Introduction

Relationship between financial instability and regulation

- A model without incorporating shadow banking predicts
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Relationship between financial instability and regulation

- This paper, which incorporates shadow banking, predicts

![Graph showing the relationship between financial instability and financial regulation. The graph indicates that as financial regulation increases, financial instability decreases. The solid line represents the trend with shadow banking, while the dashed line represents the trend without shadow banking.]
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Relationship between financial instability and regulation

- This paper, which incorporates shadow banking, predicts...
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Relationship between financial instability and regulation

- This paper, which incorporates shadow banking, predicts
This paper

- reconciles two seemingly contradictory ideas and
Contribution

This paper

- reconciles two seemly contradictory ideas and
- provides a guideline to consider financial instability in the presence of shadow banking.
Contribution

This paper

- reconciles two seemingly contradictory ideas and
- provides a guideline to consider financial instability in the presence of shadow banking.
- models shadow banking as off-balance sheet financing.
Flow of Funds I: Investment

asset

bankers
log utility
discount factor $\rho$

households
risk neutral
discount factor $r$
Flow of Funds I: Investment

- asset
- bankers
  - log utility
  - discount factor $\rho$
- households
  - risk neutral
  - discount factor $r$

- shadow banking
  - with credit limit

- regular banking
  - without credit limit

- regulatory authority
Flow of Funds I: Investment

- asset
- bankers
  - log utility
  - discount factor $\rho$
- households
  - risk neutral
  - discount factor $r$
- shadow banking
  - with credit limit
- regular banking
  - without credit limit
- no equity financing
- regulatory authority
Flow of Funds II: Return

- bankers
- log utility
- discount factor $\rho$

- households
- risk neutral
- discount factor $r$

$R < R$
Flow of Funds II: Return

- Asset
  - Bankers log utility discount factor $\rho$
  - Shadow banking with credit limit $r$
  - Regular banking without credit limit $r$

- Households risk neutral discount factor $r$

- Regulatory authority $\tau$

$R < R$
Flow of Funds II: Return

- **Asset**: Discount factor \( \rho \) for bankers and discount factor \( r \) for households.
- **Return**: \( R \) for bankers and \( R < R \) for households.
- **Shadow Banking** with credit limit: \( r \).
- **Regular Banking** without credit limit: \( r + \tau \).
- **Regulatory Authority**: \( \tau \).
Endogenous Risk and Financial Instability

\[
\kappa_1 - \kappa \equiv \kappa Q_{6/15}
\]
Endogenous Risk and Financial Instability

\[ \kappa \left( 1 - \kappa \right) \kappa q + \left( 1 - \kappa \right) \kappa q \equiv \kappa Q \]
Endogenous Risk and Financial Instability

\[ \kappa + (1 - \kappa)\kappa^q \equiv \kappa^Q \]
Balance Sheet of Bankers

\[
\frac{\ln[W]}{\rho} + h
\]
Balance Sheet of Bankers

\[
\frac{\ln[W]}{\rho} + h
\]

regular bank

shadow bank

bad shock

\[(W+S)\kappa Q\]

\[S^*\kappa Q\]
Balance Sheet of Bankers

\[
\frac{\ln[W]}{\rho} + h
\]

\[
\ln\left[W - (W + S + S^*)\kappa Q\right] + h
\]

\[
(W + S)\kappa Q
\]

\[
S^*\kappa Q
\]

\[
\rho + \hat{h}
\]
Balance Sheet of Bankers

\[
\frac{\ln[W]}{\rho} + h
\]

lose access to shadow banking

\[
\frac{\ln[W-(W+S)\kappa^Q]}{\rho} + \hat{h}
\]

\[
\frac{\ln[W-(W+S+S^*)\kappa^Q]}{\rho} + h
\]

\[
(W+S)\kappa^Q
\]

\[
(W+S)\kappa^Q
\]

\[
S^*\kappa^Q
\]

\[
S^*\kappa^Q
\]
Enforceability Constraint

\[
\frac{\ln \left[ W - (W + \tilde{S} + S^*) \kappa^Q \right]}{\rho} + h \geq \frac{\ln \left[ W - (W + \tilde{S}) \kappa^Q \right]}{\rho} + \hat{h}
\]
Enforceability Constraint

\[
\frac{\ln \left[ W - \left( W + \tilde{S} + S^* \right) \kappa Q \right]}{\rho} + h \geq \frac{\ln \left[ W - \left( W + \tilde{S} \right) \kappa Q \right]}{\rho} + \hat{h}
\]

\[\downarrow\]

\[s^* \leq \bar{s}^* \equiv (1 - \exp \left[ -\rho H \right]) \left( \frac{1}{\kappa Q} - (1 + \tilde{s}) \right),\]

where \(s^* = \frac{S^*}{W}, \tilde{s} = \frac{\tilde{S}}{W}, H = h - \hat{h}\)
Enforceability Constraint

\[
\frac{\ln \left( W - (W + \tilde{S} + S^*) \kappa^Q \right)}{\rho} + h \geq \frac{\ln \left( W - (W + \tilde{S}) \kappa^Q \right)}{\rho} + \hat{h}
\]

\[
\downarrow
\]

\[
s^* \leq \bar{s}^* \equiv (1 - \exp \left[ -\rho H \right]) \left( \frac{1}{\kappa^Q} - (1 + \tilde{s}) \right),
\]

where 
\[
s^* = \frac{S^*}{W}, \tilde{s} = \frac{\tilde{S}}{W}, H = h - \hat{h}
\]

financial instability \( \kappa^Q \) \( \downarrow \) \( \implies \) \( \bar{s}^* \) \( \uparrow \)
Enforceability Constraint

\[
\frac{\ln \left[ W - (W + \tilde{S} + S^*) \kappa^Q \right]}{\rho} + h \geq \frac{\ln \left[ W - (W + \tilde{S}) \kappa^Q \right]}{\rho} + \hat{h}
\]

\[
\Rightarrow
\]

\[
s^* \leq \bar{s}^* \equiv (1 - \exp[-\rho H]) \left( \frac{1}{\kappa^Q} - (1 + \tilde{s}) \right),
\]

where \( s^* = \frac{S^*}{W}, \tilde{s} = \frac{\tilde{S}}{W}, H = h - \hat{h} \)

financial instability \( \kappa^Q \) \( \downarrow \) \( \Rightarrow \) \( \bar{s}^* \) \( \uparrow \)
leverage for regular banking \( \tilde{s} \) \( \downarrow \) \( \Rightarrow \) \( \bar{s}^* \) \( \uparrow \)
Enforceability Constraint

\[
\frac{\ln \left[ W - (W + \tilde{S} + S^*) \kappa^Q \right]}{\rho} + h \geq \frac{\ln \left[ W - (W + \tilde{S}) \kappa^Q \right]}{\rho} + \hat{h}
\]

\[\Downarrow\]

\[s^* \leq \bar{s}^* \equiv (1 - \exp[-\rho H]) \left( \frac{1}{\kappa^Q} - (1 + \tilde{s}) \right),\]

where \(s^* = \frac{S^*}{W}, \tilde{s} = \frac{\tilde{S}}{W}, H = h - \hat{h}\)

financial instability \(\kappa^Q \downarrow \implies \bar{s}^* \uparrow\)

leverage for regular banking \(\tilde{s} \downarrow \implies \bar{s}^* \uparrow\)

opportunity cost of default \(H \uparrow \implies \bar{s}^* \uparrow\)
What is $H$

Probabilistic representation of $H$

$$H_t \equiv h_t - \hat{h}_t \cong E_t \left[ \int_t^\infty \exp \left[ - (\rho + \bar{\xi} + \chi) u \right] \frac{s_u^* \tau}{\rho} \, du \right]$$

tax rate $\tau \uparrow \implies H \uparrow \implies \bar{s}^* \uparrow$$
Regulatory Paradox

Markov equilibrium with a single state variable, bankers’ wealth share $\omega$. 
Regulatory Paradox

Markov equilibrium with a single state variable, bankers’ wealth share $\omega$. 

**Without Shadow Banking**

- $q_k^q$ vs. Bankers' Wealth Share, $\omega$

**With Shadow Banking**

- $q_k^q$ vs. Bankers' Wealth Share, $\omega$

**Cost of Default**

- $H$ vs. Bankers' Wealth Share, $\omega$

**Leverage for Shadow Banking**

- $s^*$ vs. Bankers' Wealth Share, $\omega$
Where is Conventional Wisdom Result

- does conventional wisdom result still hold in my model?
Where is Conventional Wisdom Result

- does conventional wisdom result still hold in my model?

- yes, when $s^* = 0$ in equilibrium
Feedback Loop

Enforceability constraint

\[ s^* \leq \bar{s}^* \equiv (1 - \exp[-\rho H]) \left( \frac{1}{\kappa Q} - (1 + \bar{s}) \right) \]  

(1)
Feedback Loop

Enforceability constraint

\[ s^* \leq \bar{s}^* \equiv (1 - \exp [-\rho H]) \left( \frac{1}{\kappa Q} - (1 + \bar{s}) \right) \]  

(1)

Probabilistic representation of \( H \)

\[ H_t \equiv h_t - \hat{h}_t \equiv E_t \left[ \int_t^\infty \exp \left[ - (\rho + \bar{\xi} + \chi) u \right] \frac{s^*_u \tau}{\rho} du \right] \]  

(2)
Two Equilibria

- Degenerate equilibrium (always exists):
  - where shadow banking does not exist effectively, i.e.,
  - $s_t^* = 0$ and $H_t = 0$;

- Non-degenerate equilibrium (might exist): $s_t^* > 0$ and $H_t > 0$. 
Equilibrium Uniqueness

Define mapping $\Gamma$

$$\Gamma H[\omega] \equiv E_t \left[ \int_t^\infty \exp \left[ - (\rho + \zeta + \chi) u \right] \frac{s^*[\omega u] \tau}{\rho} \, du \right| \omega_t = \omega$$

where

$$s^*[\omega] \leq \bar{s}^*[\omega],$$

and

$$\bar{s}^*[\omega] = (1 - \exp [ - \rho H[\omega] ] \left( \frac{1}{\kappa Q[\omega]} - (1 + \bar{s}[\omega]) \right).$$

Theorem

if $\tau < (\rho + \zeta + \chi) \kappa$, then $H[\omega] = 0$ is the unique fixed point of mapping $\Gamma$. 
Equilibrium Uniqueness

Define mapping $\Gamma$

$$\Gamma H[\omega] \equiv E_t \left[ \int_t^{\infty} \exp \left[ - (\rho + \zeta + \chi) u \right] \frac{s^*[\omega_u] \tau}{\rho} du \right| \omega_t = \omega$$

where

$$s^*[\omega] \leq \bar{s}^*[\omega],$$

and

$$\bar{s}^*[\omega] = (1 - \exp [-\rho H[\omega]]) \left( \frac{1}{\kappa Q[\omega]} - (1 + \bar{s}[\omega]) \right).$$

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**Equilibrium Uniqueness**

Define mapping $\Gamma$

\[ \Gamma H [\omega] \equiv E_t \left[ \int_t^\infty \exp \left[ - (\rho + \xi + \chi) u \right] \frac{s^*[\omega u] \tau}{\rho} du \mid \omega_t = \omega \right] \]

where

\[ s^*[\omega] \leq \bar{s}^*[\omega], \]

and

\[ \bar{s}^*[\omega] = (1 - \exp [-\rho H[\omega]]) \left( \frac{1}{\kappa Q[\omega]} - (1 + \bar{s}[\omega]) \right). \]

**Theorem**

*if* $\tau < (\rho + \xi + \chi) \kappa$, *then* $H[\omega] = 0$ *is the unique fixed point of mapping* $\Gamma$

\[ \tau, \downarrow \rightarrow H, \downarrow \rightarrow s^*, \downarrow \Rightarrow \begin{cases} s^* = 0, H = 0 \end{cases} \]
Financial Instability

Leverage for Shadow Banking