Bank Risk Dynamics and Distance to Default

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Financial crisis highlighted need to understand bank default risk and bank risk dynamics:

- Counterparty risk assessment
- Pricing deposit insurance
- Estimation of too-big-to-fail (TBTF) subsidies
Introduction

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  - Counterparty risk assessment
  - Pricing deposit insurance
  - Estimation of too-big-to-fail (TBTF) subsidies
- Standard approach to default risk and risk dynamics: Structural models (Merton 1974)
  - Log-normal process for firm asset value with \textit{constant} volatility
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Standard approach to default risk and risk dynamics:
Structural models (Merton 1974)
  - Log-normal process for firm asset value with constant volatility

Applications of structural models to banks
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- Standard approach to default risk and risk dynamics: Structural models (Merton 1974)
  - Log-normal process for firm asset value with \textit{constant} volatility
- Applications of structural models to banks
- Problem: Assumption of log-normal asset value could be grossly violated for banks
  - Bank assets are debt claims with limited upside. Short a put option!
  - Bank equity and debt are really options on options
Bank asset payoffs: Illustration with perfectly correlated borrower defaults

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Bank Risk Dynamics and Distance to Default
Non-linearity due to short put option in bank assets: Bank asset value conditional on borrower asset value.
Neglecting the short put option in bank assets: Underestimating risk in “good” times

Locally fitted lognormal model

True nonlinear bank asset value

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Bank Risk Dynamics and Distance to Default
Outline

- Options-on-options model of bank equity and debt
  - **Borrower** asset values log-normal with idiosyncratic and systematic risk
  - **Bank** assets as contingent claim on borrower assets
  - Bank equity and debt as contingent claims on bank asset

Calibration to bank panel data 2002 to 2012: Comparison with standard Merton model

Risk-neutral default probabilities

Bank equity risk changes conditional on negative asset value shocks
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  - Risk-neutral default probabilities
  - Bank equity risk changes conditional on negative asset value shocks
Bank issues zero-coupon loans with maturity $T$
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Maturities staggered across $N$ cohorts of borrowers
Bank issues zero-coupon loans with maturity $T$
Maturities staggered across $N$ cohorts of borrowers
Cohorts labeled by remaining maturity $\tau = T, T(N - 1)/N, \ldots, 1/N$ of their loans

Pricing of bank equity and debt

Cohort $T$
Cohort $T - 1/N$
...

$t = 0$
Bank’s debt matures

$t = H$
Each cohort contains a continuum of borrowers indexed by $i \in [0, 1]$ with mass $1/N$. 

Borrower asset values: Log-normal
Each cohort contains a continuum of borrowers indexed by $i \in [0, 1]$ with mass $1/N$.

One-factor model of borrower collateral value

$$\frac{dA_{t}^{T,i}}{A_{t}^{T,i}} = (r - \delta)dt + \sigma(\sqrt{\rho}dW_t + \sqrt{1 - \rho}dZ_{t}^{T,i}),$$

with $A_{T}^{T,i} = 1$ at initial loan origination.
Borrower asset values: Log-normal

- Each cohort contains a continuum of borrowers indexed by $i \in [0, 1]$ with mass $1/N$.
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with $A_{\tau}^{\tau,i} = 1$ at initial loan origination.
- Common factor $W$ will be the only source of stochastic shocks at the aggregate loan portfolio level
Loan payoffs to bank at cohort level

- Loan face value $F_1$ and initial loan-to-value ratio

$$\ell = F_1 e^{-\mu T},$$

with promised yield $\mu$ determined by competitive loan pricing.
Loan payoffs to bank at cohort level

- Loan face value $F_1$ and initial loan-to-value ratio
  \[ \ell = F_1 e^{-\mu T}, \]
  with promised yield $\mu$ determined by competitive loan pricing.
- Loan payoff
  \[ L_{\tau,i}^T = \min(A_{\tau,i}^T, F_1). \]
Loan payoffs to bank at cohort level

- Loan face value \( F_1 \) and initial loan-to-value ratio
  \[
  \ell = F_1 e^{-\mu T},
  \]
  with promised yield \( \mu \) determined by competitive loan pricing.
- Loan payoff
  \[
  L_{\tau,j}^\tau,i = \min(A_{\tau,j}^\tau,i, F_1).
  \]
- Loan payoff from cohort \( \tau \)
  \[
  L_\tau^\tau = \frac{1}{N} \int_0^1 L_{\tau,j}^\tau d\tau
  \]
  \[
  = \frac{1}{N} \int_0^1 A_{\tau,j}^\tau d\tau - \frac{1}{N} \int_0^1 \max(A_{\tau,j}^\tau - F_1, 0) d\tau
  \]
  \[
  = \frac{1}{N} \left[ A_{\tau}^\tau \Phi(d_1) + F_1 \Phi(d_2) \right],
  \]
At maturity, proceeds from loans invested into new loans to same cohort.
Loan portfolio payoffs aggregated across cohorts

- At maturity, proceeds from loans invested into new loans to same cohort
- Borrowers reduce or replenish collateral to get back to $LTV = \ell$. 

\[
V_H = \sum_{\tau < H} e^{-r(\tau + T - H)} E_Q[H_{\tau + T}] + \sum_{\tau \geq H} e^{-r(\tau - H)} E_Q[H_{\tau}]
\]
Loan portfolio payoffs aggregated across cohorts

- At maturity, proceeds from loans invested into new loans to same cohort
- Borrowers reduce or replenish collateral to get back to LTV = \( \ell \).
- Aggregate value of bank loan portfolio at \( t = H \),

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\]

- Simulate loan portfolio payoffs by simulating common factor \( W \) under risk-neutral distribution
Loan portfolio value conditional on aggregate borrower asset value
Bank has zero-coupon debt with face value $D$ maturing at $t = H$. 

Debt value $B_0 = e^{-rH}D - e^{-rH}E^Q_t[(D - V_H + Y_H) + ]$

Equity value (ex-dividend) $S_0 = V_0 - e^{-rH}Y_H - B_0$
Pricing bank equity and debt

- Bank has zero-coupon debt with face value $D$ maturing at $t = H$
- Bank defaults if asset value at maturity $V_H < D$

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$B_0 = e^{-r_H}D - e^{-r_H}E_{t[H]}[(D - V_H + Y_H)]$

$S_0 = V_0 - e^{-r_H}Y_H - B_0$
Bank has zero-coupon debt with face value $D$ maturing at $t = H$

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Equity value (ex-dividend)

$$S_0 = V_0 - e^{-rH}Y_H - B_0$$
Bank equity value as function of borrower asset value

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Bank Risk Dynamics and Distance to Default
Assumption: Our modified model represents the true data generating process
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Consider an analyst that calibrates a standard Merton model to data simulated from our model
- Simulated equity values and instantaneous volatility used to solve for asset volatility and asset value
- Calibration based on (false) assumption of a log-normal asset value process
Risk-neutral default probabilities as function of current loan portfolio value

Merton (red) and modified (blue)

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Key problem with standard Merton model: Underestimation of bank default risk in good times

- Standard Merton model logic: If bank equity value high and equity volatility low ... ⇒ infer that current asset volatility must be low ⇒ asset volatility will **continue** to be low in the future ⇒ **High** distance to default
Key problem with standard Merton model: Underestimation of bank default risk in good times

- Standard Merton model logic: If bank equity value high and equity volatility low ...
  ⇒ infer that current asset volatility must be low
  ⇒ asset volatility will *continue* to be low in the future
  ⇒ **High** distance to default

- But with true nonlinear risk dynamics: If bank equity value high and equity volatility low ...
  ⇒ infer that current asset volatility must be low
  ⇒ but, conditional on a bad shock in the future, asset volatility could be much **higher** in the future
  ⇒ **Low** distance to default

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*Bank Risk Dynamics and Distance to Default*
Implied credit spread

Merton (red) and modified (blue)

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Bank Risk Dynamics and Distance to Default
Value of a government guarantee

Merton (red) and modified (blue)

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Bank Risk Dynamics and Distance to Default
Empirical calibration

- Sample: Intersection of Compustat and CRSP-FRB linked dataset from 2002-2012
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- $D$ is face value of all outstanding debt (including time and demand deposits)
Empirical calibration

- Sample: Intersection of Compustat and CRSP-FRB linked dataset from 2002-2012
- Loan face value ≈ book assets of bank
- $D$ is face value of all outstanding debt (including time and demand deposits)
- Debt maturity $H = 5$. 
Empirical calibration

- Sample: Intersection of Compustat and CRSP-FRB linked dataset from 2002-2012
- Loan face value $\approx$ book assets of bank
- $D$ is face value of all outstanding debt (including time and demand deposits)
- Debt maturity $H = 5$.
- Market equity and book value of assets normalized by $D$
Empirical calibration

- Sample: Intersection of Compustat and CRSP-FRB linked dataset from 2002-2012
- Loan face value $\approx$ book assets of bank
- $D$ is face value of all outstanding debt (including time and demand deposits)
- Debt maturity $H = 5$.
- Market equity and book value of assets normalized by $D$
- Equity volatility is computed (in annualized form) from daily bank stock returns over one-year moving windows.
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- Debt maturity $H = 5$.
- Market equity and book value of assets normalized by $D$
- Equity volatility is computed (in annualized form) from daily bank stock returns over one-year moving windows.
- Calibration of our model: Every quarter, back out current borrower asset value and borrower asset volatility to match empirical bank equity value and volatility
## Table 1: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>Borrower Asset Depreciation Rate</td>
<td>0.005</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Bank payout Rate</td>
<td>0.002</td>
</tr>
<tr>
<td>$T$</td>
<td>Bank Loan Maturity</td>
<td>10 years</td>
</tr>
<tr>
<td>$H$</td>
<td>Bank Debt Maturity</td>
<td>5 years</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Borrower asset value correlation</td>
<td>0.5</td>
</tr>
<tr>
<td>$\ell$</td>
<td>Loan-to-Value Ratio</td>
<td>$0.8e^{(\mu-r)T}$</td>
</tr>
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</table>
### Model-Implied Risk-Neutral Probabilities of Default

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>sd</th>
<th>min</th>
<th>p25</th>
<th>p50</th>
<th>p75</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Merton Model RNPD</td>
<td>0.26</td>
<td>0.27</td>
<td>0.01</td>
<td>0.05</td>
<td>0.15</td>
<td>0.39</td>
<td>0.89</td>
</tr>
<tr>
<td>Modified Model RNPD</td>
<td>0.34</td>
<td>0.21</td>
<td>0.00</td>
<td>0.17</td>
<td>0.35</td>
<td>0.52</td>
<td>0.70</td>
</tr>
<tr>
<td>Observations</td>
<td>20,823</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Comparison of calibrated risk-neutral default probabilities

Cumulative RN default probabilities over 5-year horizon

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Our modified model and standard Merton model differ starkly in their predictions of how bank equity risk responds to negative asset value shocks.
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Evaluate model predictions about equity volatility conditional on a negative asset value shock.
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Evaluate model predictions about equity volatility **conditional** on a negative asset value shock.

Calibrate both models to pre-crisis data from 2006Q2.
Our modified model and standard Merton model differ starkly in their predictions of how bank equity risk responds to negative asset value shocks.

Evaluate model predictions about equity volatility **conditional** on a negative asset value shock.

Calibrate both models to pre-crisis data from 2006Q2.

Then apply negative borrower asset value shock: cumulative log change in the Federal Housing Finance Agency (FHFA) quarterly house price index (purchases only) from 2006Q2 until a subsequent quarter $t$.

- Use our model to calculate impact on bank asset value.
- Apply this bank asset value shock in Merton model and our model.
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Borrower asset volatility kept constant to focus purely on bank asset non-linearity channel.
Model-implied equity volatility dynamics

Year–Quarter
Annualized Equity Volatility

Merton Model
Modified Model

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Bank Risk Dynamics and Distance to Default
Model-implied equity volatility dynamics

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Bank Risk Dynamics and Distance to Default
Conclusions

- Bank equity and debt have options-on-options nature
  - Standard structural models underestimate bank default risk, particularly in “good times”
  - Negative shocks to asset values are more “toxic” than in standard structural models because they raise bank asset volatility
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  - Standard structural models underestimate bank default risk, particularly in “good times”
  - Negative shocks to asset values are more “toxic” than in standard structural models because they raise bank asset volatility

- Useful extensions:
  - Government as a claim-holder: Explicit and implicit government guarantees
  - Liquidity problems
  - Jumps in asset values
  - Complex maturity and seniority structures of bank debt