Perfect Competition in Markets with Adverse Selection

Eduardo Azevedo (Wharton) and Daniel Gottlieb
(Washington University in St. Louis)

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Adverse selection is considered a first-order problem in many markets, which are already heavily regulated in complicated ways:

- Mandates, community rating, risk adjustment, differential subsidies, regulation of contract characteristics.

However, standard models have trouble explaining contract characteristics.

1. Akerlof / Einav, Finkelstein and Cullen.
2. Rothschild and Stiglitz.

Goal: understand effects of different policies and optimal policies.
This paper

Develops a price-taking model of adverse selection.

- Contract characteristics are endogenous.
- Consumers can be heterogeneous in more than one dimension.
- Equilibrium always exists.

Basic idea:

- Use the same logic as price-taking models (Akerlof and Einav-Finkelstein and Cullen) to determine both prices and which contracts are traded.
Determining prices

\[ AC(q) \]

\[ D(p) \]

\[ p^* \]
Determining which contracts are traded

\[ D(p) \]

\[ AC(q) \]
Equilibrium Effects of a Mandate
Outline

1. Model
2. Competitive Equilibrium
3. Application: Equilibrium Effects of a Mandate
4. Market Failure
Model

- **Consumer** types $\theta \in \Theta$, distributed according to a probability distribution $\mu$.
- **Contracts** $x \in X$.
- Agent $\theta$ has **utility**
  \[ U(x, p, \theta) \]
  of buying $x$ at a price $p$, and the **cost** is
  \[ c(x, \theta) \geq 0. \]
Examples

- Example we understand: Akerlof / Einav Finkelstein and Cullen
  - Single exogenous contract $X = \{0, 1\}$.
  - Equilibrium given by supply and demand.

- Toy example: Rothschild and Stiglitz
  - More interesting contracts (% of loss covered) $X = [0, 1]$.
  - But simpler preferences, $\Theta = \{L, H\}$.
  - Equilibrium may not exist.

- Interesting example: Einav et al. *AER* 2013
  - Empirical model of health insurance with many dimensions of heterogeneity.
Simplified assumptions for the talk

- Quasilinear utility $U(x, p, \theta) = u(x, \theta) - p$.
- $u$ and $c$ are continuously differentiable and $\Theta$ and $X$ are compact subsets of $\mathbb{R}^n$. 
Prices and allocations

- A **price** is a measurable function $p$ over $X$. The price of contract $x$ is $p(x)$.
- An **allocation** $\alpha$ is a measure over $\Theta \times X$ such that $\alpha|\Theta = \mu$.
  - Think of “$\alpha(\theta, x)$” as the number of $\theta$ types buying contract $x$.
- $(p, \alpha)$ is **incentive compatible** if almost all consumers purchase an optimal bundle according to $\alpha$. 
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- An **allocation** \( \alpha \) is a measure over \( \Theta \times X \) such that \( \alpha|_{\Theta} = \mu \).
  - Think of \( \alpha(\theta, x) \) as the number of \( \theta \) types buying contract \( x \).
- \( (p, \alpha) \) is **incentive compatible** if almost all consumers purchase an optimal bundle according to \( \alpha \).

- Conditional moments are denoted as

\[
\mathbb{E}_x[c] = \text{expected cost of selling contract } x.
\]
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Weak equilibrium

Definition
An price-allocation pair \((p, \alpha)\) is **weak equilibrium** if

1. It is incentive compatible.
2. All contracts make 0 profits,

\[ p(x) = \mathbb{E}_x[c] \]

almost everywhere according to \(\alpha\).
Weak Equilibrium Example: Rothschild-Stiglitz

- \( X = [0, 1] \) and \( \Theta = \{L, H\} \).
But there are *many* other weak equilibria

- \( X = [0, 1] \) and \( \Theta = \{L, H\} \).
Definition: Perturbations

- A behavioral type $x$ is an agent who always demands contract $x$ and has cost 0.
- A perturbation $(\tilde{X}, \eta)$ is an economy with a finite set of contracts $\tilde{X}$ and a small extra measure $\eta$ of behavioral types, with a positive mass at every contract.
Equilibrium of a Perturbation: Example
Definition: Convergence of Perturbations

A sequence of perturbations *converges to the original economy* if the set of contracts approaches \( X \) and the total mass of behavioral types converges to 0.
Definition: Equilibrium

Consider a sequence of perturbations converging to the original economy. A sequence of weak equilibria converges to \((p^*, \alpha^*)\) if both the allocations and prices converge.

Definition

\((p^*, \alpha^*)\) is a competitive equilibrium if it is the limit of a sequence of equilibria of perturbations that get finer and finer.
Equilibrium: Example
Equilibrium: Example
Equilibrium: Example
Theorem

A competitive equilibrium exists.
Discussion: Do we like this equilibrium?

- So far we justified the equilibrium based on unraveling and free entry.
- Formal strategic foundations:
  - The equilibrium is the limit of differentiated products Bertrand models (Starc, 2014; Tebaldi, 2015; Mahoney and Weyl, 2015).
  - Can get no cross subsidies as a result.
- The equilibrium is “backwards compatible” in particular cases with some existing concepts (Akerlof, RS, Riley, Dubey and Geanakoplos, Bisin and Gottardi’s APT).
  - The key is that competitive equilibrium is well-defined and tractable in richer models.
- The equilibrium varies discontinuously with fundamentals in the RS case, unlike existing concepts with cross-subsidies.
  - But these features do not hold generically.
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Calibration: Einav et al. health insurance model

\[ u(x, \theta) = x \cdot M_\theta + \frac{x^2}{2} \cdot H_\theta + \frac{1}{2} x(2 - x) \cdot S^2_\theta A_\theta, \quad \text{and} \]
\[ c(x, \theta) = x \cdot M_\theta + x^2 \cdot H_\theta. \]

<table>
<thead>
<tr>
<th></th>
<th>( A )</th>
<th>( H )</th>
<th>( M )</th>
<th>( S )</th>
</tr>
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<tbody>
<tr>
<td>Mean</td>
<td>1.0E-5</td>
<td>1,330</td>
<td>4,340</td>
<td>24,474</td>
</tr>
</tbody>
</table>

Log covariance

\[
A \quad 0.25 \quad -0.01 \quad -0.12 \quad 0 \\
H \quad \sigma^2_{\log H} = 0.28 \quad -0.03 \quad 0 \\
M \quad 0.20 \quad 0 \\
S \quad 0.25 \]

Azevedo and Gottlieb (Wharton)
Equilibrium prices and adverse selection
Equilibrium demand profile
Equilibrium Effects of a Mandate

Equilibrium Prices

Average Loss Parameter

Contract ($)

$0 $2,000 $4,000 $6,000 $8,000

$0 $2,000 $4,000 $6,000 $8,000

0 0.2 0.4 0.6 0.8 1

Contract

Equilibrium Prices

Average Loss Parameter

$0 $2,000 $4,000 $6,000 $8,000

0 0.2 0.4 0.6 0.8 1

Equilibrium Effects of a Mandate
Equilibrium Effects of a Mandate

No Mandate – Prices
Mandate – Prices
No Mandate – Losses
Mandate – Losses
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Market Failure

- I Will cover main ideas in plain vanilla rational, Kaldor-Hicks welfare, no excess cost of public funds (as opposed to constrained Pareto efficiency).
- Simple set of contracts, $X = [0, 1]$.
- Consumers only adjust in the intensive margin to small changes.
- A benevolent government can regulate menus and prices, but has no informational advantage.
Starting point for regulation: equilibria are *inefficient*.

To see why, define

- **Marginal cost**
  
  \[ mc(x, \theta) = \partial_x c(x, \theta). \]

- **Marginal utility**
  
  \[ mu(x, \theta) = \partial_x u(x, \theta). \]

- **Intensive margin selection coefficient** by
  
  \[ S_I(x) = \partial_x (\mathbb{E}_x [c]) - \mathbb{E}_x [mc]. \]
Equilibrium is Inefficient

Private optimum where marginal utility equals $p'$.
Equilibrium is Inefficient
But social optimum where marginal utility equals marginal cost.
Equilibrium is Inefficient

Two distortions: $\mathbb{E}_x[mc] \neq p'$ and $mc_\theta \neq \mathbb{E}_x[mc]$. 
Equilibrium is Inefficient
Sources: adverse / advantageous selection and multidimensional heterogeneity.
Effectively, any regulation can be implemented setting $p(x)$. It is insightful to write a formula for the per unit subsidy the government must give firms,

$$p(x) + t(x) = E_x[c].$$

Will now find necessary conditions for optimum by perturbing a price schedule, similar to optimal taxation theory (examples: Dixit and Sandmo 1977, Saez 2001).
Optimal Regulation Formula

Optimal regulation is a modified risk adjustment formula: risk adjustment plus covariance term:

\[ t'(x) = S_l(x) - \frac{\text{Cov}_x[\epsilon, mc]}{\mathbb{E}_x[\epsilon]} \]
Regulation Example

- No Mandate − Prices
- No Mandate − Losses
- Mandate − Prices
- Mandate − Losses
Regulation Example

Equilibrium Prices
Equilibrium Losses
Optimum Prices
Optimum Losses
The mandate raises welfare by $127 per consumer.

Optimal regulation increases it by $279.

A simple policy like the mandate can increase efficiency. But also has important unintended consequences.

Contract characteristics matter.

Optimal policy also addresses selection in the intensive margin.

Exception: In some cases it is optimal to give up on screening and offer a single contract.
Discussion: Do we like this equilibrium?

- So far we justified the equilibrium based on unraveling and free entry.
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- The equilibrium varies discontinuously with fundamentals in the RS case, unlike existing concepts with cross-subsidies.
  - But these features do not hold generically.
Conclusion

- Key idea is to apply the supply and demand approach from the one-contract model to a more general case.
- Gives a simple model to explain contract characteristics.
- Numerical results show that policy can have important unintended consequences, and that regulation should also address selection on the intensive margin.
“Complete Model” and “Price Theory”

Thank You!