Generalized Social Marginal Welfare Weights for Optimal Tax Theory

Emmanuel Saez
(Berkeley)

Stefanie Stantcheva
(Harvard)

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Standard Welfarist Approach: Critiques and Puzzles

- Maximize concave function of individual utilities or weighted sum of utilities.

\[ \text{max } SWF = \text{max } \int \omega_i \cdot u_i \]

- Special case: utilitarianism, \( \omega_i = 1 \).

- Cannot capture elements important in tax practice:
  - Source of income: earned versus luck.
  - Counterfactuals: what agents *would* have done absent tax system.
  - Horizontal Equity concerns that go against “tagging.”


- Methodological and conceptual critique: Policy makers use reform-approach rather than posit and maximize objective.
A Novel Approach to Model Social Preferences

- **Tax reform approach**: weighs gains and losses from tax changes.
  
  Tax reform desirable iff: \( \int_i g_i dT_i > 0 \) with \( g_i \equiv G'(u_i)u'_i \)  

- Optimality: no budget neutral reform can increase welfare.

- Weights directly come from social welfare function, are restrictive.
A Novel Approach to Model Social Preferences

- **Tax reform approach**: weighs gains and losses from tax changes.

  \[
  \text{Change in welfare: } \int_i g_i dT_i \text{ with } g_i \equiv g(c_i, z_i; x_i^s, x_i^b).
  \]

- Replace restrictive social welfare weight by **generalized social marginal welfare weights**.
  - A “price” for $1$ consumption/ social value of $1$ transfer for each person.
  - Specified to directly capture fairness criteria.
  - Not necessarily derived from SWF (nest standard ones).
Generalized social welfare weights approach

\[ u_i = u(c_i - v(z_i; x^u_i, x^b_i)) \quad g_i = g(c_i, z_i; x^s_i, x^b_i) \]
Framework Resolves Puzzles and Unifies Alternative Approaches

- **Resolve puzzles**: Can depend on luck vs. deserved income, can capture counterfactuals (“Free Loaders”), can model horizontal equity concerns.

- Can avoid problem of utility cardinality or representation.


- **Pareto efficiency** guaranteed (locally) by non-negative weights.

- **Positive tax theory**: Can estimate weights from revealed social choices.

- Approach can be applied to other policies.
Applying Standard Formulas with Generalized Weights

- Individual weights need to be “aggregated” up to characteristics that tax system can conditioned on.
  - E.g.: If $T(z, x^b)$ possible, aggregate weights at each $(z, x^b) \rightarrow \bar{g}(z, x^b)$.
  - If standard $T(z)$, aggregate at each $z$: $\bar{G}(z)$ and $\bar{g}(z)$.

- Then apply standard formulas. Nests standard approach.

- If $g_i \geq 0 \ \forall i$, (local) Pareto efficiency guaranteed.

- Can we back out weights? Optimum $\Leftrightarrow \max \ SWF = \int_i \omega_i \cdot u_i$ with Pareto weights $\omega_i = g_i / u_{ci} \geq 0$ where $g_i$ and $u_{ci}$ are evaluated at the optimum allocation. Impossible to posit correct weights $\omega_i$ without first solving for optimum.
Aggregating Standard Weights at Each Income Level

Taxes depend on $z$ only.: express everything in terms of observable $z$.

$H(z)$: CDF of earnings

$h(z)$: PDF of earnings

Definition

$	ilde{G}(z)$ is the (relative) average social marginal welfare weight for individuals earning more than $z$:

$$
\tilde{G}(z) \equiv \frac{\int_{\{i:z_i \geq z\}} g_i}{\text{Prob}(z_i \geq z) \cdot \int_i g_i}
$$

$\bar{g}(z)$ is the average social marginal welfare weight at $z$:

$$
\bar{g}(z) = -\frac{1}{h(z)} \frac{d(\tilde{G}(z) \cdot [1 - H(z)])}{dz}
$$
Standard Tax Formula Expressed with Welfare Weights

Result

The optimal marginal tax at $z$:

$$T'(z) = \frac{1 - \bar{G}(z)}{1 - \bar{G}(z) + \alpha(z) \cdot e(z)}$$

$e(z)$: average elasticity of $z$ w.r.t $1 - T'$ at $z_i = z$

$\alpha(z)$: local Pareto parameter $zh(z) / [1 - H(z)]$.

Can invert tax formula to obtain the weights (Werning, 2007, Hendren, 2013).

Proposition

If $T'(z) < 1$ exists for all $z$, there is an unique $\bar{G}(z) < 1 + \alpha(z) \cdot e(z)$ defined by $\bar{G}(z) = [1 - T'(z)(1 + \alpha(z) \cdot e(z))] / [1 - T'(z)]$ satisfying the optimal tax formula.
Standard Linear Tax Formula Expressed with Welfare Weights

The optimal linear tax rate, such that $c_i = z_i (1 - \tau) + \tau \int_i z_i$ can also be expressed as a function of an income weighted average marginal welfare weight (Piketty and Saez, 2013).

Result

The optimal linear income tax is:

$$\tau = \frac{1 - \bar{g}}{1 - \bar{g} + e} \quad \text{with} \quad \bar{g} \equiv \frac{\int_i g_i \cdot z_i}{\int_i g_i \cdot \int_i z_i}$$

$e$: elasticity of $\int_i z_i$ w.r.t $(1 - \tau)$. 
Related Literature


Outline

1. Resolving Puzzles of the Standard Approach
2. Link With Alternative Justice Principles
3. Empirical Testing and Estimation Using Survey Data
4. Conclusion
1. Luck versus Deserved Income: Setting

- Widely perceived that fairer to tax luck income than earned income and to insure against luck shocks.

- Provides micro-foundation for weights increasing in taxes, decreasing in consumption.

- \( y^d \): deserved income due to effort

- \( y^l \): luck income, not due to effort, with average \( Ey^l \).

- \( z = y^d + y^l \): total income.

- Society believes earned income fully deserved, luck income not deserved. Captured by binary set of weights:

  \[ g_i = 1(y^l_i - Ey^l \leq z_i - c_i) \]

  \( g_i = 1 \) if taxed more than excess luck income (relative to average).

- \( x_i^s = (y^l_i, Ey^l) \), with \( Ey^l \) aggregate characteristic.
1. Luck vs. Deserved Income: Results

- If luck income observable, can condition taxes on it: \( T_i = T(z_i, y_i^l) \).
  - Aggregate weights for each \((z, y^l)\) pair.
  - Everybody gets average luck income, zero taxes on total income.
  - Example: Health care costs.

- If luck income unobservable, multiple equilibria: high and low tax.
2. Transfers and Free Loaders: Setting

- Behavioral responses closely tied to social weights: biggest complain against redistribution is “free loaders.”
- Generalized welfare weights can capture “counterfactuals.”
- Consider linear tax model where $\tau$ funds demogrant transfer.
  \[ u_i = u(c_i - v(l_i; \theta_i)) = u(c_i - \theta \cdot l) \text{ with } l \in \{0, 1\}. \]
- Individuals can choose to not work, $l = 0$, $c_i = c_0$.
- If they work ($l = 1$), earn uniform wage $w$, consume $c_1 = w \cdot (1 - \tau) + c_0$.
- Cost of work $\theta$, with cdf $P(\theta)$, is private information.
- Individual: work iff $\theta \leq c_1 - c_0 = (1 - \tau) \cdot w$.
- Fraction working: $P(w(1 - \tau))$.
- $e$: elasticity of aggregate earnings $w \cdot P(w(1 - \tau))$ w.r.t $(1 - \tau)$. 
2. Transfers and Free Loaders: Optimal Taxation

Standard Approach:

- \( g_i = u'(c_0) \) for all non-workers so that \( \bar{g}_0 = u'(c_0) \).

- Hence, approach does not allow to distinguish between the deserving poor and free loaders.

- We can only look at actual situation: work or not, not “why” one does not work.

- Contrasts with public debate and historical evolution.
2. Transfers and Free Loaders: Generalized Welfare Weights

- Distinguish people according to what would have done absent transfer.
- Welfare weights function \( g_i = g(c, z/w; \theta_i, w) \), with \( x_i^b = (\theta_i, w) \).
- **Workers**: Fraction \( P(w(1 - \tau)) \). \( g_i = u'(c_1 - \theta_i) \).
- **Deserving poor**: would not work even absent any transfer: \( \theta > w \). Fraction \( 1 - P(w) \). \( g_i = u'(c_0) \).
- **Free Loaders**: do not work because of transfer: \( w \geq \theta > w \cdot (1 - \tau) \). Fraction \( P(w) - P(w(1 - \tau)) \). \( g_i = 0 \).

- Cost of work enters weights – fair to compensate for (i.e., not laziness).
- Average weight on non-workers \( \bar{g}_0 = u'(c_0) \cdot (1 - P_0)/(1 - P) \) scales by fraction of deserving non-workers – lower than in utilitarian case.
- Reduces optimal tax rate not just through \( e \) but also through \( \bar{g}_0 \).
2. Transfers and Free Loaders: Other Applications

- **Desirability of in-work benefits** if weight on non-workers becomes low enough relative to workers.

- **Transfers over the business cycle**: composition of those out of work depends on ease of finding job.
2. Transfers and Free Loaders: Sidenote

- Ex post, possible to find suitable Pareto weights that rationalize same tax.
  - E.g.: $\omega(\theta) = 1$ for $\theta \leq w \cdot (1 - \tau^*)$ (workers)
  - $\theta \geq w$ (deserving poor)
  - $\omega(\theta) = 0$ for $w \cdot (1 - \tau^*) < \theta < w$ (free loaders).

- But: these weights depend on equilibrium tax rate $\tau^*$. 
3. Horizontal Equity: Puzzle

- Standard theory strongly recommends use of “tags” – yet not used much.
- Can capture this with generalized welfare weights by making weights dependent on level and direction of change of reform.
- Social marginal welfare weights concentrated on those suffering from horizontal inequity.
- If no horizontal inequity, a reform that creates horizontal inequity needs to be penalized.
- Horizontal inequity can be equilibrium only if helps discriminated group.
- Tagging must be Pareto improving to be desirable, limits scope for use.
- New Rawlsian criterion: “Permissible to discriminate against a group based on tags, only if discrimination improves this group’s welfare.”
Outline

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1. Equality of Opportunity: Setting

- Ability to earn is result of i) family background $B_i \in \{0, 1\}$ (which individuals not responsible for) and to merit (which individuals are responsible for), $w_i$.
- Society is willing to redistribute across background, but not across income conditional on background.
- High family background gives “unfair” earnings advantage.
- $r_i$: percentile of $i$ in earnings distribution conditional on background – measure of merit.
- Conditional on earnings, those coming from $B_i = 0$ are more meritorious.
- $\bar{c}(r) \equiv (\int_{i: r_i = r} c_i) / \text{Prob}(i : r_i = r)$: average consumption at rank $r$.
- $g_i = g(c_i; \bar{c}(r_i)) = 1(c_i \leq \bar{c}(r_i))$, with $x_i^s = \bar{c}(r_i)$, $x_i^u = B_i$ and $x_i^b$ empty.
1. Equality of Opportunity: Results

- Suppose government cannot condition taxes on background.

- $\bar{G}(z)$: **Representation index**: % from disadvantaged background earning $\geq z$ relative to % in population.

- Implied Social Welfare function as in Roemer et al. (2003).

- $\bar{G}(z)$ decreasing since harder for those from disadvantaged background to reach upper incomes.

- If at top incomes, representation is zero, revenue maximizing top tax rate.

- Justification for social welfare weights decreasing with income not due to decreasing marginal utility (utilitarianism).
1. Equality of Opportunity vs. Utilitarian Tax Rates

<table>
<thead>
<tr>
<th>Income percentile</th>
<th>Equality of Opportunity</th>
<th>Utilitarian (log-utility)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fraction from low background (=parents below median) above each percentile</td>
<td>Implied social welfare weight G(z) above each percentile</td>
</tr>
<tr>
<td>z= 25th percentile</td>
<td>44.3%</td>
<td>0.886</td>
</tr>
<tr>
<td>z= 50th percentile</td>
<td>37.3%</td>
<td>0.746</td>
</tr>
<tr>
<td>z= 75th percentile</td>
<td>30.3%</td>
<td>0.606</td>
</tr>
<tr>
<td>z= 90th percentile</td>
<td>23.6%</td>
<td>0.472</td>
</tr>
<tr>
<td>z= 99th percentile</td>
<td>17.0%</td>
<td>0.340</td>
</tr>
<tr>
<td>z= 99.9th percentile</td>
<td>16.5%</td>
<td>0.330</td>
</tr>
</tbody>
</table>

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Online Survey: Goals and Setup

Two goals of empirical application:

1. Discover notions of fairness people use to judge tax and transfer systems.
   - Focus themes addressed in theoretical part.

2. Quantitatively calibrate simple weights

Online Platform:

- Amazon mTurk (Kuziemko, Norton, Saez, Stantcheva, 2013).
- 1100 respondents with background information.
Evidence against utilitarianism

- Respondents asked to compare families w/ different combinations of $z$, $z - T(z)$, $T(z)$.
- Who is more deserving of a $1000$ tax break?
- Both disposable income and taxes paid matter for sww
  - Family earning $50K$, paying $15K$ in taxes judged more deserving than family earning $40K$, paying $5K$ in taxes
  - Family earning $40K$, paying $10K$ in taxes judged more deserving than family earning $50K$, paying $10K$ in taxes
- Frugal vs. Consumption-loving person with same net income

<table>
<thead>
<tr>
<th>Consumption-lover more deserving</th>
<th>Frugal more deserving</th>
<th>Taste for consumption irrelevant</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
<td>22%</td>
<td>74%</td>
</tr>
</tbody>
</table>
Does society care about effort to earn income?

- **Hard-working vs. Easy-going person with same net income**

  “A earns $30,000 per year, by working in two different jobs, 60 hours per week at $10/hour. She pays $6,000 in taxes and nets out $24,000. She is very hard-working but she does not have high-paying jobs so that her wage is low.”

  “B also earns the same amount, $30,000 per year, by working part-time for 20 hours per week at $30/hour. She also pays $6,000 in taxes and hence nets out $24,000. She has a good wage rate per hour, but she prefers working less and earning less to enjoy other, non-work activities.”

<table>
<thead>
<tr>
<th>Hardworking</th>
<th>Easy-going</th>
<th>Hours of work irrelevant conditional on total earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>more deserving</td>
<td>more deserving</td>
<td>54%</td>
</tr>
<tr>
<td>43%</td>
<td>3%</td>
<td></td>
</tr>
</tbody>
</table>
Do people care about “Free Loaders” and Behavioral Responses to Taxation?

Starting from same benefit level, which group most deserving of more benefits?

<table>
<thead>
<tr>
<th></th>
<th>Disabled unable to work</th>
<th>Unemployed looking for work</th>
<th>Unemployed not looking for work</th>
<th>On welfare not looking for work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average rank (1-4)</td>
<td>1.4</td>
<td>1.6</td>
<td>3.0</td>
<td>3.5</td>
</tr>
<tr>
<td>% assigned 1st rank</td>
<td>57.5%</td>
<td>37.3%</td>
<td>2.7%</td>
<td>2.5%</td>
</tr>
<tr>
<td>% assigned last rank</td>
<td>2.3%</td>
<td>2.9%</td>
<td>25%</td>
<td>70.8%</td>
</tr>
</tbody>
</table>
Calibrating Social Welfare Weights

- Calibrate $g(c, T) = g(c - \alpha T)$
- 35 fictitious families, w/ different net incomes and taxes
- Respondents rank them pair-wise (5 random pairs each)

Which of these two families is most deserving of the $1,000 tax break?

- Family earns $100,000 per year, pays $50,000 in taxes, and hence nets out $50,000
- Family earns $25,000 per year, pays $1,250 in taxes, and hence nets out $23,750

Which of these two families is most deserving of the $1,000 tax break?

- Family earns $50,000 per year, pays $2,500 in taxes, and hence nets out $47,500
- Family earns $500,000 per year, pays $170,000 in taxes, and hence nets out $330,000
Eliciting Social Preferences

Is A or B more deserving of a $1,000 tax break?
Eliciting Social Preferences

Is A or B more deserving of a $1,000 tax break?
Eliciting Social Preferences

\( S_{ijt} = 1 \) if \( i \) ranked 1st in display \( t \) for respondent \( j \), \( dT_{ijt} \) (\( dc_{ijt} \)) is difference in taxes (net income) for families in pair shown.

\[
S_{ijt} = \beta_0 + \beta_T dT_{ijt} + \beta_c dc_{ijt} \quad \alpha = \frac{dc}{dT} | S = -\frac{\beta_T}{\beta_c} = -\text{slope}
\]

\[
g(c,T) = g(c - \alpha T)
\]

indifference curves
Eliciting Social Preferences

<table>
<thead>
<tr>
<th>Sample</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full</td>
<td>Excludes cases with income $1m</td>
<td>Excludes cases with income $500K+</td>
<td>Excludes cases with income $500K+ and $10K or less</td>
<td>Liberal subjects only</td>
<td>Conservative subjects only</td>
</tr>
<tr>
<td>d(Tax)</td>
<td>0.0017***</td>
<td>0.0052***</td>
<td>0.016***</td>
<td>0.015***</td>
<td>0.00082***</td>
<td>0.0032***</td>
</tr>
<tr>
<td></td>
<td>(0.0003)</td>
<td>(0.0019)</td>
<td>(0.0019)</td>
<td>(0.0022)</td>
<td>(0.00046)</td>
<td>(0.00068)</td>
</tr>
<tr>
<td>d(Net Income)</td>
<td>-0.0046***</td>
<td>-0.0091***</td>
<td>-0.024***</td>
<td>-0.024***</td>
<td>-0.0048***</td>
<td>-0.0042***</td>
</tr>
<tr>
<td></td>
<td>(0.00012)</td>
<td>(0.00028)</td>
<td>(0.00078)</td>
<td>(0.00094)</td>
<td>(0.00018)</td>
<td>(0.00027)</td>
</tr>
<tr>
<td>Number of observations</td>
<td>11,450</td>
<td>8,368</td>
<td>5,816</td>
<td>3,702</td>
<td>5,250</td>
<td>2,540</td>
</tr>
<tr>
<td>Implied $\alpha$</td>
<td>0.37</td>
<td>0.58</td>
<td>0.65</td>
<td>0.64</td>
<td>0.17</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.07)</td>
<td>(0.09)</td>
<td>(0.12)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>Implied marginal tax rate</td>
<td>73%</td>
<td>63%</td>
<td>61%</td>
<td>61%</td>
<td>85%</td>
<td>57%</td>
</tr>
</tbody>
</table>
Outline

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Conclusion

- Generalized marginal social welfare weights are fruitful way to extend standard welfarist theory of optimal taxation.
  - Allow to dissociate individual characteristics from social criteria.
  - Which characteristics are fair to compensate for?

- Helps resolve puzzles of traditional welfarist approach.

- Unifies existing alternatives to welfarism.

- Weights can prioritize social justice principles in lexicographic form.
  1. Injustices created by tax system itself (horizontal equity)
  2. Compensation principle (health, background)
  3. Luck vs. effort or preferences for work.
  4. Utilitarian concept of decreasing marginal utility of consumption.