Who Should Pay for Credit Ratings and How?

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Motivation

• Virtually every government inquiry into the 2008 and 2009 financial crisis has assigned some blame to credit rating agencies (CRAs)
  ○ “This crisis could not have happened without the rating agencies,” — Financial Crisis Inquiry Commission
  ○ “Inaccurate AAA credit ratings introduced risk into the U.S. financial system and constituted a key cause of the financial crisis,” — U.S. Senate, Permanent Subcommittee on Investigations

• Shirking by CRAs is presumed to be one of the causes of the crisis
  ○ “Factors responsible for the inaccurate ratings include rating models that failed to include relevant mortgage performance data, unclear and subjective criteria used to produce ratings, a failure to apply updated rating models to existing rated transactions, and a failure to provide adequate staffing to perform rating and surveillance services, despite record revenues,” — U.S. Senate, Permanent Subcommittee on Investigations
Motivation

• Rating inaccuracies are also thought to be related to the ‘issuer-pays’ business model and CRAs’ battle for market share

  ○ “The business model under which firms issuing securities paid for their ratings seriously undermined the quality and integrity of those ratings; the rating agencies placed market share and profit considerations above the quality and integrity of their ratings,” — Financial Crisis Inquiry Commission

  ○ “S&P’s concerns about market share, revenues and profits drove them to issue inflated ratings, thereby misleading the public and defrauding investors. In so doing, we believe that S&P played an important role in helping to bring our economy to the brink of collapse,” — Department of Justice Complaint

• Rating errors seemed particularly severe for new securities

  ○ “Data appears to indicate that the methodologies were inadequate and suggests that the CRAs staff did not have an adequate understanding of the structured finance market, or at least in the subprime residential mortgages market,” — European Commission
Questions

• How do you compensate CRAs to avoid shirking?
• Does the issuer-pays model generate more shirking than, e.g., the investor-pays model?
• Does CRAs’ battle for market share reduce rating quality?
• Are incentives to shirk more severe for new securities?
• What determines a decision to downgrade a security?
Five Facts

We develop a parsimonious model that explains five facts about the rating business in a unified fashion:

1. Rating mistakes are in part due to insufficient effort by CRAs
   - Owusu-Ansah (2012), Dodd-Frank Act

2. Outcomes and the accuracy of ratings do differ depending on which party pays for a rating (e.g., the issuer or investors)

3. Competition between CRAs reduces the accuracy of ratings
   - Becker & Milbourn (2011)

4. Ratings mistakes are more common for newer securities with shorter histories than for more established types of securities

5. CRAs are slow in downgrading their ratings
   - Covitz & Harrison (2003), Worldcom and Enron bankruptcies
Outline

• Consider an environment where a CRA must exert effort to evaluate project quality
  ○ Effort is unobservable → need to provide incentives
• Derive the optimal compensation scheme that gives the CRA incentives to exert effort
• Compare the CRA’s effort and total surplus depending on who orders a rating (the planner, issuer, or investors)
• Extensions:
  ○ Multiple CRAs
  ○ New securities
  ○ Revising ratings (delays in downgrades)
Economy

- Static, one-period model
- One firm (issuer) with one project
- A number \((n \geq 2)\) of investors
- One CRA

All agents are risk neutral, maximize expected profits
The Firm and Investors

**Firm:** endowed with a project

- requires one unit of investment
- generates output $y > 0$ if success, $0$ if failure
- unknown quality of the project: $q \in \{g, b\}$
- success occurs with probability $p_q$, where $0 < p_b < p_g < 1$
- profitable to finance $g$ but not $b$: $-1 + p_b y < 0 < -1 + p_g y$
- prior: $\gamma \equiv \Pr\{q = g\}$
- has no funds, so it must borrow from investors

**Investors:**

- deep-pocketed
- behave competitively, make zero profits in equilibrium
The Credit Rating Agency

- The CRA privately exerts effort $e$ to obtain a signal about the project’s quality
- The signal (rating) can be high or low: $\theta \in \{h, \ell\}$
- $\Pr\{\theta = h|q = g, e\} = \Pr\{\theta = \ell|q = b, e\} = 1/2 + e$, where $e \in [0, 1/2]$
  - probability of getting a signal consistent with the true quality (signal informativeness about project quality) is $\uparrow$ in effort
- Without effort, signals are uninformative:
  - I.e., $\Pr\{\theta = h|q = i, 0\} = \Pr\{\theta = \ell|q = i, 0\} = 1/2$ for $i \in \{g, b\}$ and hence posterior after any signal = prior
- $\psi(e)$ — cost of effort
  - $\psi' > 0$, $\psi(0) = 0$, $\lim_{e \to 1/2} \psi(e) = +\infty$
- The CRA is protected by limited liability
Additional Assumptions on the Cost Function

1. $\psi'' > 0$ (strict concavity of the CRA’s problem)
2. $\psi''' \geq 0$ (strict concavity of the planner’s problem)
3. $\psi'(0) = 0$ (interior solution of the CRA’s problem)
4. $\psi''(0) = 0$ (interior solution of the planner’s problem)

Example:

- $\psi(e) = Ae^a$, $A > 0$, $a \geq 2$
Timing

1. The CRA sets fees (paid at the end, conditional on history)
   ○ We study an optimal contracting problem; hence we allow the fee structure to be as rich as possible
2. Investors announce financing terms contingent on a rating or its absence
3. X (the planner, the firm, or each investor) decides whether to order a rating
4. If a rating is ordered, the CRA exerts effort, reveals the rating to X
   ○ Assumption that can be relaxed: the CRA cannot misreport the rating
5. X decides whether to announce the rating to other agents
6. The firm decides whether to borrow from investors to finance the project
7. If the project is financed, success/failure is observed
8. The firm repays the investors, the CRA collects fees
Equilibrium

• We will study Pareto Efficient perfect Bayesian equilibria in this game, with different games for each $X$

• We will compare effort and total surplus in equilibrium. Why total surplus?
  ○ Can think of a consumer who owns both the firm and CRA, and the planner maximizes his utility
  ○ In the static environment, we will not always be able to Pareto rank equilibria depending on who orders the rating
  ○ However, features that lead to lower total surplus in the static model, lead to Pareto dominance in a repeated infinite horizon version of the model
Notation

• $\pi_1 = p_g \gamma + p_b (1 - \gamma)$ (ex-ante probability of success)

• $\pi_h(e) = \left(\frac{1}{2} + e\right)\gamma + \left(\frac{1}{2} - e\right)(1 - \gamma)$

  $\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 1-(1/2+e)$

  (probability of high rating)

• $\pi_\ell(e) = 1 - \pi_h(e)$ (probability of low rating)

• $\pi_{h1}(e) = p_g \left(\frac{1}{2} + e\right)\gamma + p_b \left(\frac{1}{2} - e\right)(1 - \gamma)$

  (probability of high rating followed by success)

• $\pi_{h0}(e) = (1 - p_g) \left(\frac{1}{2} + e\right)\gamma + (1 - p_b) \left(\frac{1}{2} - e\right)(1 - \gamma)$

  (probability of high rating followed by failure)
No Wasted Effort

**Note:** If the CRA exerts $e > 0$, the project can only be financed if the rating is high

- Suppose the financing decision does not depend on the rating
- But the rating was costly to produce
- So it cannot be optimal to pay that cost given that information is not used
Observable Effort: First Best

- In this case the total surplus doesn’t depend on who orders a rating
- Recall: investors make zero profits → all surplus is split between the firm and CRA
- First-best frontier: if the CRA’s payoff is $v$, then the firm’s payoff is $S^{FB} - v$, where

$$S^{FB} = \max \left\{ 0, -1 + \pi_1 y, \max_{e} -\psi(e) + \pi_h(e) \left[-1 + \frac{\pi_{h1}(e)}{\pi_h(e)} y \right] \right\}$$

- Denote $e^* \equiv \arg \max_e -\psi(e) - \pi_h(e) + \pi_{h1}(e)y$
  - $e^* > 0$ given our assumptions
Observable Effort: First Best

Lemma 1

\[ \exists \gamma, \bar{\gamma}, \quad 0 < \gamma < \bar{\gamma} < 1 \text{ such that} \]

- \[ e^{FB} = 0 \text{ for } \gamma \in [0, \gamma] \text{ and the project is never financed} \]
- \[ e^{FB} = 0 \text{ for } \gamma \in [\bar{\gamma}, 1] \text{ and the project is always financed} \]
- \[ e^{FB} = e^* > 0 \text{ for } \gamma \in (\gamma, \bar{\gamma}) \text{ and the project is only financed after a high rating} \]

Intuition: if the prior belief is close to zero or one, it does not pay off to acquire additional information about the project quality
Pareto Frontier with Unobservable Effort

- Denote:
  - \( f_i, \ i \in \{h1, h0, \ell\} \) — the fee paid after history \( i \)
  - \( v \) — the CRA’s payoff
  - \( u(v) \) — the firm’s payoff if finance after high rating only

- Consider the problem where the planner sets the fees.
  Pareto frontier: \( \max\{0 - v, -1 + \pi_1 y - v, u(v)\} \), where

  \[
  u(v) = \max_{e,f_{h1},f_{h0},f_{\ell}} -\pi_h(e) + \pi_{h1}(e)y - \pi_{h1}(e)f_{h1} - \pi_{h0}(e)f_{h0} - \pi_\ell(e)f_{\ell}
  \]

  s.t. \[
  -\psi(e) + \pi_{h1}(e)f_{h1} + \pi_{h0}(e)f_{h0} + \pi_\ell(e)f_{\ell} \geq v \\
  \psi'(e) = \pi'_{h1}(e)f_{h1} + \pi'_{h0}(e)f_{h0} + \pi'_\ell(e)f_{\ell} \\
  e \geq 0, \ f_{h1} \geq 0, \ f_{h0} \geq 0, \ f_{\ell} \geq 0
  \]

- **Note:** As before, \( e = 0 \) and never finance for \( \gamma \) low enough, and \( e = 0 \) and always finance for \( \gamma \) high enough
Optimal Structure of CRA Compensation

Proposition 1 (Optimal Compensation Scheme)

Suppose the project is financed only after the high rating.
Define \( \hat{\gamma} = \frac{1}{1 + \sqrt{p_g/p_b}} \).

- If \( \gamma > \hat{\gamma} \) then \( f_{h1} > 0 \) and \( f_\ell = f_{h0} = 0 \) (reward only for high rating and success when prior is high enough)
- If \( \gamma < \hat{\gamma} \) then \( f_\ell > 0 \) and \( f_{h1} = f_{h0} = 0 \) (reward only for low rating otherwise)

Proof

Intuition: Given the prior, reward the CRA for the event whose occurrence is the most consistent with the CRA exerting effort (i.e., reward for the event with the highest likelihood ratio)

Remark: If the CRA can misreport the signal, then both \( f_{h1} > 0 \) and \( f_\ell > 0 \)
Pareto Frontier when Finance after High Rating only

Proposition 2 (Pareto Frontier)

(i) \( \exists v^*: e(v) = e^* \) for all \( v \geq v^* \),
    but \( u(v) < 0 \)

(ii) \( \exists v_0 > 0: (1) \) is slack for \( v < v_0 \),
    binds for \( v \geq v_0 \), and \( e(v_0) > 0 \)

(iii) \( e(v) \) and total surplus are \( \uparrow \) in \( v \),
    strictly for \( v \in (v_0, v^*) \)

Denote \( \bar{v} \equiv \max \{ v | u(v) = 0 \} < v^* \).

The firm’s profits are maximized when CRA gets \( v_0 > 0 \), where it exerts \( e > 0 \)

- For \( v < v_0 \), both CRA and firm can be made better off by making CRA exert
  more effort. (If (1) was imposed w/ equality, \( u(v) \uparrow \) in \( v \) for \( v < v_0 \).)
Effort Under-Provision → Give Surplus to the CRA

- Total surplus and effort are highest when all surplus is given to the CRA
  - Unobservability of effort leads to its under-provision
  - To implement highest possible effort, set fees as high as possible: extract all surplus from the firm and give it to CRA
  - Implementing $e^*$ would result in negative profits to the firm
- The firm, on the other hand, would prefer a less informative rating (but still an informative one, since $e > 0$ at $v_0$)
- As long as it is optimal to finance the project only after the high rating, the CRA sets the same fees as the planner would
Social Planner Decides if Rating Should be Acquired

- Now we will compare equilibrium outcomes depending on who \( X \) has the option of ordering a rating
  - For each \( X \), given that \( X \) orders a rating, equilibrium will correspond to a different point \((v, u(v))\)
  - Whether \( X \) chooses to order a rating will also depend on \( X \)

**Proposition 3 \((X = \text{Planner})\)**

If \( X \) is the social planner, then

- The maximum total surplus is
  \[
  S^{SB} = \max \{0, -1 + \pi_1 y, \bar{v} + u(\bar{v})\}
  \]
- \( e^{SB} \leq e^{FB} \) and \( S^{SB} \leq S^{FB} \), with strict inequalities if \( e^{FB} > 0 \)
Numerical Example

Providing incentives is costly $\Rightarrow$ effort and total surplus are lower than when effort is observable, and a rating is acquired on a smaller set of priors
**Issuer Pays**

- The firm decides whether a rating should be acquired
- Recall: the CRA sets fees, and hence will charge the highest fee that the firm is willing to pay
- The firm’s willingness to pay is bounded by having no rating, $u \equiv \max\{0, -1 + \pi_1 y\}$

**Proposition 4 ($X = \text{Issuer}$)**

If $X$ is the firm, then

- The maximum total surplus is
  
  $$S^{iss} = \max\{0, -1 + \pi_1 y, v^{iss} + u(v^{iss})\},$$
  where
  
  $$v^{iss} = \max\{v | u(v) = u\} \leq \bar{v}, < \text{if } -1 + \pi_1 y > 0$$

- $e^{iss} \leq e^{SB}$ and $S^{iss} \leq S^{SB}$, $< \text{if } e^{SB} > 0 \text{ and } -1 + \pi_1 y > 0$

  (recall: CRA’s profits ↓ ⇒ effort ↓ ⇒ total surplus ↓)
When $-1 + \pi_1 y > 0$,

- Effort and total surplus are lower than under the planner due to the firm’s higher outside option
  - $e^{iss} \downarrow$ with $\gamma$ because the outside option $-1 + \pi_1 y \uparrow$ with $\gamma$
Investors Pay

• Suppose each investor decides whether to order a rating
  ◦ Recall our assumption: whoever orders a rating can choose whether to reveal it to other agents
  ◦ When investors pay, this assumption is crucial:
    - If a rating automatically becomes public, investors have incentives to free ride → \( e > 0 \) can never be implemented

• Recall: investors make competing offers for financing terms conditional on the rating
Investors Pay

Proposition 5 ($X = \text{Investors}$)

Suppose each investor decides whether to order a rating. Then

\[ S^{inv} = \max\{0, v^{inv} + u(v^{inv})\}, \]  

where $v^{iss} \leq v^{inv} \leq \bar{v}$, with strict inequalities if $-1 + \pi_1 y > 0$.

- The term $-1 + \pi_1 y$ disappears from the expression, i.e., investors ask for a rating even when it is inefficient
  - If no one orders a rating, one investor orders and makes profits
- Effort is higher than if issuer pays, lower than if planner picks
  - The option to finance w/o a rating caps interest rates (and fees):
    - the payment cap is $R = 1/\pi_1$ (solves $-1 + \pi_1 R = 0$)
    - firm pays the same $R$ as w/o rating but is financed less often, i.e., $u(v^{inv}) = \pi_{h1}(y - 1/\pi_1) < \pi_1(y - 1/\pi_1) = u(v^{iss})$
Numerical Example Continued

When $-1 + \pi_1 y > 0$,

- Investors ask for a rating when the planner (or the issuer) would not.
- The investor-pays model yields higher rating precision than the issuer-pays model, but lower than under the planner.
  - $e^{inv} \downarrow$ with $\gamma$ because the interest rate cap $R = 1/\pi_1 \downarrow$ with $\gamma$.
Summary of the Results so far

- Moral hazard $\rightarrow$ higher rating errors relative to the first best
- The issuer-pays model leads to lower rating precision than if the planner picks
  - Option to issue w/o rating reduces the firm’s willingness to pay
- The investor-pays model yields higher rating precision than when the issuer pays, but lower than under the planner
  - Option to finance w/o rating caps interest rates (and fees)
- Investors ask for a rating too often relative to the planner
  - The CRA earns profits by selling a rating to investors, while under the planner the CRA would earn zero
  - In this case, not only is the total surplus lower, but other feasible allocations Pareto dominate this one
Competition among CRAs

- Suppose there are multiple CRAs but only one rating is acquired in equilibrium

- CRAs post fees, the issuer chooses whom to ask for a rating
  - CRAs compete in fees, maximizing the issuer’s profits

- Hence $S_{iss}^{many} = \max \{0, -1 + \pi_1 y, v_0 + u(v_0)\}$, where $v_0$ was defined in Proposition 2

- With competition, desire to win business leads to lower fees, which means less accurate ratings and lower total surplus
  - $e_{iss}^{many} \leq e_{iss}^{one}$ and $S_{iss}^{many} \leq S_{iss}^{one}$, with $< \text{if } e_{iss}^{one} > 0$

- But the firm’s surplus is higher despite lower overall surplus

- Note: CRAs generate profits despite competition ($v_0 > 0$)
New Securities and the Possibility of Misreporting

• Let $\psi(e) = A\varphi(e)$, $A > 0$

• Think of a new security as one with a higher $A$, keeping everything else (e.g., $p_g$, $p_b$, $y$, and $\gamma$) the same
  - Less historical data, inadequate model $\rightarrow$ more noise in a regression

• Suppose $A$ increases to $A'$
  - If the shift in $A$ is anticipated, and fees change appropriately
    - It is optimal to implement lower effort with $A'$ than with $A$, which lowers rating precision
  - If the shift in $A$ is unanticipated, and fees remain unchanged
    - Suppose the CRA can misreport the rating
    - The additional incentive constraint with $A'$ instead of $A$ becomes violated
    - CRA exerts $e = 0$ and always reports $h$ or $\ell$, depending on $\gamma$

• In both cases, errors in ratings are larger for new securities
Delays in Downgrading

- Two periods
  - The project requires investment in both periods
  - The project quality is the same in both periods
  - The CRA exerts effort in each period, is paid at the end
- Trade-off in the optimal contract
  - The best way to provide incentives for effort in period 2 after a ‘mistake’ (a high rating followed by the project failure) is to pay fees after \{\text{high, failure, high, success}\} or \{\text{high, failure, low}\}
  - But the best way to provide incentives for effort in period 1 is to pay fees only after \{\text{high, success, high, success}\} or \{\text{low, low}\}
  - The optimal contract is designed to balance this trade-off
    - Positive 2\textsuperscript{nd}-period fees after a mistake reduce 1\textsuperscript{st}-period effort
    - Desire to support 1\textsuperscript{st}-period effort makes 2\textsuperscript{nd}-period fees/effort after a mistake too low ex post ⇒ the probability of not downgrading a bad-quality project is too high ex post
Conclusions

• We develop a simple optimal contracting model that addresses multiple issues regarding rating performance
• Rating mistakes are in part due to unobservability of CRAs’ effort
• The issuer-pays model generates more rating inaccuracies relative to what the planner could attain
• The investor-pays model yields higher rating precision than the issuer-pays model, but investors ask for a rating inefficiently often
• Battle for market share by competing CRAs leads to less accurate ratings, but this is applauded by the firm
• Rating errors are larger for new securities
• Optimal provision of incentives for initial rating and revision naturally generates delays in downgrading
Firm Has Private Information About Its Quality

- Suppose, as in our model, the firm has no internal funds
- The firm knows its type perfectly
  - The firm only makes payment after $h1 \rightarrow$ either both types agree to this payment or neither one does $\rightarrow$ cannot separate types
  - The only possible type of equilibrium is pooling, and analysis is the same as in our original model
- The firm has private info about its project but does not know its type perfectly
  - A separating equilibrium may exist: different types choose different fees and thus different rating precision
  - With two types, in a separating equilibrium where both types get rated, the firm that has a lower prior about its quality must receive a more precise rating.
Related Literature

- Empirical papers that document our motivating facts:
  - Owusu-Ansah (2012)
  - Becker & Milbourn (2011)
  - Covitz & Harrison (2003)

- Theoretical papers:
  - Strausz (2005)
  - Faure-Grimaud, Peyrache & Quesada (2009), Goel & Thakor (2011)

- Policy papers:
  - Summary by Medvedev and Fennell (2011)
Most Relevant Theoretical Papers

• Opp, Opp, and Harris (2012)
  ○ Explain rating inflation using a model where ratings not only provide info to investors but are used for regulatory purposes
  ○ Effort affects rating precision, but the CRA can commit to it (∼ observable effort)
  ○ Do not study optimal contracts

• Bolton, Freixas, and Shapiro (2012)
  ○ Some investors are naive → incentives for the CRA, paid by the issuer, to inflate (misreport) ratings
  ○ The CRA’s signal precision and reputation costs are exogenous
  ○ The CRA is more likely to inflate (misreport) ratings in booms, when there are more naive investors, and/or when the risks of failure which could damage CRA reputation are lower
  ○ Competition may reduce efficiency due to more rating shopping
Sketch of the Proof of Lemma 1

Recall $S^{FB} = \max\{0, -1 + \pi_1 y, \max_e -\psi(e) - \pi_h(e) + \pi_{h1}(e)y\}$

- At $\gamma = 0$, third term < first term:
  \[\max_e -\psi(e) + (1/2 - e)(-1 + p_b y) < 0\]

- At $\gamma = 1$, third term < second term:
  \[\max_e -\psi(e) + (1/2 + e)(-1 + p_g y) < -1 + p_g y = -1 + \pi_1 y\]

- At $\gamma = \gamma^*$: $-1 + \pi_1 y = 0$, third term > first term = second term
  - FOC: $\psi'(e) = -(-1 + p_g y)(-1 + p_b y)/[y(p_g - p_b)] > 0$
  - This has a unique solution $e > 0$
  - Zero surplus could be obtained by choosing $e = 0$
  - Since the problem is strictly concave in effort,
    \[-\psi(e) - \pi_h(e) + \pi_{h1}(e)y \text{ must be } > 0 \text{ at the optimal } e\]

- $\max_e -\psi(e) - \pi_h(e) + \pi_{h1}(e)y$ is strictly increasing and convex in $\gamma$
  \[\Rightarrow \text{ it must single-cross 0 at } \gamma \in (0, \gamma^*) \text{ and } -1 + \pi_1 y \text{ at } \bar{\gamma} \in (\gamma^*, 1)\]
Derivation of the Firm’s Payoff

• Suppose that investors pay fees to the CRA

• Denote the loan repayment amount after a high rating by $R_h$ (no loan is made if a low rating)

• Payoff to the firm:
  \[ \pi_{h1}(y - R_h) \]

• Payoff to investors:
  \[ -\pi_h + \pi_{h1}R_h - \pi_{h1}f_{h1} - \pi_{h0}f_{h0} - \pi_{\ell}f_{\ell} = 0 \]

• Substituting, the payoff to the firm becomes
  \[ -\pi_h + \pi_{h1}y - \pi_{h1}f_{h1} - \pi_{h0}f_{h0} - \pi_{\ell}f_{\ell} \]

• Since investors break even, it is as if the firm pays fees and makes investment itself
Sketch of the Proof of Proposition 1

• Let $\lambda$ and $\mu$ denote Lagrange multipliers on constraints (PC firm) and (IC)

$(-1 + \lambda)\pi_i + \mu\pi'_i \leq 0$, with equality if $f_i > 0$

• FOC with respect to $f_i$, $i \in \{h1, h0, \ell\}$:

$(−1 + \lambda)\pi_i + \mu\pi'_i \leq 0$, with equality if $f_i > 0$

• The one with highest likelihood ratio $\pi'_i/\pi_i$ holds w/ equality

• $\pi'_{h1}/\pi_{h1} > \pi'_{h0}/\pi_{h0}$ always, $\pi'_{h1}/\pi_{h1} > \pi'_{\ell}/\pi_{\ell}$ iff $\gamma > \hat{\gamma}$
If the CRA Can Misreport a Rating

• If the CRA can misreport the signal, the optimal lie is a double deviation: exert no effort, pick a rating and stick to it

• Impose an additional incentive constraint:

\[-\psi(e) + \pi_{h_1}(e)f_{h_1} + \pi_{\ell}(e)f_{\ell} \geq \max\{\pi_1 f_{h_1}, f_{\ell}\}\]

exert effort \(e\) and report truthfully       exert zero effort, always report \(h\) or \(\ell\)

Lemma 2

Suppose the project is financed only after the high rating. Then (IC\(_2\)) binds:

\[ -\psi(e) + \pi_{h_1}(e)f_{h_1} + \pi_{\ell}(e)f_{\ell} \geq \pi_1 f_{h_1} \text{ binds when } \gamma > \hat{\gamma} \]

\[ -\psi(e) + \pi_{h_1}(e)f_{h_1} + \pi_{\ell}(e)f_{\ell} \geq f_{\ell} \text{ binds when } \gamma < \hat{\gamma} \]

• It follows that both \(f_{h_1} > 0\) and \(f_{\ell} > 0\)
Proof of Lemma 2

Without (IC$_2$), either $f_{h1} > 0 = f_{\ell}$ or $f_{\ell} > 0 = f_{h1}$ ⇒

$-\psi(e) + \pi_{h1}(e)f_{h1} \leq \pi_{h1}(e)f_{h1} < [\pi_{h1}(e) + \pi_{h0}(e)]f_{h1} = \pi_1 f_{h1}$ or

$-\psi(e) + \pi_{\ell}(e)f_{\ell} \leq \pi_{\ell}(e)f_{\ell} < [\pi_{h}(e) + \pi_{\ell}(e)]f_{\ell} = f_{\ell}$

⇒ (IC$_2$) would be violated
Proof of Proposition 2 (i)

• Consider the case $f_{h1} > 0 = f_\ell$ (the other case is analogous).
• Define $v^* \equiv -\psi(e^*) + \pi_{h1}(e^*)f_{h1}^*$, where $f_{h1}^* = \psi'(e^*)/\pi_{h1}'(e^*)$.

For $v > v^*$, pay $f_{h1}^*$ plus an upfront fee $v - v^*$.

• Proof of $u(v^*) < 0$:
  ◦ First best: the firm solves $\max_e -\psi(e) + \pi_{h1}(e)(y - R(e))$, where $-\pi_h(e) + \pi_{h1}(e)R(e) = 0$ (investors break even)
    - $R(e) \downarrow$ in $e$ because $1/R(e) \equiv \pi_{1|h}(e) = \pi_{h1}(e)/\pi_{h}(e)$, conditional probability of success given the high rating, $\uparrow$ in $e$
    - The firm takes into account that effort increases both the probability that a payoff is generated, $\pi_{h1}(e)$, and the size of the payoff, $y - R(e)$
  ◦ Unobservable effort: the CRA solves $\max_e -\psi(e) + \pi_{h1}(e)f_{h1}$
    - $f_{h1}$ cannot depend on $e$
    - Thus, to induce $e^{FB}$ must have $f_{h1} > y - R(e) \Rightarrow \pi_{h1}(e)(y - R(e) - f_{h1}) < 0$
Proof of Proposition 2 (ii)

- Consider the case $f_{h_1} > 0 = f_\ell$ (the other case is analogous). From CRA’s IC, $f_{h_1} = \psi'(e)/\pi'_{h_1}(e)$.
- The firm’s payoff is $-\pi_h(e) + \pi_{h_1}(e)y - \pi_{h_1}(e)f_{h_1} = (-1+p_gy)(1/2+e)\gamma + (-1+p_by)(1/2-e)(1-\gamma) - \pi_{h_1}(e)(\psi'/\pi'_{h_1})'(e)$.
- Maximizing it wrt $e$ without imposing (1) yields FOC

$$0 = \left[\begin{array}{c}
(1 - 1 + p_g y)\gamma - (1 - 1 + p_b y)(1 - \gamma) \\
> 0 & < 0
\end{array}\right]$$

$$-\psi'(e) - \psi''(e)\pi_{h_1}(e)/(p_b\gamma - p_g(1 - \gamma)),$$

where the right-hand side is $> 0$ at $e = 0$.

- Thus $e = 0$ cannot maximize the firm’s profits.
- The CRA’s payoff is $\Pi(e) = -\psi(e) + \pi_{h_1}(e)\psi'(e)/\pi'_{h_1}(e)$, $\Pi(0) = 0$, $\Pi'(e) = \psi''(e)\pi_{h_1}(e)/(p_b\gamma - p_g(1 - \gamma)) \geq 0$, with $> 0$ for $e > 0$.
Combining Three Frontiers

• Positions of curves depend on $\gamma$
  ○ For intermediate $\gamma$, $u(v) + v > \max\{0, -1 + \pi_1y\}$ for all or some $v$
  ○ For high $\gamma$, $-1 + \pi_1y > u(v) + v \ \forall v \Rightarrow e = 0$ and always finance
  ○ For low $\gamma$, $u(v) + v = 0 > -1 + \pi_1y \ \forall v \Rightarrow e = 0$ and never finance
Investors Pay: Sketch of Proof of Proposition 5

1. Why does the term $-1 + \pi_1 y$ disappear from the expression?

- Suppose $-1 + \pi_1 y > 0$ and no rating is acquired
- CRA and investors make zero profits while firm captures all surplus
- Suppose the CRA tries to sell a rating, but no one orders it
- Consider a deviation by one investor:
  - Order a rating, only invest if it is high, ask for the same (or a slightly lower) return as the uninformed investor
  - Net of fees, this generates zero revenue if the rating is low, strictly positive revenue if the rating is high.
  - Is paying the fee worth it? Yes, at least if it implements a low enough effort:
    - At $e = 0$, profits $\Pi(0) = 0$, and using Ass. 3–4, $\Pi'(0) > 0$
  - That is, ordering an arbitrarily uninformative rating, a deviating investor can generate positive profits
Investors Pay: Sketch of Proof of Proposition 5

2. Why is \( v^{inv} < \bar{v} \) when \(-1 + \pi_1 y > 0\)? Because when \(-1 + \pi_1 y > 0\), there is an additional restriction on the problem:
   
   - Investors can’t charge a return higher than \( R \): \(-1 + \pi_1 R = 0\) (the return where investors break even if they always finance)
     
     - Suppose investors charge \( R_h > R \)
     - Then there is a profitable deviation by one investor:
       
       - do not order a rating and offer \( R' \in (R, R_h) \)
       - the firm prefers \( R' \) to \( R_h \) (lower loan repayment)
       - the investor makes positive profits with \( R' \)

   - Suppose this restriction \( R_h \leq R \) does not bind
     
     - To maximize total surplus, push the firm’s payoff to zero:
       \[
       \pi_{h1}(y - R_h) = 0 \Rightarrow R_h = y > R, \text{ a contradiction}
       \]
Corollary 1

1. Suppose $-1 + \pi_1 y \leq 0$. Then $S^{inv} = S^{iss} = S^{SB}$ and
   $e^{inv} = e^{iss} = e^{SB}$

2. Suppose $-1 + \pi_1 y > 0$. Then
   - $e^{inv} < e^{SB}$ if $e^{SB} > 0$, and $e^{inv} > e^{SB}$ if $e^{SB} = 0$
   - $e^{inv} \to 0$ and $S^{inv} \to S^{SB}/2$ as $\gamma \to 1$
   - $S^{inv} < S^{SB}$
   - $e^{iss} \leq e^{inv}$, with strict inequality if $e^{inv} > 0$
   - $S^{iss} < S^{inv}$ if $S^{inv} \geq -1 + \pi_1 y$, and $S^{iss} > S^{inv}$ otherwise
Multiple Ratings

If multiple ratings are acquired in equilibrium, the problem becomes much harder

- Contracts might depend on CRAs’ relative performance
  - Compensate Moody’s in part based on S&P’s rating
  - Ask for the second rating just to cross check the first one?
- It might not be reasonable to model CRAs’ signals as conditionally independent
- It might be possible to generate ‘rating shopping’:
  - Look at sequential rating acquisition and suppose all ratings are published at the end
  - Firms with the first low rating might shop for a second rating, hoping investors would not be able to distinguish them from firms with the first high rating
  - Technical difficulty: adverse selection arises (unobservable first rating = type of the issuer)