliefs are exogenous)

• Trade occurs.
  • New equilibrium prices $q(t)$ are determined.
  • Each household rebalances its portfolio based on its new beliefs (and new prices).
  • The rebalanced portfolio shares are the endowment $e^h(t + 1)$ for next round.
• At time 0, each household receives an exogenous signal $\pi^h(0)$

• There is a stochastic matrix $T \in \mathbb{R}^{H \times H}$ (the trust matrix) and an attention parameter $\kappa \in (0,1)$ so that beliefs at time $t$ are given by

$$
\pi^h(t) = (1 - \kappa) \sum_{j=1}^{H} T_{hj} \cdot \pi^j(t - 1) + \kappa q(t - 1)
$$
DeGroot Learning
DeGroot Learning With Prices
DeGroot Learning With Prices

• Beliefs are formed via DeGroot learning so that

\[ \pi^h(t) = (1 - \kappa) \sum_{j=1}^{H} T_{hj} \cdot \pi^j(t - 1) + \kappa q(t - 1) \]

• With some extra work (not on the slide), we can define an equilibrium so that

\[ q(t - 1) = \sum_{j=1}^{H} w^j(t - 1) \pi^j(t - 1) \]

• From the two equations above, we get that

\[ \pi^h(t) = \sum_{j=1}^{H} ((1 - \kappa)T_{hj} + \kappa w^j(t - 1)) \cdot \pi^j(t - 1) \]
DeGroot Learning With Prices

- Beliefs are formed so that
  \[ \pi^h(t) = (1 - \kappa) \sum_{j=1}^{H} (T_{hj} + \kappa w^j(t - 1)) \cdot \pi^j(t - 1) \]

- Let \( W(t - 1) \) be a matrix where all the rows are equal to
  \[ w(t - 1) = (w^1(t - 1), \ldots, w^H(t - 1)) \]

- We can write the belief update rule as
  \[ \pi(t) = ((1 - \kappa) T + \kappa W(t - 1)) \cdot \pi(t - 1) \]
Main Results
Social Learning Questions

The following questions should be asked of any social learning model

1. Will agents eventually achieve consensus?
2. How quickly will the process converge?
3. Will consensus answer be the correct answer?
Proposition 1

For any parameters \((T, \kappa, \pi(0), e(0))\) of the economy, long-run equilibrium beliefs will converge to a consensus \(\pi^* = \lim_{t \to \infty} \pi(t)\). In addition, price and wealth vectors will also converge

\[ q^* = \lim_{t \to \infty} q(t), \quad w^* = \lim_{t \to \infty} w(t) \]
DeGroot Learning With Prices
Correctness of Consensus?

• In Classical DeGroot learning, the consensus will be correct if each agent’s influence vanishes as $H \rightarrow \infty$ (Golub and Jackson 2010)

• But in the model with markets, agents with large long-run wealth may have much long-run influence

• So consensus beliefs may be biased towards the initial beliefs of the wealthy
Speed of Convergence

- The duration of disagreement is governed by the second eigenvalue of $T$.

For large enough $t$:

$$||\pi(\infty) - \pi(t)||_2 \leq C (1 - \kappa)^t \lambda_2^t$$

$$||\pi(\infty) - \pi(t)||_1 \leq C \sqrt{n} (1 - \kappa)^t \lambda_2^t$$
Conclusion

• I developed a theory that explains a CNBC effect for social media

• Main Results: Prices converge, but they can be biased by the beliefs of well-connected and wealthy investors

• Concurrent Work: Testing predictions on empirical data