Sovereign Defaults in the Data

- Sovereign defaults (suspension of payments) are recurrent but infrequent events

- Associated with:
  - Severe output and consumption losses
  - Large fall in imports of intermediate goods
  - Maturity of debt shortens as default is more likely
Conventional Approach

Incomplete market approach to sovereign debt:

▷ Sovereign borrower can issue only non-contingent debt
▷ Sovereign borrower cannot commit to fully repay its debt

Typically:

▷ Exogenous maturity composition of debt
▷ Exogenous cost of default
▷ Markov equilibrium
Conventional View of Debt Crises

- Pervasive inefficiencies
  - Defaults due to incomplete contracts

- Excessive reliance on short-term debt causes crises
My Approach

As in conventional approach:

- Sovereign borrower can issue only non-contingent debt
- Sovereign borrower cannot commit to fully repay its debt

Extend by allowing for:

- Endogenous maturity composition of debt
- Endogenous cost of default (Production economy)
- Best equilibrium
My View of Debt Crises

- Best equilibrium outcome is constrained efficient
  - Solution to an optimal contracting problem with:
    - Lack of commitment by sovereign borrower
    - Private information
    - Non-contingent defaultable debt of multiple maturities sufficient to implement efficient outcome
  - High reliance on short-term when default is likely part of the efficient arrangement
    - Symptom, *not* cause
Features of the Best Outcome

Recurrent but infrequent defaults associated with:

▶ Output and consumption losses

▶ Fall in imports of intermediate goods

▶ Maturity of debt shortens as indebtedness increases before default
Contribution to the Literature

Incomplete market literature on sovereign default:


Extend by allowing for:

- Endogenous maturity composition of debt
- Endogenous cost of default (Production economy)
- Best equilibrium

Develop efficiency benchmark useful for policy analysis
Optimal dynamic contracting literature:

- Atkeson (1991) and Ales, Maziero, and Yared (2012)

Implementation: Relate efficient outcome to data on default, bond prices, maturity composition of debt
Outline

- Physical Environment

- Sovereign Debt Game
  - Best Equilibrium Outcome is Efficient

- Characterization of Efficient Allocation

- Implementation
  - Default, Bond Prices, and Maturity Composition of Debt
PHYSICAL ENVIRONMENT
Taste Shock Economy

- $t = 0, 1, \ldots, \infty$

- 2 types of agents:
  - Domestic agents (government)
  - Foreign lenders

- 2 goods:
  - Final good
  - Intermediate good
Foreign Lenders

- Risk neutral, discount factor $q \in (0, 1)$
- Value consumption of the final good
- Large endowment of the intermediate good
- Technology of the foreign lenders is such that relative price between intermediate and final good is one
Domestic Agents

- Preferences over final good:

\[
\sum_{t=0}^{\infty} \sum_{\theta^t} \beta^t \Pr(\theta^t) \theta_t U(c(\theta^t))
\]

with \(U\) strictly increasing, strictly concave and \(\beta \leq q\)

- Taste shock: \(\theta_t \in \{\theta_L, \theta_H\}\) distributed according to \(\mu\), iid

- Endowed with 1 unit of labor
Domestic Technology

- Final good produced using
  - $\ell$: domestic labor
  - $m$: foreign intermediate good

\[ Y = F(m, \ell) \]

- $F$ CRS, $F(0, 1) > 0$, $\lim_{m \to 0} F_m(m, \ell) = \infty$

- Let $f(m) = F(m, 1)$
SOVEREIGN DEBT GAME
Timing

Foreign exporters choose $p_t, m_t$ (private info)

Firms choose $\theta_t$ is realized

Gov’t chooses policy, $\pi_t$

Foreign lenders choose debt prices, $q_t$
Government Policy, $\pi = (\tau, b_S, b_L, \delta)$

- Government taxes payment by firms to foreign exporters at rate $\tau$

- Government issues two non-contingent defaultable bonds
  - Short-term: 1 period, $b_S$
  - Long-term: Consol, $b_L$

- Government can choose three levels of payment for existing debt:
  - $\delta = 1$: Full payment
  - $\delta = r \in (0, 1)$: Partial payment
  - $\delta = 0$: Suspension of payments in current period
Government Budget Constraint

- If there is full payment, $\delta = 1$:

$$b_s + b_L + c \leq Y(\tau) + q_S(h_{\pi}^t, \pi_t)b'_S + q_L(h_{\pi}^t, \pi_t)(b'_L - b_L)$$

- If there is partial payment, $\delta = r \in (0, 1)$:

$$rb_s + \frac{r}{1-q}b_L + c \leq Y(\tau) + q_S(h_{\pi}^t, \pi_t)b'_S + q_L(h_{\pi}^t, \pi_t)b'_L$$

- If there is suspension of payments, $\delta = 0$:

$$c \leq Y(\tau) \quad \text{and} \quad (b'_S, b'_L) = (b_S, b_L)$$

where $Y(\tau) = f(m_t) - (1 - \tau)p_t m_t$

and $h_{\pi}^t$ is the public history when the government acts
Definition of Sustainable Equilibrium

A *sustainable equilibrium* is \((\sigma, p, m, q_S, q_L)\) such that for all histories

- Given private strategies, the government’s strategy, \(\sigma\), maximizes domestic agents utility subject to budget constraints

- Given the government’s strategy:
  - \(p\) and \(m\) are consistent with foreign exporters and firms optimality conditions
  - \(q_S\) and \(q_L\) are consistent with lenders’ no-arbitrage condition
BEST SUSTAINABLE EQUILIBRIUM OUTCOME IS CONSTRAINED EFFICIENT
Incentive Compatibility and Sustainability Constraint

- In any sustainable equilibrium outcome the government must have no incentive to conduct *undetectable deviations*

- Incentive Compatibility Constraint

\[
\theta_t U(c(\theta^t)) + \beta v(\theta^t) \geq \theta_t U(c(\theta^{t-1}, \theta')) + \beta v(\theta^{t-1}, \theta') \quad \text{(IC)}
\]

where \(v(\theta^t)\) = continuation value for the domestic agent
Incentive Compatibility and Sustainability Constraint

- In any sustainable equilibrium outcome the government must have no incentive to conduct *detectable deviations*.

- Sustainability Constraint

\[
\theta_t U(c(\theta^t)) + \beta v(\theta^t) \geq \theta_t U(f(m(\theta^{t-1}))) + \beta v_a \quad (\text{SUST})
\]

where \( v_a = \text{value of autarky (worst equilibrium)} \)
Best Equilibrium Outcome is Constrained Efficient

Main Proposition of the Paper

The best sustainable equilibrium outcome is such that

\[ x = \{ m(\theta^{t-1}), c(\theta^t) \}_{t=0}^{\infty} \] solves the following optimal contracting problem:

\[
J(v_0) = \max_x \sum_{t=0}^{\infty} \sum_{\theta^t} q^t \Pr(\theta^t) \left[ -m(\theta^{t-1}) + f \left( m(\theta^{t-1}) \right) - c(\theta^t) \right]
\]

subject to (IC), (SUST) and

\[
\sum_{t=0}^{\infty} \sum_{\theta^t} \beta^t \Pr(\theta^t) \theta_t U(c(\theta^t)) \geq v_0 \quad \text{(PC')}
\]
CHARACTERIZATION OF THE CONSTRAINED EFFICIENT ALLOCATION
Efficient Allocation Solves Recursive Problem for $t \geq 1$

- $B$: Lenders’ value (total market value of debt)
- $v$: Borrower’s value

The efficient allocation solves

$$B(v) = \max_{m, \{c_s, v'_s\}_{s=H,L}} \sum_{s \in \{L, H\}} \mu_s \left[ -m + f(m) - c_s + qB(v'_s) \right]$$

subject to

$$\mu_H \left[ \theta_H U(c_H) + \beta v'_H \right] + \mu_L \left[ \theta_L U(c_L) + \beta v'_L \right] = v \quad \text{(PKC)}$$

$$\theta_L U(c_L) + \beta v'_L \geq \theta_L U(c_H) + \beta v'_H \quad \text{(IC)}$$

$$\theta_H U(c_H) + \beta v'_H \geq \theta_H U(f(m)) + \beta v_a \quad \text{(SUST)}$$

$$v'_H, v'_L \geq v_a \quad \text{(SUST')},$$
PROPERTIES

- Region with ex-post inefficiencies
  - Lack of commitment plays critical role

- Transit in and out of the region with ex-post inefficiencies
  - Private information plays critical role

- Low borrower values are associated with low imports of intermediates and low output
Region of Ex-Post Inefficiencies

When borrower’s value is low, can make both borrower and lenders better off ex-post
Any Efficient Allocation Starts on the Efficient Region

Region with Ex-Post Inefficiencies

Efficient Region

Lenders’ Value

B

Borrower’s Value

$\tilde{v}$

$v_a$
Starting from $v_0$, a sequence of high taste shocks pushes the economy to the region with ex-post inefficiencies.
After the realization of a high taste shock, the continuation value is lower than the current one: $v^\prime_H(v) < v$
Two Countervailing Forces

- Incentive force: Want to spread continuation values
  - Cheapest way to provide utility
    - Make $c_H$ large and $c_L$ small
    - Spread out continuation values
  - Desirable to make $v'_H$ low

- Commitment force: Want to back-load borrower payoff
  - By back-loading, relax future sustainability constraint
  - Low production distortions in the future
  - Want high $v'_H$
Two Countervailing Forces

- Incentive force: Want to spread continuation values
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If either (i) $\theta_H - \theta_L$ sufficiently high or (ii) $\mu_H$ sufficiently low the incentive force outweighs commitment force $\Rightarrow v'_H(v) < v$
Transits to the Region with Ex-Post Inefficiencies

Starting from $v_0$, a sequence of high taste shocks pushes the economy to the region with ex-post inefficiencies
What Happens When Reach Autarky?

Bounce up the first time $\theta_L$ is realized
Low $v$ Associated with Low Output and Intermediates

$m^* = \text{statically efficient amount of intermediates}, \ f'(m^*) = 1$
Recap

- A sequence of high taste shocks pushes the economy to the region with ex-post inefficiencies
- This path is associated with falling imports of intermediates and output
- Autarky is a reflecting point, not absorbing
IMPLEMENTATION:
DEFAULTS, BOND PRICES, AND MATURITY COMPOSITION
Equilibrium Payment Policy

- If $v > v_a$: Full payment, $\delta = 1$

- If $v = v_a$:
  - If $\theta = \theta_H$: Suspension of payments, $\delta = 0$
  - If $\theta = \theta_L$: Partial payment, $\delta = r$
Equilibrium Bond Prices

Given the equilibrium payment policy, prices consistent with lenders’ arbitrage conditions

\[
q_S(v) = \begin{cases} 
q & \text{if } v \in (v_a, \bar{v}] \\
q\bar{r} & \text{if } v = v_a
\end{cases}
\]

\[
q_L(v) = \begin{cases} 
q \sum_{s=L,H} \mu_j \left[1 + q_L(v'_s(v))\right] & \text{if } v \in (v_a, \bar{v}] \\
q \frac{\bar{r}}{1-q} & \text{if } v = v_a
\end{cases}
\]

where \( \bar{r} \) is the expected recovery rate

LT bond price increasing in borrower continuation value
From the contracting problem, total value of debt is:

\[ b(v, \theta_s) \equiv f(m(v)) - m(v) - c_s(v) + qB(v_s'(v)) \]

When \( \delta = 1 \), given prices, \( b_L(v) \) and \( b_S(v) \) must solve

\[ b(v, \theta_L) = b_S(v) + b_L(v) \left[ 1 + q_L (v'_L(v)) \right] \]

\[ b(v, \theta_H) = b_S(v) + b_L(v) \left[ 1 + q_L (v'_H(v)) \right] \]
How is Insurance Provided?

When there is default (only when $v = v_a$):

- Suspension and partial payments provide insurance

When there is no default:

- After $\theta_H$: Debt dilution
  - Borrower’s continuation value decreases
  - Higher probability of default in the near future
  - Long-term debt price falls $\Rightarrow$ capital loss for lenders
  - Wealth transfer from lender to the borrower

- After $\theta_L$: Debt buyback
  - Borrower’s continuation value increases
  - Lower probability of default in the near future
  - Long-term debt price rises $\Rightarrow$ capital gain for lenders
  - Wealth transfer from the borrower to the lenders
Default On-Path is Necessary

- Variation in price of long-term debt from variation in future default probability
- Not from variation in equilibrium SDF
  - Idiosyncratic shocks (SOE)

  - Aggregate shocks
  - Variation in equilibrium SDF
MATURITY OF DEBT SHORTENS AS DEFAULT IS MORE LIKELY
Maturity of Debt Shortens as Default is More Likely

Recall: $b_L(v)$ and $b_S(v)$ solve

\[
b_S(v) + b_L(v) \left[ 1 + q_L \left( v'_L(v) \right) \right] = b(v, \theta_L)
\]

\[
b_S(v) + b_L(v) \left[ 1 + q_L \left( v'_H(v) \right) \right] = b(v, \theta_H)
\]

When indebtedness is high (future default is likely):

- Long-term bond prices more sensitive to shocks
- Can obtain needed insurance with small amount of long-term debt
- Overall indebtedness is high so short-term debt must be high
Conclusion

- Key aspects of sovereign debt and default rationalized as best outcome of a sovereign debt game

- Best outcome is constrained efficient
  - It solves optimal contracting problem with informational and commitment frictions

- Default is *not* driven by
  - Market incompleteness
  - The high reliance on short-term debt

But by the underlying frictions

- Method to implement efficient allocation likely generalize to other contracting problems
Dynamics Around Default Episodes

- **GDP**
  - % deviation from trend vs. Year after Default
  - Mean

- **Consumption**
  - % deviation from trend vs. Year after Default
  - Mean

- **Intermediate Imports**
  - % deviation from trend vs. Year after Default
  - Mean

Sample of Defaults Episodes from Mendoza and Yue (2012)

Source: WDI, UN Comtrade and Feenstra
Pricing Function: Short-Term Bond

Consistent with lenders’ arbitrage condition

\[ q_S(h^t) = q\mathbb{E}\left[\chi_S(h^{t+1})|h^t\right] \]

where

\[ \chi_S(h^{t+1}) = \begin{cases} 
1 & \text{if } \delta_{t+1} = 1 \\
r & \text{if } \delta_{t+1} = r \\
q\mathbb{E}\left[\chi_S(h^{t+2})|h^{t+1}\right] & \text{if } \delta_{t+1} = 0 \end{cases} \]
Pricing Function: Long-Term Bond

Consistent with lenders’ arbitrage condition

\[ q_L(h^t) = q \mathbb{E} [\chi_L(h^{t+1}) | h^t] \]

where

\[ \chi_L(h^{t+1}) = \begin{cases} 
1 + q_L(h^{t+1}) & \text{if } \delta_{t+1} = 1 \\
\frac{r}{1-q} & \text{if } \delta_{t+1} = r \\
q \mathbb{E} [\chi_L(h^{t+2}) | h^{t+1}] & \text{if } \delta_{t+1} = 0
\end{cases} \]
Foreign exporters no-arbitrage condition:

\[ 1 = \mathbb{E} \left[ p_t(h^{t-1}) \left( 1 - \tau(h^t_g, \theta_t) \right) \big| h^{t-1} \right] \]

Firms’ optimality:

\[ f'(m(h^t_m)) = p(h^{t-1}) \]
Equilibrium Capital Controls and Imports Price

- From the contracting problem, I get $m(v)$

- Construct $p(v)$ and $\tau(v)$ consistent with firms optimality and foreign exporters no-arbitrage conditions:

\[
f'(m(v)) = p(v)
\]

\[
1 = p(v)(1 - \tau(v))
\]