A Welfare Criterion for Models with Distorted Beliefs

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Belief distortions have no effect on recent financial regulations

This research: Welfare criterion for models with \textit{heterogeneously} distorted beliefs.
A true story of famous economists

Joe Stiglitz
Believes natural, 90%
Has $100

Bob Wilson
Believes synthetic, 90%
Has $100

Pillow worth: $50
They decide to bet, at some cost

Side bet, total $200
Destroy pillow to find out!

Pillow worth: $50
They realize that this is Pareto optimal.

Expected return from the bet:

\[ 0.9 \times $100 - 0.1 \times $100 - 1/2 \times $50 = $55 \]

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Pillow worth: $50
The end of the story is unknown

Nothing is produced, a perfect pillow is destroyed

A negative-sum game

Pareto criterion seems unsatisfactory
Heterogeneous beliefs might come from two distinct sources

**Subjective beliefs (Differences in priors):**
- Savage: Beliefs as representation of preferences.
- Spurious unanimity problem (Mongin, Gilboa, Schmeidler)

**Distorted beliefs (Mistakes in Bayesian updating):**
- Biases that distort people’s beliefs: Overconfidence, representativeness, limited attention...
- Pareto not appropriate. Need to use **objective belief**.

**Challenge:** Planner might not know the objective belief.
Our solution: Listen to all beliefs

**Chinese proverb**: Listen to all, pluck a feather from every passing goose, but, follow no one absolutely.

Our **belief-neutral criterion**: An allocation is (in)efficient, if it is (in)efficient under any reasonable belief.

- **Reasonable**: Any convex combination of agents’ beliefs.
- Same belief must be used to evaluate everyone’s utility.

Criterion is necessarily incomplete but works surprisingly well!

**Outline of the talk**: Formalization and applications.
Consider a generic dynamic economy

- Dynamic model with periods \( t \in \{0, 1, \ldots, T\} \).
- Uncertainty captured by state, \( s_t \).
- Agents \( i \in \{1, \ldots, N\} \) with **different beliefs** \( \Pi^i = \{\pi^i_{t,s}\} \).
- A social allocation: \( x = \{x^i_T(s_T)\} \).
- Agent \( i \)’s expected utility under own belief: \( E^i_0 [u_i (s_T, x^i_T(s_T))] \).

**Set of reasonable beliefs:** Any convex combination of agents’ beliefs: 
\( \Pi^h = \sum_i h^i \Pi^i \), where \( h^i \geq 0 \) and \( \sum_i h^i = 1 \).

- Agent \( i \)’s expected utility under belief \( h \): \( E^h_0 [u_i (s_T, x^i_T(s_T))] \).
- We offer two versions of our criterion...
Belief-neutral criterion: Welfare function version

- Suppose the planner uses a fixed social welfare function:

\[ W \left( E^h_0 [u_1], E^h_0 [u_2], \ldots, E^h_0 [u_N] \right). \]

E.g.: Bergsonian: \( \sum \lambda_i E^h_0 [u_i] \) with \( \lambda_i \geq 0 \), Utilitarian: \( \sum E^h_0 [u_i] \).

**Definition (V1: Expected social welfare)**

An allocation \( x \) is **belief-neutral inferior** to another allocation \( \hat{x} \) if it leads to a lower social welfare under any reasonable belief \( \Pi^h \), i.e.:

\[
W \left( E^h_0 \left[ u_1 \left( s_T, x^1_T (s_T) \right) \right], \ldots, E^h_0 \left[ u_N \left( s_T, x^N_T (s_T) \right) \right] \right) \\
\leq W \left( E^h_0 \left[ u_1 \left( s_T, \hat{x}^1_T (s_T) \right) \right], \ldots, E^h_0 \left[ u_N \left( s_T, \hat{x}^N_T (s_T) \right) \right] \right).
\]
Belief-neutral criterion: Welfare function version

Back to the story:

- Suppose planner uses utilitarian function: \( W = E^h [u_{Joe}] + E^h [u_{Bob}] \).
- Planner assumes probability of natural is between 0.1 and 0.9.
- For any belief:
  - Expected welfare from status quo = $100 + $100.
  - Expected welfare from the bet = $100 + T^h + $100 - T^h - $50.
- The bet is belief-neutral inferior to the status quo.

**Key insight:** Transfer has no effect on the sum \( \Longrightarrow \) Negative sum.
Belief-neutral criterion: Pareto version

- Suppose the planner uses a generalized Pareto criterion.

**Definition (V2: Belief-neutral Pareto)**

An allocation $x$ is belief-neutral Pareto inefficient (resp. efficient) if, under any reasonable belief $\prod^h$, there exists (resp. does not exist) an alternative feasible allocation $\hat{x}$ s.t.

$$E^h_0 \left[u_i \left(s_T, x^i_T (s_T) \right) \right] \leq E^h_0 \left[u_i \left(s_T, \hat{x}^i_T (s_T) \right) \right] \quad \text{for each agent } i,$$

with strict inequality for at least one $i$. 
Belief-neutral criterion: Pareto version

Back to the story:

- Under Joe’s belief:
  - Bet: Expected transfer of $80 from Bob to Joe and a destroyed pillow.
  - Alternative: Transfer $80 from Bob to Joe and save the pillow.

- Under any reasonable belief: There is a (belief-dependent) transfer that could make everyone better off.

- The bet is belief-neutral Pareto inefficient.

Key insight: There is no belief that can rationalize the destroyed pillow.
The two versions are closely related

- **Common element across V1-V2:** Consider all reasonable beliefs. V1 fixes a social welfare function.
- Enables to compare allocations directly.
- V2 considers all social welfare functions.

- **Proposition:** \( x \) is belief-neutral efficient (inefficient) iff under any reasonable belief \( \Pi^h \), there exists (does not exist) Pareto weights s.t.:

\[
x \in \arg \max_{\hat{x} \text{ feasible}} \sum_{i=1}^{N} \lambda_i E_0^h \left[ u_i \left( s_T, \hat{x}_T^i \left( s_T \right) \right) \right].
\]

- With V2, no direct comparisons. But **categorizes** allocations:
  1. Belief-neutral inefficient...
  2. Belief-neutral efficient...
  3. Neither...
Comments on the criterion

- We separate distorted beliefs from preferences.
  - Move subjective beliefs into state-dependent utility.

- We evaluate welfare with *heterogeneously distorted beliefs*.
  - Not applicable to aggregate mistakes.
  - But *negative-sum speculation* prevalent in many applications.
  - We detect also *positive-sum speculation* in relevant applications.

**Next:** Brief discussion of various (negative and positive) applications.
Speculative bubbles: Harrison-Kreps (1978), Scheinkman-Xiong (2003)...

- Overvaluation and trading costs (brokerage fees, bid-ask spreads).
  → Equilibrium is belief-neutral inefficient.
- Active trading severely undercuts portfolio returns (Barber-Odean...)
Application 2: Overinvestment in bubble models

S. bubbles also drive **overinvestment**: Bolton-Scheinkman-Xiong (2006)...

- Overvaluation with **endogenous supply**.
- Excessive investment **according to all beliefs**.
  - Equilibrium is **belief-neutral inefficient**.

Related app: Cons./savings distortions in macro models (Sims, 2008).
Speculation can also be **positive-sum in lemons models**.

- Overvaluation induces sellers with quality goods to sell.
- Mitigates the lemons problem. **Additional gains from trade.**

\[ \implies \text{Equilibrium is belief-neutral superior to no-trade status quo.} \]
Leverage cycles: Geanakoplos (2003, 2009), Simsek (2010)...

- Optimists use **collateralized debt** to finance their purchases.
- Might choose **risky debt** despite **bankruptcy costs** in bad state.
- Costs incurred with positive probability **according to all beliefs**.

→ Equilibrium is **belief-neutral inefficient**.
Application 5: Excessive risk taking with speculation

Speculative trading models: Many examples in asset pricing.

- Previous models assume risk-neutrality for simplicity.
- But trading increases agents’ consumption risks.
- With risk-averse agents, negative-sum in expected utilities.

⇒ Equilibrium is belief-neutral inefficient.
Application 5.5: Speculation and risk sharing trade-off

There can also be a risk sharing motive for trade (Simsek, 2012):

- Asset with payoff $v$, where $v \sim N(0, 1)$.
- Two traders with MV prefs and endowments: $e + v$ and $e - v$.

**Common beliefs** $\implies$ Trading reduces risks.

- Trader 1’s position and net worth:
  $$x_1 = -1 \text{ and } n_1 = n_2 = e.$$

**Heterogeneous beliefs:** Trader 1: $v \sim N(\varepsilon, 1)$, trader 2: $v \sim N(-\varepsilon, 1)$.

- Trader 1’s position and net worth:
  $$x_1 = -1 + \frac{\varepsilon}{\theta} \text{ and } n_1 = e + \frac{\varepsilon}{\theta}v.$$

**Large disagreements, $\varepsilon > \theta$** $\implies$ Trading increases risks.

Equilibrium is **belief-neutral inefficient** whenever trading increases risks.
Conclusion: A new welfare criterion

Welfare criterion for models with heterogeneously distorted beliefs:

- **Belief-neutral (in)efficiency ⇔ Listen to all beliefs.**
- Incomplete ranking, but surprisingly powerful.

Opens normative analysis for financial regulation.

- Avoid negative-sum speculation (trading costs, over-investment, excessive risks...).
- Also detects positive-sum speculation (mitigate the lemons problem...).