What is Cyclical in Credit Cycles?

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Introduction

- Credit cycles are **growth cycles**
  - Cyclicality in the amount of new credit
  - Explanations: collateral constraints, equity constraints, leverage constraints

- Credit cycles are also **risk cycles**
  - Cyclicality in the distribution of new credit — credit quality of the marginal borrowers
  - Modeling production heterogeneity is essential

- Today: A general equilibrium model with a banking sector featuring the comovement in the quantity and quality of credit
Credit cycles are not only growth cycles, they are also risk cycles.
Mechanism

(0) Current banking sector balance sheet determines effective discount rates
Bankers are the only marginal agent on the loan market by assumption
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(2) Capital producers respond to fluctuating asset prices in their production decisions
Asset prices movements shift the production frontier of the aggregate economy
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(3) Once financed, these projects stay and accumulate on banks balance sheets
Fully solved general equilibrium model to extract dynamic implications
Results

- Interaction between production heterogeneity and financial frictions generates fundamental economic forces that leads to endogeneous boom-bust cycles
  - A risks buildup process
  - A slow recovery process

- Negative correlation in financial volatility and growth in real volatility
  - New perspective on “volatility paradox” that typically focuses on financial volatility
Set up

- Three types of agents: households, bankers and capital producers.

- Risk neutral households can consume and make deposits with bankers, they maximize

  \[
  E \left[ \int_0^\infty \exp(-\rho t) \, dC_t^H \right]
  \]

- Bankers hold all risky capital. I impose that bankers consume \( \lambda N dt \) (\( N \) is bankers networth). They maximize

  \[
  E \left[ \int_0^\infty \exp(-\lambda t) \log(\lambda N_t) \, dt \right]
  \]

- This is a continuous time adoption of Kiyotaki-Gertler model, but with fixed risk free rate \( \rho \) and simplified effective bankers’ pricing kernel \( \theta_B^t = \exp(-\lambda t) \frac{\lambda}{N_t} \).
Two types of capital producers producing $K_j \in \{A, B\}$, both capital produces cash flow at rate $AK_j dt$, they depreciate at rate $\delta$.

But they have differential exposure to the systematic shock, in aggregate

$$\frac{dK^j}{K^j} = (\Phi_j(i_j) - \delta) \ dt + \bar{\sigma}_j dZ_t$$

"Quality" is captured by $\bar{\sigma}_A < \bar{\sigma}_B$. Cash flow from type B projects are more sensitive to macroeconomic shocks than type A projects.
Capital Producers

- Capital producers are owned by household, but can only sell their capital to bankers. The production function of type $j$ capital is

$$\Phi_j(i_j) = \sqrt{\frac{2i_j}{\kappa_j}}$$

- Key assumption: $\kappa_A > \kappa_B$. Supply of high quality projects are limited.

- Key endogeneous variable is the risk adjusted present value of the cash flow (net of investment) produced by type $j$ capital

$$q_j = PV_j = E \left[ \int_0^{\infty} A \frac{K_t^j}{K_0^j} \theta_B^t dt \right]$$

where

$$\frac{dK_t^j}{K_0^j} = -\delta dt + \bar{\sigma}_j dZ_t$$
Frictionless Benchmark

- Less Risky Capital
  - $K_A$ Producers
- More Risky Capital
  - $K_B$ Producers

 Rebate Profit

 Own

 Households
- Less Risky Capital
- More Risky Capital
Model Scheme

- Less Risky Capital $K_A$ Producers
- More Risky Capital $K_B$ Producers

Buy Capital
Sell Capital
Rebate Profit
Own

Intermediaries
- Less Risky Loans
- Net Worth $N$
- More Risky Loans
- Short-Term Debt

Households
- Deposits

Borrow $R_f$, Pay Dividend
Deposit $R_f$
Capital Producers’ Problem

- Given $q_A, q_B$, capital producers solve a static problem

$$\max_{i_j} \Phi_j (i_j) K_j q_j - i_j K_j$$

- Optimal investment follows

$$\Phi_j^* (i_j^*) = \frac{q_j}{\kappa_j}$$
Bankers Problem

Given their preference, bankers solves a portfolio problem that resembles standard mean-variance efficient investors

$$
\max_{\alpha_A, \alpha_B} \mathbb{E} \left[ \int_0^\infty \exp(-\lambda t) \log(\lambda N_t) \, dt \right]
$$

st.

$$
\frac{dN_t}{N_t} = -\lambda dt + (\alpha_A \pi_A + \alpha_B \pi_B + (1 - \alpha_A - \alpha_B) r_f) \, dt + (\alpha_A \sigma_A + \alpha_B \sigma_B) \, dZ_t
$$

where $\alpha_A, \alpha_B$ are portfolio shares, $\pi_A, \pi_B$ are excess returns by investing in $K_A, K_B$; $\sigma_A, \sigma_B$ are return volatilities for $K_A, K_B$
Equilibrium Definition

An **equilibrium** of this economy consists of prices processes \((q_A, q_B, r_f)\), and decisions, \((c_H, \alpha_A, \alpha_B, i_A, i_B)\), such that

1. Given prices, households, bankers and capital producers solve their optimization problems.

2. Given decisions, markets for risky capital \((K_A, K_B)\) and risk-free bond clears. This pins down bankers’ portfolio choices \(\alpha_A, \alpha_B\)

3. Market for goods clear

\[
A (K_A + K_B) = i_A K_A + i_B K_B + C_H
\]
Solving the Model

1. Conjecture the model has two scaled state variables: “size” and “quality” of intermediaries balance sheet

   \[
   \eta = \frac{N}{q_A K_A + q_B K_B} \\
   s = \frac{K_B}{K_A + K_B}
   \]

2. Write down a system of PDEs that \( q_A \) and \( q_B \) must satisfy as functions of \( \eta \) and \( s \)

3. Above equations solved on \([\eta, s] \in [\epsilon, 1 - \epsilon] \times [0, 1]\). Boundary conditions
   3.1 \( s = 0, 1 \rightarrow \) Single technology economy, solved in \( ODE \)
   3.2 \( \eta = \epsilon \), impose \( q_{\eta} = 0 \)
   3.3 \( \eta = 1 - \epsilon \), reduce to a system of lower order equations

4. Numerically, I use projection method (5-7th order Chebyshev polynomials) to minimize PDE error over a grid.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Full Model</th>
<th>Simple Model</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>Household Time Discount Rate</td>
<td>0.01</td>
<td>0.01</td>
<td>Risk Free Rate</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Bankers' Time Discount Rate</td>
<td>0.15</td>
<td>0.19</td>
<td>Unconditional Moments</td>
</tr>
<tr>
<td>( \bar{\sigma}_A )</td>
<td>Cash Flow Volatility of ( K_A )</td>
<td>0.02</td>
<td>0.046</td>
<td>Output Volatility</td>
</tr>
<tr>
<td>( \bar{\sigma}_B )</td>
<td>Cash Flow Volatility of ( K_B )</td>
<td>0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \kappa_A )</td>
<td>Adjustment Cost of ( K_A )</td>
<td>10.00</td>
<td>9.10</td>
<td>Investment Volatility</td>
</tr>
<tr>
<td>( \kappa_B )</td>
<td>Adjustment Cost of ( K_B )</td>
<td>7.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A )</td>
<td>Productivity</td>
<td>0.16</td>
<td>0.16</td>
<td>Investment-Capital Ratio</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Depreciation</td>
<td>0.10</td>
<td>0.10</td>
<td>Literature</td>
</tr>
</tbody>
</table>

Model Solution

Figure: Solid blue line corresponds to the solution for median $s$. Shaded area plots the solution corresponding to 25% – 75% distribution of $s$. Median output volatility = 0.046, top to bottom quartile of the distribution of output volatility is [0.025, 0.071].
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## Unconditional Moments

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<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_Y$</td>
<td>Median Output Volatility(%)</td>
<td>4.60</td>
<td>4.60</td>
<td>2.0 $\sim$5.0</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>Return Volatility of $K_A$(%)</td>
<td>5.11</td>
<td>8.54</td>
<td>19.00</td>
</tr>
<tr>
<td>$\sigma_B$</td>
<td>Return Volatility of $K_B$(%)</td>
<td>15.23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$SR$</td>
<td>Sharpe Ratio</td>
<td>0.33</td>
<td>0.35</td>
<td>0.50</td>
</tr>
<tr>
<td>$\mu_c$</td>
<td>Consumption Growth(%)</td>
<td>1.65</td>
<td>1.77</td>
<td>2.00</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Consumption Growth Volatility(%)</td>
<td>2.45</td>
<td>2.31</td>
<td>1.90</td>
</tr>
<tr>
<td>$\sigma_{\Phi(i_A)}$</td>
<td>Investment Volatility of $K_A$(%)</td>
<td>3.70</td>
<td>6.53</td>
<td>8.13</td>
</tr>
<tr>
<td>$\sigma_{\Phi(i_B)}$</td>
<td>Investment Volatility of $K_B$(%)</td>
<td>10.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i_A$</td>
<td>Investment / Capital Ratio for $K_A$(%)</td>
<td>10.9</td>
<td>11.20</td>
<td>11.40</td>
</tr>
<tr>
<td>$i_B$</td>
<td>Investment / Capital Ratio for $K_B$(%)</td>
<td>12.2</td>
<td></td>
<td></td>
</tr>
</tbody>
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Conditional Implications

- Risks “Buildup”
  - Without production heterogeneity, positive shocks always push the economy away from crisis state
  - Therefore, well capitalized banks (higher $\eta$) are associated with lower risks of entering a crisis
  - In my framework, well capitalized banks have strong incentive to take on additional risks — this will show up in the term structure of crisis probability
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- “Slow” Recovery
  - In the model, bank equity grows by earning this risk premium associated with its asset
  - Risk premium is higher in crisis state, so return on equity is high $\rightarrow$ recovery is fast
  - When risk taking is endogeous, banks substitute risky, high-yield projects with safe, low-yield ones $\rightarrow$ return on equity $\downarrow$ in crisis
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Figure: Left Panel: Investment Ratio as a function of $\eta$ and $s$. Right Panel: Drifts of the state variable when starting from $\eta = 0.6$ and median $s$. 
Risks Buildup

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Figure: Left Panel: Homogeneous Production. Right Panel: Heterogeneous Production. I plot the conditional probability of hitting the top 25% of the Sharpe Ratio when starting from $\eta = 0.6$. 
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Recovery Dynamics

**Figure:** Left Panel: Investment Ratio as a function of $\eta$ and $s$. Right Panel: Drifts of the state variable when starting from $\eta = 0.2$ and median $s$. 

\[ \Phi(i_b)/\Phi(i_a) \] 

$\eta$

Median $s$

Top $s$ quartile

Bottom $s$ quartile
Recovery Dynamics

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Figure: Left Panel: Homogeneous Production. Right Panel: Heterogeneous Production. I plot the conditional probability of staying in the top 25% of the Sharpe Ratio when starting from $\eta = 0.2$. 
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- My model endogenize both fundamental and financial volatilities
  - Low financial volatility symptomatic of lower risk prices
  - Riskier projects come into the money and get financed
  - Negative correlation between financial volatility and growth in fundamental volatility
  - Accumulation of riskier project tend to coincide with a period of low financial volatility and pushes economy closer to a crisis
Figure: Left Panel: Homogeneous Production. Right Panel: Heterogeneous Production. Simulated 200 years.
Conclusion

- Financial sector’s optimal financing decision determined the production mix in the real economy

- Credit quality of the marginal borrowers vary systematically over the credit cycles.

- A model to keep track of both asset and liability side of the financial sector.

- Extract model’s conditional implications from the term structure of distress probabilities.