Banking, Liquidity, and Bank Runs

in an

Infinite Horizon Economy

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Motivation

Banking distress and the real economy: Two complementary approaches:

1. "Macro" (e.g. Gertler and Kiyotaki, 2011)
   (a) Bank balance sheets affect the cost of bank credit.
   (b) Losses of bank capital in a downturn raises intermediation costs.

2. "Micro" (e.g. Diamond and Dybvig, 1983)
   (a) Maturity mismatch opens up the possibility of runs.
   (b) Runs lead to inefficient asset liquidation and loss of banking services.
Motivation (con’t)

• During the crisis both "macro" and "micro" phenomena were at work.
  – (Gorton, 2010, Bernanke, 2010).

• Starting point: Losses on sub-prime related assets depleted bank capital
  – Forced a contraction of many financial institutions.
  – Bank credit costs sky-rocketed
  – Some of the major investment and money funds experienced runs
Motivation (con’t)

• Macro models of banking distress:
  – Emphasize balance sheet/financial accelerator effects
  – Bank runs are excluded.

• Micro models of banks
  – Highly stylized; e.g. two periods
  – Runs often unrelated to health of the macroeconomy.
What We Do

• Develop a simple macro model of banking instability that features both
  – Balance sheet financial/accelerator effects
  – Banks runs

• The model emphasizes the complementary nature of the mechanisms
  – Balance sheet conditions affect whether runs are feasible
  – Two key variables:
    * Bank leverage ratio (affects degree of maturity mismatch)
    * Liquidation prices
  – Both depend on macroeconomics conditions
Model Overview

- Baseline Model: Infinite horizon endowment economy with fixed capital
  - Households
  - Bankers
    - Assume bankers issue short term non-contingent debt
      * Leads to maturity mismatch

- Extended Model: adds idiosyncratic household liquidity risks as in DD
  - Households face uncertain need to make extra expenditures.
  - A way motivate short term demandable bank debt (as in DD)
Intermediated vs. Directly Held Capital

- Capital Allocation

\[ K_t^b + K_t^h = \overline{K} = 1 \]

- \( K_t^b \equiv \) intermediated capital
- \( K_t^h \equiv \) capital directly held by households
Intermediated vs. Directly Held Capital (con’t)

- Technology for intermediated capital

\[
\begin{align*}
\text{date } t & \\
K^b_t \text{ capital} & \rightarrow \begin{cases}
K^b_t \text{ capital} \\
Z_{t+1}K^b_t \text{ output}
\end{cases} \\
\text{date } t+1 &
\end{align*}
\]

- Rate of return on intermediated capital

\[
R^b_{t+1} = \frac{Z_{t+1} + Q_{t+1}}{Q_t}
\]
Intermediated vs. Directly Held Capital (con’t)

- Technology for capital directly held by households

\[ date \ t \quad \quad \quad \quad \quad \quad date \ t+1 \]

\[
\begin{align*}
K_t^h & \quad \text{capital} \\
\{ f(K_t^h) \quad \text{goods} & \quad \rightarrow \quad \{ \quad K_t^h \quad \text{capital} \\
\end{align*}
\]

\[ Z_{t+1}K_t^h \quad \text{output} \]

\[ f(K_t^h) \equiv \text{management cost; } f' > 0, \ f'' \geq 0. \]

- Rate of return on directly held capital

\[
R_{t+1}^h = \frac{Z_{t+1} + Q_{t+1}}{Q_t + f'(K_t^h)}
\]

- Households directly hold capital due to financial constraints on banks.
### NO BANK RUN EQUILIBRIUM

#### BANKS

<table>
<thead>
<tr>
<th>ASSETS</th>
<th>LIABILITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_t K_t^b$</td>
<td>$D_t$</td>
</tr>
<tr>
<td>$N_t$</td>
<td></td>
</tr>
</tbody>
</table>

#### DIRECT CAPITAL HOLDING

$Q_t K_t^h$

### BANK RUN EQUILIBRIUM

#### CAPITAL

$K$

$Q_t^* K$

#### HOUSEHOLDS
Households

- Deposit contract:
  - Short term (one period)
  - Non-contingent return $R_{t+1}$ (absent a bank run)
  - Sequential service constraint (as in Diamond/Dybvig)
    * In the event of a run, payoff either $R_{t+1}$ or 0
    * Depends on place in line.

- Bank runs completely unanticipated.
Households (con’t)

- choose \( \{C_t^h, D_t, K_t^h\} \) to max:

\[
U_t = E_t \left( \sum_{i=0}^{\infty} \beta^i \ln C_{t+i}^h \right)
\]

- subject to:

\[
C_t^h + D_t + Q_t K_t^h + f(K_t^h) = Z_t W_t^h + R_t D_{t-1} + R_{t+1}^h Q_{t-1} K_{t-1}^h
\]

- fonc yield standard euler equations for \( D_t \) and \( K_t^h \).
Bankers

• A measure unity of bankers

• Each has an i.i.d. survival probability of $\sigma$
  
  ⇒ expected horizon is $\frac{1}{1-\sigma}$

• Banker consumes wealth upon exit

• Preferences are linear in "terminal" consumption $c_{t+i}^b$

\[
V_t = E_t \left[ \sum_{i=1}^{\infty} \beta [(1 - \sigma)c_{t+1}^b + \sigma \beta V_{t+1}] \right]
\]

• Each exiting banker replaced by a new banker.
  
  - Starts with an endowment $w^b$. 
Bankers (con’t)

• Bank balance sheet

\[ Q_t k_t^b = d_t + n_t \]

• Net worth \( n_t \) for surviving bankers

\[ n_t = R_t Q_{t-1} k_{t-1}^b - R_t d_{t-1}. \]

• \( n_t \) for new bankers

\[ n_t = w^b \]

• \( c_t^b \) for exiting bankers

\[ c_t^b = n_t \]
Limits to Bank Arbitrage

- Agency Problem:
  - After the banker borrows funds at the end of period $t$, it may divert: a fraction of $\theta$ of loans
  - If the bank does not honor its debt, creditors can
    * recover the residual funds and
    * shut the bank down.

⇒

- Incentive constraint

$$\theta Q_t k^b_t \leq V_t$$
Solution

- "Leverage" constraint

\[
\frac{Q_t k_t^b}{n_t} \leq \phi_t
\]

- \( \phi_t \) is
  - decreasing in \( \theta \)
  - increasing in \( \mu_t \)

\[
\mu_t = \beta E_t[(R_{t+1}^b - R_{t+1})\Omega_{t+1}]
\]

where \( \Omega_{t+1} > 1 \) is the banker’s expected shadow value of \( n_{t+1} \)

- \( \mu_t \) is countercyclical \( \Rightarrow \) \( \phi_t \) is countercyclical.
Aggregation

- Aggregate leverage constraint

\[ Q_t K_t^b = \phi_t N_t \]

- Aggregate net worth

\[ N_t = \sigma \left[ (R_t^b - R_t) \phi_{t-1} + R_t \right] N_{t-1} + W^b \]

- Volatility of \( N_t \) depends on \( \phi_{t-1} \) and volatility of \( R_t^b \).
Bank Runs

- Ex ante zero probability of a run.

- Consider the possibility of a run ex post:

- Ex post a "bank run" equilibrium" is possible if:
  
  - Individual depositors believe that if other households do not roll over their deposits, the bank may not be able to meet its obligations on the remaining deposits.
Conditions for a Bank Run Equilibrium (BRE)

- Timing of events:
  - At the beginning of period $t$, depositors decide whether to roll over their deposits with the bank.
  - If they choose to "run", the bank liquidates its capital and it sells it to households who hold it with their less efficient technology.

- A run is then possible if:

  $$R^*_b Q_{t-1} K^b_{t-1} < R_tD_{t-1}$$

  $R^*_b \equiv$ rate of return on bank assets conditional on liquidation of bank assets

  $$R^*_b = \frac{(Z_t + Q^*_t)}{Q_{t-1}}$$

  $Q^*_t \equiv$ the liquidation price of a unit of the bank’s assets.
Conditions for a Bank Run Equilibrium (BREC) (con’t)

- We can simplify the condition for a BRE:

\[ R^b_t \prec R_t \cdot \frac{D_{t-1}}{Q_{t-1} K^b_t} = R_t \left(1 - \frac{1}{\phi_{t-1}}\right) \]

with

\[ R^b_t = \frac{(Z_t + Q^*_t)}{Q_{t-1}} \]

- Whether a BRE exists depends on \((Q^*_t, \phi_{t-1}, R_t)\).

- \(Q^*_t\) is procyclical and \(\phi_t\) is highly countercyclical \(\Rightarrow\) the likelihood of a BRE is countercyclical.
Liquidation Price

- After a bank run at $t$:

$$K_{t+i}^h = \overline{K} = 1 \nabla i$$

- Household euler equation for direct capital holding

$$E_t\{\Lambda_{t,t+1} R_{t+1}^{h*}\} = 1$$

with

$$R_{t+1}^{h*} = \frac{Z_{t+1} + Q_{t+1}^*}{Q_t^* + f'(1)}$$

where $f'(1)$ is the marginal management cost which as at a maximum at $K_t^h = 1$. 
Household Liquidity Risks:

- Suppose the representative family has a continuum of members of measure unity.

- With probability $\pi$ a member has a need for emergency expenditures.

- Let $c^m_t$ be emergency expenditures by an individual, with $\pi c^m_t = C^m_t$ total expenditures by the family.

  - For an individual with emergency expenditures needs momentary utility is:
    \[ \log C^h_t + \kappa \log c^m_t \]

  - For someone without:
    \[ \log C^h_t \]
Household Liquidity Risk (con’t)

Timing of Events:

- The family chooses $C_t^h$ and its portfolio before learning of the realization of the liquidity risk.
- After choosing $D_t$, the household divides it evenly amongst its members.
- Emergency expenditures must be financed by deposits:
  \[
  c_t^m \leq D_t
  \]
- Those who do not use their deposits return them to the family.
- The household also sells any unused endowment to other households for deposits
  - by l.l.n. outflows of $D_t$ equal inflows during $t$. 
Figure 1: A Recession in the Baseline Model: No Bank Run Case
Figure 3: Ex Post Bank Run in the Baseline Model
Figure 2: A Recession in the Liquidity Risk Model: No Bank Run Case
Figure 4: Ex Post Bank Run in the Liquidity Risk Model
Some Remarks About Policy

- As in Diamond/Dybvig a role for deposit insurance.
  - Eliminates bank run equilibrium
  - But may have moral hazard effects on risk-taking.

- Can offset with capital requirements
  - Reduces risk-taking
  - Reduces the likelihood of a bank run equilibrium
  - But if bank equity capital costly to raise, can increase intermediation costs.

- Alternative: commitment to lender-of-last resort policies
  - Stabilizing liquidation prices reduces likelihood of bank runs
  - Examples: lending against good collateral
  - Asset purchases a good quality securities (e.g. AMBS)
### Table 1: Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline Model</th>
<th>Additional Parameters for Liquidity Model</th>
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<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>$\kappa$</td>
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<tr>
<td>$\sigma$</td>
<td>0.95</td>
<td>$c^{m}$</td>
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<tr>
<td>$\alpha$</td>
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<td>$\gamma_{L}$</td>
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<tr>
<td>$\tilde{K}^{h}$</td>
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<td>Preference weight on $c_{m}$</td>
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<td>Threshold for $c_{m}$</td>
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<td>$\rho$</td>
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<td>Probability of a liquidity shock</td>
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<tr>
<td>$Z$</td>
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<td>Fraction of depositors that can run</td>
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<td>$\omega^b$</td>
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<td>Steady state productivity</td>
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<tr>
<td>$\omega^h$</td>
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<td>Bankers endowment</td>
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<td>Threshold for $c_{m}$</td>
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<td>$\gamma_{L}$</td>
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<td>Probability of a liquidity shock</td>
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Table 2: Steady State Values

<table>
<thead>
<tr>
<th>Steady State for No Bank-Run Equilibrium</th>
<th>Baseline</th>
<th>Liquidity</th>
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</thead>
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<th>Baseline</th>
<th>Liquidity</th>
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<tbody>
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<td>K</td>
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<tr>
<td>$R$</td>
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<td></td>
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</table>