CONFRONTING RISK AND AMBIGUITY IN MACROECONOMICS AND FINANCE

Lars Peter Hansen

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Uncertainty as Source for Caution

- Uncertainty influences how we choose to invest for the future.
- Uncertainty induces “uncertainty premia” in market prices. Exposure to uncertainty requires market compensation.

**Aim:** To develop models with quantitative/empirical implications to further our understanding of this behavior.
APPRAOCH

- Explore the dynamic economic consequences of
  - alternative statistical models with the challenge of learning and
  - robust learning by the private sector and government (planner)
  within an equilibrium model.

- In models in which agents acknowledge the potential for model
  misspecification, there are new channels for equilibrium
  economic outcomes and valuations to fluctuate.

- This work expands the discussion of policy analysis in a
  structured and formal way.
Use a recursive utility model (see Koopmans, Kreps & Porteus...) to highlight how uncertainty about future events affects asset valuation.

Explore ways in which expectations and uncertainty about future growth rates influence risky claims to consumption.

Investigate how beliefs about the future are reflected in current-period assessments, through continuation values of prospective consumption processes. The forward-looking nature of the recursive utility model provides an additional channel through which sentiments about the future matter.
Consider the following choices:

1. A coin is flipped in *each* future date. If heads you get a high consumption payoff and if tails a low one.

2. A coin is flipped once. If heads you get a high consumption payoff in all future dates and if tails you get a low one in *all* future dates.

Which do you prefer?

With standard discounted expected utility, you are indifferent.

With recursive utility, you may well prefer the first option.
Consider the homogeneous-of-degree-one aggregator specified in terms of $C_t$ the current period consumption and $V_t$ the continuation value:

$$V_t = \left[ (C_t)^{1-\rho} + \exp(-\delta) \left[ R_t(V_{t+1}) \right]^{1-\rho} \right]^{1-\rho}.$$

where

$$R_t(V_{t+1}) = \left( E \left[ (V_{t+1})^{1-\gamma} \mid \mathcal{F}_t \right] \right)^{1-\gamma}$$

adjusts the continuation value $V_{t+1}$ for risk. With these preferences, $\frac{1}{\rho}$ is the elasticity of intertemporal substitution and $\delta$ is a subjective discount rate.

Special case: Discounted, power utility sets $\rho = \gamma$.

General case: Care about the intertemporal composition of risk.
Recursive utility observations

- Preferences become more sensitive to the intertemporal characterization of an uncertain future.
- Their use typically relies on “large” risk aversion.
- Alters magnitudes but does not contribute to our understanding of fluctuations in equilibrium values.
Decision theory II: Uncertain Urns

Consider the following Urns:

1. Urn has 50 red balls and 50 blue balls.
2. Urn has 100 balls with an unknown number of red and blue balls

A ball is drawn from each urn. You get to pick the urn and the choice of balls to bet on. You get a positive payoff if you are correct and a zero payoff if you are not. Which do you prefer?

Under standard expected utility you are indifferent.

Under ambiguity aversion you prefer the first urn.
ICONS OF STATISTICAL DECISION THEORY

FRANK KNIGHT

LEONARD JIMMIE SAVAGE

ABRAHAM WALD
ECONOMETRICS AS A TWO-STAGE LOTTERY

- Model of the future

\[ \phi(y^*|x, \theta) \]

\( \phi \) is a probability density function for \( Y_{t+1} - Y_t \) conditioned on an observable state vector \( X_t = x \) and a parameter \( \theta \).

- The parameter \( \theta \) is unknown and estimated given past data. Let \( \pi(\theta) \) represent a (possible) specification of a probability density function for \( \theta \).

Form average over \( \theta \)'s using \( \pi \):

\[
\int \phi(y^*|x, \theta) \pi(\theta) \ d\theta
\]

risk ambiguity

to construct a “hyper” model.

Two-stage lottery: Draw \( \theta \) and then draw \( Y_{t+1} - Y_t \) conditioned on \( x \) and \( \theta \).
MODELING SKEPTICISM

FIGURE: La Tour, *The Cheat*
ROBUSTNESS AND AMBIGUITY AVERSION

- Decision makers that are averse to ambiguity often consider a family of models.
- Decision makers that are concerned about robustness start with a benchmark model or a family of such models and consider families of alternatives or distorted models.
TWO APPROACHES TO AMBIGUITY

Two-stage lottery:

\[ \int \phi(y^*|x, \theta) \pi(\theta) \ d\theta \]

risk ambiguity

- Skeptical about the model \( \phi(y^*|x, \theta) \) - explore the utility consequence of mis-specifying \( \phi \). No scope for learning over time.

- Skeptical about the “prior” \( \pi(\theta) \) - explore the utility consequence of mis-specifying \( \pi \). Scope for learning about \( \theta \) over time.
Bayesian approach remains a valuable benchmark.

- $\pi_t$ is the date $t$ prior for $\theta$.
- Observe signal $Y_{t+1} - Y_t$ that depends on $\theta$ through a model $\phi(y^*|X_t, \theta)$.
- Update using Bayes rule to form date $t + 1$ prior:

$$\pi_{t+1}(\theta) \propto \phi(Y_{t+1} - Y_t|X_t, \theta)\pi_t(\theta)$$

Rescale the right-hand side to integrate to one.
AN IMPORTANT ROLE FOR ECONOMETRICIANS

Explore the dynamic economic consequences of

- alternative statistical models with the challenge of learning and
- robust learning by the private sector or public sector (social planner) within an equilibrium model.
EXAMPLE FORMALIZATION

Static counterpart:

\[
\min_{M \geq 0, EM = 1} EM \log V + \theta EM \log M
\]

where \( M \) is a likelihood ratio between an alternative probabiltiy distribution and a benchmark one.

Solution entails exponential tilting:

\[
M \propto \exp \left( -\frac{1}{\theta} \log V \right) = V^{-\frac{1}{\theta}}.
\]

Construct a specification of preferences as in

- Hansen-Sargent (AER) and
- Maccheroni-Marinacci-Rustichini (Econometrica, JET)

Imitate risk aversion from recursive utility: \( \theta = \frac{1}{\gamma - 1} \).
ECONOMIC AGENTS AS ECONOMETRICIANS

Opportunities that investors face depend on the underlying macroeconomy. When making their investment decisions they confront macroeconomic uncertainty about future growth prospects.

▶ Some components of the macroeconomy may be sufficiently subtle that they are hard to isolate from historical data.
▶ Study pricing in such an environment when investors are skeptical of their model of the macroeconomy and of their historical estimates of model parameters. Standard Bayesian approaches have limited success in capturing this skepticism.
▶ Confront an empirical challenge to get movements in “risk” prices over time.
Long-run risk model of consumption

- Extend calculations from Hansen (AER) and Hansen and Sargent (QE)
- Explore a model motivated by Bansal and Yaron.

$$Y_{t+1} - Y_t = \mu + X_t + GW_{t+1}$$

$$X_{t+1} = AX_t + BW_{t+1}$$

where $X$ is a scalar hidden state and $\mu$ an unknown parameter governing growth and $W_{t+1}$ is a shock distributed as a bivariate standard normal. The coefficients $\mu$, $A$ and $B$ are unknown by investors and posteriors are solved recursively.
PRICE OF UNCERTAINTY, A, B AND $\mu$ UNKNOWN
Some Extensions:

- **Robust Ramsey planner**: Ramsey planner is a stand-in for the benevolent policy maker. The introduction of this entity is used as a device to determine “optimal” policies. Robust concerns from private and/or public sectors alter the design of good policies. Policy maker engages in “managing” or “monitoring” expectations and may be cautious because of model uncertainty. (See Hansen-Sargent, 2012, Carnegie, Rochester, NYU Conference paper published in the JME for a more complete discussion.)

- **Heterogeneous beliefs**: This framework “endogenizes” expectations through the computation of “worst-case models.” Such models depend on the decision problem, and hence generates *ex post* heterogeneous beliefs.

- **Simple rules**: When are “simple” policy rules more robust?