Resource Allocation in the Brain

Ricardo Alonso  
USC Marshall

Juan D. Carrillo  
USC and CEPR

Isabelle Brocas  
USC and CEPR

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Use evidence from neuroscience to revisit economic theories of decision-making

- Examples of neuroscience evidence include:
  Existence of multiple brain systems, interactions between systems, physiological constraints, etc.

- Revisiting theories of decision-making includes:
  Building models of bounded rationality based not on inspiration or casual observation but on the physiological constraints of our brain
  → derive behavioral biases from brain limitations

The brain is, so it should be modeled as, a multi-system organization
This paper (1)

Build a model of constrained optimal behavior in multi task decision-making based on evidence from neuroscience:

i. Different brain systems are responsible for different tasks. Neurons in a system respond exclusively to features of that particular task.

ii. The brain allocates resources (oxygen, glucose) to systems. Resources are transformed into energy that make neurons fire (fMRI measure blood oxygenation, PET measure changes in blood flows, etc.).

iii. More complex tasks necessitate more resources. Performance suffers if resources needs are not filled.

iv. Resources are scarce: “biological mechanisms place an upper bound on the amount of cortical tissue that can be activated at any given time”.
v. Central Executive System (CES) coordinates the allocation of resources:
   - Active when two tasks are performed simultaneously.
   - Not active if only one task, if two sequential tasks, or if two tasks but subject instructed to focus on only one.

vi. Asymmetric information in the brain: neuronal connectivity is very limited → information carried by neurotransmitters reaches some systems but not others.
The model

- Three systems (0, 1, 2) perform three types of tasks:
  - System 0 (S₀) controls motor skill functions. Needs θ₀ are known
  - Systems 1 and 2 (S₁ and S₂) control higher order cognitive functions (mental rotation, auditory comprehension, face recognition)
    Needs θ₁ and θ₂ depend on task complexity and are privately known
  - Distributions F₁(θ₁) and F₂(θ₂) satisfy Increasing Hazard Rate (IHR)

- Another system, Central Executive System (CES), allocates resources \{x₀, x₁, x₂\} to S₀, S₁, S₂

- Performance of system \(l\) is \(U_l(\theta_l) = -\frac{1}{\beta_l}(x_l - \theta_l)^2\) with \(l \in \{0,1,2\}\).
  Systems are tuned to respond only to their task

- CES maximizes (weighted) sum of performances of systems: \(U₀ + U₁ + U₂\)

- Resources are scarce and bounded at \(k\): \(x₀ + x₁ + x₂ \leq k\)

- Each system requires a minimum of resources to operate: \(x_l \geq 0\)
The model

CES
central executive system

\[ x_0 + x_1 + x_2 < k \]

\[ U_0 + U_1 + U_2 \]

- \( U_0(x_0, \theta_0) = -\frac{1}{\beta_0} (x_0 - \theta_0)^2 \)
- \( U_1(x_1, \theta_1) = -\frac{1}{\beta_1} (x_1 - \theta_1)^2 \)
- \( U_2(x_2, \theta_2) = -\frac{1}{\beta_2} (x_2 - \theta_2)^2 \)

Motor function
\( \theta_0 \) “public”

Cognitive functions 1 and 2
\( \theta_1 \) and \( \theta_2 \) “private”

- \( S_0 \) (lifting)
- \( S_1 \) (rotation)
- \( S_2 \) (spelling)
Benchmark case: full information

\[
\max_{x_0, x_1, x_2} \quad \frac{1}{\beta_0} U_0(x_0(\theta_1, \theta_2), \theta_0) + \frac{1}{\beta_1} U_1(x_1(\theta_1, \theta_2), \theta_1) + \frac{1}{\beta_2} U_2(x_2(\theta_1, \theta_2), \theta_2)
\]

s.t. \( x_0(\theta_1, \theta_2) + x_1(\theta_1, \theta_2) + x_2(\theta_1, \theta_2) \leq k \)  \hspace{1cm} (R)

\( x_0(\theta_1, \theta_2) \geq 0, x_1(\theta_1, \theta_2) \geq 0, x_2(\theta_1, \theta_2) \geq 0 \)  \hspace{1cm} (F)
Benchmark case: full information

\[
\max_{x_0, x_1, x_2} \frac{1}{\beta_0} U_0(x_0(\theta_1, \theta_2), \theta_0) + \frac{1}{\beta_1} U_1(x_1(\theta_1, \theta_2), \theta_1) + \frac{1}{\beta_2} U_2(x_2(\theta_1, \theta_2), \theta_2)
\]

\[
\text{s.t.} \quad x_0(\theta_1, \theta_2) + x_1(\theta_1, \theta_2) + x_2(\theta_1, \theta_2) \leq k \quad \text{(R)}
\]
\[
\quad x_0(\theta_1, \theta_2) \geq 0, x_1(\theta_1, \theta_2) \geq 0, x_2(\theta_1, \theta_2) \geq 0 \quad \text{(F)}
\]

Solution under full information (assuming (R) binds and (F) does not):

\[
x_l^F = \theta_l - \frac{\beta_l}{\sum \beta} \left( \sum \theta - k \right)
\]

Distribute \( k \) according to needs \((\theta_0, \theta_1, \theta_2)\) weighed by importance \((\beta_0, \beta_1, \beta_2)\)

\[
U_l^F(x_l^F; \theta_l) = -\frac{\beta_l}{(\sum \beta)^2} \left( \sum \theta - k \right)^2
\]

Utility of a system depends on total needs (sum of \(\theta\)) and relative importance \((\beta_l)\) but not on how needs are distributed among the systems \((\theta_1 \text{ v. } \theta_2)\)
1. **Normative approach**: optimal allocation given private information if CES could use any conceivable communication mechanism
   - General properties
   - Comparative statics

2. **Positive approach**: can this allocation be implemented using a physiologically plausible mechanism?

3. **Applications**
   - Task inertia
   - Task separation
1. **Normative approach**: optimal allocation given private information if **CES** could use any conceivable communication mechanism
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The optimization problem

Optimal allocation of resources when needs of systems 1 and 2 are unknown and CES can use any mechanism. Using the revelation principle:

\[
\max_{x_0, x_1, x_2} \int \int \frac{1}{\beta_0} U_0(x_0(\theta_1, \theta_2), \theta_0) + \frac{1}{\beta_1} U_1(x_1(\theta_1, \theta_2), \theta_1) + \frac{1}{\beta_2} U_2(x_2(\theta_1, \theta_2), \theta_2) dF_1(\theta_1) dF_2(\theta_2)
\]

s.t. \( U_i(x_i(\theta_i, \tilde{\theta}_i, \theta_i), \theta_i) \geq U_i(x_i(\tilde{\theta}_i, \theta_j), \theta_i) \) \quad \forall i, j \in \{1, 2\}, \forall \theta_i, \tilde{\theta}_i, \theta_j \quad \text{(IC)}

\( x_0(\theta_1, \theta_2) + x_1(\theta_1, \theta_2) + x_2(\theta_1, \theta_2) \leq k \) \quad \forall \theta_1, \theta_2 \quad \text{(R)}

\( x_0(\theta_1, \theta_2) \geq 0, x_1(\theta_1, \theta_2) \geq 0, x_2(\theta_1, \theta_2) \geq 0 \) \quad \forall \theta_1, \theta_2 \quad \text{(F)}
The solution

• Optimal mechanism $\mathcal{M}$:
  - resource cap $\overline{x}_1(\theta_2)$ for $S_1$
  - resource cap $\overline{x}_2(\theta_1)$ for $S_2$

• Equilibrium allocation under $\mathcal{M}$:
  - $x_1^*(\theta_1, \theta_2) = \min \{ \theta_1, \overline{x}_1(\theta_2) \}$
  - $x_2^*(\theta_1, \theta_2) = \min \{ \theta_2, \overline{x}_2(\theta_1) \}$
  - $x_0^*(\theta_1, \theta_2) = k - x_1(\theta_1, \theta_2) - x_2(\theta_1, \theta_2)$

• What are the optimal caps $\overline{x}_1(\theta_2)$ and $\overline{x}_2(\theta_1)$?
The solution

Sketch of proof

1. Derive optimal allocation with only 2 systems (1 with private info.)

2. Use it to derive “priority mechanism”:
   - $\mathbf{P}_1$: optimal mechanism under the requirement that $S_1$ always obtains the resources it requests
   - $\mathbf{P}_2$: optimal mechanism under the requirement that $S_2$ always obtains the resources it requests.

3. Compare $\mathbf{P}_1$ and $\mathbf{P}_2$. Show that optimum is a hybrid of both: it behaves like $\mathbf{P}_1$ for certain $(\theta_1, \theta_2)$ and like $\mathbf{P}_2$ for some other $(\theta_1, \theta_2)$. 
Optimal Resource Allocation

Optimal caps $\bar{x}_1(\theta_2)$ and $\bar{x}_2(\theta_1)$ are first strictly decreasing and then constant in the needs of the other system.

\[
\bar{x}_1(\theta_2) = y_1(k - \theta_2) \quad \text{if} \quad \theta_2 < k_2 \\
k_1 \quad \text{if} \quad \theta_2 \geq k_2
\]

\[
\bar{x}_2(\theta_1) = y_2(k - \theta_1) \quad \text{if} \quad \theta_1 < k_1 \\
k_2 \quad \text{if} \quad \theta_1 \geq k_1
\]
Optimal Resource Allocation

Optimal caps $\bar{x}_1(\theta_2)$ and $\bar{x}_2(\theta_1)$ are first strictly decreasing and then constant in the needs of the other system

$$\frac{1}{\beta_1} (E[\theta_1 | \theta_1 > k_1] - k_1) = \frac{1}{\beta_2} (E[\theta_2 | \theta_2 > k_2] - k_2) = \frac{1}{\beta_0} (\theta_0 - (k - k_1 - k_2))$$
Equilibrium allocation \((x_1^*(\theta_1, \theta_2), x_2^*(\theta_1, \theta_2))\)

→ unconstrained for “small” needs and fixed for “large” needs
(with \(x_0^*(\theta_1, \theta_2) = k - x_1^*(\theta_1, \theta_2) - x_2^*(\theta_1, \theta_2)\))
Properties

- Equilibrium is unique (under Increasing Hazard Rate)
- $k_1, k_2, k - k_1 - k_2$ are guaranteed resources for $S_1, S_2, S_0$
- Resource monotonicity: if $\theta_1 \downarrow$, both $x_2 \uparrow$ and $x_0 \uparrow$
- Comparative statics. Same monotonicity principle:
  - If $\beta_2 \downarrow$ ($S_2$ more important), then $x_2^* \uparrow, x_1^* \downarrow, x_0^* \downarrow$
  - If $k \uparrow$ (more resource), then $x_0^* \uparrow, x_1^* \uparrow, x_2^* \uparrow$

**Implication 1.** Let $\beta_1 = \beta_2$. Fix $\theta_1 + \theta_2$ with $\theta_1 > \theta_2$
- Full information: $U_1^F = U_2^F$
- Private information: $U_1^* \leq U_2^*$
  Better performance in easy tasks than in difficult tasks
1. Normative approach: optimal allocation given private information if CES could use any conceivable communication mechanism
   - General properties
   - Comparative statics

2. **Positive approach**: can this allocation be implemented using a physiologically plausible mechanism?

3. Applications
   - Task inertia
   - Task separation
Implementation

So far, abstract revelation mechanism: “announce $\theta'_i$, receive $x_i(\theta'_i, \theta'_j)$”

Can CES implement the optimal mechanism in a “simple” way and, most importantly, in a way **compatible with the physiology** of the brain?

- CES sends oxygen to $S_1$, $S_2$, $S_0$ at rates $k_1/k$, $k_2/k$, $(k - k_1 - k_2)/k$.
- Systems deplete oxygen to produce energy. CES observes depletion which is a signal that more resources are needed (autoregulation).
- If $S_i$ stops consumption, oxygen is redirected to $S_j$ and $S_0$ at a new rate.
- If both $S_i$ and $S_j$ stop consumption, the remaining oxygen is sent to $S_0$.

→ $S_i$ grabs incoming resources up to satiation or up to constraint
→ $S_i$ doesn’t need to know needs or even existence of $S_j$
→ $S_i$ doesn’t need to know its own needs $\theta_i$ until they are hit
→ CES must be able to redirect resources and change the rates
If $\theta_1 > k_1$ and $\theta_2 > k_2$

If $\theta_1 = \theta'_1$ and $\theta_2 = \theta'_2$

If $\theta_1 > x_1(\theta'_2)$ and $\theta_2 = \theta'_2$
1. Normative approach: optimal allocation given private information if CES could use any conceivable communication mechanism
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Application 1: task inertia

- **CES** has imperfect knowledge of the distribution of needs
- **CES** gradually learns the distribution through observation of past needs
- How should **CES** adjust current allocation rules based on past needs?
- Learning:
  - Distributions $F_i(\theta_i|s_i)$ depends on unknown but fixed state $s_i$
    (is this task usually complex or simple?)
  - Prior belief of state is $p_i(s_i)$
  - Realization of $\theta_{it}$ conditionally independent across periods
  - After $S_i$ reports $\theta_{it}$ in period $t$, **CES** updates belief over $s_i$
  - Assume $F_i(\theta_i|s_i)$ satisfies MLRP: needs are likely to be high ($\theta_i$ high) when task is usually complex ($s_i$ high).

→ **Lemma**: $G_i(\theta_{i,t+1}|\theta_{i,t})$ satisfies MLRP: high $\theta_{i,t}$ implies that $s_i$ is likely to be high which implies that $\theta_{i,t+1}$ is also likely to be high
Application 1: task inertia

Assume $s_i$ unknown and compare public info. ($\theta_1^t, \theta_2^t$ known by CES at $t$) with private info. ($\theta_1^t, \theta_2^t$ unknown by CES at $t$)

**Implication 2.** Inertia and path-dependence of the allocation rule.

Under private info. and conditional on present needs, allocation of $S_i$ is higher if past needs were high:

If $\theta_2^{t-1} \uparrow$, then $x_2^t(\theta_1^t, \theta_2^t) \uparrow$, $x_1^t(\theta_1^t, \theta_2^t) \downarrow$, $x_0^t(\theta_1^t, \theta_2^t) \downarrow$

→ consistent with neuroscience evidence on “task switching cost”.

→ consistent with neuroscience evidence on “task switching cost”.
Application 2: task separation

- Is it better to have an integrated system responsible for tasks 1 and 2 or two separate systems each responsible for one task?

- **Trade-off:**
  - Integrated system allocates more efficiently its resources between tasks 1 and 2
  - Separated systems require less “informational rents”: cap of $S_1$ can depend on announcement of $S_2$

**Implication 3.**
- Integration of $S_1$ and $S_2$ dominates when motor task is important (“low” $\beta_0$)
- Separation of $S_1$ and $S_2$ dominates when cognitive tasks are important (“low” $\beta_1$ and $\beta_2$)
Conclusions

- The brain is a multi-system organization.
- **Bounded rationality model** based not on inspiration but on physiological constraints of the brain → derive behaviors from brain limitations
- **Optimal resource allocation**: each system has guaranteed resources ($k_0, k_1, k_2$). More resources are available only if others are satiated.
- **Physiologically plausible implementation**.
- Resource allocation under capacity constraint and asymmetric information provides an informational rationale for (not a model built to explain):
  - Better performance in easier tasks
  - Task inertia and task switching cost
  - Conditions for integration v. separation of functions
- Model can be straightforwardly applied to standard organization problems:
  - Allocation of resources between research, marketing and production
  - Market split of colluding firms (no transfers!), etc.