Motivation

- Countries impose rules to constrain governments’ policy decisions
  - Fiscal rules in place in 92 countries in 2015, up from 7 in 1990

- Credible enforcement mechanisms are critical for institution of rules
  - E.g., Chile feared breaking fiscal rule in 2007 would set bad precedent
  - Rule broken in 2009 due to extraordinary circumstances
  - Lax policy persisted, including under next govt; rule reinstated in 2011

What is an optimal fiscal rule when enforcement is limited?
- How restrictive should a fiscal rule be?
- Should we expect governments to occasionally violate their rules?
- What is the optimal structure of penalties for violating rules?
Motivation

- Countries impose rules to constrain governments’ policy decisions
  - Fiscal rules in place in 92 countries in 2015, up from 7 in 1990

- Credible enforcement mechanisms are critical for institution of rules
  - E.g., Chile feared breaking fiscal rule in 2007 would set bad precedent
  - Rule broken in 2009 due to extraordinary circumstances
  - Lax policy persisted, including under next govt; rule reinstated in 2011

- What is an optimal fiscal rule when enforcement is limited?
  - How restrictive should a fiscal rule be?
  - Should we expect governments to occasionally violate their rules?
  - What is the optimal structure of penalties for violating rules?
Tradeoffs Behind Rules

- **Benefit:** Rules can fix commitment problems
  - Governments are present-biased $\implies$ Excessive deficits or spending

- **Cost:** Rules reduce flexibility. Some discretion can be desirable
  - Not all contingencies/shocks are contractible or observable
Tradeoffs Behind Rules

- **Benefit**: Rules can fix commitment problems
  - Governments are present-biased \( \implies \) Excessive deficits or spending

- **Cost**: Rules reduce flexibility. Some discretion can be desirable
  - Not all contingencies/shocks are contractible or observable

- **Model tradeoff in self-control framework using mechanism design**
  - Government privately observes shock. Must truthfully report it
  - Literature assumes perfect enforcement of rules
    - E.g., Amador-Werning-Angeletos 2006 (AWA), Halac-Yared 2014
  - **This paper**: Fiscal rules must be self-enforcing
What We Do

- Small open economy. Government chooses debt and spending
  - Marginal value of public spending is stochastic
What We Do

- Small open economy. Government chooses debt and spending
  - Marginal value of public spending is stochastic

- Frictions
  - Government is present-biased towards public spending
  - Marginal value of spending privately observed by government
  - New: Government has full policy discretion
What We Do

- Small open economy. Government chooses debt and spending
  - Marginal value of public spending is stochastic

- Frictions
  - Government is present-biased towards public spending
  - Marginal value of spending privately observed by government
  - New: Government has full policy discretion

- Rule must satisfy private information and self-enforcement constraints
  - Perfect enforcement: Optimal rule is a deficit limit (AWA)
  - Self-enforcement: Observable deviations punished off path
    - Worst continuation equilibrium sustains optimum (Abreu 1988)
Results

- Optimal fiscal rule is a maximally enforced deficit limit
  - Any on- or off-path violation leads to the worst punishment
  - Unlike under perfect enforcement, potential for on-path punishment
  - Key technical result: *Bang-bang* dynamic incentives
Results

- Optimal fiscal rule is a **maximally enforced deficit limit**
  - Any on- or off-path violation leads to the worst punishment
  - Unlike under perfect enforcement, potential for on-path punishment
  - Key technical result: *Bang-bang* dynamic incentives

- Necessary and sufficient conditions for violation of deficit limit
  - Severe deficit bias, rare extreme shocks
Results

- Optimal fiscal rule is a **maximally enforced deficit limit**
  - Any on- or off-path violation leads to the worst punishment
  - Unlike under perfect enforcement, potential for on-path punishment
  - Key technical result: **Bang-bang** dynamic incentives

- Necessary and sufficient conditions for violation of deficit limit
  - Severe deficit bias, rare extreme shocks

- Worst punishment takes form of temporary overspending
  - Maximally enforced surplus limit. Deficit limit eventually reinstated
  - Periods of fiscal rectitude and fiscal profligacy sustain each other
Related Literature

- Commitment versus flexibility

- Political economy of fiscal policy

- Hyperbolic discounting and commitment devices

- Price wars and bang-bang equilibria
Environment

- $t = \{0, 1, \ldots\}$, i.i.d. shock $\theta_t \in [\underline{\theta}, \bar{\theta}]$, $f(\theta_t) > 0$
Environment

- $t = \{0, 1, \ldots\}$, i.i.d. shock $\theta_t \in [\underline{\theta}, \bar{\theta}]$, $f(\theta_t) > 0$

- Resource constraint at time $t$:

$$G_t = \tau + \frac{B_{t+1}}{1 + r} - B_t$$
Environment

- \( t = \{0, 1, \ldots\} \), i.i.d. shock \( \theta_t \in [\underline{\theta}, \bar{\theta}] \), \( f(\theta_t) > 0 \)

- Resource constraint at time \( t \):

\[
G_t = \tau + \frac{B_{t+1}}{1+r} - B_t
\]

- Welfare at time \( t \):

  - Society:

\[
\sum_{k=0}^{\infty} \delta^k \mathbb{E}[\theta_{t+k} U(G_{t+k})]
\]

  - Government after \( \theta_t \)'s realization, when choosing policy:

\[
\theta_t U(G_t) + \beta \sum_{k=1}^{\infty} \delta^k \mathbb{E}[\theta_{t+k} U(G_{t+k})], \text{ where } \beta \in (0, 1)
\]
Interpretation

Suppose two-period economy

- First best: $\theta_0 U'(G_0) = \delta(1 + r) \mathbb{E}[\theta_1] U'(G_1)$
- Full flexibility: $\theta_0 U'(G_0) = \beta \delta(1 + r) \mathbb{E}[\theta_1] U'(G_1)$
  - Government overborrows since $\beta < 1$
Interpretation

- Suppose two-period economy
  - First best: \( \theta_0 U'(G_0) = \delta (1 + r) \mathbb{E}[\theta_1] U'(G_1) \)
  - Full flexibility: \( \theta_0 U'(G_0) = \beta \delta (1 + r) \mathbb{E}[\theta_1] U'(G_1) \)
    - Government overborrows since \( \beta < 1 \)

- Politicians temporarily in charge of budget overweigh spending
  - Arises in political economy setting with turnover
  - Arises under aggregation of heterogeneous preferences (Jackson-Yariv)
Interpretation

- Suppose two-period economy
  - First best: \( \theta_0 U'(G_0) = \delta (1 + r) \mathbb{E}[\theta_1] U'(G_1) \)
  - Full flexibility: \( \theta_0 U'(G_0) = \beta \delta (1 + r) \mathbb{E}[\theta_1] U'(G_1) \)
    - Government overborrows since \( \beta < 1 \)

- Politicians temporarily in charge of budget overweigh spending
  - Arises in political economy setting with turnover
  - Arises under aggregation of heterogeneous preferences (Jackson-Yariv)

- Realization of \( \theta_t \) is private information
  - Rules cannot depend on \( \theta_t \) explicitly
  - Heterogeneous citizen preferences; government sees aggregate
  - Exact cost of public goods not perfectly observed
Assumption on Preferences

- **Assumption**: $U(G_t) = \log(G_t)$
  - Spending rate $g_t = G_t/((1 + r)\tau/r - B_t)$, savings rate $x_t = 1 - g_t$
  - Welfare from savings $W(x_t) = \delta \mathbb{E} [\theta_t] U(x_t)/(1 - \delta)$
Assumption on Preferences

- **Assumption:** \( U(G_t) = \log(G_t) \)
  - Spending rate \( g_t = G_t/((1 + r)\tau / r - B_t) \), savings rate \( x_t = 1 - g_t \)
  - Welfare from savings \( W(x_t) = \delta \mathbb{E}[^{\theta_t}][U(x_t)]/(1 - \delta) \)

- Implies welfare is separable with respect to debt

Society: \[ \sum_{k=0}^{\infty} \delta^k \mathbb{E}\left[ \theta_{t+k} U(g_{t+k}) + W(x_{t+k}) \right] + \chi(B_t) \]

Govt: \[ \theta_t U(g_t) + \beta W(x_t) + \beta \sum_{k=1}^{\infty} \delta^k \mathbb{E}\left[ \theta_{t+k} U(g_{t+k}) + W(x_{t+k}) \right] + \varphi(B_t) \]
Assumption on Preferences

**Assumption:** \( U(G_t) = \log(G_t) \)

- Spending rate \( g_t = G_t / ((1 + r)\tau/r - B_t) \), savings rate \( x_t = 1 - g_t \)
- Welfare from savings \( W(x_t) = \delta \mathbb{E}[\theta_t] U(x_t) / (1 - \delta) \)

Implies welfare is separable with respect to debt

\[
\text{Society: } \sum_{k=0}^{\infty} \delta^k \mathbb{E}[\theta_{t+k} U(g_{t+k}) + W(x_{t+k})] + \chi(B_t)
\]

\[
\text{Govt: } \theta_t U(g_t) + \beta W(x_t) + \beta \sum_{k=1}^{\infty} \delta^k \mathbb{E}[\theta_{t+k} U(g_{t+k}) + W(x_{t+k})] + \varphi(B_t)
\]

**Two benchmarks**

- First best: \( \theta_t U'(g^{fb}(\theta_t)) = W'(x^{fb}(\theta_t)) \)
- Full flexibility: \( \theta_t U'(g^f(\theta_t)) = \beta W'(x^f(\theta_t)) \)
Perfect public equilibria

Government chooses $g_t$ given $\{g_0, g_1, \ldots, g_{t-1}\}$ and private info $\theta_t$
Self-Enforcing Rules

- Perfect public equilibria
  - Government chooses $g_t$ given $\{g_0, g_1, \ldots, g_{t-1}\}$ and private info $\theta_t$

- Strategy profile implies spending sequence $\{\{g_t(\theta^t)\}_{\theta^t \in \Theta}^{\infty}\}_{t=0}$
  - Continuation value at $t$ (normalized by debt) is
    \[
    V_t(\theta^{t-1}) = \sum_{k=0}^{\infty} \delta^k \mathbb{E} \left[ \theta_{t+k} U(g_{t+k}(\theta^{t+k-1}, \theta_{t+k})) + W(x_{t+k}(\theta^{t+k-1}, \theta_{t+k})) \right]
    \]
Self-Enforcing Rules

- Perfect public equilibria
  - Government chooses $g_t$ given $\{g_0, g_1, \ldots, g_{t-1}\}$ and private info $\theta_t$

- Strategy profile implies spending sequence $\{\{g_t(\theta^t)\}_{\theta^t \in \Theta}^\infty\}_{t=0}^\infty$
  - Continuation value at $t$ (normalized by debt) is
    \[
    V_t(\theta^{t-1}) = \sum_{k=0}^{\infty} \delta^k \mathbb{E}[\theta_{t+k} U(g_{t+k}(\theta^{t+k-1}, \theta_{t+k})) + W(x_{t+k}(\theta^{t+k-1}, \theta_{t+k}))]
    \]

- Equilibrium iff govt at $\theta^{t-1}$ prefers $\{g_t(\theta^{t-1}, \theta_t), V_{t+1}(\theta^{t-1}, \theta_t)\}$ to:
  - Unobservable deviation: $\{g_t(\theta^{t-1}, \theta'), V_{t+1}(\theta^{t-1}, \theta')\}$ for $\theta' \neq \theta_t$
  - Observable deviation: $\{g^f(\theta_t), V\}$

  - Where $V$ is lowest value supported by equilibrium strategies
Optimal Self-Enforcing Rule

\[
\bar{V} = \max_{\{g(\theta), x(\theta), V(\theta)\}_{\theta \in \Theta}} \mathbb{E}\left[\theta U(g(\theta)) + W(x(\theta)) + \delta V(\theta)\right]
\]

subject to

\[
\theta U(g(\theta)) + \beta W(x(\theta)) + \beta \delta V(\theta) \geq \theta U(g(\theta')) + \beta W(x(\theta')) + \beta \delta V(\theta')
\]

(private information constraint)

\[
\theta U(g(\theta)) + \beta W(x(\theta)) + \beta \delta V(\theta) \geq \theta U(g^f(\theta)) + \beta W(x^f(\theta)) + \beta \delta V
\]

(self-enforcement constraint)

\[
g(\theta) + x(\theta) = 1 \text{ and } V(\theta) \in [\underline{V}, \overline{V}]
\]

(feasibility)
Definition: Maximally Enforced Deficit Limit

\[ \theta^* \in [0, \bar{\theta}) \text{ and } \theta^{**} > \max\{\theta^*, \bar{\theta}\} \]
Preliminaries

- Envelope condition: Government welfare given $\theta$ equals

$$\theta U(g(\theta)) + \beta W(x(\theta)) + \beta \delta V(\theta) + \int_{\theta}^{\bar{\theta}} U(g(\tilde{\theta})) d\tilde{\theta} \quad (1)$$

- $(1)$ and $g(\theta)$ rising $\implies$ private information constraint satisfied

Social welfare (normalized by debt) equal to

$$\frac{1}{\beta} \theta U(g(\theta)) + W(x(\theta)) + \delta V(\theta) + \int_{\theta}^{\bar{\theta}} U(g(\tilde{\theta})) d\tilde{\theta}$$

where $Q(\theta) \equiv \frac{1}{\beta} - F(\theta) - \theta f(\theta)(1 - \beta)$

$Q(\theta)$: Weight on allowing spending distortions by type

- Higher $Q(\theta)$ = Lower social welfare cost of distorting $\theta$’s spending
Preliminaries

- Envelope condition: Government welfare given $\theta$ equals

$$\theta U(g(\theta)) + \beta W(x(\theta)) + \beta \delta V(\theta) + \int_{\theta}^{\bar{\theta}} U(g(\tilde{\theta})) d\tilde{\theta} \quad (1)$$

  - (1) and $g(\theta)$ rising $\implies$ private information constraint satisfied

- Social welfare (normalized by debt) equal to

$$\frac{1}{\beta} \theta U(g(\theta)) + W(x(\theta)) + \delta V(\theta) + \frac{1}{\beta} \int_{\theta}^{\bar{\theta}} U(g(\theta)) Q(\theta) d\theta$$

  where $Q(\theta) \equiv 1 - F(\theta) - \theta f(\theta)(1 - \beta)$
Preliminaries

- Envelope condition: Government welfare given $\theta$ equals

$$\theta U(g(\theta)) + \beta W(x(\theta)) + \beta \delta V(\theta) + \int_{\theta}^{\theta} U(g(\tilde{\theta})) d\tilde{\theta} \quad (1)$$

  - (1) and $g(\theta)$ rising $\implies$ private information constraint satisfied

- Social welfare (normalized by debt) equal to

$$\frac{1}{\beta} \theta U(g(\theta)) + W(x(\theta)) + \delta V(\theta) + \frac{1}{\beta} \int_{\theta}^{\theta} U(g(\tilde{\theta})) Q(\tilde{\theta}) d\tilde{\theta}$$

  where $Q(\theta) \equiv 1 - F(\theta) - \theta f(\theta)(1 - \beta)$

- $Q(\theta)$: Weight on allowing spending distortions by type $\theta$
  - Higher $Q(\theta) \implies$ Lower social welfare cost of distorting $\theta$'s spending
Bang-Bang Incentives

**Proposition:** Suppose \( Q(\theta) \) satisfies these generic properties:

1. \( Q'(\theta) \neq 0 \) almost everywhere

2. If \( Q(\theta^L) = Q(\theta^H) = \hat{Q} \), then \( \int_{\theta^L}^{\theta^H} Q(\theta) \, d\theta \neq \int_{\theta^L}^{\theta^H} \hat{Q} \, d\theta \)

Then in any optimal rule, \( V(\theta) \in \{ V, \hat{V} \} \) for all \( \theta \in (\underline{\theta}, \bar{\theta}) \)
Proposition: Suppose $Q(\theta)$ satisfies these generic properties:

1. $Q'(\theta) \neq 0$ almost everywhere

2. If $Q(\theta^L) = Q(\theta^H) = \hat{Q}$, then $\int_{\theta^L}^{\theta^H} Q(\theta) d\theta \neq \int_{\theta^L}^{\theta^H} \hat{Q} d\theta$

Then in any optimal rule, $V(\theta) \in \{V, \overline{V}\}$ for all $\theta \in (\bar{\theta}, \bar{\theta})$

Bang-bang property is necessary for optimality

- Intuition: Rich info structure $\implies$ distortions by steepening incentives
- Result also applies to perfect enforcement
Bang-Bang Incentives

**Proposition:** Suppose $Q(\theta)$ satisfies these generic properties:

1. $Q'(\theta) \neq 0$ almost everywhere

2. If $Q(\theta^L) = Q(\theta^H) = \hat{Q}$, then $\int_{\theta^L}^{\theta^H} Q(\theta) d\theta \neq \int_{\theta^L}^{\theta^H} \hat{Q} d\theta$

Then in any optimal rule, $V(\theta) \in \{V, \bar{V}\}$ for all $\theta \in (\underline{\theta}, \bar{\theta})$

- Bang-bang property is necessary for optimality
  - Intuition: Rich info structure $\Rightarrow$ distortions by steepening incentives
  - Result also applies to perfect enforcement

- Relationship to Abreu-Pearce-Stacchetti 1990
  - Instead of moral hazard, we study adverse selection with self-control:
    - Continuation value set is one-dimensional
    - Locally spreading out continuation values may not be IC or beneficial
Sketch of Proof: Three Steps

- **Step 1:** $V(\theta)$ is a step function (no local dynamic incentives)
- **Step 2:** $V(\theta) \in \{\underline{V}, \overline{V}\}$ whenever $g(\theta)$ is strictly increasing
- **Step 3:** $V(\theta) \in \{\underline{V}, \overline{V}\}$ whenever $g(\theta)$ is constant
Step 1: Rule Out Local Dynamic Incentives

- Suppose $V'(\theta) < 0$ with $g'(\theta) > 0$

- If $Q'(\theta) < 0$, flattening perturbation increases welfare

- If $Q'(\theta) > 0$, steepening perturbation increases welfare
Step 1: Rule Out Local Dynamic Incentives

- Suppose $V'(\theta) < 0$ with $g'(\theta) > 0$
- If $Q'(\theta) < 0$, flattening perturbation increases welfare
- If $Q'(\theta) > 0$, steepening perturbation increases welfare
Step 1: Rule Out Local Dynamic Incentives

- Suppose \( V'(\theta) < 0 \) with \( g'(\theta) > 0 \)
- If \( Q'(\theta) < 0 \), flattening perturbation increases welfare
- If \( Q'(\theta) > 0 \), steepening perturbation increases welfare
Step 2: Rule Out Interior Values under Rising Spending

- Suppose $V(\theta) \in (\underline{V}, \overline{V})$ with $g'(\theta) > 0$
- If $Q'(\theta) < 0$, flattening perturbation increases welfare
- If $Q'(\theta) > 0$, steepening perturbation increases welfare
Suppose 
\[ V(\theta) \in (\underline{V}, \overline{V}) \]
with \( g'(\theta) = 0 \)

If \( Q(\theta^L) > \int_{\theta^L}^{\theta^H} Q(\theta) \, d\theta \), segment shifting perturbation increases welfare

Analogous perturbations under different conditions
Step 3: Rule Out Interior Values under Constant Spending

- Suppose $V(\theta) \in (\underline{V}, \overline{V})$ with $g'(\theta) = 0$

- If $Q(\theta^L) > \int_{\theta^L}^{\theta^H} Q(\theta) d\theta$, segment-shifting perturbation increases welfare

- Analogous perturbations under different conditions
Monotonic Incentives

- **Assumption**: There exists $\hat{\theta} \in \Theta$ such that

$$Q'(\theta) < (> ) 0 \text{ if } \theta < (> ) \hat{\theta}$$

- Satisfied for uniform, exponential, log-normal, gamma, among others
Monotonic Incentives

- **Assumption:** There exists $\hat{\theta} \in \Theta$ such that

$$Q'(\theta) < (>) 0 \text{ if } \theta < (>) \hat{\theta}$$

- Satisfied for uniform, exponential, log-normal, gamma, among others

- **Lemma:** In any optimal rule, $V(\theta)$ is weakly decreasing, $V(\theta) = \overline{V}$

- Implies either $V(\theta) = \overline{V} \forall \theta$ or $V(\theta)$ jumps down to $\overline{V}$ at interior $\theta^{**}$
- $\theta > \hat{\theta} \implies$ Load spending distortions at top, high-powered incentives
- $\theta < \hat{\theta} \implies$ Load spending distortions at bottom, low-powered incentives

Proof makes use of perturbations like previous ones
Monotonic Incentives

- **Assumption:** There exists $\hat{\theta} \in \Theta$ such that

\[ Q'(\theta) < (>) 0 \text{ if } \theta < (>) \hat{\theta} \]

- Satisfied for uniform, exponential, log-normal, gamma, among others

- **Lemma:** In any optimal rule, $V(\theta)$ is weakly decreasing, $V(\underline{\theta}) = \overline{V}$

- Implies either $V(\theta) = \overline{V} \forall \theta$ or $V(\theta)$ jumps down to $\underline{V}$ at interior $\theta^{**}$

- $\theta > \hat{\theta} \implies$ Load spending distortions at top, high-powered incentives

- $\theta < \hat{\theta} \implies$ Load spending distortions at bottom, low-powered incentives

- Proof makes use of perturbations like previous ones
Sketch of Proof: Three Steps

■ **Step 1:** If $V(\theta) = \underline{V}$, then $\theta \geq \hat{\theta}$
  - Otherwise, $Q'(\theta) < 0$, and $g(\theta) = g^f(\theta)$ by self-enforcement
  - Improve w/flattening perturbation

■ **Step 2:** If $V(\theta') = \underline{V}$, then $V(\theta) = \underline{V}$ for all $\theta \geq \theta'$
  - Otherwise, $V(\theta) = \underline{V}$ and $Q'(\theta) > 0$ over $[\theta_L, \theta_H]$, $\theta_L > \theta'$
  - Improve w/steepening ($g'(\theta) > 0$) or segment-shifting ($g'(\theta) = 0$) perturbation

■ **Step 3:** $V(\theta) = \underline{V}$
  - Otherwise, $V(\theta) = \underline{V}$ for all $\theta \in \Theta$
  - Improve w/global perturbation that increases $V(\theta)$ for all $\theta \in \Theta$
Sketch of Proof: Three Steps

- **Step 1**: If $V(\theta) = V$, then $\theta \geq \hat{\theta}$
  - Otherwise, $Q'(\theta) < 0$, and $g(\theta) = g^f(\theta)$ by self-enforcement
  - Improve w/flattening perturbation

- **Step 2**: If $V(\theta') = V$, then $V(\theta) = V$ for all $\theta \geq \theta'$
  - Otherwise, $V(\theta) = V$ and $Q'(\theta) > 0$ over $[\theta^L, \theta^H]$, $\theta^L > \theta'$
  - Improve w/steepeining ($g' > 0$) or segment-shifting ($g' = 0$) perturbation
Sketch of Proof: Three Steps

- **Step 1:** If $V(\theta) = \underline{V}$, then $\theta \geq \hat{\theta}$
  - Otherwise, $Q'(\theta) < 0$, and $g(\theta) = g^f(\theta)$ by self-enforcement
  - Improve w/flattening perturbation

- **Step 2:** If $V(\theta') = \underline{V}$, then $V(\theta) = \underline{V}$ for all $\theta \geq \theta'$
  - Otherwise, $V(\theta) = \overline{V}$ and $Q'(\theta) > 0$ over $[\theta^L, \theta^H]$, $\theta^L > \theta'$
  - Improve w/steepeening ($g' > 0$) or segment-shifting ($g' = 0$) perturbation

- **Step 3:** $V(\theta) = \overline{V}$
  - Otherwise, $V(\theta) = \underline{V}$ for all $\theta \in \Theta$
  - Improve w/global perturbation that increases $V(\theta)$ for all $\theta \in \Theta$
**Proposition**: Any optimal rule is a maximally enforced deficit limit

- Follows from previous results, after proving $g(\theta)$ must be continuous
- If $g(\theta)$ is discontinuous, improve with perturbation that closes “hole”
**Proposition:** Any optimal rule is a maximally enforced deficit limit

- Follows from previous results, after proving $g(\theta)$ must be continuous
- If $g(\theta)$ is discontinuous, improve with perturbation that closes “hole”

Under perfect enforcement, optimum is deficit limit $\theta_e$, $V(\theta) = \overline{V}$ $\forall \theta$

$$\int_{\theta_e}^{\bar{\theta}} Q(\theta) = 0$$
**Optimal Self-Enforcing Fiscal Rule**

- **Proposition**: Any optimal rule is a maximally enforced deficit limit
  - Follows from previous results, after proving \( g(\theta) \) must be continuous
  - If \( g(\theta) \) is discontinuous, improve with perturbation that closes “hole”

- Under perfect enforcement, optimum is deficit limit \( \theta_e \), \( V(\theta) = \bar{V} \ \forall \theta \)

\[
\int_{\theta_e}^{\bar{\theta}} Q(\theta) = 0
\]

- **Corollary**: Suppose \( \theta_e \) is self-enforcing, that is:

\[
\bar{\theta} U(g^f(\theta_e)) + \beta W(x^f(\theta_e)) + \beta \delta \bar{V} \geq \bar{\theta} U(g^f(\bar{\theta})) + \beta W(x^f(\bar{\theta})) + \beta \delta \bar{V}
\]

Then \( \theta^* = \theta_e \) and \( \theta^{**} \geq \bar{\theta} \). No dynamic incentives
Use of Punishment

- If $\theta_e$ is not self-enforcing, define $\theta_b$:

$$\bar{\theta} U(g^f(\theta_b)) + \beta W(x^f(\theta_b)) + \beta \delta \bar{V} = \bar{\theta} U(g^f(\bar{\theta})) + \beta W(x^f(\bar{\theta})) + \beta \delta \bar{V}$$
Use of Punishment

- If $\theta_e$ is not self-enforcing, define $\theta_b$:

$$
\bar{\theta} U\left(g^f(\theta_b)\right) + \beta W\left(x^f(\theta_b)\right) + \beta \delta \bar{V} = \bar{\theta} U\left(g^f(\bar{\theta})\right) + \beta W\left(x^f(\bar{\theta})\right) + \beta \delta \bar{V}
$$

- **Proposition**: Suppose $\theta_e$ is not self-enforcing. Optimal rule is unique:

  1. If $\int_{\theta_b}^{\bar{\theta}} (Q(\theta) - Q(\bar{\theta})) d\theta \geq 0$, then $\theta^* = \theta_b$ and $\theta^{**} = \bar{\theta}$
  2. Otherwise, $\theta^* \in (\theta_e, \theta_b)$ and $\theta^{**} < \bar{\theta}$
Use of Punishment

- If $\theta_e$ is not self-enforcing, define $\theta_b$:

$$
\bar{\theta} U(g^f(\theta_b)) + \beta W(x^f(\theta_b)) + \beta \delta \bar{V} = \bar{\theta} U(g^f(\bar{\theta})) + \beta W(x^f(\bar{\theta})) + \beta \delta \bar{V}
$$

**Proposition:** Suppose $\theta_e$ is not self-enforcing. Optimal rule is unique:

1. If $\int_{\theta_b}^{\bar{\theta}} (Q(\theta) - Q(\bar{\theta})) d\theta \geq 0$, then $\theta^* = \theta_b$ and $\theta^{**} = \bar{\theta}$

2. Otherwise, $\theta^* \in (\theta_e, \theta_b)$ and $\theta^{**} < \bar{\theta}$

Condition reflects benefits and costs of dynamic incentives

- Discipline lower types; no discipline and dynamic costs for higher types
- For any $\bar{V} - \underline{V}$, no punishment if $Q'(\theta) < 0 \ \forall \theta$. True if $f'(\theta) \geq 0 \ \forall \theta$
- For any $\bar{V} - \underline{V}$, punishment if $f(\theta) \to 0$ as $\theta \to \bar{\theta}$
Definition: Maximally Enforced Surplus Limit

\[ \theta_p^* > \theta \text{ and } \theta_p^{**} \in [\theta, \min\{\theta_p^*, \bar{\theta}\}] \]
Characterization of Punishment

**Proposition**: Worst punishment is a maximally enforced surplus limit
Characterization of Punishment

**Proposition:** Worst punishment is a maximally enforced surplus limit

**Intuition:** Government cares more about present spending than society
- Inducing overspending relaxes self-enforcement and minimizes welfare
- Best incentives achieved with maximal reward and punishment
Characterization of Punishment

- **Proposition:** *Worst punishment is a maximally enforced surplus limit*

- **Intuition:** Government cares more about present spending than society
  - Inducing overspending relaxes self-enforcement and minimizes welfare
  - Best incentives achieved with maximal reward and punishment

- **Proof:** Characterizing $V$ analogous to characterizing $\overline{V}$ (but reverse)
  - Key step establishes that punishment is not absorbing
  - Rewarding overspending by high types reduces welfare
Bang-Bang Dynamics

- Optimal fiscal rule is solution to two problems:
  - \( \{ \theta^*, \theta^{**} \} \) which yield maximum social welfare given \( \bar{V} - V \)
  - \( \{ \theta_p^*, \theta_p^{**} \} \) which yield minimum social welfare given \( \bar{V} - V \)
  - Larger \( \bar{V} - V \) \( \implies \) Higher max and lower min
Bang-Bang Dynamics

- Optimal fiscal rule is solution to two problems:
  - \( \{\theta^*, \theta^{**}\} \) which yield maximum social welfare given \( \overline{V} - V \)
  - \( \{\theta_p^*, \theta_p^{**}\} \) which yield minimum social welfare given \( \overline{V} - V \)
  - Larger \( \overline{V} - V \) \(\implies\) Higher max and lower min

- Conditions for dynamics incentives
  - Self-enforcement constraint sufficiently binding
  - \( \int_{\theta_b^*}^{\bar{\theta}} (Q(\theta) - Q(\bar{\theta})) d\theta < 0 \)
  - Both hold if \( \bar{\theta} \) is sufficiently extreme
Bang-Bang Dynamics

- Optimal fiscal rule is solution to two problems:
  - \(\{\theta^*, \theta^{**}\}\) which yield maximum social welfare given \(\overline{V} - \underline{V}\)
  - \(\{\theta_p^*, \theta_p^{**}\}\) which yield minimum social welfare given \(\overline{V} - \underline{V}\)
  - Larger \(\overline{V} - \underline{V}\) \(\implies\) Higher max and lower min

- Conditions for dynamics incentives
  - Self-enforcement constraint sufficiently binding
    - \(\int_{\theta_b}^{\overline{\theta}} (Q(\theta) - Q(\overline{\theta})) d\theta < 0\)
  - Both hold if \(\overline{\theta}\) is sufficiently extreme

- Phases of fiscal rectitude and fiscal profligacy sustain each other
Conclusion

- Characterization of optimal self-enforcing fiscal rule
  - Maximally enforced deficit limit
  - Conditions for limit to be violated following high shocks
  - Punishment in the form of temporary overspending

Some possible extensions
- General hyperbolic preferences
- Enforcement of common rules in groups of countries and federations

Other applications
- Self-control and going off the wagon
- Regulation with socially costly penalties
Conclusion

- Characterization of optimal self-enforcing fiscal rule
  - Maximally enforced deficit limit
  - Conditions for limit to be violated following high shocks
  - Punishment in the form of temporary overspending

- Some possible extensions
  - General hyperbolic preferences
  - Enforcement of common rules in groups of countries and federations
Conclusion

- Characterization of optimal self-enforcing fiscal rule
  - Maximally enforced deficit limit
  - Conditions for limit to be violated following high shocks
  - Punishment in the form of temporary overspending

- Some possible extensions
  - General hyperbolic preferences
  - Enforcement of common rules in groups of countries and federations

- Other applications
  - Self-control and going off the wagon
  - Regulation with socially costly penalties
Thank you!
Recall distributional assumption: There exists $\hat{\theta} \in \Theta$ such that

$$Q'(\theta) < (>) 0 \text{ if } \theta < (>) \hat{\theta}$$
Discussion of Distributional Assumptions

- Recall distributional assumption: There exists $\hat{\theta} \in \Theta$ such that

  \[ Q'(\theta) < (>) 0 \text{ if } \theta < (>) \hat{\theta} \]

- **Proposition**: If assumption is violated, there exist \{\(\underline{V}, \overline{V}\)\} for which a maximally enforced deficit limit is strictly suboptimal
Recall distributional assumption: There exists $\hat{\theta} \in \Theta$ such that

\[ Q'(\theta) < (>) 0 \text{ if } \theta < (>) \hat{\theta} \]

**Proposition**: If assumption is violated, there exist \( \{V, \bar{V}\} \) for which a maximally enforced deficit limit is strictly suboptimal.

- Implies our distributional assumption is necessary for characterization
  - Weaker distributional assumption needed under perfect enforcement