Technology-Skill Complementarity

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Introduction

This paper studies a simple general equilibrium model with complementarity between technology and human capital.

There are two main motivations.

1. Wage inequality has displayed large and long-lived shifts over the last century: see Katz and Goldin (2007).

In recent years, wage inequality has grown, in the U.S. and elsewhere.

Empirical work suggests that most of this change is an increase in **between-firm** inequality, with very little increase in **within-firm** inequality.

Changes in technology are an obvious candidate to explain these large shifts in the wage structure.

Introduction

2. There is an extensive literature that uses search models to study employment and wages in settings with heterogeneous firms.

Relative to this literature, the contribution of the present paper is to micro-found the surplus function.

Here each firm faces a downward sloping demand curve for its product.

This demand curve determines the quantity of labor the firm wants to employ, as a function of its productivity.

Thus, the surplus generated by any worker depends on total employment within the firm.

Related literature

Wage inequality: Song, Price, Guvenen, et. al. (2015) for the U.S.; Faggio, Salvanes, and van Reenen (2007) for the U.K.; Card, Heining, and Kline (2013) for Germany; Hakanson () for Sweden; Helpman, et al. for Brazil.

Search models: Burdett and Mortensen (1998), Jovanovic (1998), Moscarini and Postel-Vinay (2013), Lise and Robin (2013), etc.

Introduction

The distinction between human capital and technology is not clear.

Some would argue that technology is simply a form of human capital.

Here, human capital is an asset that belongs to a single worker,

who is the only one that can employ it in production.

Hence it is a “rival” input.

Technology is an asset that belongs to a firm. The firm can employ multiple workers, and technology is a “nonrival” input used by all of the workers. The fact that it is nonrival, within the firm, also distinguishes it from physical capital.

Introduction

Here technology and human capital are inputs in a CES production function.

They are complements: the substitution elasticity is less than unity.

Labor markets are assumed to be frictionless.

The low substitution elasticity means that the market (and efficient) allocation of labor across firms displays positively assortative matching.

Outline

1. The model
2. The competitive equilibrium
3. Technical change: does a rising tide lift all boats?
4. A multi-sector extension: revisit the rising tide question
5. Conclusions

1. Model: final goods

A single final good is produced competitively, with CRS, using differentiated goods as inputs.

Producers of differentiated goods are indexed by technology $x_j > 0$, which determines their price p_j.

All differentiated goods enter symmetrically,

$$Y_F = \left(N \sum_{j=1}^{J} \gamma_j y_j^{(\rho-1)/\rho} \right)^{\rho/(\rho-1)},$$

where $\rho > 1$ is the substitution elasticity, $\{\gamma_j\}_{j=1}^{J}$ are shares for technologies $\{x_j\}_{j=1}^{J}$, and N is the number (mass) of firms.

1. Model: final goods

The price of the final good is

$$p_F \equiv 1 = \left(N \sum_{j=1}^{J} \gamma_j p_j^{1-\rho} \right)^{1/(1-\rho)},$$

and demands for differentiated goods are

$$y_j = \left(\frac{p_j}{p_F} \right)^{-\rho} Y_F, \qquad \text{all } j.$$

1. Model: differentiated goods

Labor, differentiated by human capital level h, is the only input.

The output of a firm depends on the size and quality of its workforce,

as well as its technology.

A firm with technology x_j that employs workers with various

human capital levels, $\ell(h) \geq 0$, all h, has output

$$y_j = \int \ell_j(h)\phi(h, x_j)dh, \qquad \text{all } j,$$

where $\phi(h, x)$ is a CES function with elasticity $\eta < 1$.

1. Model: wages, labor allocation

A firm employs labor types h that minimize unit cost $w(h)/\phi(h, x_j)$.

Since $\eta < 1$, efficiency requires positively assortative matching.

Hence equilibrium is characterized by cutoff levels $\{b_j\}_{j=1}^{J-1}$, where

workers with $h \in (b_{j-1}, b_j]$ work for firms of type j,

with $b_0 = h_{\min}$, and $b_J = h_{\max}$.

Hence the equilibrium wage function $w(h)$ satisfies

$$\frac{w'(h)}{w(h)} = \frac{\phi_h(h, x_j)}{\phi(h, x_j)}, \qquad h \in (b_{j-1}, b_j), \qquad \text{all } j,$$

with kinks at the points b_j, $j = 1, ..., J-1$.

1. Model: differentiated good prices, output levels

Price is the usual markup over unit cost,

$$p_j = \frac{\rho}{\rho - 1} \frac{w(h)}{\phi(h, x_j)}, \qquad h \in (b_{j-1}, b_j], \qquad \text{all } j.$$

Since x_j and x_{j+1} are both willing to hire workers b_j,

$$\begin{aligned} \frac{p_{j+1}}{p_j} &= \frac{\phi(b_j, x_j)}{\phi(b_j, x_{j+1})}, \\ \frac{y_{j+1}}{y_j} &= \left(\frac{\phi(b_j, x_{j+1})}{\phi(b_j, x_j)} \right)^{\rho}, \qquad j = 1, ..., J-1. \end{aligned}$$

Firms with higher x_j have lower cost and price, $p_{j+1} < p_j$.

and they have higher output, revenue, profits, $y_{j+1} > y_j$.

The labor allocation across firms with the same technology x_j is

not entirely pinned down, but all have the same price and output.

2. Competitive equilibrium

Define Ψ_j as "total labor productivity" at firms of type j,

$$\Psi_j \equiv \int_{b_{j-1}}^{b_j} \phi(h, x_j) g(h) dh, \qquad j = 1, ..., J.$$

Labor market clearing requires

$$L\Psi_j = N\gamma_j y_j, \qquad j = 1, ..., J. \qquad \text{(LMC)}$$

CE is characterized by $\{b_j\}_{j=1}^{J-1}$ satisfying (LMC) and

$$\frac{y_{j+1}}{y_j} = \left(\frac{\phi(b_j, x_{j+1})}{\phi(b_j, x_j)}\right)^{\rho}, \qquad j = 1, ..., J-1,$$

with $b_0 = h_{\min}$ and $b_J = h_{\max}$.

A solution exists and it is unique.

2. Competitive equilibrium: an example

The distribution of firm types in the computed example is continuous.

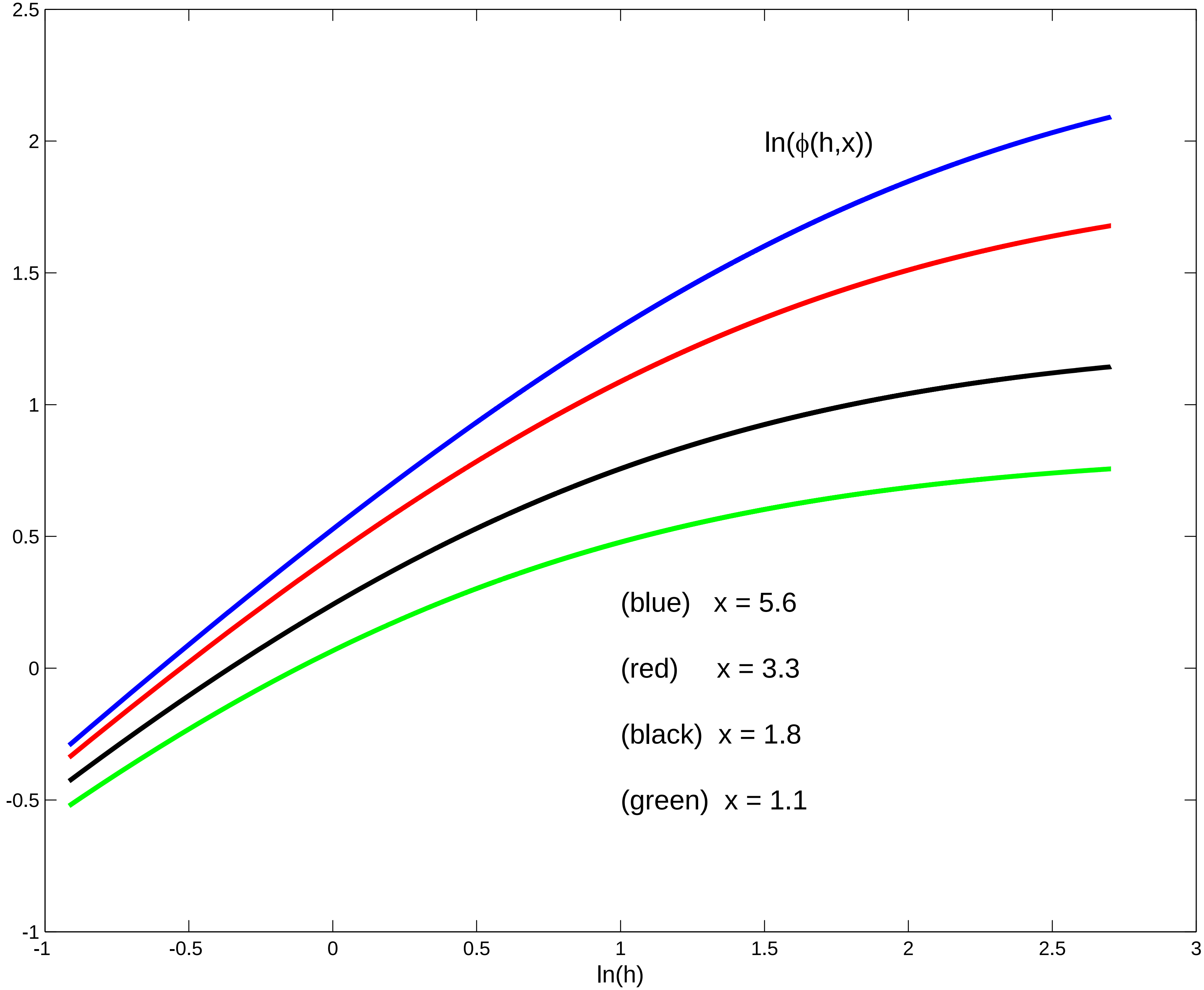
h has a (truncated) lognormal distribution, with parameters (μ_h, σ_h).

x has a (truncated) Pareto distribution, with shape parameter λ_F.

The parameters are

$$\begin{aligned}
\omega &= 0.5, & \eta = 0.5, && \rho = 6, && \\
\lambda_F &= 1.04, & x^{\min} = 1, && x^{\max} = 8, && N = 5, \\
\mu_h &= 1, & \sigma_h = 1, && && \\
h^{\min} &= 0.4, & h^{\max} = 15, && L = 100. &&
\end{aligned}$$

Fig A1. labor productivity $\phi(h,x)$, various x-values, log-log scale



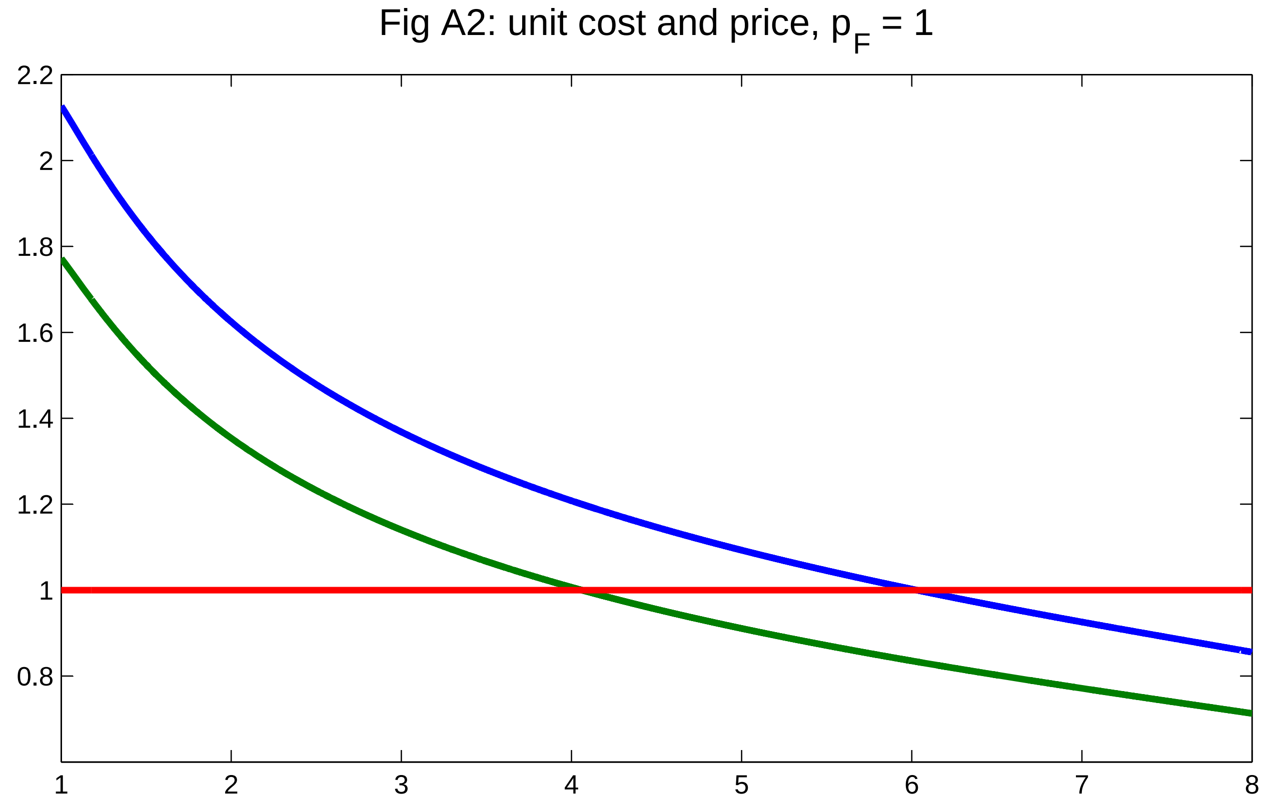
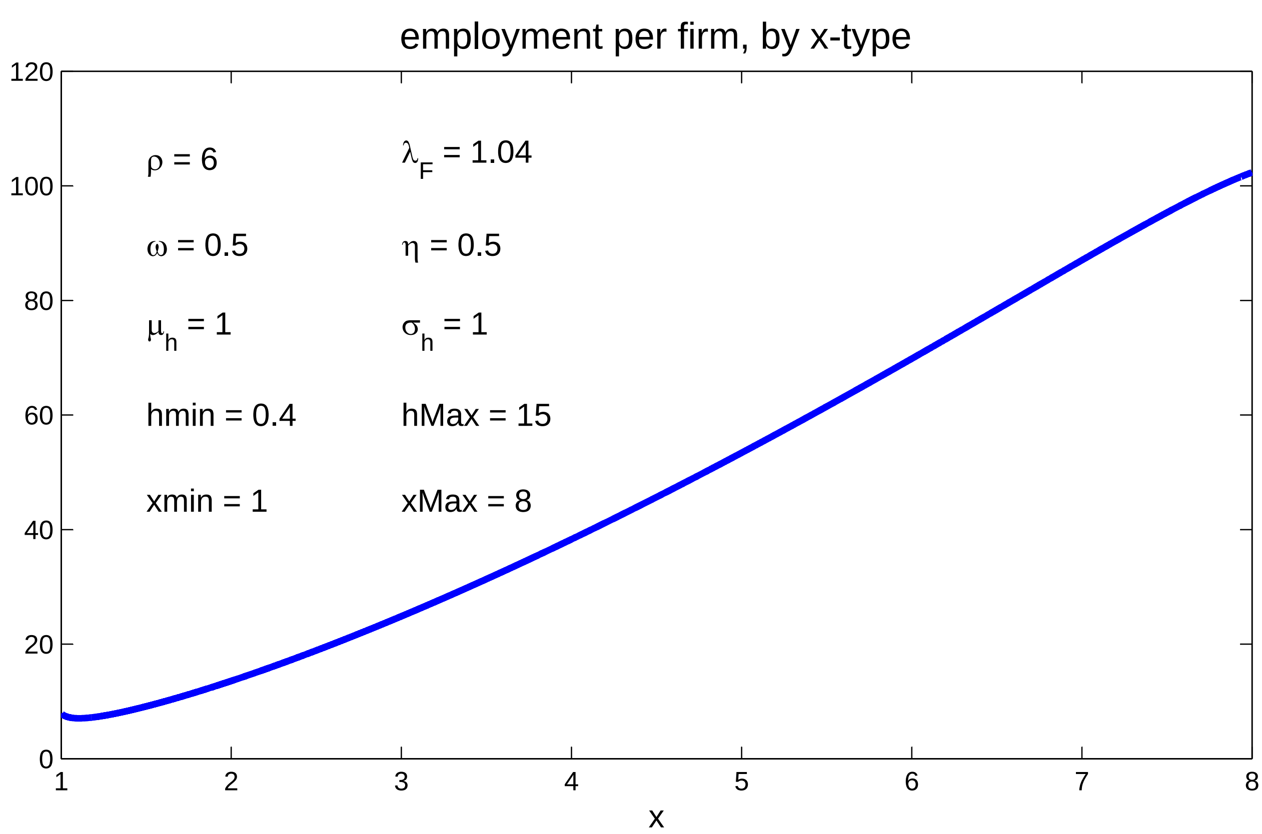


Fig A2: unit cost and price, p_F = 1
2.2
2
1.8
1.6
1.4
1.2
1
0.8
1
2
3
4
5
6
7
8



employment per firm, by x-type
120
100
80
60
40
20
0
ρ = 6
λ_F = 1.04
ω = 0.5
η = 0.5
μ_h = 1
σ_h = 1
hmin = 0.4
hMax = 15
xmin = 1
xMax = 8
1
2
3
4
5
6
7
8
x

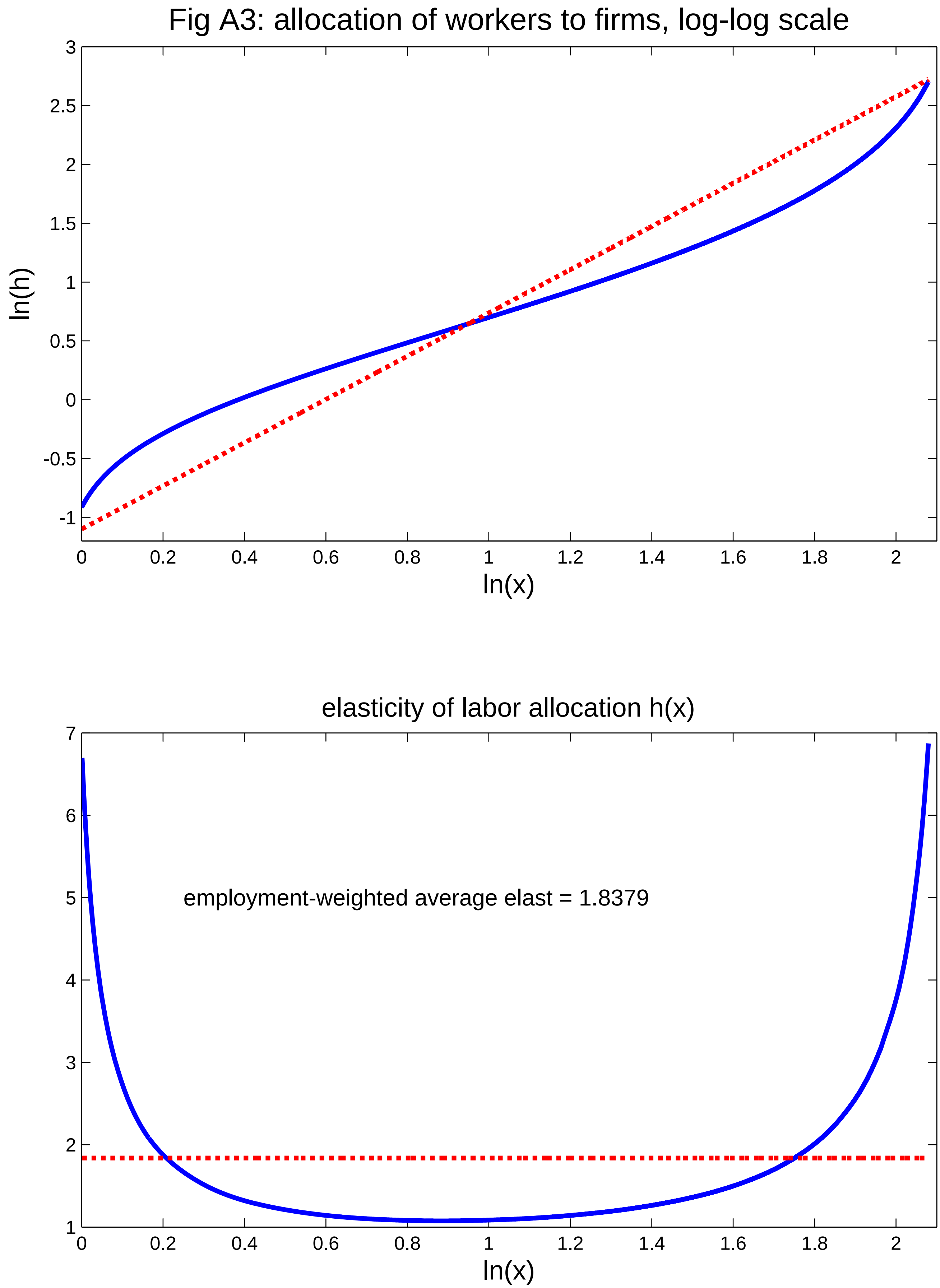
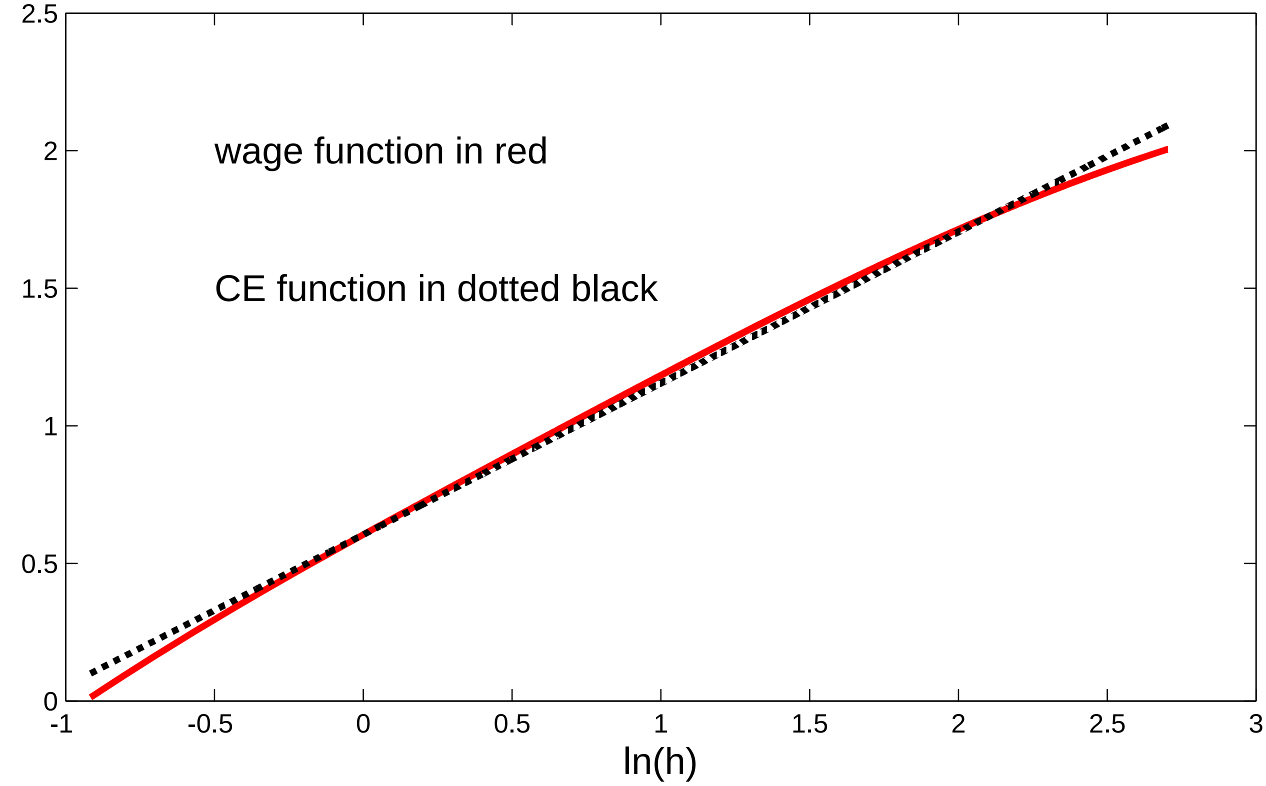
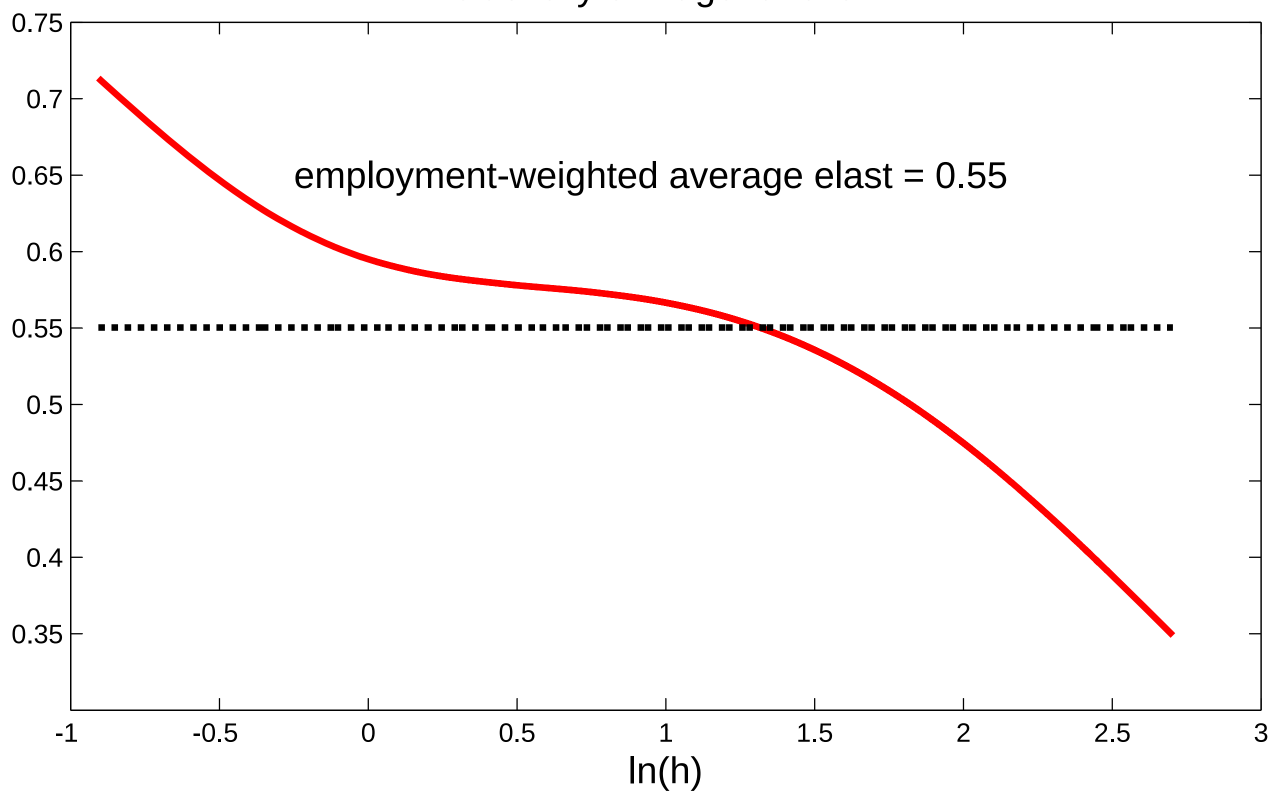


Fig A3: allocation of workers to firms, log-log scale
ln(h)
ln(x)
elasticity of labor allocation h(x)
employment-weighted average elast = 1.8379
ln(x)

Fig A4: wage function and CE function, log-log scale



elasticity of wage function



3. Technology improvements

Can a technology improvement $dx_k = \varepsilon > 0$ reduce wages for some workers? Or does a rising tide lift all boats?

Questions:

1. What are the short run (SR) effects on outputs y_j, Y_F, and prices p_j while labor is immobile?
2. What are the long run (LR) effects, when labor adjusts?
3. What are the LR effects on employment, wages?

3. Technology improvements

Let "hats" denote proportionate changes from the perturbation.

For both SR and LR, the change in final output is

$$\hat{Y}_F = \sum_{j-1}^{J} \nu_j \hat{y}_j,$$

where the weights are expenditure shares

$$\nu_j \equiv \frac{N\gamma_j}{Y_F} p_j y_j, \quad \text{all } j, \qquad \text{with} \quad \sum_{j-1}^{J} \nu_j = 1.$$

3. Technology improvements: Short Run

In the SR output changes only through the direct effect of technology,

$$\hat{y}_k^{SR} = \hat{\Psi}_k^{SR} > 0,$$

and $\hat{y}_j^{SR} = 0$, for $j \neq k$.

Final output changes by

$$\hat{Y}_F^{SR} = \nu_k \hat{y}_k^{SR} > 0.$$

The price changes for differentiated goods (with $p_F = 1$ fixed) are

$$\hat{p}_j^{SR} = \frac{1}{\rho}\left(\hat{Y}_F^{SR} - \hat{y}_j^{SR}\right), \qquad \text{all } j,$$

so $\hat{p}_k < 0$ and $\hat{p}_j > 0$, $j \neq k$.

3. Technology improvements: long run

In the long run firms adjust the quantity and quality of labor, but ....

PROPOSITION: To a first-order approximation, the reallocation of labor across firms has no effect on output of the final good, $\hat{Y}_F^{LR} = \hat{Y}_F^{SR}$.

The proof uses the Envelop Condition.

Since labor markets are competitive, the original (CE) allocation maximizes final output.

Hence for a small perturbation to technologies, reallocating labor has no first-order effect on final output.

But it does affect individual differentiated good outputs and prices, and it affects wages.

3. Technology improvements: long run

Let $dx_k = \varepsilon$, and let $\{b_j(\varepsilon)\}_{j=1}^{J-1}$ be the thresholds.

Differentiate the CE condition to characterize the b_j''s.

The signs depend on

$$\begin{aligned} -\left[\rho\hat{\phi}_x(b_{k-1}, x_k) - \hat{\Psi}_k\right], \\ \left[\rho\hat{\phi}_x(b_k, x_k) - \hat{\Psi}_k\right]. \end{aligned}$$

The reasoning is illustrated by looking at two special cases,

two technologies, $J = 2$. Either x_1 or x_2 is improved.

The size of the price decline $\hat{p}_k$ is proportional $\hat{\Psi}_k/\rho$.

Before the change, both x_1 and x_2 are willing to employ $h = b_1$.

3. Technology improvements: long run

If $x_k = x_1$, then $h = b_1$ is the *highest* skilled worker at his firm, so his

so his productivity rises *more* than the average for the firm,

$$\hat{\Upsilon}_1 < \hat{\phi}_x(b_1, x_1) < \rho\hat{\phi}_x(b_1, x_1).$$

Hence in the long run, x_1 firms expand employment,

$$b_1' = \left[\rho\hat{\phi}_x(b_k, x_k) - \hat{\Upsilon}_k\right] \times \text{positive terms} > 0,$$

reinforcing the original pattern of price changes.

All workers get wage increases,

$$\begin{aligned} \hat{w}(h) &= \hat{p}_1^{LR} + \hat{\phi}_x(h, x_1) > 0, && \text{at } x_1, \\ \hat{w}(h) &= \hat{p}_2^{LR} > 0, && \text{at } x_2, \end{aligned}$$

reinforcing the original pattern of price changes.

3. Technology improvements: long run

If $x_k = x_2$, then $h = b_1$ is the *lowest* skilled worker at his firm, so his productivity rises *less* than the average for the firm,

$$\hat{\Psi}_2 > \hat{\phi}_x(b_1, x_2).$$

Nevertheless, since $\rho > 1$, if the gap is not too large, then

$$\hat{\Psi}_2 < \rho\hat{\phi}_x(b_1, x_1),$$

so $b_1' > 0$, and x_2 firms expand employment, reinforcing the original pattern of price changes.

As before, all workers get wage increases.

CONJECTURE: The same logic holds for $J > 2$.

Wages may rise for all workers even if the condition above fails.

Increasing the supply of some differentiated inputs increases the demand for all others, through the effect on final output Y_F.

Hence their prices are bid up, and wages rise.

3. Technology improvements: a simulation

Suppose the top 10% of firms are affected.

The top 5% of firms get a 20% increase in productivity.

The next 5% get smaller increases (to keep the distribution smooth).

Because firms at the top hire more labor,

about a third of the workforce is directly affected.

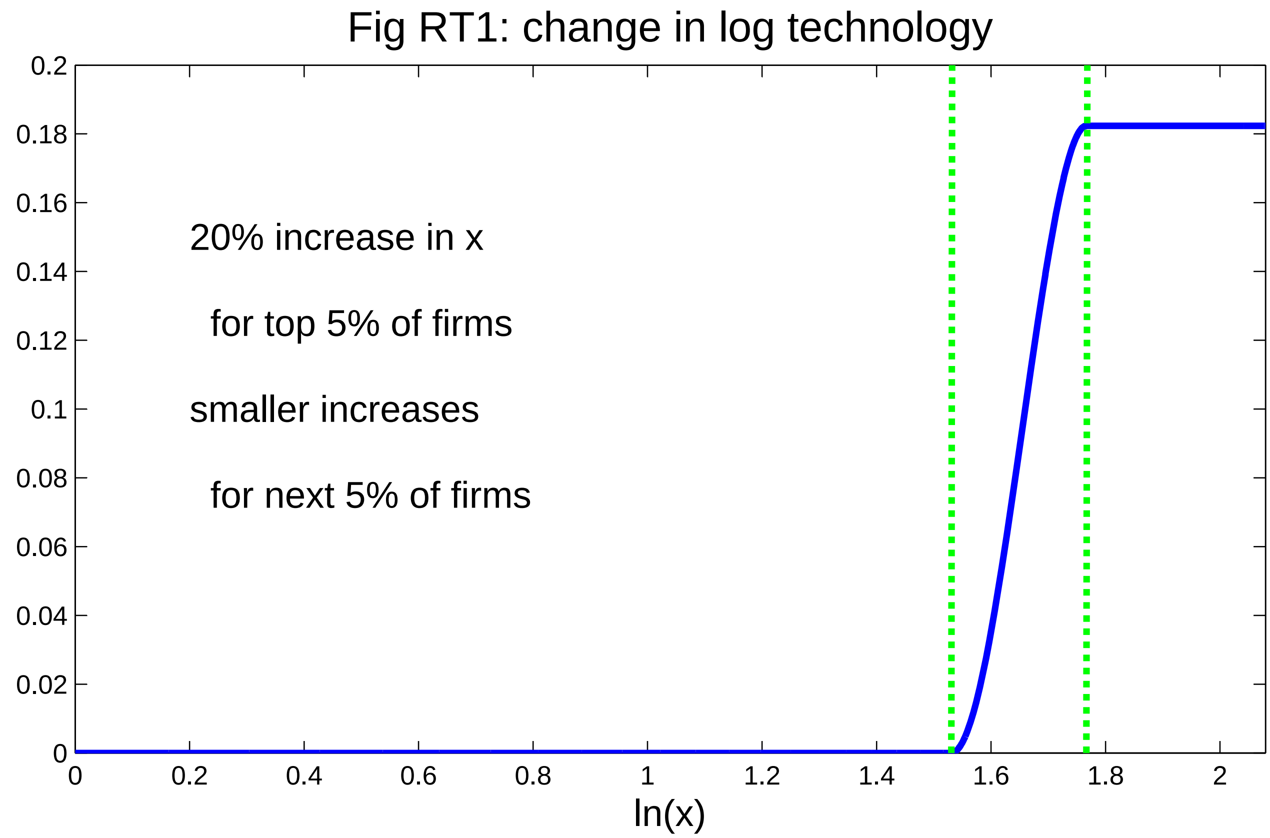
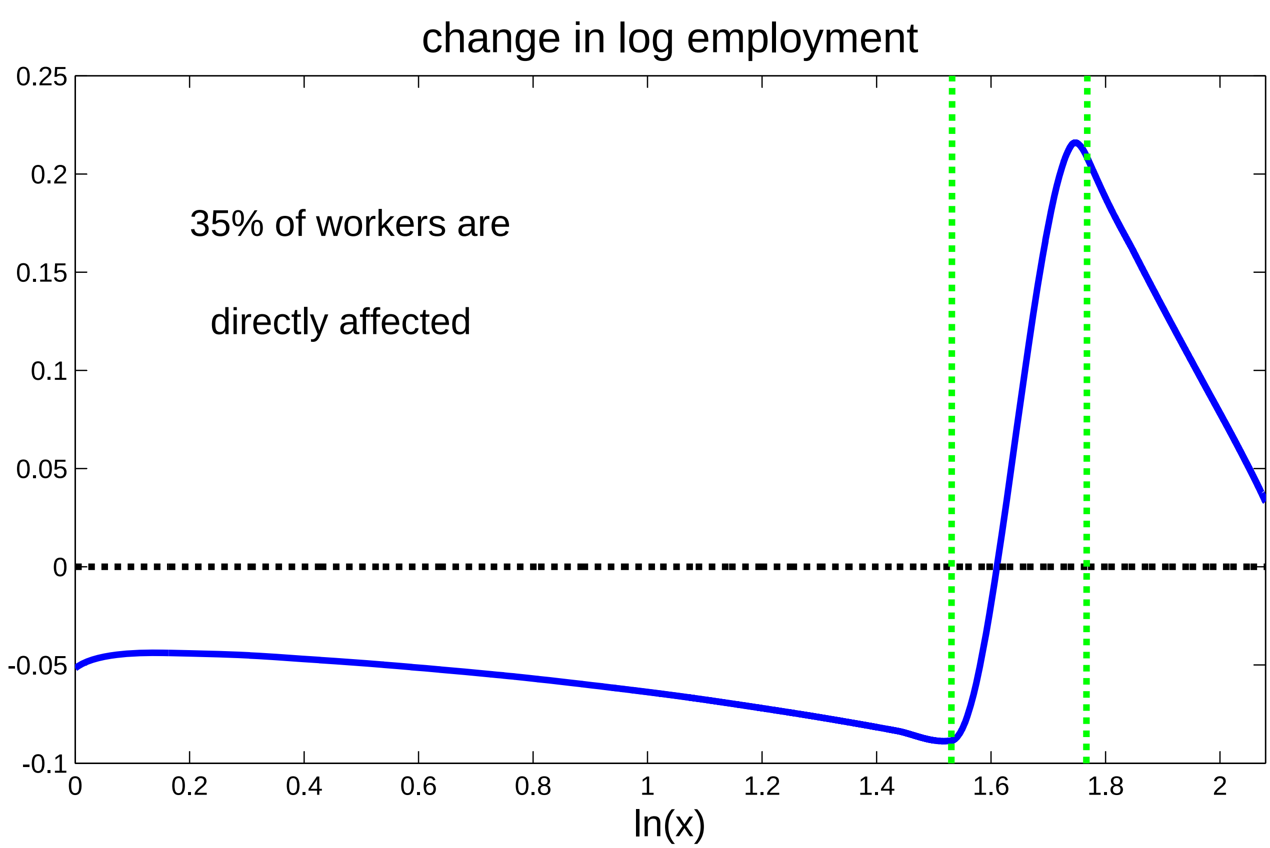


Fig RT1: change in log technology
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
20% increase in x
for top 5% of firms
smaller increases
for next 5% of firms
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
ln(x)


change in log employment
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
35% of workers are
directly affected
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
ln(x)

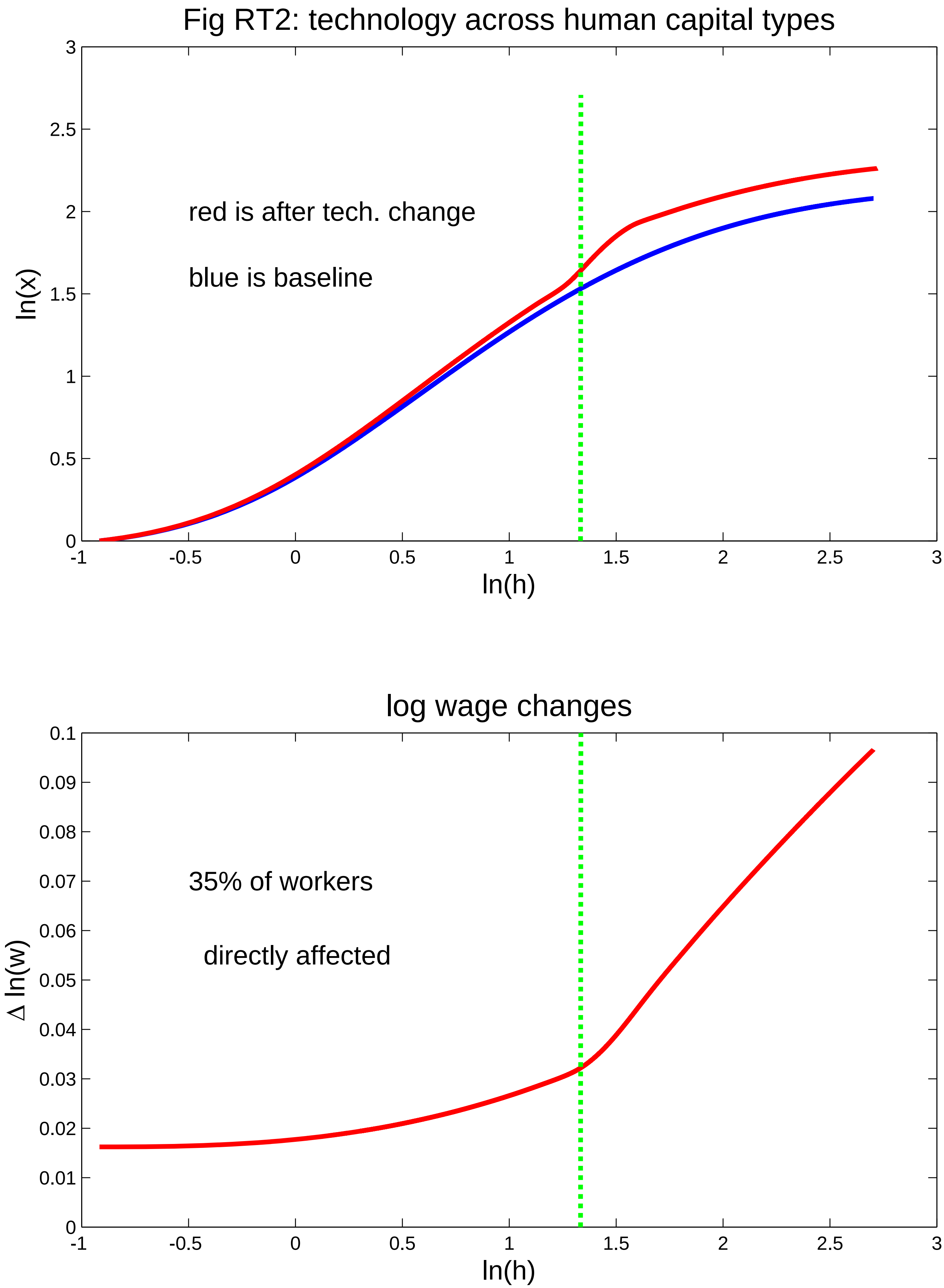


Fig RT2: technology across human capital types
red is after tech. change
blue is baseline
ln(x)
ln(h)
3
2.5
2
1.5
1
0.5
0
-1
-0.5
0
0.5
1
1.5
2
2.5
3
log wage changes
35% of workers
directly affected
Δ ln(w)
ln(h)
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-1
-0.5
0
0.5
1
1.5
2
2.5
3

4. A multi-sector extension

Can the answer to the “rising tide” question be reversed?

Consider a model with two tiers in production.

In the upper tier sectoral aggregates are used to produce final goods, and lower tiers, one for each sector, differentiated goods are used to produce the aggregates.

Each tier uses a CES aggregator, and the lower tiers have a higher elasticity of substitution.

The price effects of a limited technical shift are quite different in this setting.

4. A multi-sector extension

Suppose the final good technology is Cobb-Douglas, $\sigma = 1$,

$$Y_F = \prod_{s=1}^{S} Y_s^{\theta_s}, \qquad \sum_{s=1}^{S} \theta_s = 1.$$

The price of final output is

$$P_F \equiv 1 = \left[\prod_{s=1}^{S} \left(\frac{\theta_s}{P_s} \right)^{\theta_s} \right]^{-1}.$$

Demands for sectoral intermediates are

$$Y_s = Y_F \frac{\theta_s P_F}{P_s}, \qquad \text{all } s.$$

4. A multi-sector extension

Each sector has its own set of differentiated inputs $\{y_{sj}\}$.

The shares $\{\gamma_{sj}\}_{j=1}^{J}$ and number of firms N_s can vary across sectors.

The technologies and prices for sectoral intermediates are as before,

$$\begin{aligned} Y_s &= \left(N_s \sum_{j=1}^{J} \gamma_{sj} y_{sj}^{(\rho-1)/\rho} \right)^{\rho/(\rho-1)}, \\ P_s &= \left(N_s \sum_{j=1}^{J} \gamma_{sj} p_{sj}^{1-\rho} \right)^{1/(1-\rho)}, \qquad \text{all } s, \end{aligned}$$

Key assumption: $\rho > \sigma$. Goods within a sector are more substitutable

than are intermediates across sectors.

Equilibrium conditions: similar to the earlier model.

4. A multi-sector extension

Demands for differentiated inputs are

$$y_{sj} = Y_s \left(\frac{p_{sj}}{P_s} \right)^{-\rho}, \qquad \text{all } j, s,$$

so y_{sj} is increasing in Y_s and in P_s.

But Y_s, P_s are also linked through demand by final goods producers, so

$$y_{sj} = Y_s \theta_s^{\rho} p_{sj}^{-\rho} \left(\frac{Y_s}{Y_F} \right)^{-\rho/\sigma}.$$

With $\rho > \sigma$, Y_s has a **stronger effect through price than directly**.

An increase in Y_s reduces price P_s so sharply that demand y_{sj} falls.

Final output

$$Y_F = Y_1^{1/2} Y_2^{1/2}.$$

3 technology levels and 3 skill levels.

Firms: $N_1 = N_2 = 1$, and

all firms in sector 2 have $x = x_H$.

among firms in sector 1, share $\gamma \in (0,1)$ have x_M,

and $(1-\gamma)$ have x_L.

Labor supplies: $L_H = 1, \;\; L_M = \gamma, \;\; L_L = 1 - \gamma.$

Each firm employs one worker, and x_j employs h_j, so

$$y_j = \phi_j = \phi(h_j, x_j), \qquad j = L, M, H.$$

Sector-level aggregates are:

$$\begin{aligned} Y_1 &= \left[(1-\gamma)\, y_L^{(\rho-1)/\rho} + \gamma y_M^{(\rho-1)/\rho}\right]^{\rho/(\rho-1)} \\ Y_2 &= y_H. \end{aligned}$$

Since $Y_2 = y_H$, prices are

$$
\begin{aligned}
p_{2H} &= \frac{1}{2}\left(\frac{Y_1}{y_H}\right)^{1/2}, \\
p_{1j} &= \frac{1}{2}\left(\frac{y_H}{Y_1}\right)^{1/2}\left(\frac{y_{1j}}{Y_1}\right)^{-1/\rho}, \qquad j = L, M.
\end{aligned}
$$

Wages are proportional to revenue product,

$$
w_H = \frac{\rho - 1}{\rho} p_{2H} y_H, \qquad w_j = \frac{\rho - 1}{\rho} p_{1j} y_{1j}, \qquad j = L, M.
$$

Consider technical change that increases x_M. Then

$$\hat{Y}_1 = \nu \hat{y}_M,$$

where $\nu \in (0,1)$ is the cost share for x_M goods.

All wages increase if $\rho > 2$,

$$\begin{aligned}
\hat{w}_H &= \hat{p}_{2H} = \frac{1}{2}\hat{Y}_1 > 0, \\
\hat{w}_L &= \hat{p}_{1L} = \left(\frac{1}{\rho} - \frac{1}{2}\right)\hat{Y}_1, \qquad \hat{w}_L < 0 \quad \Longleftrightarrow \quad \rho > 2, \\
\hat{w}_M &= \hat{p}_{1M} + \hat{y}_M = \left(\frac{1}{\rho} - \frac{1}{2}\right)\hat{Y}_1 - \frac{1}{\rho}\hat{y}_M + \hat{y}_M \\
&= \left(\frac{1}{\rho} - \frac{1}{2}\right)\nu\hat{y}_M + \frac{\rho - 1}{\rho}\hat{y}_M, \\
\hat{w}_M &> 0 \quad \Longleftrightarrow \quad \rho + (\rho - 2)(1 - v) > 0.
\end{aligned}$$

4. Multi-sector model: a numerical example

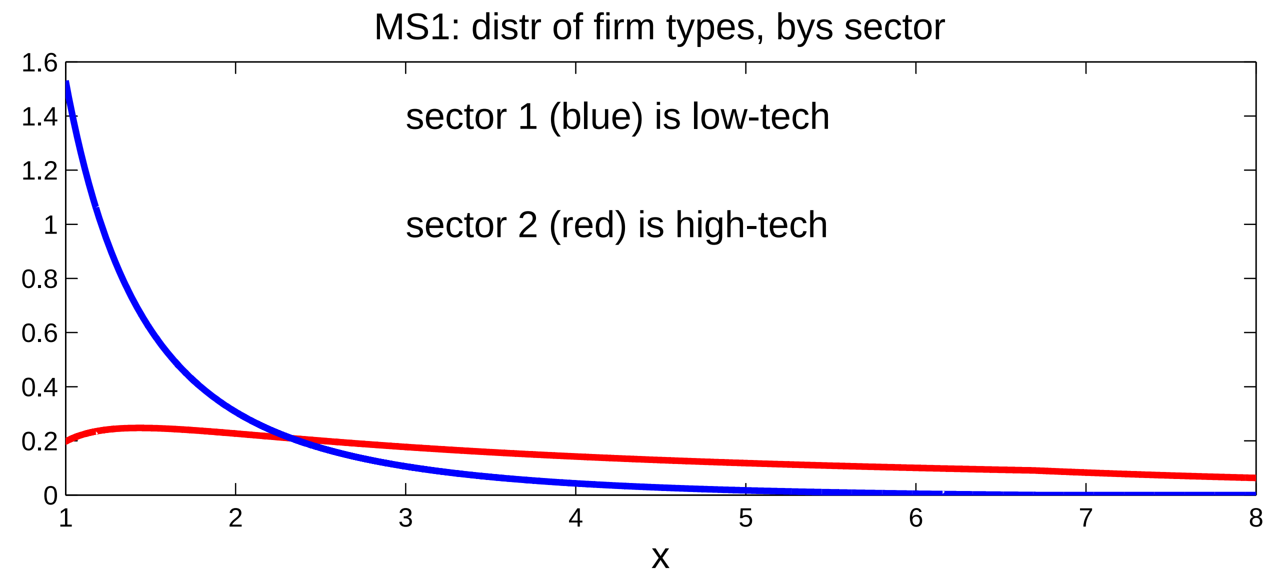
Keep all of the parameters from before, including N, L, G.

Two sectors, $S = 2$. For final goods

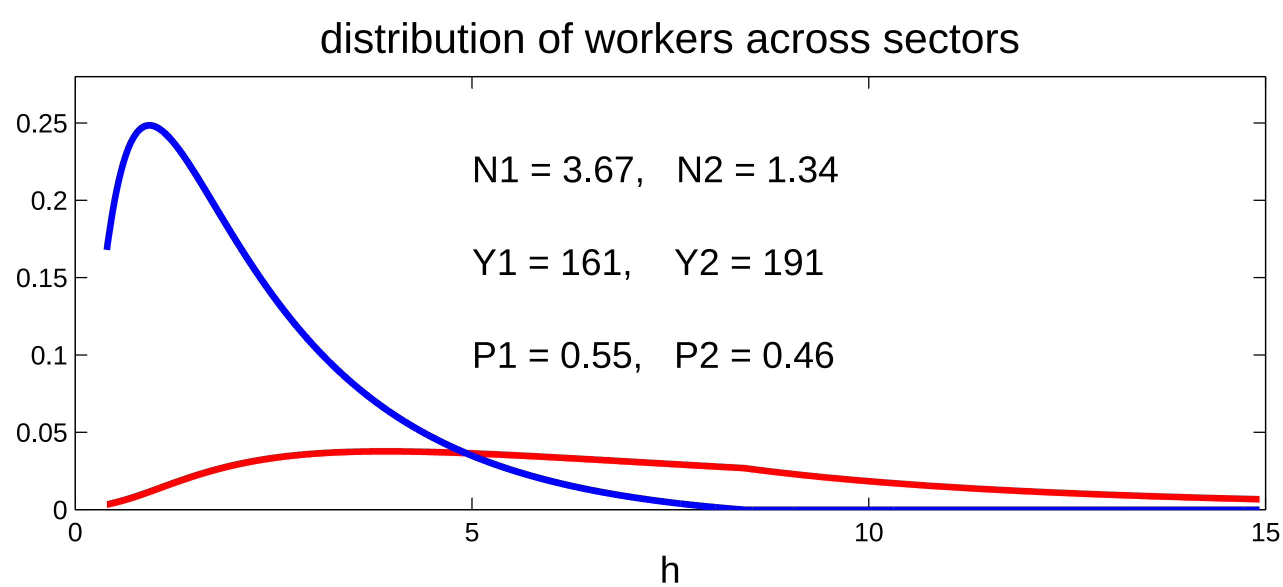
$$\sigma = 1, \qquad \theta_1 = \theta_2 = 1/2.$$

Sector 2 is high-tech.

The technology shift affects firms in sector 1, in the middle range of x.


MS1: distr of firm types, bys sector
sector 1 (blue) is low-tech
sector 2 (red) is high-tech
x


employment per firm
x


distribution of workers across sectors
N1 = 3.67,   N2 = 1.34
Y1 = 161,   Y2 = 191
P1 = 0.55,   P2 = 0.46
h

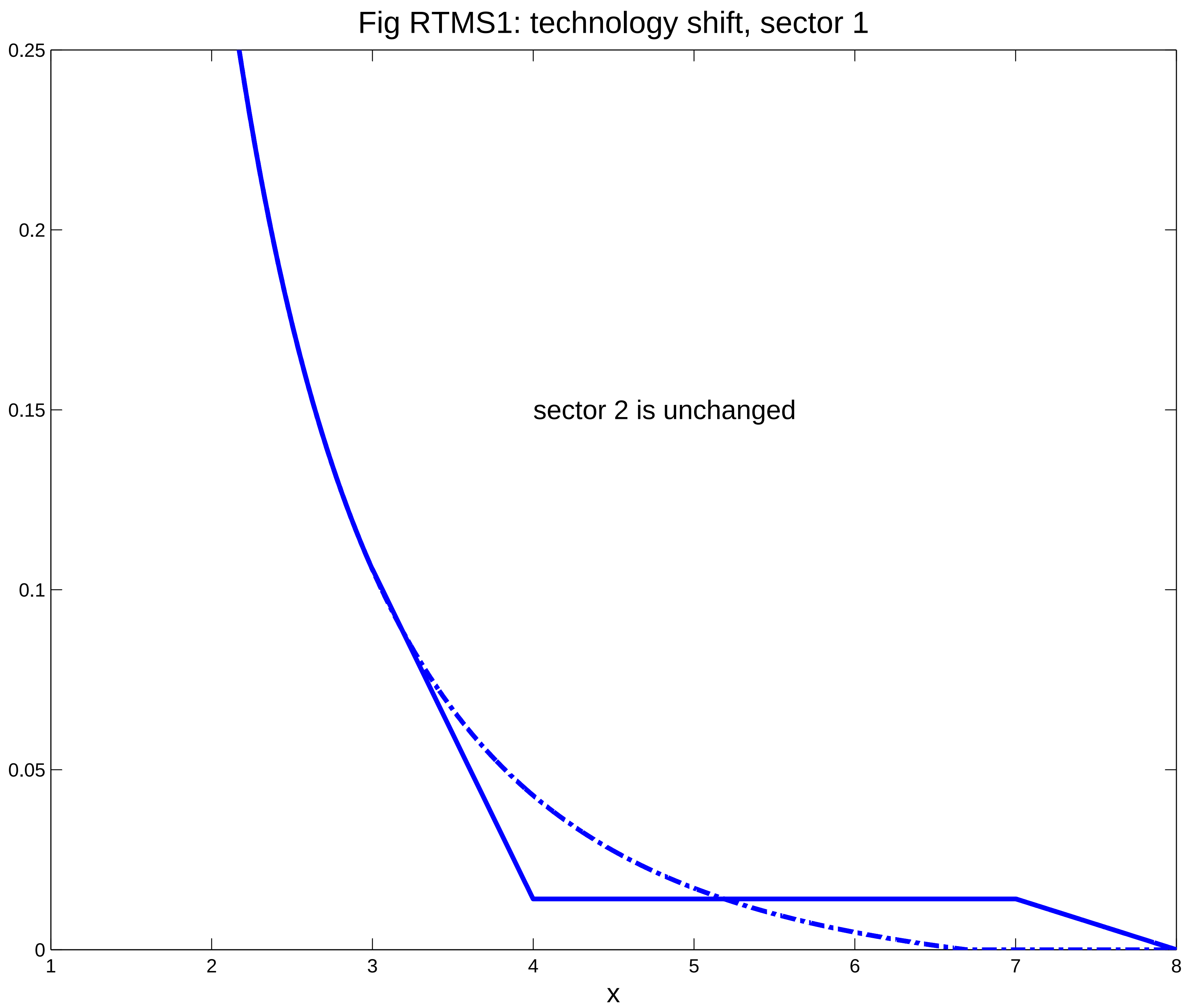
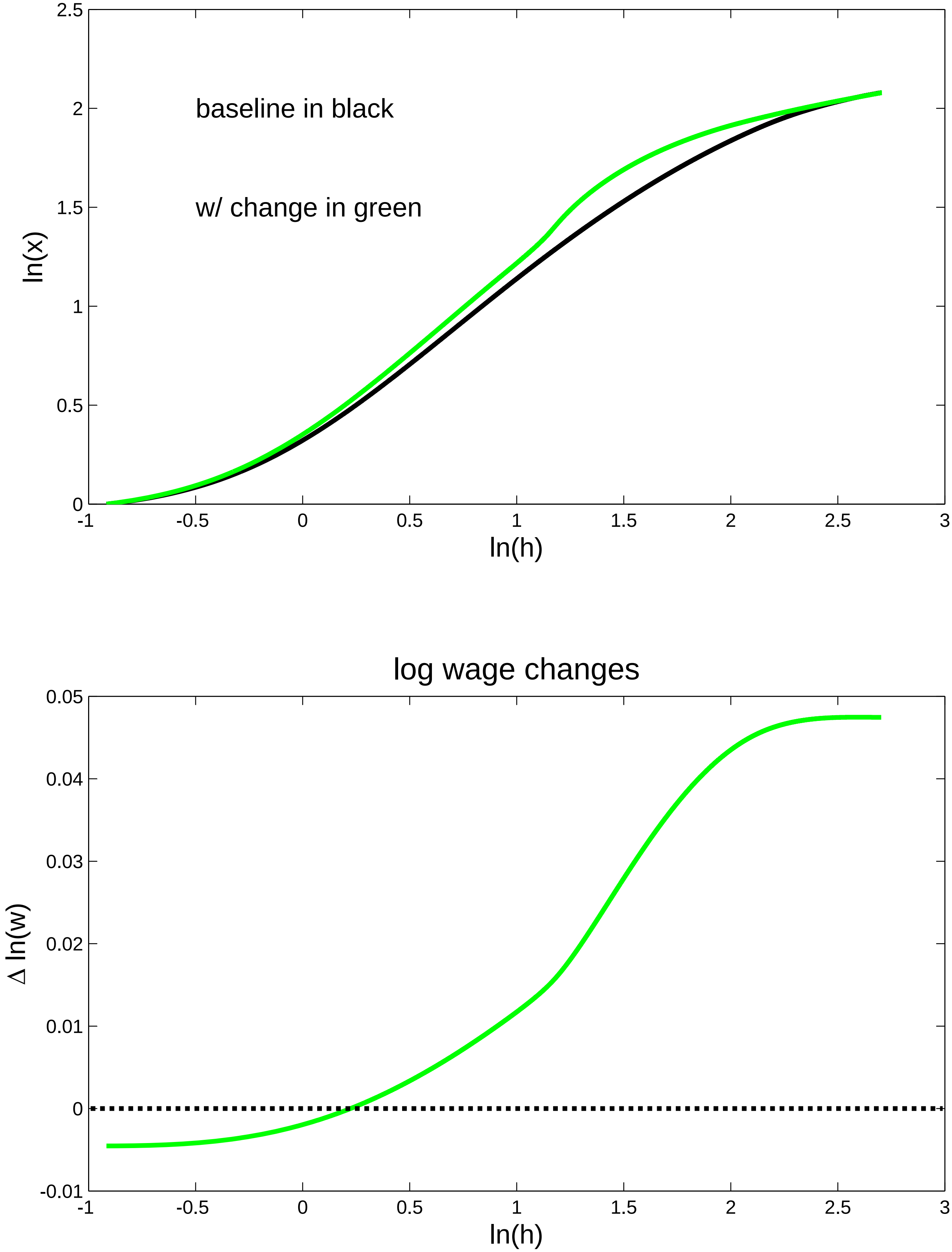


Fig RTMS1: technology shift, sector 1
sector 2 is unchanged
0.25
0.2
0.15
0.1
0.05
0
1
2
3
4
5
6
7
8
x

Fig RTMS2: technology across human capital types



5. Conclusions

To understand long-run changes in wage inequality, we need better models connecting wage rates to changes in technology at the level of firms and industries.