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Insider Investor and Information*

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Abstract

We model dynamic financing of innovative projects where relational financiers observe entrepreneurs' experimentation and thus information they endogenously produce, before deciding on continued financing. We show that entrepreneurs' optimal information productions follow threshold strategies. Insider financiers extract little or no interim rent even with full information monopoly, hindering efficient financing ex ante. Independent experimentation by insiders mitigate the problem, but can be either a complement or substitute to entrepreneurs' information production. Moderate competition also facilitates information production and relationship financing. We characterize how investors' relative sophistication and competition interact to produce the non-monotone empirical patterns linking relationship lending and competition. Our model also allows general forms of security, timing of experimentation and offer negotiation, and applies to venture financing. Finally, entrepreneurs' optimal security design is more efficient, and entails issuing convertible securities to sophisticated early insiders, and residual claims to outsiders.

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Keywords: Information and Security Design, Bayesian Persuasion, Relationship Banking, Information Hold-up, Venture Financing, Information Disclosure, Competition.

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1 Introduction

What is the benefit of getting finance from intermediaries a firm repeatedly interacts with, instead of issuing securities in the public market? As it is well-known in the literature on relationship banking, certain intermediaries such as banks have special abilities to monitor, gather or interpret information (e.g., Diamond (1984, 1991), Fama (1985)), and can mitigate informational asymmetry and mitigate moral hazard when they become “insider” financiers (e.g. Ramakrishnan and Thakor (1984)). Yet, with insider’s informational advantage, they may hold up the entrepreneur (Sharpe (1990) and Rajan (1992)). Existing studies typically focus on agent’s actions that shape project’s cash flow and abstract away from those that alter the informational environment. Moreover, insider investors’ informational advantage mostly comes from information that is exogenously given. Several questions naturally arise: Do information’s sources and endogenous production matter for bank orientation? Do they change our understanding of the benefit and costs of insider financing? What are their implications for designing securities that an entrepreneur issues to financiers who invest at different stages?

This paper aims to answer these questions, and by doing so, represents a first attempt to endogenize information design in relationship financing and information hold-up problems. Specifically, we clarify the type and extent of information production that mitigates information asymmetry, and the type and extent of information that leads to holdup problems. The economic insights of our theory apply to two major areas of finance. First, they help explain empirical patterns of relationship lending and bank competition that extant theories do not fully account for. Second, it proves robustness of the popular use of convertible securities and equities in financing innovative projects. To our knowledge, this paper is also the first to derive the optimal joint security design in multi-stage financing, which entails issuing convertible securities to early insiders, and residual claims such as equity to later outsiders.

Our baseline model considers a capital-constrained entrepreneur with a project that requires two rounds of financing. The first round requires a fixed investment that enables the entrepreneur to “experiment”—broadly interpreted as conducting early-stage activities such as hiring key personnels, acquiring initial users, and developing product prototypes. An investor in the first round, whom we call insider, does not have expertise on how the entrepreneur chooses from an unrestricted set of the experiments, and thus cannot fully contract on it. But by monitoring and having insider access, she receives signals from the experiment about the eventual profitability of the project, which we call “*unsupervised information*”.

To fix ideas and relate to the literature on relationship lending, we first focus on banks as insider investors before generalizing the results beyond debt financing and discussing security design.

The entrepreneur enjoys private benefit of continuing the project, and issues more debts in the second round by borrowing more or rolling over existing loans from the insider bank. Because the insider banker has information monopoly, it is well-known that she can more efficiently continue or terminate the project, relative to arms-length investors (outsiders), if there exists any. She can also hold up the entrepreneur to extract more rents, which distorts the latter's incentives.

We first show that when the entrepreneur endogenously produces unsupervised information, the insider investor cannot extract any rent in the second round or distort entrepreneur incentives. Akin to the soft-budget constraint problem, through designing the informational environment and providing vague signals, the entrepreneur makes the insider investor break even with no profit to recover the investment in the first round. This holdup by the entrepreneur implies that insider information does not necessarily mitigate informational asymmetry concerning project continuation because the project may not get financed in the first round to start with. These results are in sharp contrast to existing theories on bank monitoring that ignore the endogenous experimentation and thus the endogenous informational environment.

What if an insider investor can contract on the experimentation to some extent or can also conduct independent research to acquire "*supervised information*"? We show that in industries requiring less entrepreneur-specific knowledge, or when the insider investors are very skilled at evaluating a project using information beyond an entrepreneur's provision, an insider investor can extract positive rent in the second round, restoring the feasibility of relationship financing. We also show that the endogenous information design has a nested-interval structure found in Guo and Shmaya (2017): the set of states that lead to continuation is an interval for a given piece of supervised information; the interval for more positive supervised information contains that for the less positive. Furthermore, for investors with lower ability in producing supervised information, productions of supervised and unsupervised information are complements, but for investors with high ability, there is a "crowding-out effect on unsupervised information production. The intuition is that if the investor can very accurately evaluate the profitability of the project, the entrepreneur has the incentive to provide less clear information to increase the chance of inefficient continuation for his private benefit. Overall, the social efficiency is non-monotone in the investors ability

in producing supervised information.¹

Relationship lending also becomes viable through interim competition, even without supervised information. For example, outsiders in proximity of the startup may observe partial information about the experiment over time, which can be captured in reduced-form by an exogenously given level of competition (Petersen and Rajan (1995)). Alternatively, the investor may endogenously contract in the first-round a covenant allowing the entrepreneur to directly issue a fraction of the securities in a competitive market among outsider investors or passive syndicates. Such commitment to moderate competition can be particularly useful for financing projects that are technically advanced or those that insider investors do not possess the relevant experience and expertise to evaluate. While introducing competition decreases the insider’s share from the total profit produced in the later rounds of the project, it incentivizes the entrepreneur to design an experiment that produces more information on project profitability, which facilitates better continuation decisions.

Because an insider investor risks losing the initial investment if the project is not continued or turns out to be low quality, the maximum first-round financing an investor willingly affords naturally proxies for the ease of forming relationship financing or its length and scope. We find interesting interactions between bank competition/concentration (reflected through how much profit share an insider investor gets) and her level of sophistication (captured by her ability in supervised information production), which jointly impact the dependence of relationship financing on competition. In particular, for sufficiently sophisticated insider investors, this capacity is decreasing in the level of competition, as predicted by Petersen and Rajan (1995) and Dell’Ariccia and Marquez (2004). However, for a wide range of intermediate levels of sophistication, the model predicts a U-shaped relationship — consistent with empirical findings that extant theories cannot explain. Finally, for low sophistication, relationship lending is weakly increasing in competition, consistent with Boot and Thakor (2000), while less sophisticated ones rely on competition to encourage more unsupervised information production on project profitability. Because higher sophistication crowds out unsupervised information production, achieving the same informational environment requires greater competition, which hurts the insider’s profit share. For intermediate sophistication, intermediate competition hurts insider’s second round surplus sharing, without improving information production sufficiently much to increase total surplus significantly, yielding the

¹Similarly, Kolotilin (2017) derives a general version of the non-monotone effect of the receiver’s precision on sender and receiver’s utilities. We provide an exact characterization of the non-monotonicity on social welfare as well as agents’ payoffs.

non-monotone pattern.

The aforementioned results and insights apply to other forms of security as well, and to staged venture financing. Since it is hard to find private outsiders, an insider VC resorts to leading syndicates for follow-up rounds. We take a step further and solve for the optimal security and information production. Recall that due to the sequential nature of financing and contract incompleteness on experimentation, the commitment and timing of security design proceeds that of the information design. This fact partially breaks the indeterminacy of optimal design in Szydlowski (2016). An optimal security balances the benefits from increased information sensitivity upon project continuation, and the entrepreneur's desire (bias) for continued control, reminiscent of the use of callable debts in Diamond (1993).

Specifically, the entrepreneur is biased towards continuation and overall gets all the social surplus, so he wants to commit to efficient information production and continuation. He therefore uses a security that gives him the full downside exposure, which forces him to internalize the cost of inefficient continuation. At the same time, he needs a security with enough upside exposure for the insider so as to leave the insider enough rent, in order to get financing in the initial round. Consequently, when the entrepreneur can only design a single security or anticipates a complete lock-in with an insider financier, equity is optimal; otherwise, he optimally issues convertible securities to early insiders and residual securities to outsiders. These are consistent with the common usage of convertible securities and equities in venture financing of innovative projects.

Finally, we remark how our results are robust to security design by the insider and to its timing relative to signal realization. In many cases, allocating the design right to entrepreneurs is more socially efficient because financiers do not internalize the entrepreneurs' private benefit of project continuation and become overly concerned with the surplus split in the second stage of financing. In addition, the interval nature of information design also calls for rethinking the conventional theoretical assumptions on information disclosure and speculation in financial markets.

Literature

Our paper is foremost related to the enormous literature on relationship lending, but differs in explicitly distinguishing learning technology and endogenizing information production. Boot (2000) and Srinivasan et al. (2014) provide comprehensive surveys. Theoretical studies on the role of institutional private lenders such as banks focus on information production and control functions (e.g., Diamond (1984, 1991) and Fama (1985)). By mitigating adverse

selection and moral hazard problems and by providing flexibility through re-contracting, bank financing can be less expensive than borrowing from public (arm's length) lenders and more efficient (e.g., Petersen and Rajan (1994)).

Relationship financiers naturally have access to private information (Berger and Udell (1995) and Petersen and Rajan (2002)), or have the expertise and incentive to monitor and interpret certain information (Diamond (1991), Rajan (1992), and Gorton and Winton (2003)). This informational advantage leads to potential hold-ups (e.g., Santos and Winton (2008) and Schenone (2010)), which is not necessarily detrimental (Bharath, Dahiya, Saunders, and Srinivasan (2009)). We add to the debate on hold-up in relationship financing by providing a sharper characterization under different types of projects, informational environment, and learning technology. We clarify that insider information does not necessarily lead to information holdup. Moreover, we show that in addition to finding the right proxies for banks' information (Petersen and Rajan (1994, 1995); Berger and Udell (1995); Degryse and Van Cayseele (2000); Ongena and Smith (2001)), it is equally important to distinguish the type and production technology of information. In particular, information generation through monitoring and independent experimentation has different equilibrium implications. Our discussion and insights on insider financier's information and hold-up also apply to venture funds and private equity (Burkart, Gromb, and Panunzi (1997) and Hochberg, Ljungqvist, and Vissing-Jørgensen (2013)).

The effect of competition on bank orientation has received significant attention but is ambiguous on a theoretical level. The investment theory (e.g., Petersen and Rajan (1995) and Dell'Araccia and Marquez (2004)) argues that as the credit market concentration decreases, the firms borrowing options expand, rendering banks less capable to recoup in the course of the lending relationship the initial investments in building relationship, which hinders relationship banking; the strategic theory (e.g., Boot and Thakor (2000) and Dinc (2000)) says fiercer interbank competition drives local lenders to take advantage of their competitive edge and reorient lending activities towards relational-based lending to small, local firms, which strengthens relationship banking. Others (e.g., Yafeh and Yosha (2001) and Anand and Galetovic (2006)) suggest that competition can have ambiguous effects on lending relationships, but typically predict an inverted U-shape pattern. Yet empirically, Elsas (2005) and Degryse and Ongena (2007) document a U-shaped effect of market concentration on relationship lending which cannot be explained by extant theory. These two studies stand out because they measure relationship banking directly in terms of duration and scope of interactions, thus improve upon and complement indirect measures such as loan rate (Petersen

and Rajan (1995)) or credit availability over firms' life time (Black and Strahan (2002)), for which the impact of competition could be ambiguous in equilibrium (Boot and Thakor (2000)). Presbitero and Zazzaro (2011) suggest that this non-monotonicity can be explained by looking at the organizational structure of local credit markets.² Our model offers an explanation for the empirical pattern based on distinguishing the type of insider information and endogenizing borrowers' endogenous response to monitoring and investor learning. We also highlight the integral role of the investor's relative sophistication.

Our paper is also broadly related to the role of intermediaries (Kortum and Lerner (2001)) and security design in financing innovation (Da Rin, Hellmann, and Puri (2011)). Herrera and Minetti (2007) uses duration of credit relationship to proxy information of firms' main banks, and documents that informed financier promotes product innovation. We zoom in further to show how different information channels shape the efficient continuation of viable projects, and characterize its interaction with investor competition. In particular, the insider investor's independent information can have non-monotone effects on overall informational efficiency.

The extensive use of convertible securities in venture financing is well-documented (Gompers (1997); Kaplan and Strömberg (2003); Kaplan and Strömberg (2004)). Hellmann (2006) provides a comprehensive description of earlier theories on the optimality of convertible securities. We show convertible securities are also optimal under endogenous information design, thus demonstrating the robustness of earlier findings. We also derive the novel result that issuing convertible securities to early insider investors and residual claims to later outsider investors is optimal and robust. These findings are related to the use of callable debt contracts in Diamond (1993), but with general security design space and endogenous interim information production.

Our solution methodology involves a linear programming approach similar to Bergemann and Morris (2017) who examine cases with finite payoff-relevant states. We complement by demonstrating that the method of Lagrange multiplier applies even for the case of infinite payoff-relevant states, because for most Bayesian persuasion settings (e.g. Kamenica and Gentzkow (2011)) the function space of signals (typically conditional probabilities) is a Banach space. In addition to having a continuum of types for the sender, we also allow the receiver to be privately informed (Section 3). Our discussion is thus related to Kolotilin

²They provide evidence that marginal increases in interbank competition are detrimental to relationship lending in markets where large and out-of-market banks are predominant. By contrast, where relational lending technologies are already widely in use in the market by a large group of small mutual banks, an increase in competition may drive banks to further cultivate their extensive ties with customers.

(2017), Gentzkow and Kamenica (2016), and Guo and Shmaya (2017). We add to the literature by relaxing in our specific application the assumption that the sender’s utility from a message completely depends on the expected state (Kolotilin (2017)), or the sender’s payoff over the receiver’s actions is independent of the state (Gentzkow and Kamenica (2016)). Also featuring both sender’s and receiver’s payoffs dependent on the state and receiver’s type, Guo and Shmaya (2017) characterize sender-optimal mechanism under more general Bayesian-persuasion settings. None of these studies concern information hold-up or security design. In particular, security choices effectively endogenize the sender’s type distribution, which is new to the literature.

Applications of information design in finance have been relatively limited but growing. Our paper most closely relates to Szydlowski (2016) which derives an entrepreneur’s optimal joint design of disclosure and security: in general, disclosure policy follows a threshold strategy and security choice is irrelevant. Trigilia (2017) finds that under costly-state-verification, firms under-disclose and firm leverage is negatively correlated to transparency. Goldstein and Huang (2016) and Cong, Grenadier, and Hu (2017) analyze information design in coordination games with government interventions. Bouvard, Chaigneau, and Motta (2015), Goldstein and Leitner (2015), and Orlov, Zryumov, and Skrzypacz (2017) discuss stress test and optimal disclosure. Our paper is the first to apply information design (and jointly with security design) to relationship financing and information holdup. Moreover, different from Szydlowski (2016), we focus on pre-IPO financing where investors are generally not fully competitive, and on information holdup and joint security design for sequential investors.

2 Insider Financier with Monitoring

This section describes the basic economic environment and re-examines two key results from the existing literature concerning insider investors: mitigation of informational asymmetry, which leads to more efficient financing, and distortion due to information hold-up. We show that under a richer environment for information production and design, the conventional wisdom warrants modification.

2.1 Model Set-up

A risk-neutral entrepreneur has a project that requires a fixed investment $I \in (0, 1)$, and produces an uncertain cash-flow $X \in [0, 1]$ with a prior distribution denoted by a continuous

and atomless pdf $f(X)$. To finance the project, the entrepreneur can issue debt at face value $D > I$ to an investor. We allow general form of security and discuss optimal design later in Section 5. Moreover, we assume the entrepreneur receives private benefit $\varepsilon > 0$ if the project is financed, which could correspond to his utility from control (“nonassignable control rent” in Diamond (1993), see also Winton and Yerramilli (2008); Szydlowski (2016)), or payoff from assets- or business-in-place (Myers and Majluf (1984)). There is no time discounting.

To best illustrate our economic mechanism and match reality for early business startups and financing of innovation, we assume the project has negative NPV ex-ante, i.e. $\mathbb{E}[X - I] < 0$. In other words, the project would not be financed directly by arms-length financiers ex ante, a case for which relationship financing and informational considerations are the most important.³ However, with investment $K > 0$ the project can go through an initial stage of development – experimentation that generates more information about the distribution of final cash-flow. With exogenous experimentation, the initial stage generates information that an insider investor can use to more efficiently decide whether to continue financing. Her information monopoly allows her to extract rent, which may distort the entrepreneur’s effort provision as in Rajan (1992).

One may think of $K + I$ as the total investment needed, but raised in stages. For simplicity, we assume the seed stage with investment K generates negligible initial cash flows compared to the final payoff expected by the investor and the entrepreneur, which is similar to normalizing the liquidation value to zero (Diamond (1993)) and can be equivalently interpreted as K representing the investment net of the initial cash flows. The entrepreneur cannot divert funds to his private account or for his consumption. Furthermore, $\mathbb{E}[\max\{X - I + \varepsilon, 0\}] > K$, i.e., it is socially efficient to run an experiment that reveals whether $X \geq I$ or not.

Departing from existing studies, we allow full flexibility in terms of the informative signals the endogenous experiment generates. In other words, the entrepreneur can design an experiment that generates a signal $z \in \mathcal{Z}$, whose distribution follows from a mapping $\pi : [0, 1] \rightarrow \Delta(\mathcal{Z})$. Therefore, (\mathcal{Z}, π) fully characterizes the experiment, and $\pi(z|X)$ shows the conditional probability that z is realized when the state is X . The fact that π is a probability distribution over \mathcal{Z} automatically implies that Bayes Plausibility Constraint holds, though we do not need to analyze the posterior.

³Our results on efficient financing by relationship investors, insider information hold-up and rent extraction are robust to this assumption; our results on optimal security design depend on this assumption, and thus apply more to the financing of business startups and innovative projects.

In the baseline case, we assume only the entrepreneur has the relevant skill and endogenously designs the experiment after raising K .⁴ Importantly, the design space is rich and complex such that the entrepreneur cannot commit to the design ex ante due to contract incompleteness.⁵ That said, intermediaries such as banks have the ability to monitor the experiment and interpret the results once they finance the entrepreneur initially, giving them insider information monopoly.

We use “**unsupervised information**” to denote information endogenously produced by the entrepreneur, directly or indirectly, and autonomous of investors’ direction. For example, all the reports that the entrepreneur provides on the performance of the project in the earlier rounds are considered as unsupervised information productions. In section 3, we introduce “**supervised information**” as the set of information that the investor generates actively and independently from the entrepreneur’s actions and experiments. In both cases, because the financier collects the information through relationship interactions with the entrepreneur over time, it is more natural to interpret the information as soft information that only financiers with access (being an insider) and expertise (such as banks) have.⁶

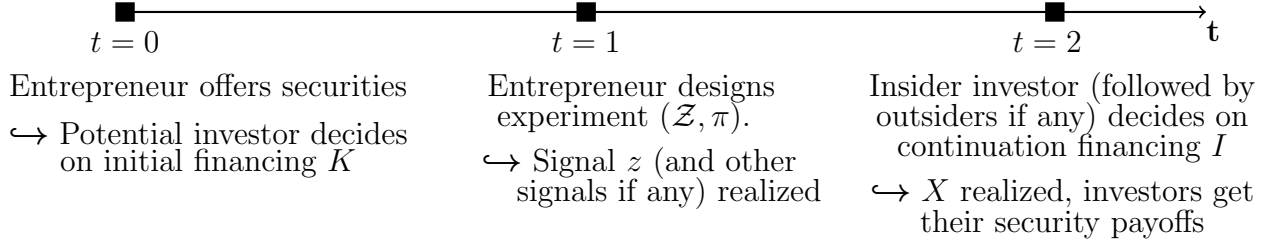
The game has three periods, $t = 0, 1, 2$. In period 0, the investor decides whether to finance the experiment and become an insider. If the investor invests K in period 0, the entrepreneur in period 1 chooses an observable experiment (\mathcal{Z}, π) which delivers an observable signal $z \in \mathcal{Z}$ about the profitability of the project. Then, the investor decides whether to invest I to continue financing the project. The final cash-flow X is realized in period 2. The time-line is illustrated in figure 1.

⁴As we discuss later, this situation can be thought as a situation where the lender either has no previous experience on the project or it is too costly for him to extract information, e.g. the firm is located in a hardly accessible location, or the investor has no relevant expertise to generate independent signals.

⁵For example, an angel investor does not know what kinds of team members a founder is going to assemble, or what kind of field-specific experiments to conduct. Carroll (2015) considers a similar situation but with the alternative assumption that investors care about robustness, and shows very generally the optimal contract is linear.

⁶Hard information, on the other hand, is typically reducible to numbers and its collection does not affect its use. That said, our analysis can be applied to hard information that is not public or verifiable by a third party as well, but is accessible by an insider financier. Liberti and Petersen (2017) provides a detailed discussion on hard versus soft information. Diamond (1984, 1991); Ramakrishnan and Thakor (1984) describe banks’ superior ability in collecting and processing information.

Figure 1: Timeline of the game.



2.2 Equilibrium

We solve for the equilibrium backward. Given an experiment (\mathcal{Z}, π) , the investor decides to invest after observing $z \in \mathcal{Z}$ if

$$\mathbb{E}[\min\{D, X\}|z] \geq I \quad (1)$$

Let \mathcal{Z}^+ denote the set of all signals that induce investment, i.e.

$$\mathcal{Z}^+ = \{z \in \mathcal{Z} | \mathbb{E}[\min\{D, X\}|z] \geq I\}$$

For each possible value of outcome X , we can find the probability of investment implied by (\mathcal{Z}, π)

$$I(X) = \int_{z \in \mathcal{Z}} 1\{z \in \mathcal{Z}^+\} \pi(z|X) dz$$

We refer to $I(\cdot)$ as the “*investment function*”, then the expected payoff for the entrepreneur from this experiment is:

$$U(\mathcal{Z}, \pi) = \int_0^1 [\varepsilon + \max\{X - D, 0\}] I(X) f(X) dX \quad (2)$$

In (2), we see the experiment enters the entrepreneur’s utility function only through the investment function. In other words, the entrepreneur aims to implement an investment function that gives the highest expected payoff. We denote such an investment function as the entrepreneur’s “*optimal investment function*”. Lemma 1 characterizes the optimal design of the experiment and the investment function it implements:

Lemma 1. (a) *There is an optimal experiment in which the entrepreneur only chooses two*

signals, i.e. $|\mathcal{Z}| = 2$. Denote $\mathcal{Z} = \{l, h\}$, and let \bar{X} be the solution to

$$\mathbb{E}[\min\{X, D\} | X \geq \bar{X}] - I = 0 \quad (3)$$

Then the optimal design uses threshold \bar{X} : the experiment generates high signal h and induces investment if $X > \bar{X}$; otherwise, it generates a low signal l leading to no investment. Therefore, $\pi^*(\cdot)$ and $I^*(\cdot)$ can be written as follows:

$$\pi^*(X) = \begin{cases} h & X \in [\bar{X}, 1] \\ l & X \in [0, \bar{X}] \end{cases} \quad I^*(X) = \begin{cases} 1 & X \in [\bar{X}, 1] \\ 0 & X \in [0, \bar{X}] \end{cases} \quad (4)$$

The equilibrium payoffs are as follows:

$$U^E(\{h, l\}, \pi^*) = \mathbb{E}[\max\{X - D, 0\} I^*(X)] = \int_{\bar{X}}^1 \max\{X - D, 0\} f(X) dX \quad (5)$$

$$U^I(\{h, l\}, \pi^*) = \mathbb{E}[(\min\{X, D\} - I) I^*(X)] = \int_{\bar{X}}^1 (\min\{X, D\} - I) f(X) dX = 0 \quad (6)$$

(b) All optimal experiments implement the same investment function and equilibrium payoffs for the investor and the entrepreneur, almost surely. Hence the equilibrium is essentially unique.

Lemma 1 characterizes the equilibrium investment function if the investor funds the experiment at $t = 0$. It shows in equilibrium only good enough projects are financed. However, it is clear from (3) that $\bar{X} < I$. Therefore, while all profitable projects are financed, some inefficient ones might be invested as well. Figure 2 illustrates this optimal experiment design. Corollary 1 concludes that for small enough values of K , the implemented investment function improves upon the case without experimentation and learning.

Corollary 1. *If $\mathbb{E}[\max\{X - D, 0\}] > K$, then $\mathbb{E}[(X - I + \varepsilon) I^*(X)] > K$.*

This result is a reminder that relationship lending can improve efficiency, because the lender acquires specific information about the borrower in earlier rounds of financing.

That said, Lemma 1 reveals that the investor gets 0 payoff in the subsequent game if she pays K at $t = 0$. Intuitively, the entrepreneur designs the high signal such that the investor breaks even in expectation, since the entrepreneur always strictly prefers continuing the project. Anticipating this subgame payoff, the investor has no incentive to pay $K > 0$ to start with. Therefore the only equilibrium in the benchmark case involves no experimentation

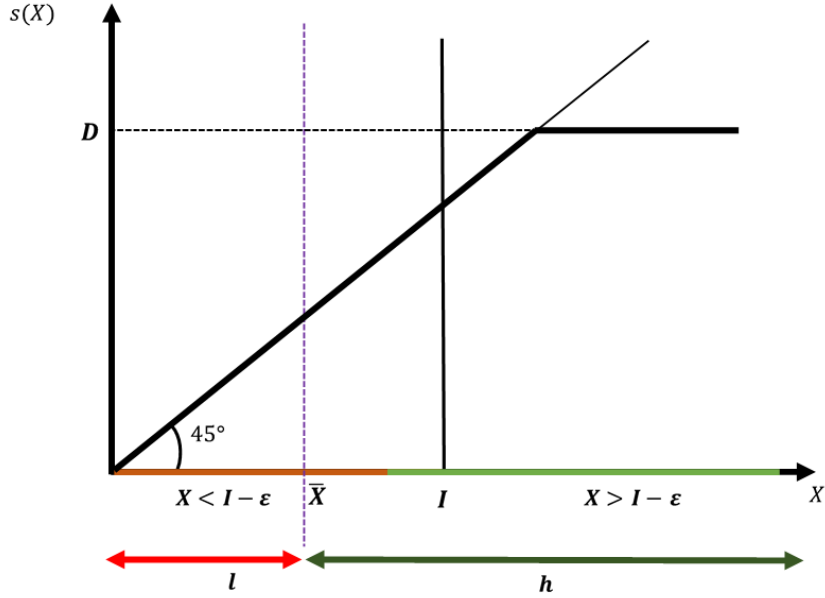


Figure 2: The optimal information design in the baseline scenario. Lighter green line ($X > I - \epsilon$) indicates the range of outputs that investment is socially optimal. Darker green line ($X > \bar{X}$) indicates the the range of outputs that the investment takes place in the subgame equilibrium at $t = 1$.

and consequently no project financing, unless $K = 0$. Proposition 1 summarizes the main results. In the proof in Appendix, we show that the result does not depend on the type of security. In Section 6.3, we discuss how our findings are robust to scalable investment, and how diminishing returns to scale potentially yields a less extreme outcome.

Proposition 1. *The unique equilibrium entails no project being financed in the initial round at $t = 0$ unless $K = 0$.*

Surprisingly, even with information monopoly, it is the insider financier that is held-up, resembling a problem of soft-budget constraint (e.g., Dewatripont and Maskin (1995)). Proposition 1 emphasizes the possible inefficiency that arises from the agency problem in information design and it cannot be resolved by debt financing from insider banks. Note that this result is in sharp contrast with the common belief in the literature that the investor can collect positive rent as she has information monopoly in the continuation decision. This result shows that even if the investor acquires information through monitoring and the entrepreneur is completely held-up by the investor, if the entrepreneur can control the structure of the information disclosed in the earlier stages, no rent would be left to the investor. In Section 6.2, we show that this result is robust to the timing of the security design and the allocation

of security design right.

It is apparent now that the type of insider information matters in relational financing. When insider investors such as banks are simply monitoring the borrowers, they do not distort borrower incentives through information hold-up. However, the benefit of mitigating anticipated interim informational asymmetry does not obtain if establishing the relationship is costly ($K > 0$) because no project is financed and no relationship is established in the first place.

The following sections present two possible resolutions. In Section 3, the insider investor gets positive payoff in the second round if she can acquire “supervised information”. In Section 4.1, the same happens when the entrepreneur and insider investor can bring in some outsider financiers, or face outsider competition. In both cases, projects with smaller K are easier to enter into relationship financing, often in terms of durability and scope in real life. K thus serves as a metric for the ease of relationship financing.

3 Supervised Information Production

Empirically, it is a robust result that the investor’s ability to monitor strongly predicts the formation of relationship financing. As an example, physical proximity between the lender and the borrower facilitates information collection and relational lending (e.g., Agarwal and Hauswald (2010); Mian (2006)). In this regard, proposition 1 demonstrates in a general setting that a sole insider investor without such ability to produce independent signal extracts no rent in later rounds of investment, so she does not finance the project to start with.

In this section, we introduce “supervised information”, which for simplicity corresponds to the information the investor can actively acquire in period 1 before making the investment decision, independently of entrepreneur’s unsupervised experiment, or by partially controlling the experiment. For example, the investor may be able to contract on the experiment to some extent, or use her business experience to better predict the market demand of the product, or project valuation in future financing rounds.

We derive three important results. First, supervised information generates a positive rent for the insider, which increases the chance of forming a lending relationship. Second, if the investor is not so sophisticated in producing high-quality signal, in equilibrium she discards her supervised information and invests based only on the signal generated by the entrepreneur’s experiment. Third, social welfare is non-monotone in the investor’s ability (which we call sophistication) in producing supervised information. If the investor is too

sophisticated, the entrepreneur may prefer sending a vaguer signal to increase his chance of project continuation which may be inefficient. The last two combine to imply that supervised and unsupervised information can be either complements or substitutes.

Equilibrium Characterization

Suppose the insider investor costlessly receives supervised information – signal $y \in \{\tilde{h}, \tilde{l}\}$ with specific signal structure $(\{\tilde{h}, \tilde{l}\}, \omega_q)$ for some $q \in (\frac{1}{2}, 1)$, where

$$\omega_q(\tilde{h}|X) = \begin{cases} q & I \leq X \leq 1 \\ 1 - q & 0 \leq X < I. \end{cases}$$

In words, if the project is profitable, the investor receives a high signal $y = \tilde{h}$ with probability $q > \frac{1}{2}$.⁷ q captures the investor’s ability in independent experimentation, namely the degree of the investor’s sophistication *relative* to the entrepreneur’s and the fundamental uncertainty. q can be related to the physical distance of the investor from the borrower or the extent to which the project is field-specific or technical. A higher q is naturally associated with relatively less advanced technologies or technologies the insider investor has expertise on.

Altogether, the investor receives two signals before the continuation decision. One is z generated by the entrepreneur’s experiment (\mathcal{Z}, π) and the other one is y . She invests if

$$\mathbb{E}[\min\{X, D\}|z, y] \geq I$$

One can see that the investor invests after receiving (z, \tilde{h}) , for some $z \in \mathcal{Z}$, if

$$(1 - q) \int_0^I (\min\{X, D\} - I)\pi(z|X)f(X)dX + q \int_I^1 (\min\{X, D\} - I)\pi(z|X)f(X)dX \geq 0;$$

and after receiving (z, \tilde{l}) , she invests if

$$q \int_0^I (\min\{X, D\} - I)\pi(z|X)f(X)dX + (1 - q) \int_I^1 (\min\{X, D\} - I)\pi(z|X)f(X)dX \geq 0.$$

Under this information structure the investor gets positive rent in the second round of the

⁷The form of supervised information is simple and natural. Richer forms constitute interesting future studies, but is beyond this paper’s focus.

project, making her more willing to bear the initial cost K . The intuition is the following: in the baseline case, the entrepreneur designs the experiment to make the investor exactly break even if they induce investment. Now the realization of y and its information structure are outside the entrepreneur's control. For example, if (\mathcal{Z}, π) is designed in a way that for some $z \in \mathcal{Z}$ the investor invests if she receives (z, \tilde{l}) , then she would invest and get positive expected profit if (z, \tilde{h}) is realized, since

$$\mathbb{E}[\min\{X, D\}|z, \tilde{h}] > \mathbb{E}[\min\{X, D\}|z, \tilde{l}] \geq I$$

Therefore, if the investor invests with positive probability after receiving \tilde{l} , she gets positive expected profit in the second round.

We now characterize the endogenous experiment designed by the entrepreneur, the investment function and the equilibrium payoffs, for the subgame after the investor pays K .

Lemma 2.

a) *There exists an experiment (\mathcal{Z}, π) , where $|\mathcal{Z}| \leq 3$ and implements an optimal investment function. In particular, it has at most one signal like $h \in \mathcal{Z}$ that both (h, \tilde{h}) and (h, \tilde{l}) induce investment, at most one like $m \in \mathcal{Z}$ that only (m, \tilde{h}) induces investment, and at most one like $l \in \mathcal{Z}$ that never induces investment. Moreover, all optimal experiments induce the same investment function, almost surely.*

b) *Let $\bar{X}(q)$ be the solution to*

$$q \int_{\bar{X}(q)}^I (\min\{X, D\} - I)f(X)dX + (1 - q) \int_I^1 (\min\{X, D\} - I)f(X)dX = 0. \quad (7)$$

Then there exists $\varepsilon^{max} > 0$ such that if $q(1 + q) < 1$ and $\varepsilon \leq \varepsilon^{max}$, the entrepreneur uses a two-signal experiment $(\{h, l\}, \pi_q^)$, where π_q^* has a threshold scheme with threshold value $\bar{X}(q)$.*

In this equilibrium, the investor invests if and only if she receives $z = h$. Correspondingly, the equilibrium experiment and payoffs are given by

$$\pi_q^*(X) = \begin{cases} h & X \in [\bar{X}(q), 1] \\ l & X \in [0, \bar{X}(q)] \end{cases} \quad I_q^*(X) = \begin{cases} 1 & X \in [\bar{X}(q), 1] \\ 0 & X \in [0, \bar{X}(q)] \end{cases} \quad (8)$$

$$U_q^E(\{h, l\}, \pi_q^*) = \int_{\bar{X}(q)}^1 (\varepsilon + \max\{X - D, 0\})f(X)dX \quad (9)$$

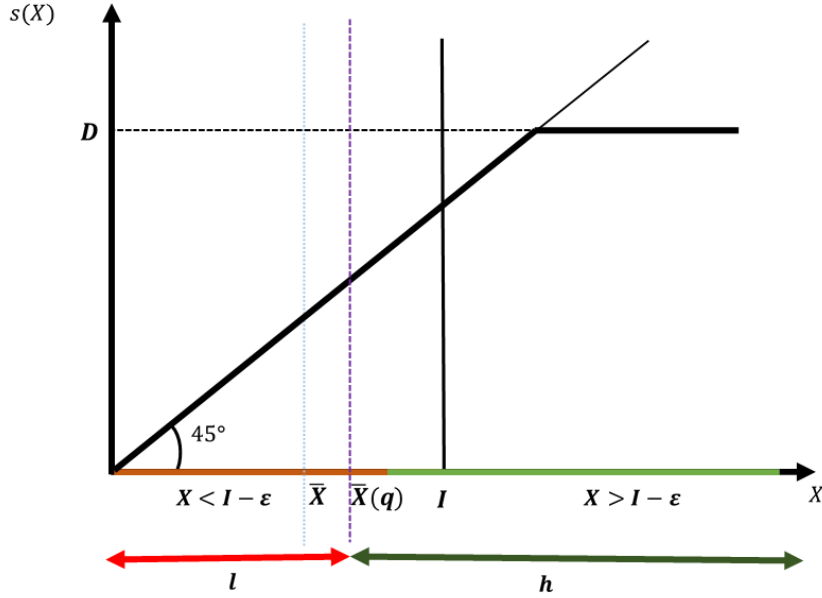


Figure 3: The optimal information design with supervised learning, for $q(1-q) < 1$. Lighter green line ($X > I - \epsilon$) indicates the range of outputs that investment is socially optimal. Darker green line ($X > \bar{X}(q)$) indicates the the range of outputs that the investment takes place in the subgame equilibrium at $t = 1$.

$$U_q^I(\{h, l\}, \pi_q^*) = \int_{\bar{X}(q)}^1 (\min\{X, D\} - I)f(X)dX > 0 \quad (10)$$

c) There exists $q^* \in (\frac{1}{2}, 1)$, where $q^*(1 + q^*) \geq 1$ and for $q \geq q^*$, the optimal investment function $I_q^*(X) < 1$ for a positive measure of $X \in [I, 1]$. Consequently, in equilibrium the project is inefficiently terminated with positive probability. The optimal disclosure policy has an interval structure. For $q < q^*$, the equilibrium specified in (b) holds.

d) Define $\bar{K}_Q(q) \equiv \mathbb{E}[I_q^*(X)(\min\{X, D\} - I)]$ as the highest cost that the investor can bear in the initial stage to start a relationship lending. Then, $\bar{K}_Q(q)$ is strictly increasing in $[0, q^*]$ and $[q^*, 1]$, while it drops discontinuously at q^* .

Lemma 2 contains a clear message that the investor's supervised information generates positive profit and up to some level, the more sophisticated the insider investor is (the more able in supervised information acquisition), the more socially efficient investment decisions are. Note that even if the information the investor produces is publicly available, namely it is obtained from some credit rating agency that is available to all competitor investors, it increases the investor's rent in the second round, which increases the possibility of relationship lending. Figure 3 illustrates the optimal endogenous experimentation in the presence

of supervised information.

To understand the main mechanism, consider a case in which $q(1+q) < 1$ and $\mathbb{E}[s(X) - I|\tilde{h}] < 0$. No investor can earn positive profit on the project just based on unsupervised information. According to the part (b) of Lemma 2, the investor’s decision is independent of the realization of her independent signal y , since she invests if and only if she receives a high signal from the entrepreneur’s experiment. It is thus intriguing how this supervised information generates positive rent for the insider, given it does not affect project output directly, nor does it generate any incremental value to the unsupervised information produced by the entrepreneur in equilibrium.

The key insight is that the existence of signal y influences the way the entrepreneur designs the experiment. In particular, the entrepreneur sends a more accurate information to preempt the risk of losing high payoffs in bad states (\tilde{l}). Note that when q is low, the risk of such loss is higher.⁸ Because the distance between borrowers and lenders is negatively correlated with banks’ ability to generate basic supervised information (Hauswald and Marquez (2006)), our model helps to explain the empirical patterns documented in previous studies such as Degryse and Ongena (2007).

Part (c) of Lemma 2 provides a counter-intuitive result that the investor’s payoff is not increasing in the accuracy of the investor’s information. In fact, it shows that if the information the investor receives is very accurate, then the entrepreneur adopts a less informative experiment to increase the chance of inefficient continuations with the cost of positive probability of inefficient terminations. In this regard, part (d) shows that while the possibility of relationship investment is locally increasing in the investor’s ability in information extraction, it is not true globally.

Another implication of Lemma 2 is that the errors in the investor’s evaluation of the entrepreneur, if there are any, have “second order” effect on the efficiency in continuation decision, especially when the investor’s supervised information is less accurate. It may explain the commonly-observed voluntary disclosure of information to the banks. However, similar to Proposition 5 in Kolotilin (2017), our lemma highlights a “crowding-out effect” for information provision, for higher levels of the investor’s sophistication.

⁸To see this effect more clearly, consider another information structure for the investor’s information, in which she receives a perfectly informative signal with probability $r > 0$ and an uninformative signal with probability $1-r$. In this case, the entrepreneur knows if a perfectly informative signal is realized, the investor does not use the signal provided by the entrepreneur’s experiment. In other words, he is not afraid of losing a high profit as a result of an error in the investor’s signal. Therefore, the entrepreneur only targets the case that the investor receives no informative signal. Therefore, the experiment he designs is no different from the one described in Lemma 1.

Who and to what extent should have the right to collect information on the profitability of a project? Proposition 1 shows that if the entrepreneur alone has this right, projects may not receive funding in the first place. Lemma 2 shows that the investors or some agents on their behalf should have the right to acquire independent information, but only to a moderate extent so that it does not “crowd out” entrepreneur’s endogenous information production.

4 Introducing Financier Competition

4.1 Competition under Unsupervised Information

So far, we have in the second round shut down inter-bank or credit market competition that influences relationship lending. We now introduce investor competition, most naturally interpreted as interim inter-bank competition, without supervised information production. As mentioned in Section 2, this situation corresponds to financing technically advanced or innovative projects, or financing by investors so inexperienced in evaluating the project that entrepreneurs fully control information production for the continuation decision.

Earlier studies typically model bank competition in reduced-form. For example, in Petersen and Rajan (1995), a bank’s market power directly relates to the interest rate it charges; in Rajan (1992), a bank’s “bargaining power” pins down the share of the unallocated surplus it gets upon continuation; in Hauswald and Marquez (2003), other outsider financiers may freely observe part of any interim information generated with small cost or even free. In what follows, we also model an insider investor’s market power as the share of the surplus μ it gets upon continuation, as in Rajan (1992). One interpretation is that due to exogenous inter-bank competition (Boot and Thakor (2000), the insider only gets to finance μI of the capital required, possibly because the entrepreneur establishes some observable track records (Diamond (1991)). In other words, a fraction of the private information of the insider lender becomes public, and the larger that fraction, the more competitive is the market, and the lower are the rents the inside lender gets. The setup is a one-to-one map to the reduced-form model of competition in Petersen and Rajan (1995) and would be consistent with a situation where there is information leakage and bank competition is exogenous (Hauswald and Marquez (2003)).

Sometimes the level of competition μ is not entirely exogenous (Dell’Ariccia and Marquez (2004)), for example, in the case of investor syndication or commitment to credit market competition. We can interpret the spillover described in Hauswald and Marquez (2003) as

endogenized by either the insider financier or the entrepreneur through choosing what type of financial intermediary to associate with. To model syndication, we can think of the insider as being capable to credibly communicate to other members of a syndicate and share her information about the entrepreneur. Specifically, at day 0, the investor can secure fraction μ of the issued security $s(X) = \min\{X, D\}$ and decide whether to pay μI after receiving the entrepreneur’s signal. Then, she lets the entrepreneur auction the rest of the security ($s^{1-\mu}(\cdot) = (1 - \mu)s(\cdot)$) to fellow syndicate members in order to raise the rest of the required fund ($(1 - \mu)I$) for continuation. We keep referring to the lead investor as the insider, and other syndicate members as outsiders as they only join in the second round. This way, at day 1 after the entrepreneur designs the experiment and the the signal is realized to all syndicate members (potentially through the lead insider), the members of the syndicate bid for the fraction $1 - \mu$ of the issued security and at the same time, the lead insider decides whether to pay μI to the entrepreneur.⁹ Syndication in some way also captures the “multi-market contact” measure in Degryse and Ongena (2007), which they show fosters relationship banking.

An alternative interpretation of this setup is that the entrepreneur raises $1 - \mu$ fraction of I from a competitive (public) market, with a modification on timing: after realization of the entrepreneur’s signal z , the entrepreneur gives the investor a take-it-or-leave-it offer of $\mu s(X)$ for upfront payment μI . Competitive outsiders in the credit market bid for the rest of the issues security $(1 - \mu)s(X)$ after observing the insider’s decision – a case in point for the “certification” function of bank lending (Diamond (1991)). For simplicity, we assume that the insider cannot participate in the auction.¹⁰

In any of the above interpretations, μ captures the extent the entrepreneur is locked with the insider’s investor for later rounds of investing. This parameter corresponds to, for example, the credit market characteristics or mutual agreement by the entrepreneur and insider in the earlier rounds. It is also possible that the insider investor may request collateral with value μI to secure her share from the profit in the later rounds of investment. The

⁹As the experiment’s signal has binary realization, an equivalent setting is that only the insider investor observes the signal first and decides whether to pay μI to the entrepreneur, then the syndicates, after observing the insider investor’s action, decide how much to bid.

¹⁰One way to think about this situation is that suppose after the experiment, the project is spoiled after T periods of time and the cash-flow generated from the project goes to 0. For perfectly patient agents, one equilibrium is that the insider investor makes investment decision immediately after the realization of the entrepreneur’s signal and the market participants bid competitively. Even if the lead investor can participate in the auction, one can imagine that it is a sealed-bid second price auction to isolate the effect of her private information, rather than what is conveyed from her first decision.

importance of collaterals in relationship lending is shown in Besanko and Thakor (1987), Inderset and Mueller (2007), and Jimnez, Salas, and Saurina (2009).

If the experiment (\mathcal{Z}, π) is designed and signal $z \in \mathcal{Z}$ is generated, the unique equilibrium for the investment decision part is that security $s^{1-\mu}(\cdot)$ is sold at $(1 - \mu)\mathbb{E}[s(X)|z]$ and the investor pays μI , if $\mathbb{E}[s(X) - I|z] \geq 0$. Therefore, the expected payoffs for the entrepreneur and the insider investor after the realization of signal $z \in \mathcal{Z}$ are the following:

$$\begin{aligned} U^E(z; (\mathcal{Z}, \pi)) &= \{\mathbb{E}[\varepsilon + X - s(X)|z] + (1 - \mu)\mathbb{E}[s(X) - I|z]\} \mathbf{1}_{\{\mathbb{E}[s(X)|z] \geq I\}} \\ U^I(z; (\mathcal{Z}, \pi)) &= \mu\mathbb{E}[s(X) - I|z] \mathbf{1}_{\{\mathbb{E}[s(X)|z] \geq I\}} \end{aligned} \quad (11)$$

Equation (11) shows that μ determines how the surplus from the issued security, $\mathbb{E}[s(X) - I|z]$ is divided between the insider investor and the entrepreneur. In other words, μ shows the fraction of the surplus generated by the issued security net of the investment cost is secured for the insider investor. In this regard, $1 - \mu$ can be equivalently interpreted as the strength of competition in the credit market.

Note that the insider investor needs to earn positive profit in expectation to recover the initial cost of earlier rounds of financing. Nevertheless, we saw in Lemma 1 that she recovers nothing in expectation when $\mu = 1$, i.e. namely highest degree of bargaining power. In spite of this result, in Proposition 2, we show that the insider investor invests K in period 0 for an interior value of μ .

When μ is endogenous, this counter-intuitive result is reminiscent of the literature on second-sourcing. For example, Farrell and Gallini (1988) show that if a buyer needs to make an initial investment to buy from a monopolistic firm, the firm may want to attract some competitors to direct the buyer's incentives to a more socially efficient level of demand. In fact, the monopolist may want to forsake some part of her monopolistic rent to let the buyer internalize some part of the total surplus generated by her decisions. Similarly, Bouckaert and Degryse (2004) show that banks spontaneously provide information for strategic reasons as increasing a rival's second-period profits lowers overall competition.

Here, the investor commits a fraction $1 - \mu$ of the possible surplus from the security to the entrepreneur to let him internalize the benefit of an experiment that better differentiates project opportunities.

Proposition 2.

a) Suppose $\varepsilon < I - \bar{X}$ (where \bar{X} is specified in (3)) and $\bar{\mu} = 1 - \frac{\varepsilon}{I - \bar{X}}$. Then for $\mu \in [0, \bar{\mu}]$,

the equilibrium investment function corresponding to μ is

$$I_{\mu,\varepsilon}^*(X) = \begin{cases} 1 & X \in [I - \frac{\varepsilon}{1-\mu}, 1] \\ 0 & X \in [0, I - \frac{\varepsilon}{1-\mu}) \end{cases} \quad (12)$$

Now define:

$$\bar{K}_S^\varepsilon(\mu) = \max\{\mu \int_{I - \frac{\varepsilon}{1-\mu}}^1 (\min\{X, D\} - I) f(X) dX, 0\} \quad (13)$$

Then, the insider investor invests at day 0 if and only if $K \leq \bar{K}_S^\varepsilon(\mu)$. Note that similar to $\bar{K}_Q(q)$, $\bar{K}_S^\varepsilon(\mu)$ shows the possibility of forming a lending relationship for (ε, μ) . Moreover, let $\mu^*(\varepsilon)$ be the value of μ that maximizes (13).

b) For $\varepsilon > 0$, $\mu^*(\varepsilon) < 1$ and it is decreasing. Moreover, $\bar{K}_S^\varepsilon(\mu)$ is weakly monotone up to $\mu^*(\varepsilon)$ and it is decreasing afterward.

c) The social efficiency $\mathcal{E}(\varepsilon) = \mathbb{E}[(X + \varepsilon - I)I_{\mu^*(\varepsilon),\varepsilon}^*(X)]$ and the highest bearable cost of initial round $\bar{K}_S(\varepsilon)$ are decreasing in ε .

d) As ε goes to 0, $\mu^*(\varepsilon)$ goes to 1 and the social efficiency $\mathcal{E}(\varepsilon)$ converges to its first-best value $\mathbb{E}[\max\{X + \varepsilon - I, 0\}]$.

Proposition 2 shows how the insider investor by inducing competition can start a relationship financing without using supervised information. The idea is that by introducing competition in later rounds, the entrepreneur faces a trade off between less informative information structure that increases the chance of continuation and a more informative information structure that can bring better terms of financing (possibly by offering lower face value¹¹).

Note that the insider investor cannot commit to offer better terms after receiving more promising news about the profitability of the project. Therefore, in equilibrium, the terms of further lending are not sensitive to the prospect or the information available on the profitability, as long as it is a positive NPV opportunity. In other words, the investor commits to better terms of financing by introducing competition in later rounds.

¹¹What we have in the model that competing investors in the syndicate offer higher upfront payments for the same face value D . One can simply relabel the contract and show it is equivalent by offering lower face value for the same upfront value I .

In addition, Proposition 2 shows that for an entrepreneur with higher private benefit of continuation, more competition should be introduced to induce the entrepreneur to provide an experiment with a better information structure. As the private benefit goes to 0, less competition is required, the investor can bear higher cost of experiment in earlier rounds and the outcome converges to its first-best scenario.

4.2 Investor Competition and Supervised Information

In Section 4.1, we see that the competition can induce better information production and higher chance of relationship lending when the insider investor only has unsupervised information. We now consider the case with both supervised and unsupervised information, i.e., the investor is able to independently evaluate the profitability of the project. We show any level of competition can reduce her incentives in starting a new lending relationship, as predicted by Petersen and Rajan (1995), but has to be considered together with supervised information because their interactions in general lead to non-monotone effects on bank orientation.

Suppose the level of competition is μ , which could be exogenous. Or similar to the previous case, the entrepreneur can sell fraction $1 - \mu$ of the issued security $s(X) = \min\{X, D\}$ in a competitive market or to competitive outsider syndicate members.

Proposition 3. *a) If $q(1+q) < 1$ and $\varepsilon + (1 - \mu)(\bar{X}(q) - I) \geq 0$ ($\bar{X}(q)$ is introduced in (7)), then the equilibrium designed experiment and investment function is the same as the one introduced in Lemma 2, part (b). Therefore, the equilibrium payoffs for the insider investor and the entrepreneur are:*

$$U_{q,\mu}^E = \mathbb{E}[(\varepsilon + X - \mu \min\{X, D\} - (1 - \mu)I)\mathbf{1}\{X \geq \bar{X}(q)\}] \quad (14)$$

$$U_{q,\mu}^I = \mu \mathbb{E}[(\min\{X, D\} - I)\mathbf{1}\{X \geq \bar{X}(q)\}] \quad (15)$$

b) For any $q \in (\frac{1}{2}, 1]$, if $\varepsilon + (1 - \mu)(\bar{X}(q) - I) < 0$, then the designed experiment has a threshold scheme with threshold $I - \frac{\varepsilon}{1-\mu}$, similar to that in proposition 2, part (a).

c) For any $q \in (\frac{1}{2}, 1]$ define $\bar{K}_C^\varepsilon(\mu; q)$ as follows:

$$\bar{K}_C^\varepsilon(\mu; q) = \mu \mathbb{E}[(\min\{X, D\} - I)I_{q,\mu,\varepsilon}^*(X)]$$

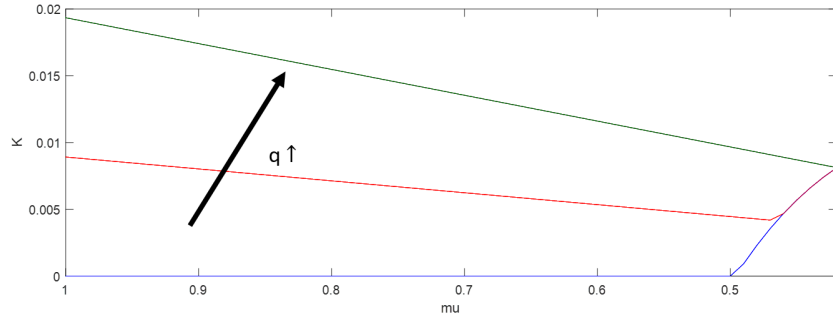
where $I_{q,\mu,\varepsilon}^*(X)$ is the equilibrium investment function for the triple (q, μ, ε) . If $q(1+q) < 1$, then there exists a decreasing function $\varepsilon^*(q)$ such that $\bar{K}_C^\varepsilon(\mu; q)$ is decreasing in μ if $\varepsilon > \varepsilon^*(q)$ and it is U-shape over a range $[\mu^*(\varepsilon, q), 1]$ for $\varepsilon < \varepsilon^*(q)$.

d) Suppose as ε goes to 0, function $\bar{K}_C^\varepsilon(\mu; q)$ converges to $\bar{K}_C^0(\mu; q)$. Then, $\bar{K}_C^0(\mu; q)$ is a decreasing function in μ

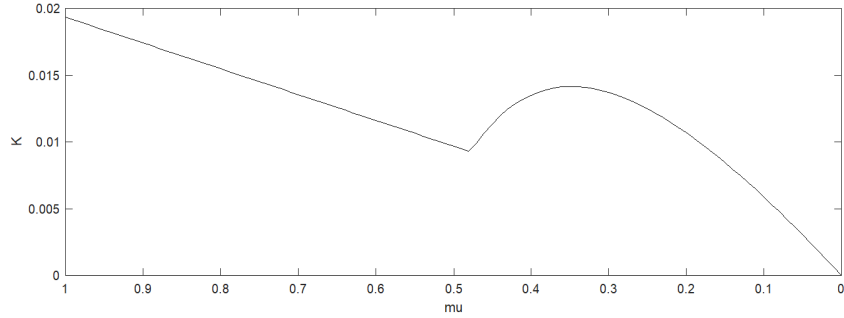
Proposition 3 shows how the level of sophistication of the lender about the project, q , and the entrepreneur's private benefit ε interact in shaping the investor's preference over different levels of competition μ . First of all, part (d) shows that without any private benefit toward continuation by the entrepreneur, investors prefer complete lock-in to all other levels of competition, since the investor can easily convince the entrepreneur to produce more informative signals about the profitability of the project, so she has no incentive to share her rents to anyone else. It is consistent with Petersen and Rajan (1995). But in general with supervised information, Boot and Thakor (2000)'s predictions also manifest.

However, as ε increases, the entrepreneur has more incentive to choose vaguer signals to increase the probability of inefficient continuation. In this regard, the investor can counteract this incentive by actively acquiring information or by introducing competition. Yet intermediate levels of competition might not induce better information provision of the entrepreneur. Therefore, the investor needs to decide between completely locking the entrepreneur in the relationship and inducing a high level of competition, for which she needs to forsake a big share of her rent.

Figure 4 Panel (a) illustrates the relationship between K , the proxy for lending formation, and μ , the measure of competition. In particular, when q takes intermediate values and μ is exogenous, the model offers a possible explanation for the finding of Elsas (2005) and Degryse and Ongena (2007) that there is a U-shape relationship between likelihood of the relationship lending and the level of competition in the credit market. On the one hand, for a fixed level of private benefit of continuation, lower levels of competition is preferred by the sophisticated investors that are able enough in producing supervised information. On the other hand, the higher levels of competition is preferred by the less sophisticated investors. Therefore, intermediate values are not preferred by any investor. Moreover, Berger, Miller, Petersen, Rajan, and Stein (2005) provide evidence consistent with small banks being better able to collect and act on soft information whereas large banks rely on hard information. To the extent that regions where large banks dominate tend to be regions with credit bureau and alternative sources that provide hard information (higher q), our model would predict



(a) The “U-shaped” relationship between K and μ



(b) Non-monotone relationship between K and μ

Figure 4: Illustration of the equilibrium capacity of the financing in the initial round as a function of the level of ex-post competition

that competition would reduce relationship formation, and the opposite holds for regions with mostly small banks and mutual banks, giving an alternative interpretation of what Presbitero and Zazzaro (2011) document empirically.

The model also predicts a more complex non-monotonicity: as the market becomes extremely competitive (μ gets much closer to 1), relationship formation eventually decreases in competition, as seen in Figure 4 Panel (b).

5 Security Design under Endogenous Experimentation

Investment banks, private equity, etc. all have the ability to acquire information through relational intermediation, and negotiate with the firms.¹² As such, our analysis extends to cases beyond relationship banking. In this section we first consider financing through securities other than debts, and then discuss the optimal security design under endogenous

¹²Firms make repeated issues and some investment banks act as delegated monitors for long-term investors (Rajan (1992)), making investment banks potential insider investors.

information production.

5.1 Venture Financing for Innovative Projects

Although our discussion thus far has focused on relationship lending by banks, the insights and intuitions regarding interactions among endogenous experimentation, independent information production, and investor competition apply more generally to other forms of “regular” securities typically considered in the security design literature, such as Nachman and Noe (1994); DeMarzo and Duffie (1999); DeMarzo, Kremer, and Skrzypacz (2005) and more recently, Cong (2017).

Definition: A *regular security* is described by $s(X) \in [0, X]$, such that s and $X - s(X)$ are both weakly increasing.

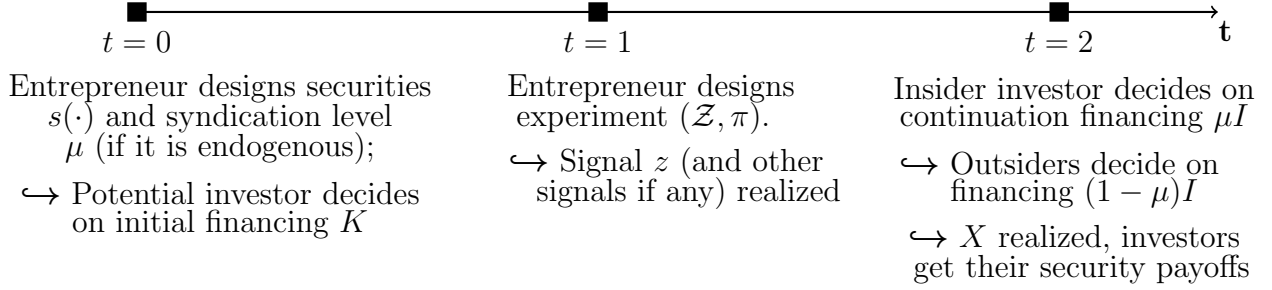
This is also the space within which we consider optimal security design in later discussion. As is standard in the literature, the security depends on the true state of the world, but is independent of the experimentation (\mathcal{Z}, π) , which is consistent with the assumption that the entrepreneur cannot commit to any experiment at the initial fundraising.

Corollary 2. *Lemmas 1,2, and Propositions 1-3 generalize to all regular securities.*

No matter what regular security is used, insider investor extracts zero rent in the second round when the entrepreneur’s information production is endogenous. This reduces insider investor’s rent from informational monopoly, but makes initial financing more difficult. Supervised information production and investor competition can mitigate the problem.

Our analysis thus applies to financing early innovative projects in general, and to venture financing in particular. Investment K captures seed capital, and I is later-stage investment or even public offering. In this regard, our model captures the essence of staged financing and the informational advantage that venture capitalists have. The natural questions are then: why, in addition to bank debts, are early startups typically financed by equity or convertible securities? What is the optimal security design under the current setting? We first answer the question in the case where the entrepreneur designs a one-size-fits-all security for all investors.

Figure 5: Timeline of the game with optimal security design.



5.2 Optimal Design of a Single Security

Suppose due to regulatory concerns, issuance costs, or lack of outsider competition in the later rounds, the entrepreneur designs a single form of security. First, consider the case that the project is so innovative or technologically advanced that the insider investor has no supervised information ($q = \frac{1}{2}$). This characteristic makes many investors reluctant to invest. In fact, without any competition in the later round ($\mu = 1$), Proposition 1 and Corollary 2 show that the project cannot be financed. Consequently, the investors may have more power over the security design, a situation we also discuss.

Proposition 4 characterizes the equilibrium security, optimal level of commitment to outsider competition, and the optimal information production, when the entrepreneur designs the security. Figure 5 displays the updated timing of the interactions.

Proposition 4.

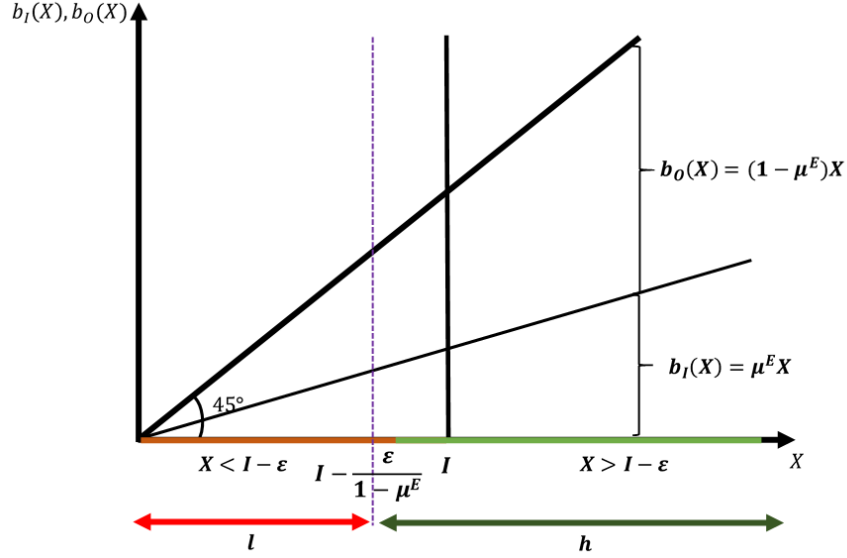
a) For initial cost of financing K , there exists a pair of security and the level of syndication $(s(\cdot), \mu)$ for which the insider investor accepts to start relationship financing if and only if condition (16) holds.

$$\max_{\mu \in [0,1]} \mu \mathbb{E}[(X - I) \mathbf{1}\{X \geq I - \frac{\varepsilon}{1 - \mu}\}] \geq K \quad (16)$$

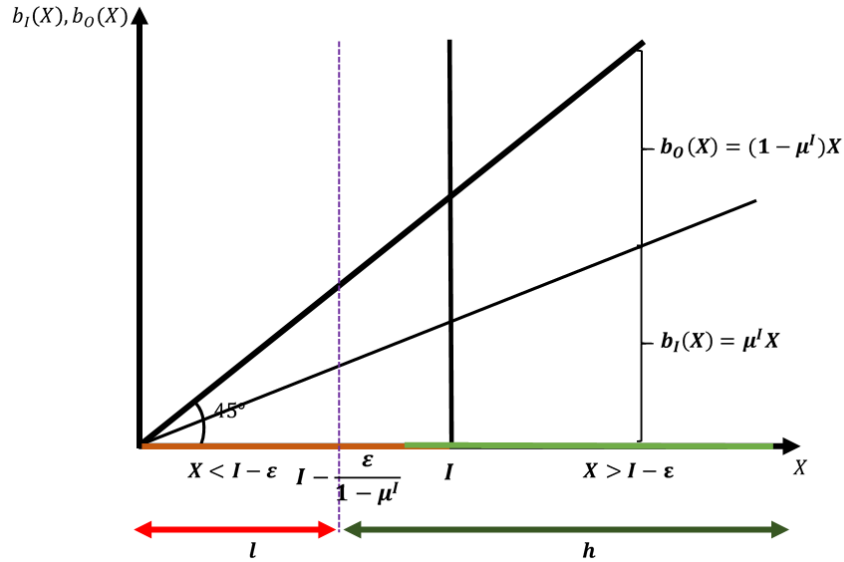
b) For a given value of $\mu \in [0, 1]$ that satisfies (16), the entrepreneur optimally issues equity, i.e., the insider investor holds in aggregate $b_I(X) = \mu X$ and the entrepreneur finances $(1 - \mu)I$ fraction of the second-round investment by selling $b_O(X) = (1 - \mu)X$ to outsider financiers.

c) The optimal syndication level for the entrepreneur is given by:

$$\mu^E = \inf_{\mu \in [0,1]} \mu \mathbb{E}[(X - I) \mathbf{1}\{X \geq I - \frac{\varepsilon}{1 - \mu}\}] \geq K$$



(a) The optimal security for the entrepreneur



(b) The optimal security for the insider investor

Figure 6: The optimal securities for the entrepreneur and the insider investor under one-security condition and without supervised learning.

The main result is that under unsupervised information and single-security condition, the optimal security is equity, no matter who designs the security. The intuition is the following: because the outsiders extract no rent, $1 - \mu$ share of the payoff all accrue to

the entrepreneur and outsiders do not distort the entrepreneur's information production. Obviously, the entrepreneur needs to give the insider investor enough rent in the second round to ensure successful initial financing. That said, due to ex-ante competition, insiders break even overall. Therefore, an optimal security design ideally maximizes social efficiency. That said, the entrepreneur's private benefit of continuation implies that he may not produce information that leads to the most efficient continuation. Security design ex ante helps him to commit to conducting experiments that improve social efficiency of financing.

Because the inefficiency lies in the continuation of bad projects, what matters is how the security allocates the downside exposure (when $X + \epsilon < I$) to the insider and the entrepreneur (through outsider investors if they are present). There are two forces at play: giving more exposure to the insider makes her less willing to continue the project, which on the margin reduces inefficient continuation (*continuation channel*); giving more exposure to the entrepreneur helps him to internalize the cost of inefficient continuation through reducing his payoff upon continuation (*payoff channel*). If continuation channel is at work (the insider marginally decides to continue), the entrepreneur always designs an experiment to generate enough inefficient continuation to extract all insider rents in the second round, which makes the project unable to get financed in the first round. Therefore, the payoff channel must be at work in equilibrium, which implies the insider must expect enough rent based on the signal to continue the project. Indeed, the insider extracts rent because the information the experiment generates leads to less inefficient continuation, which improves financing efficiency.

As such, the optimal security should give the entrepreneur and outsiders the maximum exposure to downside risk relative to the insider. With a single security, we cannot increase the relative exposure through the shape of security directly because both parties get the same security. However, by reducing μ , we naturally reduce the insider's exposure relative to the entrepreneur. Equity is thus optimal because it gives the insider the largest upside, thus allowing her to get enough rent to cover K with the smallest μ . The intuition for the case that the insider designs the security is similar, which we discuss in Section 6.2 when we analyze the social welfare effect of the allocation of security design right.

Figure 6 displays the optimal security for the entrepreneur (6a) and the insider investor (6b). The regions for the socially efficient and inefficient projects are depicted by the green and the red line in the x-axis, respectively. Moreover, the arrows specify the information structure of the signal generated by the entrepreneur. The figure shows that because the insider investor enjoys larger share and consequently more monopolistic rent ex-post, the en-

trepreneur's information production is less socially efficient, as is the project's continuation.

Next, we solve the same problem with supervised information and show that equity is still uniquely optimal. Note that the entrepreneur solves the following maximization problem in designing the security at the time of initial fundraising:

$$\begin{aligned} & \max_{s(\cdot)} \int_{\hat{X}}^1 (\varepsilon + X - s(X))f(X)dX \\ \text{s.t.} \quad & q \int_{\hat{X}}^I (s(X) - I)f(X)dX + (1 - q) \int_I^1 (s(X) - I)f(X)dX \geq 0 \\ & \int_{\hat{X}}^1 (s(X) - I)f(X)dX \geq K \end{aligned} \quad (17)$$

Proposition 5. *Consider an entrepreneur that designs regular security $s(X)$ before the initial round of investment and $q(1 + q) < 1$.*

a) *There exists a solution for (17) if and only if*

$$\frac{qK}{2q - 1} \leq \mathbb{E}[\max\{X - I, 0\}] \quad (18)$$

b) *Suppose $\varepsilon(F(I) - F(I - \varepsilon)) < \frac{(1-q)K}{2q-1}$, (18) holds and $N \geq 0$ is such that*

$$\frac{qK}{2q - 1} = \mathbb{E}[(X - N - I)\mathbf{1}\{X - I \geq 0\}], \quad (19)$$

where $F(\cdot)$ is the cdf of X . Then, the optimal security for the entrepreneur is $s(X) = \max\{X - N, 0\}$.

c) *Fixing any μ , exactly one of the following two is true:*

i- *The project is not financed under any regular security.*

ii- *The optimal security is equity.*

d) *Suppose for security $s_{X-N}(X) = \max\{X - N, 0\}$, $\hat{X}(s_{X-N}, 1)$ is the threshold above which the investment is implemented, $I_{X-N}(X) = \mathbf{1}\{X \geq \hat{X}(s_{X-N}, 1)\}$. If $\hat{X}(s_{X-N}, 1) < I - \frac{\varepsilon}{1-\mu^E}$, $\mu = \mu^E$ and the optimal security is as specified in Proposition 4, part (b). Otherwise, $\mu = 1$ and the optimal security is as specified in part (b) in this proposition.*

Proposition 5, in part (b), shows that equity is uniquely optimal under supervised single security design, regardless the level of competition. With supervised information, it is possible that the continuation channel is at work because supervised information augments the insider's ability for rent extraction in the second round, foiling the entrepreneur's ex-

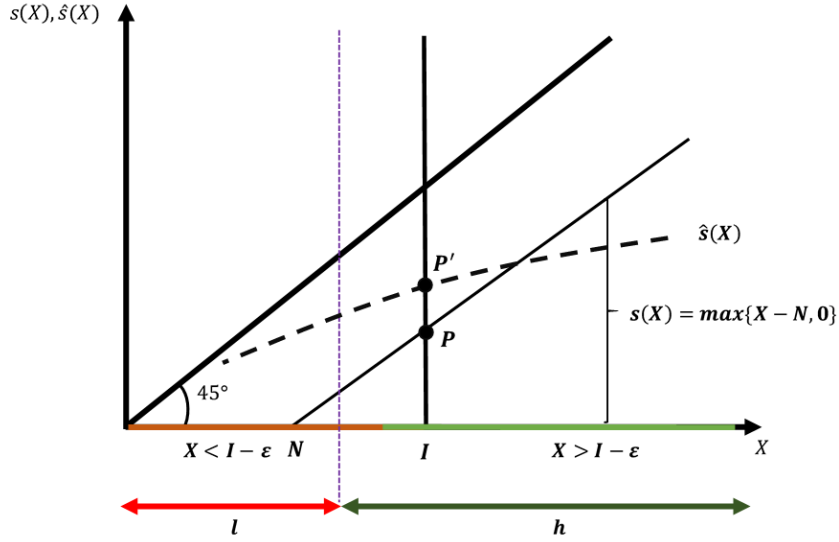


Figure 7: The optimal security under supervised learning and without outsider investors

periment design to make her break even. The intuition is provided in figure 7. Consider another feasible security $\hat{s}(X)$ that satisfies the constraints in (17). Note that $\hat{s}(X)$ is weakly “flatter” than equity $s(X)$ at every $X \geq I$, due to the regularity condition (double monotonicity). Given this, in order for two securities $s(X)$ and \hat{s} provided the same upside payoff, $\hat{s}(I)$ necessarily pays more at $X = I$, and $\hat{s}(X) \geq s(X)$ for every $X \leq I$ again due to the regularity condition. As such, $\hat{s}(X)$ enables the insider to bear more loss in the downside, rendering the continuation decision less sensitive to interim signal. Anticipating this, the entrepreneur designs a less informative interim experiment, which decreases the possibility of the relationship financing, ex ante.

Even so, for exogenous μ equity again gives her largest exposure to the downside. When μ is endogenous, issuing equity and setting $\mu = 1$ still result in the maximal exposure. Combined with the fact that if the payoff channel is at work, μ is set to μ^E , we obtain part (d).

Therefore, two possible outcomes ensue. If the best feasible level of information provision induced by competition, which is introduced in proposition 4, is more informative than what is achieved without it, the entrepreneur prefers to have the highest possible syndication or competition (lowest μ). Otherwise, the entrepreneur prefers not to have any syndication at all ($\mu = 1$) to expose the insider investor to downside risk as much as possible.

In a nutshell, equity comes out as the optimal security because when there is no ex-ante information asymmetry, information sensitivity of the security commits the entrepreneur to

informative experimentation.

5.3 Robust Optimal Security Design for Insiders and Outsiders

A salient constraint in proposition 4 and 5 is that the entrepreneur uses the same type of security for both the initial insider and outsiders. Once we relax this assumption, the type of security becomes indeterminate. However, we show the optimal design generally exhibits features of and can be implemented by convertible securities. Proposition 6 formalizes the point.

Consider the entrepreneur aims to decide on securities $b_I(X)$ for the insider investor, $b_O(X)$ for the outsider investors, and μI , the fraction of the cost of the second round of investment is financed by the insider. An optimal design then solves the following maximization problem:

$$\begin{aligned}
& \max_{b_I(\cdot), b_O(\cdot), \mu} \mathbb{E}[(\varepsilon + X - b_I(X) - (1 - \mu)I)\mathbf{1}\{\varepsilon + X - b_I(X) - (1 - \mu)I \geq 0\}] \\
& \text{s.t.} \quad \mathbb{E}[(b_I(X) - \mu I)\mathbf{1}\{\varepsilon + X - b_I(X) - (1 - \mu)I \geq 0\}] \geq K \\
& \quad \mathbb{E}[(b_O(X) - (1 - \mu)I)\mathbf{1}\{\varepsilon + X - b_I(X) - (1 - \mu)I \geq 0\}] \geq 0 \\
& \quad b_I(X) + B_O(X) \leq X \quad \forall X \in [0, 1]
\end{aligned} \tag{20}$$

Proposition 6.

a) An optimal design exists if $\mathbb{E}[(X - I)\mathbf{1}\{\varepsilon + X - I \geq 0\}] \geq K$, in which case all optimal designs implement the first-best social outcome.

b) The equilibrium disclosure policy and investment function for every optimal design are as follows:

$$\pi_F^*(X) = \begin{cases} h & X \in [I - \varepsilon, 1] \\ l & X \in [0, I - \varepsilon] \end{cases} \quad I_F^*(X) = \begin{cases} 1 & X \in [I - \varepsilon, 1] \\ 0 & X \in [0, I - \varepsilon] \end{cases} \tag{21}$$

c) Suppose $\bar{\varepsilon}$ satisfies condition (16). We deem an optimal design “robust to ε ” if it implements the optimal design for every $\varepsilon \in [0, \bar{\varepsilon}]$. Then, the set of all robust optimal security designs includes all $(b_I(\cdot), b_O(\cdot), \mu)$ that satisfy the following conditions:

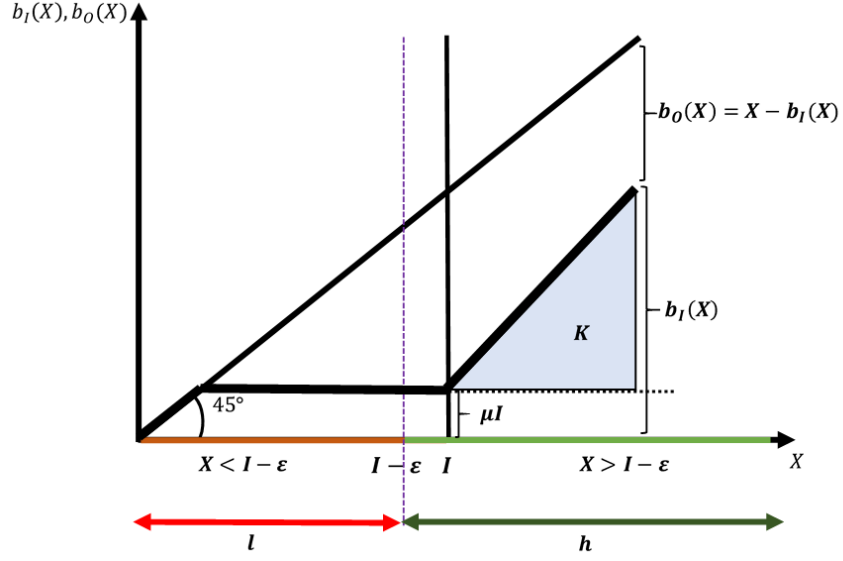


Figure 8: The optimal security under flexible security design.

$$\begin{aligned}
 b_I(X) &= \mu I \quad \forall X \in [I - \bar{\epsilon}, I] \\
 \mathbb{E}[(b_I(X) - \mu I)\mathbf{1}\{X \geq I\}] &= K \\
 0 \leq b_O(X) &\leq X - b_I(X) \quad X \in [0, 1]
 \end{aligned}$$

In short, all robust optimal securities are some form of convertible securities.

d) Even if the investment is supervised (the insider receives an independent private signal himself while it is not observed by the outsiders), the set of robust optimal designs doesn't change. In other words, the set of securities introduced in part (c) is robust to q as well.

e) When μ is exogenous, relationship financing without supervised learning is feasible if and only if $\mu \leq 1 - \frac{\bar{\epsilon}}{I}$. In this case, there exists a corresponding convertible security satisfying the conditions in part (c) that is optimal.

Comparing this result to the proposition 5, we see that while issuing equity is optimal under single security, the more efficient arrangement is to completely shift the downside risk to the outsider investors, and as a result, indirectly to the entrepreneur, which more effectively allocates exposure to downside risk to the entrepreneur relative to the insider. As a result, the entrepreneur chooses more informative experiments. The intuition is the same as that for Proposition 5 part (b)-(d), with the caveat that insiders and outsiders (thus the entrepreneur) do not need to hold the same security. Convertible securities allocate

the downside to the entrepreneur and can make him fully internalize the cost of inefficient continuation, making them optimal because they result in the first-best outcome.¹³ The allocation of the upside exposure only needs to ensure the insider earns enough from the second round to break even. Figure 8 provides the intuition: when the entrepreneur becomes exposed to the whole down-side risk, the signal becomes perfectly informative about the social efficiency of the project. As a result, the first-best continuation decision is implemented.

Could an alternative security design achieve the first-best through the continuation channel, which could be dominant in the presence of supervised information? We cannot rule this out, but for small values of ϵ , we can show that for an interval around $X = I - \epsilon$, the insider's decision is to continue after observing the equilibrium unsupervised information. The equilibrium outcome cannot be the first-best. Therefore, the optimal security designs that are robust to all values of ϵ and q are the ones spelled out in part (c).

Although the optimal security is indeterminate, the conditions in part (c) essentially pin down unsupervised information in equilibrium, leading to a unique informational environment that is also socially optimal as long as ϵ is not too big.

Our goal here is not to introduce an alternative mechanism or competing theory for the use of convertible securities or to claim the information design channel is dominant; for one, we have not considered voting or control (e.g. Harris and Raviv (1989)). So beyond deriving the optimal security under our setting, Proposition 6 indicates that earlier studies' conclusions are robust to introducing endogenous and flexible information production. Moreover, extant studies do not characterize the joint optimal securities for both early insider investors and outsiders – a phenomenon observed in real life.¹⁴ Our emphasis is on insider versus outsider, and should be distinguished from designing multiple classes of securities (e.g. Boot and Thakor (1993)).

¹³We remind the reader of the assumption $\mathbb{E}[X - I] < 0$, which drives our focus on the downside. Consistent with Winton and Yerramilli (2008), convertible securities from venture financing is more optimal than bank debts for projects for which aggressive continuation is not too profitable, business uncertainty is high, and cash flow is highly skewed.

¹⁴In many models of optimal security design in venture financing, convertible security is indistinguishable from a simple combination of debt and equity. Depending on the exact convertible security used, our optimal security may not be replicated with debt and equity. In that sense, we join Hellmann (2006) in showing why convertible security is needed.

6 Discussion

6.1 Timing of Information Design and Security Determination

One important assumption in our baseline setting is that the insider has limited ability in dictating the entrepreneur’s experimentation at the time of initial financing, but can observe it after becoming an insider. In other words, the entrepreneur cannot commit to an experimentation when raising K . Otherwise, the insider can extract rent (as seen in Section 3), and the problem in Section 5 becomes a joint optimization problem on security design and information disclosure for a total issuance of $K + I$, which Szydlowski (2016) addresses. He shows that both equity and debt can implement the optimal design.

In essence, the commitment or timing of the experiment design in our setting (partially) breaks the indeterminacy in Szydlowski (2016), in which investors break even in a single round of financing, and thus are not held up. In our setting, projects are initially financed by selling securities to the insider before the firm’s experiment. Therefore, the type of security in part contributes the entrepreneur’s endogenous commitment to future transparency.

For illustration, suppose we use debt security to finance the project under unsupervised information, the insider investor receives less reward from high realizations. Therefore, the entrepreneur promises a larger μ to the insider in the second round to compensate for her initial investment K . The entrepreneur, less exposed to the downside (smaller $1 - \mu$), then chooses less informative signals ex post, which decreases the financing capacity in the first round. Clearly, the choice of security ex-ante affects the choice of information design ex-post.

That said, as long as experimentation design happens after the initial financing, our earlier results regarding mitigating inefficient financing and information holdup are robust to the timing of security design or negotiation on security prices and quantities. To see this, suppose the security is offered with the experiment after the first round of financing. In this case, according to Corollary 2, with only unsupervised information the insider investor makes no profit for any choice of security, which means there would be no ex-ante investment. The same result holds even with supervised information and security design after signal realization.¹⁵

We have already remarked earlier how the partial determinacy of optimal security uniquely pins down the informational environment, which is indeterminate in Szydlowski (2016). What about investors’ payoff? In our setup, insider investors make positive profit in the

¹⁵To see this, a debt contract at face value I and full disclosure can guarantee the investment ex-post, while the insider receives no profit.

second round due to the informational advantage. But neither insider investor nor outsiders make positive profit if we consider all rounds of financing, because ex ante the lenders are competitive and no one has insider advantage yet at the initial round of financing.

6.2 Security Design Right

In Section 5, we mainly focus on the entrepreneur's optimal security design problem. In this part, we show that the insider investor designs optimal security differently from what the entrepreneur does, and in a less socially efficient manner.

Proposition 7. *Under both supervised and unsupervised information acquisition and single and flexible security design, it is more socially efficient that the entrepreneur designs the security.*

The intuition for proposition 7 is the following: in order to raise the initial K , the entrepreneur understands he has to provide at least a minimum amount of expected cash flow to the insider investor. Therefore among all designs that generates this amount, he chooses the one that makes him commit to the most informative disclosure policy, which maximizes the total surplus. In other words, ex ante he is the residual claimant and his incentives are more aligned with a social planner.

The insider would choose a less socially optimal security design for two reasons: first, she does not consider the private benefit of the entrepreneur; second, she not only weighs the total expected cash-flow, but also cares for her share from the output which distorts the entrepreneur's incentives. A comparison between the two panels in Figure 6 graphically reveals that the insider's design of μ_I leads to a continuation decision further away from what is socially optimal.

Insider Rent From Information Hold-up

Proposition 1 shows that the insider investor cannot extract any rent in the second round, absent supervised information and competition. We now show that even if the investor designs the security after the signal realization, which would advantage her the most, the result still holds.

Note that the investor has no competitor once the entrepreneur is locked in. The insider optimally asks for $s(X) = X$ if the project appears profitable ($\mathbb{E}[X - I|z] \geq 0$) and otherwise terminates the project. In response, according to Lemma 1, the optimal experiment and the

equilibrium investment function are as follows:

$$\pi^*(X) = \begin{cases} h & X \in [\tilde{X}, 1] \\ l & X \in [0, \tilde{X}] \end{cases} \quad I^*(X) = \begin{cases} 1 & X \in [\tilde{X}, 1] \\ 0 & X \in [0, \tilde{X}] \end{cases} \quad (22)$$

where \tilde{X} is defined as:

$$\mathbb{E}[X - I|X \geq \tilde{X}] = 0$$

One can show that for any value of $D < 1$, $\tilde{X} < \bar{X}$ (the value defined in (3)). Therefore, not only the investor still receives 0 rent in equilibrium, but the social efficiency drops.

6.3 Scale of Investment

So far, we have assumed binary investment decision for project. In this section, we enlarge the set of investment opportunities by considering the case that the investor can determine the scale of investment after observing the experimentation outcome.

In particular, we assume there exists weakly increasing function $r(\cdot) : [0, 1] \rightarrow [0, 1]$, such that if the investor decides to invest αI , the project generates random cash-flow $r(\alpha)X$. For the sake of consistency with the previous parts, we assume $r(0) = 0$ and $r(1) = 1$; Therefore, the setting in the previous parts corresponds to the function $r(\alpha) = 0$ for $\alpha < 1$ and $r(1) = 1$. Without any interim competition, the final payoffs following the investment level α and realization of cashflow $r(\alpha)X$ are $u^E = r(\alpha)X - \tilde{s}(r(\alpha), r(\alpha)X) + \varepsilon(r(\alpha))$ and $u^I = \tilde{s}(r(\alpha), r(\alpha)X) - \alpha I$ respectively, where \tilde{s} is a security payment that is generally contingent both on the project scale (or equivalently, level of investment) and the final cashflow, and private benefit of continuation depends on project scale. As an illustration, we consider the case $\tilde{s}(r(\alpha), r(\alpha)X) = r(\alpha)s(X)$ and $\varepsilon(r(\alpha)) = r(\alpha)\epsilon$, where $s(X)$ is a regular security. Because X is the profitability of the project at full-scale, and the investment only enables $r(\alpha)$ level of the full potential, the investor only gets a $r(\alpha)$ fraction of the security payoff for financing the project full-scale.

6.3.1 Projects with Full-scale Benefit

Proposition 8. *Suppose $r(\alpha) \leq \alpha$ for all $\alpha \in [0, 1]$. Then, the investor optimally chooses from $\alpha \in \{0, 1\}$. Moreover, in absence of supervised information and interim competition ($q = \frac{1}{2}$ and $\mu = 1$), the result of proposition 1 holds.*

Proposition 8 shows that for projects with full-scale benefit, which includes projects with increasing and constant returns to scale, the insider investor optimally chooses one of the extreme values for the scale of the investment.¹⁶ Intuitively, if the investor continues the project at all, she finds full-scale investment optimal. On the other hand, if the project generates loss in expectation for her, scaling up does not help, therefore she completely terminates the project.

This fact shows that the investor's and the entrepreneur's expected payoffs do not change upon expanding the investor's action space. The optimal experiment and the equilibrium payoffs are hence as introduced in Lemma 1. In particular, the insider investor does not get positive rent ex-post, which prevents the possibility of relationship financing ex ante.

In reality, developments of novel technologies and innovative startups often require a critical amount of investment and coordination, and exhibit constant or increasing returns to scale up to that level. In that sense, we argue that our model implications apply directly even when the scale of investment is a choice variable. That said, we discuss a case of diminishing returns to scale and show that insider can retain some rent from informational anomaly and investment level is interior, though the basic economic tradeoffs remain intact.

6.3.2 Diminishing Return to Scale Projects

In this case, we examine the equilibrium endogenous experimentation and the payoffs when the project is diminishing return to scale ($r'(\alpha) > 0$, $r''(\alpha) < 0$).¹⁷ We show two important results. First, the insider investor gets positive payoff under mild conditions, different from the extreme “no-rent” result in lemma 1. Secondly, the entrepreneur still optimally provides a binary signal, for a wide range of securities including equity. For debt, at most 3 signals is needed. However, there are securities that the entrepreneur provides infinite number of signals.

Throughout the following analysis, we assume $r'(0) < \frac{I}{\mathbb{E}[X]}$ to preclude positive investment without interim information. Following the solution strategy in Section 2.2, we first solve for the investor's decision on the level of investment after observing the provided signal z .

Lemma 3. *Suppose the investor receives signal z , which implements posterior $f(X|z)$. Denote the implied level of investment by $\alpha^*(z)$.*

¹⁶We call a project is increasing return to scale if for every $0 < \alpha_1 \leq \alpha_2$, we have $\frac{r(\alpha_2)}{\alpha_2} \geq \frac{r(\alpha_1)}{\alpha_1}$. Constant and decreasing return to scale projects are defined, accordingly.

¹⁷For simplicity of exposition, we assume the second derivative exists for the production function over $(0, 1)$. For $\alpha = 0$, right second derivative and for $\alpha = 1$, left second derivative exist.

a) The investor's strategy is given by:

$$\alpha^*(z) = \begin{cases} 0 & r'(0) \leq \frac{I}{\mathbb{E}[s(X)|z]} \\ (r')^{-1}\left(\frac{I}{\mathbb{E}[s(X)|z]}\right) & r'(1) \leq \frac{I}{\mathbb{E}[s(X)|z]} < r'(0) \\ 1 & r'(1) > \frac{I}{\mathbb{E}[s(X)|z]} \end{cases} \quad (23)$$

b) Observing signal z , the investor receives strictly positive payoff if $\alpha^*(z) > 0$.

Corollary 3.

a) Under the assumptions of this section, if experiment (\mathcal{Z}, π) implements investment with positive probability, then the investor receives strictly positive payoff in expectation.

b) There is positive probability of investment iff $\frac{I}{s(1)} < r'(0)$.

The intuition for corollary 3 is as follows: if the investor makes a positive investment, she should at least break even for the marginal amount of investment. Since the production function is DRS, then the investor should receive a strictly positive average return over the investment. It ensures that the investor receives strictly positive expected payoff if the entrepreneur's endogenous experimentation induces investment with positive probability. Moreover, note that the entrepreneur gets strictly positive payoff from investment. Therefore, he provides good enough signal to induce investment with positive probability. Condition $\frac{I}{s(1)} < r'(0)$ provides a sufficient and necessary condition for the existence of posterior, like $f(X|z)$, that induces positive amount of investment.

Now we solve for the optimal information design. Note that the investor's decision only depends on expected payoff she gets from the security, $\mathbb{E}[s(x)|z]$. However, the entrepreneur's payoff depends on both expected state $\mathbb{E}[X|z]$ and the expected payoff generated by the security $\mathbb{E}[s(X)|z]$. As such, we cannot directly use the approach suggested in Gentzkow and Kamenica (2016) and Dworzack and Martini (2017). To overcome this difficulty, we define a multivariate distribution $\tilde{f}(x, s) \equiv f(x)\delta(s(x))$, where $\delta(\cdot)$ is a Dirac function. The entrepreneur's maximization problem then reduces to

$$\max_{g(\cdot, \cdot)} \int_0^1 \int_0^1 r\left((r')^{-1}\left(\frac{I}{s}\right)\right)(x - s + \varepsilon)g(x, s)dsdx \quad (24)$$

Subject to the condition that $\tilde{f}(\cdot, \cdot)$ is a mean-preserving spread of $g(\cdot, \cdot)$.

Proposition 9. Suppose $\frac{I}{s(1)} < r'(0) \leq \frac{I}{\mathbb{E}[s(X)]}$. Moreover, suppose $r'''(\cdot) \leq 0$ in $\alpha \in (0, 1)$.

Suppose the entrepreneur is only allowed to choose among Monotone Partitional Signals, specified in Dworzack and Martini (2017)¹⁸. Then,

a) If $s(X)$ is quasi-convex in X , then the entrepreneur needs exactly two signals, namely a binary signal, for implementing the optimal disclosure policy.

b) If $s(X)$ is debt, i.e. $s(X) = \min\{X, D\}$ for some D , then the entrepreneur needs two or three signals.

c) There exists regular securities for which the entrepreneur uses infinite number of signals.

In proposition 9, Condition $r'''(\cdot) \leq 0$ ensures that the project becomes even harder to scale as it becomes bigger and condition $\frac{I}{s(1)} < r'(0) \leq \frac{I}{E[s(X)]}$ rules out the optimality of no-disclosure.

Essentially, the entrepreneur considers pooling vs separating different projects: his payoff is weakly increasing in X , he thus may prefer to separate projects with higher and lower X to induce more investment for the better projects. However, it is at the expense of less investment for worse projects. Therefore, his decision depends on the extent that his payoff and the size of investment are sensitive to the profitability of the project. For quasi-convex securities, because the payoff is not sensitive enough, he strictly prefers pooling different quality of projects. However, there exists concave securities that this effect is strong enough to fully separate the projects in a subset of values of X .

The key takeaway from the above analysis is that with diminishing returns to scale and a continuum of investment levels, investment decisions are no longer binary (investing I or terminating), and the investor derives partial rent from insider's informational monopoly. That said, endogenous information production still leads to reduced rent which reduces the distortion of entrepreneur incentives but potentially renders relationship financing infeasible, consistent with Lemma 1 and Proposition 1.

6.4 Optimal Contracting and Information Production

We can view the insider's lack of control over entrepreneur's experimentation (and thus information structure design) in the first round as a moral hazard problem. In particular, when μ is close to 1, competition is not playing a big role. Proposition 5 is in sharp contrast

¹⁸An experiment satisfies Monotone Partitionality condition (MP) if there exists an increasing sequence of real numbers $\{x_i\}_{i=0}^N \in [0, 1]$, $x_0 = 0$ and $x_N = 1$, and for every $0 \leq i \leq N - 1$, the entrepreneur either completely pool or separate signals in $[x_i, x_{i+1}]$.

with the celebrated result on optimal contracting that for risk-neutral agents, the optimal security is debt (Innes (1990)). More generally, we know that the entrepreneur needs to be exposed to risk to mitigate the moral hazard of effort provision (Holmstrom (1979)). It is then puzzling why in Propositions 5 and 4, it is optimal to expose the principal (insider investor) to risk instead.

In fact, the less risk the investor is exposed to, the less cost she bears from downward realizations, which decreases the entrepreneur’s incentive in generating informative signals, as he cannot fully internalize the cost of investment. In other words, even absent any cost of effort, we see a negative relation between the entrepreneur’s exposure to risk and expected output.

This contrast derives from two subtle differences between our setting and the ones in Holmstrom (1979) and Innes (1990). First, the principal also takes an action—the decision on whether to continue financing, which implies that the agent’s effort can affect his final payoff through affecting the principal’s continuation decision. By designing a security that makes the principal’s continuation decision, and thus the agent’s payoff, more sensitive to the agent’s effort, we can also better align the agent’s incentives. Second, the agent’s effort in our model affects information production, but not the final output X , therefore security design cannot make agent’s final payoff conditional on continuation directly sensitive to his effort, which shuts down the traditional channel in earlier studies.

These findings imply that incentive contracting for information production is very different from that for increasing output, a potentially interesting avenue for future research. Note that according to proposition 4 and part (d) in proposition 5, introducing outsider competition restores bargaining power to the entrepreneur over the split of interim rent, which in turn restores the link between his final payoff conditional on continuation and his information production effort. In other words, the entrepreneur can internalize the cost of investment better. Therefore, more risk exposure of the entrepreneur improves his information provision, which is consistent with the earlier theories.

6.5 Threshold Strategy and Disclosure Form

Corporate disclosure policy in the literature is often assumed to take certain forms. For example, Diamond (1985) and Diamond and Verrecchia (1991), and more recently Goldstein and Yang (2017) use normal distributions for disclosure policy. While it is not our main focus, our model has important implications for the form of corporate disclosure.

For example, together with Szydlowski (2016), our model highlights that when the entrepreneur’s (or a firm’s) endogenous information production is the only source of information available to market participants, the optimal disclosure policy follows a threshold strategy. Moreover, in Section 3, we show that even if the investors receive independent signal (richer information structure), the optimal disclosure may still involve threshold strategy. In fact, it is unclear that even under more complicated structure for the investors’ supervised information, the optimal disclosure policy becomes more complicated.

More generally, with endogenous information design corporate disclosures may follow interval strategies (Guo and Shmaya (2017)), leading to binary or categorical distributions instead of continuous distributions. The methodology provided in our paper can be used for further investigation in this regard.

7 Conclusion

We model multi-stage financing of innovative projects where relational financiers observe information produced through entrepreneurs’ experimentation, and decide whether to continue financing. We show that the entrepreneur’s endogenous information production typically follows threshold strategies, and typically leaves no rent to the insider investor despite the latter’s informational monopoly. This fact hinders the financing of projects, but can be mitigated by the insider’s independent information production and commitment to interim competition in the second round. We then characterize how investors’ relative sophistication and competition interact to produce patterns on relationship formation documented in the empirical literature, including the U-shaped link between relationship lending and competition. Our model allows general forms of security design and also applies to venture financing. In particular, we derive the optimal security design under endogenous experimentation which involves issuing equities when we restrict our attention to a single uniform security for all investors, and convertible securities to sophisticated early insiders and residual claims to outsiders, under less restricted settings.

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Appendix: Derivations and Proofs

A Technical Lemma

The entrepreneur endogenously designs the experiment to maximize his payoff, subject to satisfying the insider investors' second round participation constraint. With finite state space, the signal space as the range of a deterministic mapping from the state space is necessarily finite. Consequently, we can apply the method of Lagrange multipliers directly. But alas, we are dealing with infinite dimensional state space, and in unrestricted signal generation space, we cannot always apply the method of Lagrange multipliers.

That said, the optimal experimentation function is given by the characteristic function of a countable sup-level set of payoff densities, for some cutoff value “multiplier”. In other words, the experimentation we look at are conditional probabilities and therefore their space is a Banach space. With this insight, we first prove a technical lemma that allows us to use the method of Lagrange multipliers in the proofs of our lemmas and propositions.¹⁹

Lemma (A1). *Suppose $w_i(x), m_i(x) : [0, 1] \rightarrow \Re$ ($1 \leq i \leq N$) are continuous and bounded functions. Consider the following maximization problem:*

$$\begin{aligned} \max_{\alpha_i(\cdot) \in \mathcal{A}} \int_0^1 \sum_{i=1}^N w_i(x) \alpha_i(x) dx \\ \text{s.t.} \quad \int_0^1 m_i(x) \alpha_i(x) dx \geq 0 \quad \forall 1 \leq i \leq N, \\ \sum_{i=1}^N \alpha_i(x) \leq 1 \quad \forall x \in [0, 1], \end{aligned} \tag{25}$$

where \mathcal{A} is the set of all measurable functions that are bounded to $[0, 1]$. Then, there exist non-negative real numbers $\{\mu_i\}_{i=1}^N$, such that the solution to the following maximization problem is a solution to (25):

$$\begin{aligned} \max_{\alpha_i(\cdot) \in \mathcal{A}} \int_0^1 \sum_{i=1}^N (w_i(x) + \mu_i m_i(x)) \alpha_i(x) dx \\ \sum_{i=1}^N \alpha_i(x) \leq 1 \quad \forall x \in [0, 1] \end{aligned} \tag{26}$$

Proof. Let $\tilde{\mathcal{A}}^N$ be the set of all N -tuples of functions $(\alpha_1(\cdot), \dots, \alpha_N(\cdot))$ in \mathcal{A} that satisfies the following condition:

$$\sum_{i=1}^N \alpha_i(x) \leq 1$$

Since all functions are bounded and measurable, then it is easy to check that $\tilde{\mathcal{A}}$ constitutes a closed set

¹⁹The result is proven in greater abstraction and generalizations in, for example, Ito (2016).

in \mathcal{L}^1 . Therefore, the following maximization problem is well-defined

$$\begin{aligned} \max_{(\alpha_1(\cdot), \dots, \alpha_N(\cdot)) \in \tilde{\mathcal{A}}^N} & \int_0^1 \sum_{i=1}^N w_i(x) \alpha_i(x) dx \\ & \int_0^1 m_i(x) \alpha_i(x) dx \geq 0 \quad \forall 1 \leq i \leq N \end{aligned} \quad (27)$$

Suppose $a^* \in \tilde{A}^N$ is the solution to the problems (25) and (27). It is easy to see that Slater condition, and correspondingly, strong duality holds. Therefore, there exists a vector of non-negative real numbers $\{\mu_i\}_{i=1}^N$ such that a^* solves the following maximization problem as well:

$$\max_{(\alpha_1(\cdot), \dots, \alpha_N(\cdot)) \in \tilde{\mathcal{A}}^N} \int_0^1 \sum_{i=1}^N w_i(x) \alpha_i(x) dx + \sum_{i=1}^N \mu_i \int_0^1 m_i(x) \alpha_i(x) dx \quad (28)$$

Note that (26) is equivalent to (28). It completes the proof. \square

In fact, for most Bayesian persuasion settings studied in the literature, the signal generation function corresponds to mapping to probability space that is bounded, and therefore lies in a Banach space. This allows us to apply the approach in Bergemann and Morris (2017) beyond finite-state-space settings.

Proof of Lemma 1

We prove the lemma for general “regular” securities (defined in Section 5.1): securities $s(X)$ and $X - s(X)$ both weakly increasing. Double monotonicity is a standard requirement in the security design literature.

First, we show that two signals are enough to implement the optimal information design. To do this, we show that for every experiment (\mathcal{Z}, π) , there exists an experiment with two signals like $(\{z_1, z_2\}, \tilde{\pi})$ that implements the same investment function, i.e. $I(X) = \tilde{I}(X)$ a.s, where $I(\cdot)$ and $\tilde{I}(\cdot)$ are corresponding investment functions. Given this, we only search over experiments with two signals to find an optimal experiment.

For experiment (\mathcal{Z}, π) , define \mathcal{Z}^+ and \mathcal{Z}^- as the signals induce investment and not investment respectively, i.e.

$$\mathcal{Z}^+ = \{z \in \mathcal{Z} | \mathbb{E}[s(X)|z] \geq I\}$$

$$\mathcal{Z}^- = \{z \in \mathcal{Z} | \mathbb{E}[s(X)|z] < I\}$$

Note that for every $z \in \mathcal{Z}^+$ we have:

$$\mathbb{E}[s(X) - I|z] \geq 0 \iff \int_0^1 (s(X) - I) \pi(z|X) f(X) \geq 0$$

Integrating the recent relation over all $z \in \mathcal{Z}^+$ gives:

$$\int_0^1 (s(X) - I) \left[\int_{\mathcal{Z}^+} \pi(z|X) dz \right] f(X) \geq 0$$

Similarly, we have:

$$\int_0^1 (s(X) - I) \left[\int_{Z^-} \pi(z|X) dz \right] f(X) < 0$$

Now define a new experiment $(\{z_1, z_2\}, \tilde{\pi})$ as follows:

$$\tilde{\pi}(z_1|X) = \int_{Z^+} \pi(z|X) dz \quad \tilde{\pi}(z_2|X) = \int_{Z^-} \pi(z|X) dz$$

It is clear that the investor invests if she receives z_1 and she does not if z_2 is received. Now suppose $I(\cdot)$ and $\tilde{I}(\cdot)$ are investment functions for experiments (Z, π) and $(\{z_1, z_2\}, \tilde{\pi})$, respectively. Then, for the investment functions we have the following

$$\tilde{I}(X) = \tilde{\pi}(z_1|X) = \int_{Z^+} \pi(z|X) dz = I(X) \quad (29)$$

Equation (2) shows that both experiments imply the same expected payoff for the entrepreneur. Therefore, to find the equilibrium payoffs, we only need to consider experiments with only two signals. In other words, we can solve the optimization over all $\tilde{\pi}(z_1|X) \in [0, 1]$. The optimal experiment solves the following maximization problem:

$$\begin{aligned} \max_{\tilde{\pi}(z_1|X)} \int_0^1 [\varepsilon + X - s(X)] \tilde{\pi}(z_1|X) f(X) dX & \quad (30) \\ \text{s.t. } \int_0^1 (s(X) - I) \tilde{\pi}(z_1|X) f(X) dX \geq 0 & \\ \tilde{\pi}(z_1|X) \in [0, 1] & \end{aligned}$$

Using Lemma A1 with $N = 1$, let λ be the corresponding multiplier μ . Then the optimal experiment $\pi^*(z_1|X)$ needs to solve the following optimization problem:

$$\max_{\tilde{\pi}(z_1|X)} [\varepsilon + X - s(X) + \lambda(s(X) - I)] \tilde{\pi}(z_1|X) f(X) \quad (31)$$

Note that the bracket term is strictly increasing in X . Therefore, the optimal experiment implements a threshold strategy. In fact, for $\bar{X}_\varepsilon \in [0, 1]$ that satisfies $\varepsilon + \bar{X}_\varepsilon - s(\bar{X}_\varepsilon) + \lambda(s(\bar{X}_\varepsilon) - I) = 0$, $\tilde{I}(X) = 1$ if $X \geq \bar{X}$ and $\tilde{I}(X) = 0$ if $X < \bar{X}$.

Note that $\lambda > 0$, since otherwise $\pi^*(z_1|X) = 1$ for all $X \in [0, 1]$, while we know $\mathbb{E}[s(X)] \leq \mathbb{E}[X] < I$ and it means no investment, which is a contradiction. Therefore, the constraint in (30) is binding. It implies:

$$\int_{\bar{X}_\varepsilon}^1 (s(X) - I) f(X) dX = 0 \quad (32)$$

Define $\bar{X} \in [0, 1]$ such that:

$$\int_{\bar{X}}^1 (s(X) - I) f(X) dX = 0 \quad (33)$$

By comparing (32) and (33), we see $\bar{X}_\varepsilon = \bar{X}$ for all $\varepsilon > 0$. Hence, the optimal two-signal experiment

and its investment function is independent of ε , and they can be characterized as:

$$\pi^*(X) = \begin{cases} h & X \in [\bar{X}, 1] \\ l & X \in [0, \bar{X}] \end{cases} \quad I^*(X) = \begin{cases} 1 & X \in [\bar{X}, 1] \\ 0 & X \in [0, \bar{X}] \end{cases}$$

This completes the proof for part (a). The expected payoffs for this equilibrium as $\varepsilon \rightarrow 0$ are:

$$U^E(\{h, l\}, \pi^*) = \mathbb{E}[(X - s(X))I^*(X)] = \int_{\bar{X}}^1 (X - s(X))f(X)dX$$

$$U^I(\{h, l\}, \pi^*) = \mathbb{E}[(s(X) - I)I^*(X)] = \int_{\bar{X}}^1 (s(X) - I)f(X)dX = 0$$

where the last equality is derived from equation (33).

For part (b), note that the optimization problem (30) has a unique solution, given that we have shown the solution should be a threshold strategy and (4) uniquely implements the highest payoff for the entrepreneur among all such experiments.

Since the corresponding two-signal experiment of all optimal experiments are optimal themselves, their corresponding two-signal experiment is the ones specified in (4). Consequently, as we showed earlier, they share the same investment function. It implies all the optimal experiments induce the same pair of equilibrium payoffs. QED.

Proof of Corollary 1

$$\begin{aligned} \mathbb{E}[(X - I)I^*(X)] &= \int_{\bar{X}}^1 Xf(X)dX = \int_{\bar{X}}^1 (\min\{X, D\} - I)f(X)dX + \int_{\bar{X}}^1 \max\{X - D, 0\}f(X)dX \\ &= \int_{\bar{X}}^1 \max\{X - D, 0\}f(X)dX = \int_0^1 \max\{X - D, 0\}f(X)dX > K \end{aligned}$$

The first and the second equalities are simple rearrangements. The third equality holds by (3). The last equality holds since $D > I > \bar{X}$.

Proof of Proposition 1

As it is shown in Lemma 1, the investor gets 0 in expectation and cannot recover anything if she decides to pay K at day 0. Therefore she prefers not to pay the monitoring cost at day 0.

Proof of Lemma 2

Similar to the proof of Lemma 1, we prove the lemma for weakly increasing securities $s(X)$. For simplicity in expositions, we don't consider the securities that $\mathbb{E}[s(X) - I|X > I] \leq 0$. One can show that in this case, the investor's signal y is not used and the equilibrium experiments, investment functions and payoffs are the same as the ones provided in Lemma 1.

a) the proof is similar to the equivalence section of the proof in Lemma 1.

b) We consider three cases that the set of signals are $\{m, l\}$, $\{h, l\}$ or $\{m, h, l\}$. We show that for small enough values of ε and for $q \in [0, 1]$ such that $q(1 + q) < 1$, then only the second case can be optimal.

Lemma (A2). *If $s(I) < I$, then no experiment with two signals like $(\{l, m\}, \pi)$, where the investor invests iff she receives (m, \tilde{h}) , is optimal. If $s(I) = I$, this result holds for small enough values of ε .*

Proof. Suppose the contrary and suppose that there exists an optimal experiment like $(\{m, l\}, \pi_M^*)$. Therefore, π_M^* solves the following maximization problem:

$$\begin{aligned} \max_{\pi(m|X)} \quad & (1 - q) \int_0^I (\varepsilon + X - s(X)) \pi(m|X) f(X) dX + q \int_I^1 (\varepsilon + X - s(X)) \pi(m|X) f(X) dX \\ \text{s.t.} \quad & (1 - q) \int_0^I (s(X) - I) \pi(m|X) f(X) dX + q \int_I^1 (s(X) - I) \pi(m|X) f(X) dX \geq 0 \\ & \pi(m|X) \in [0, 1] \quad \forall X \in [0, 1] \end{aligned}$$

Let λ be the multiplier corresponding to the constraint. π_M^* then maximizes the following objective function, given the constraint $\pi_M^*(m|\cdot) \in [0, 1]$.

$$\begin{aligned} \max_{\pi(m|X)} \quad & (1 - q) \int_0^I [\varepsilon + X - s(X) + \lambda(s(X) - I)] \pi(m|X) f(X) dX \\ & + q \int_I^1 [\varepsilon + X - s(X) + \lambda(s(X) - I)] \pi(m|X) f(X) dX \end{aligned}$$

Since both $X - s(X)$ and $s(X) - I$ are weakly increasing functions, there is a threshold value $\underline{X} \in [0, 1]$ such that for all $X \geq \underline{X}$, the expression in the bracket is non-negative. It shows the optimal experiment among those that only implement signals m and l has a threshold scheme, where the threshold \underline{X} satisfies the following:

$$(1 - q) \int_{\underline{X}}^I (s(X) - I) f(X) dX + q \int_I^1 (s(X) - I) f(X) dX = 0$$

In this case, the expected utility of the entrepreneur from $(\{m, l\}, \pi_M^*)$ is:

$$U^E(\{m, l\}, \pi_M^*) = (1 - q) \int_{\underline{X}}^I (\varepsilon + X - s(X)) f(X) dX + q \int_I^1 (\varepsilon + X - s(X)) f(X) dX \quad (34)$$

By comparing the recent equality with (7), it is easy to see that $\underline{X} < \bar{X}(q)$. Now, we show that how the entrepreneur can improve upon π_M^* by introducing signal h (a signal that induces investment regardless of the signal the investor receives). To show this, we consider two cases:

- $s(I) < I$: In this case, we can find a subset $A \subset [I, 1]$ such that $\int_A (s(X) - I) f(X) dX = 0$. Then an experiment that sends h for the members of A ($\pi(h|X) = 1$ iff $X \in A$) and sends m for $\underline{X}, 1 \setminus A$ implements higher payoff for the entrepreneur by $(1 - q) \int_A (\varepsilon + X - s(X)) f(X) dX$.
- $s(I) = I$: Since $X - s(X)$ is a weakly increasing function, it implies that $s(X) = X$ for all $X \leq I$.

Consider small positive values $\eta_1, \eta_2 \geq 0$ that satisfy the following:

$$(1 - q) \int_{\underline{X}}^{\underline{X} + \eta_1} (s(X) - I) f(X) dX + q \int_{1 - \eta_2}^1 (s(X) - I) f(X) dX = 0$$

and introduce the following alternative experiment $(\{l, m, h\}, \tilde{\pi}_M)$:

$$\tilde{\pi}_M(h|X) = \begin{cases} 1 & X \in [1 - \eta_2, 1] \\ 0 & X \in [0, 1 - \eta_2) \end{cases} \quad \tilde{\pi}_M(m|X) = \begin{cases} 1 & X \in [\underline{X} + \eta_1, 1 - \eta_2) \\ 0 & X \in [0, \underline{X} + \eta_1) \cup [1 - \eta_2, 1] \end{cases}$$

It is easy to verify that the experiment $\tilde{\pi}_M$ is designed in a way that the investor invests iff she receives one of (m, \tilde{l}) , (h, \tilde{l}) or (h, \tilde{h}) . Now, the difference in expected payoffs for the entrepreneur is given by:

$$U^E(\{m, l\}, \tilde{\pi}_M) - U^E(\{m, l\}, \pi_M^*) = (1 - q) \left[\int_{1 - \eta_2}^1 (\varepsilon + X - s(X)) f(X) dX - \int_{\underline{X}}^{\underline{X} + \eta_1} (\varepsilon + X - s(X)) f(X) dX \right]$$

Now consider the contrary, that the introduced experiment is an optimal experiment. Then $\eta_1^* = \eta_2^* = 0$ should satisfy the first order conditions for the following maximization problem:

$$\begin{aligned} & \max_{\eta_1, \eta_2} \int_{1 - \eta_2}^1 (\varepsilon + X - s(X)) f(X) dX - \int_{\underline{X}}^{\underline{X} + \eta_1} (\varepsilon + X - s(X)) f(X) dX \\ \text{s.t.} \quad & (1 - q) \int_{\underline{X}}^{\underline{X} + \eta_1} (s(X) - I) f(X) dX + q \int_{1 - \eta_2}^1 (s(X) - I) f(X) dX = 0 \\ & \eta_1, \eta_2 \geq 0 \end{aligned}$$

Let μ be the multiplier for the first constraint. Then by using $s(\underline{X}) = \underline{X}$,

$$\Rightarrow [\eta_1]_{\eta_1=0} : \varepsilon f(\underline{X}) + (1 - q)\mu(\underline{X} - I)f(\underline{X}) \leq 0 \quad (35)$$

$$[\eta_2]_{\eta_2=0} : (\varepsilon + 1 - s(1))f(1) + q\mu(s(1) - I)f(1) \leq 0 \quad (36)$$

By comparing (35) and (36) we see:

$$\varepsilon \geq \frac{(1 + \varepsilon - s(1))(1 - q)(I - \underline{X})}{q(s(1) - I)} > \frac{1 - q}{q} \frac{1 - s(1)}{s(1) - I} (I - \bar{X}(q))$$

In the last equation, the right hand side is independent of ε . As a result, for small enough values of ε this inequality cannot hold, which is a contradiction for this set of values. Lemma A2 follows. \square

Lemma (A3). *If $q(1 + q) < 1$, then no three-signal experiment like $(\{l, m, h\}, \pi)$ that signals m and h are both sent with positive probability is optimal.*

Proof. Suppose the contrary and there exists a three-signal optimal experiment like $(\{l, m, h\}, \pi_{HM}^*)$, where the investor invests iff she receives one of (m, \tilde{l}) , (h, \tilde{l}) and (h, \tilde{h}) . Then $\pi_{HM}^*(m|X)$ and $\pi_{HM}^*(h|X)$ solve

the following optimization problem.

$$\begin{aligned}
& \max_{\pi(h|X), \pi(m|X)} \int_0^I (\varepsilon + X - s(X))(\pi(h|X) + (1-q)\pi(m|X))f(X)dX \\
& \quad + \int_I^1 (\varepsilon + X - s(X))(\pi(h|X) + q\pi(m|X))f(X)dX \\
s.t. \quad & q \int_0^I (s(X) - I)\pi(h|X)f(X)dX + (1-q) \int_I^1 (s(X) - I)\pi(h|X)f(X)dX \geq 0 \\
& (1-q) \int_0^I (s(X) - I)\pi(m|X)f(X)dX + q \int_I^1 (s(X) - I)\pi(m|X)f(X)dX \geq 0 \\
& \pi(h|X), \pi(m|X) \in [0, 1]
\end{aligned} \tag{37}$$

Let λ^h and λ^m be the multipliers for the first two restrictions, respectively. Define $c_m(X)$ and $c_h(X)$ as follows:

$$\begin{aligned}
c_h(X) &= \begin{cases} \varepsilon + X - s(X) + q\lambda^h(s(X) - I) & 0 \leq X < I \\ \varepsilon + X - s(X) + (1-q)\lambda^h(s(X) - I) & I \leq X \leq 1 \end{cases} \\
c_m(X) &= \begin{cases} (1-q)(\varepsilon + X - s(X) + \lambda^m(s(X) - I)) & 0 \leq X < I \\ q(\varepsilon + X - s(X) + \lambda^m(s(X) - I)) & I \leq X \leq 1 \end{cases}
\end{aligned}$$

Then $\pi(h|X)$ and $\pi(m|X)$ solves the following optimization problem subject to $0 \leq \pi(h|X), \pi(m|X) \leq 1$

$$\max_{\pi(h|X), \pi(m|X) \in [0,1]} \int_0^1 [c_h(X)\pi(h|X) + c_m(X)\pi(m|X)]f(X)dX \tag{38}$$

Note that the optimization problem (38) is linear in $\pi(h|X)$ and $\pi(m|X)$. Moreover, it is easy to see that their multipliers are equal at most in a measure-zero subset of $[0, 1]$. Therefore, $\pi(h|X), \pi(m|X) \in \{0, 1\}$ *a.s.* Correspondingly, we define their identifier sets as H_1, H_2, M_1 and M_2 . For example, $M_1(M_2)$ is the subset of $[0, I]$ ($[I, 1]$) for which $\pi(m|X) = 1$.

If M_1 is empty, then signal m is just sent for a subset of $X \in [I, 1]$. In this case, the investor invests even if she receives (m, \tilde{l}) , which is in contrast with the definition of signal m . Therefore, suppose M_1 is non-empty. For $X \in M_1$ we have:

$$c_m(X) \geq \max\{c_h(X), 0\}$$

By some rearranging in the expression of $c_h(X)$ and $c_m(X)$, we have

$$\frac{q\lambda^h - (1-q)\lambda^m}{q} \geq \frac{\varepsilon + X - s(X)}{I - s(X)} \geq \lambda^m \Rightarrow q\lambda^h \geq \lambda^m \tag{39}$$

Moreover, note that if M_2 is empty, then the investor does not ever invest when she receives m , which is a contradiction with the definition of m . Therefore, M_2 is not empty and there exists $X \in M_2$. For X , we have:

$$c_m(X) \geq c_h(X)$$

$$\begin{aligned} \Rightarrow (q\lambda^m - (1-q)\lambda^h)(s(X) - I) &\geq (1-q)(\varepsilon + X - s(X)) \\ \Rightarrow q\lambda^m - (1-q)\lambda^h &> 0 \end{aligned} \tag{40}$$

By combining (39) and (40), we get:

$$q\lambda^h > \frac{1-q}{q}\lambda^h \Rightarrow q(1+q) > 1$$

which contradicts our assumption for the value of q . It completes the proof. \square

Since part (a) shows that all experiments induce the same investment function as a two-signal or three-signal experiment, Lemmas A2 and A3 imply that the a two-signal experiment with $\{h, l\}$ implements the optimal investment function. In other words, in this experiment, the investor completely disregards her own signal.

It is easy to see that the optimal two-signal experiment has a threshold scheme, where the threshold $\bar{X}(q)$ should satisfy (7). The rest of the results for this part are easy to verify.

c) Suppose $\tilde{\lambda}$ is the multiplier for the optimization problem (30). We show if $\frac{q}{1-q} > \frac{1}{\tilde{\lambda}} \frac{\varepsilon+1-s(1)}{s(1)-I}$, then the threshold strategy provided in part (b) cannot be optimal.

Proof of Proposition 2

Proof. **a)** According to equation (11), the entrepreneur's utility from experiment (\mathcal{Z}, π) is given by:

$$U^E(\mathcal{Z}, \pi) = \Sigma_{z \in \mathcal{Z}} \mathbb{E}[\varepsilon + X - s(X) + (1-\mu)(s(X) - I) | z] \mathbf{1}\{\mathbb{E}[s(X) | z] \geq I\} \tag{41}$$

We define $\bar{X}(\mu) \equiv I - \frac{\varepsilon}{1-\mu}$. First, note that the expression in the first expectation becomes negative for $X < \bar{X}(\mu)$. Therefore, it is optimal to induce not investment for $X < \bar{X}(\mu)$. It is easy to see that If $\mu \leq \bar{\mu}$, then $\bar{X}(\mu) > \bar{X}$. Therefore, the experiment provided in (4) is no longer optimal. Moreover, note that inducing investment for $X \geq \bar{X}(\mu)$ is feasible since:

$$\int_{\bar{X}(\mu)}^1 (s(X) - I) f(X) dx \geq \int_{\bar{X}}^1 (s(X) - I) f(X) dX = 0$$

Therefore, the optimal experiment is given by (12).

b) The FOC for (13) implies the following holds for $\mu^*(\varepsilon)$:

$$\int_{I - \frac{\varepsilon}{1-\mu^*(\varepsilon)}}^1 (\min\{X, D\} - I) f(X) dX - \frac{\mu^*(\varepsilon)\varepsilon^2}{(1-\mu^*(\varepsilon))^3} = 0 \tag{42}$$

The LHS is decreasing in both μ and ε . Therefore, implicit function theorem implies that $\mu^*(\varepsilon)$ is decreasing. Moreover, $\bar{K}_S^\varepsilon(\cdot)$ has non-positive second derivative at every point that the second derivative is available. Since it is a continuous function, it implies it is quasi-concave, as well.

c) First we need to show that $\frac{\varepsilon}{1-\mu^*(\varepsilon)}$ is increasing in ε . To show this, suppose the contrary. Therefore, there should be $\varepsilon > 0$ at which the first term in (42) is increasing in ε , while the second term is decreasing (Note that $\mu^*(\varepsilon)$ is decreasing in ε), which is the contrary. Therefore, since $\bar{X}(\mu^*(\varepsilon)) < I - \varepsilon$, $\mathcal{E}(\varepsilon)$ is decreasing in ε . The second part is given by the fact that the expression in (13) is weakly decreasing in ε , for every value of μ .

d) As ε goes to zero, the LHS in (42) converges to a positive number. Moreover, note that the continuity of $\mu^*(\varepsilon)$ implies that it is converging to a point in $[0, 1]$. The only possibly value to keep the limit of RHS positive is $\lim_{\varepsilon \rightarrow 0} \mu^*(\varepsilon) = 1$. It completes the proof. \square

Proof of Proposition 3

Proof. a),b) The proof is similar to the proof of part (a) in proposition 2.

c) We can rewrite $\bar{K}_C^\varepsilon(\mu; q)$ as follows:

$$\bar{K}_C^\varepsilon(\mu; q) = \mu \mathbb{E}[(\min\{X, D\} - I) \mathbf{1}\{X \geq \max\{\bar{X}(q), I - \frac{\varepsilon}{1-\mu}\}\}]$$

Since $\bar{X}(q) < I - \varepsilon$, there exists $\hat{\mu}(\varepsilon; q)$ such that $\bar{X}(q) = I - \frac{\varepsilon}{1-\hat{\mu}(\varepsilon; q)}$. If $\hat{\mu}(\varepsilon; q) < \mu^*(\varepsilon)$ (introduced in proposition 2), then part (c) in proposition implies that the $\bar{K}_C^\varepsilon(\mu; q)$ is decreasing over $[0, \hat{\mu}(\varepsilon; q)]$. If that is not the case, then the function is inverse U-shape over this interval, which picks at $\mu^*(\varepsilon)$. Since $\bar{X}(q)$ is independent of μ , it is clear that the function is decreasing over $[\hat{\mu}(\varepsilon; q), 1]$. Therefore, if we define $\bar{X}(q) = \mu^*(\varepsilon^*(q))$, this function satisfies the conditions provided in the proposition.

d) The proof is similar to the proof of part (d) in proposition 2. \square

Proof of Proposition 4

Proof. First, we prove the following useful lemma:

Lemma (A4). *For a given security $s(\cdot)$ and ex-post bargaining power of the insider investor μ , exactly one of the following two holds:*

1. *The equilibrium investment function is:*

$$I(X) = \mathbf{1}\{\varepsilon + X - \mu s(X) - (1 - \mu)I \geq 0\}$$

2. *The investor receives 0 in equilibrium.*

Proof. Note that the entrepreneur solves the following maximization problem, after receiving the first round of financing:

$$\max_{I(\cdot)} \mathbb{E}[(\varepsilon + X - \mu s(X) - (1 - \mu)I)I(X)]$$

$$s.t. \quad \mu \mathbb{E}[(s(X) - I)I(X)] \geq 0$$

If 2 does not hold, it means that the constraint in the maximization problem does not bind. Consequently, $I(\cdot)$ is a global maximizer of the objective function. It implies (1). \square

Corollary (A5). *For a given pair of $(s(\cdot), \mu)$, the investor's expected payoff is given by:*

$$\max\{\mu\mathbb{E}[(s(X) - I)\mathbf{1}\{\varepsilon + X - \mu s(X) - (1 - \mu)I \geq 0\}] - K, 0\}$$

a) Now, we prove the proposition. For the “if” part, it is clear that there exists $\mu \in [0, 1]$ such that for $(s(\cdot), \mu)$, the insider recovers initial cost K .

For the “only if” part. Suppose the contrary and there exists a pair $(\hat{s}(\cdot), \hat{\mu})$ for which the insider recovers the initial cost. According to the Corollary 7, we should have:

$$K \leq \hat{\mu}\mathbb{E}[(\hat{s}(X) - I)\mathbf{1}\{\varepsilon + X - \hat{\mu}\hat{s}(X) - (1 - \hat{\mu})I \geq 0\}] \leq \hat{\mu}\mathbb{E}[(X - I)\mathbf{1}\{\varepsilon + X - \hat{\mu}X - (1 - \hat{\mu})I \geq 0\}]$$

which is in contrary with the condition (16). The result follows.

b), c) Lemma A4 implies that the entrepreneur solves the following maximization problem for the security design:

$$\begin{aligned} & \max_{s(\cdot), \mu} \mathbb{E}[(\varepsilon + X - \mu s(X) - (1 - \mu)I)\mathbf{1}\{\varepsilon + X - \mu s(X) - (1 - \mu)I \geq 0\}] \\ & \text{s.t. } \mu\mathbb{E}[(s(X) - I)\mathbf{1}\{\varepsilon + X - \mu s(X) - (1 - \mu)I \geq 0\}] \geq K \end{aligned}$$

Note that the objective function is bounded by $\mathbb{E}[\max\{X - I, 0\}]$ and the constraint constitutes a closed subset in a L^1 -space. Therefore, the maximum exists. Now, suppose $(s_1(\cdot), \mu_1)$ is a pair that satisfies the constraint. I denote the security $s(X) = X$, by $s_X(\cdot)$. Moreover, we define $\hat{X}(s(\cdot), \mu)$ satisfies

$$\varepsilon + \hat{X}(s(\cdot), \mu) - \mu s(\hat{X}(s(\cdot), \mu)) - (1 - \mu)I = 0$$

In fact, $\hat{X}(s(\cdot), \mu)$ is the threshold that the entrepreneur uses if the pair $(s(\cdot), \mu)$ is chosen. Since $s(X) \leq X$ and $\varepsilon > 0$, $\hat{X}(s(\cdot), \mu) < I$.

One can easily see that $\hat{X}(s(\cdot), \mu)$ is weakly increasing in the security (If $s_2(X) \geq s_3(X)$ for some $s_2(\cdot), s_3(\cdot)$ and $X \in [0, 1]$, then $\hat{X}(s_2(\cdot), \mu) \leq \hat{X}(s_3(\cdot), \mu)$ for every $\mu \in [0, 1]$) and decreasing μ . Therefore:

$$\mu_1\mathbb{E}[(s_1(X) - I)\mathbf{1}\{X \geq \hat{X}(s_1(\cdot), \mu_1)\}] \leq \mu_1\mathbb{E}[(X - I)\mathbf{1}\{X \geq \hat{X}(s_X(\cdot), \mu_1)\}] \quad (43)$$

Then there exists $\mu_X \leq \mu_1$ such that

$$\mu_X\mathbb{E}[(X - I)\mathbf{1}\{X \geq \hat{X}(s_X(\cdot), \mu_X)\}] = K$$

Now, we can show that the objective function is larger for (s_X, μ_X) than (s_1, μ_1) :

$$\begin{aligned}
& \mathbb{E}[(\varepsilon + (1 - \mu_X)(X - I))\mathbf{1}\{X \geq \hat{X}(s_X(\cdot), \mu_X)\}] = \\
& \mathbb{E}[(\varepsilon + X - I)\mathbf{1}\{X \geq \hat{X}(s_X(\cdot), \mu_X)\}] - K \geq \\
& \mathbb{E}[(\varepsilon + X - I)\mathbf{1}\{X \geq \hat{X}(s_1(\cdot), \mu_1)\}] - \mu_1 \mathbb{E}[(s_1(X) - I)\mathbf{1}\{X \geq \hat{X}(s_1(\cdot), \mu_1)\}] = \\
& \mathbb{E}[(\varepsilon + X - \mu_1 s_1(X) - (1 - \mu_1)I)\mathbf{1}\{X \geq \hat{X}(s_1(\cdot), \mu_1)\}]
\end{aligned}$$

□

Proof of Proposition 5

b) The “only if” part derives from solving for $\int_I^1 (s(X) - I)f(X)dX$ with the two constraints. Note that it gives a lower bound on $\mathbb{E}[\max\{X - I, 0\}]$. For the “if” part, we provide an example in part (c).

c) Suppose for a security that satisfies the constraints in (17), $X(\hat{s}(\cdot))$ is the solution to the following:

$$\int_{X(\hat{s}(\cdot))}^I (I - s(X))f(X)dX = \frac{(1 - q)K}{2q - 1}$$

Note that $\varepsilon(F(I) - F(I - \varepsilon)) < \frac{(1 - q)K}{2q - 1}$ implies that for every security that satisfies the constraints in (17), $\hat{X}(s(\cdot)) < I - \varepsilon$.

One can show that the entrepreneur’s expected payoff from the security $s(\cdot)$ is given by:

$$V(s(\cdot)) = \mathbb{E}[(\varepsilon + X - s(X))\mathbf{1}\{X \geq \hat{X}(s(\cdot))\}]$$

Therefore, by using the second constraint

$$V(s(\cdot)) \leq \mathbb{E}[(\varepsilon + X - I)\mathbf{1}\{X \geq \hat{X}(s(\cdot))\}] - K$$

We show that the $s_N(X) = \max\{X - N, 0\}$ has the highest \hat{X} among all securities that satisfy the constraint, and the constraints bind for this security.

First note that $X - s(X)$ is a weakly increasing function. Therefore,

$$\begin{aligned}
& \int_I^1 (I - s(I))f(X)dX \leq \int_I^1 (X - s(X))f(X)dX \\
& = \int_I^1 (X - I)f(X)dX - \int_I^1 (s(X) - I)f(X)dX \\
& \leq \mathbb{E}[\max\{X - I, 0\}] - \frac{qK}{2q - 1}
\end{aligned}$$

Therefore, $I - s(I) \leq N$. It implies that $s(X) \geq X - N$ for every $X \leq I$ and every security that satisfies the constraints. In other words, $s(X) \geq s_N(X)$ for $X \leq I$. Moreover, note that $s_N(X)$ binds both constraints. The result follows.

d) The entrepreneur solves the following maximization problem:

$$\begin{aligned}
& \max_{s(\cdot), \mu, \hat{X}} \mathbb{E}[(\varepsilon + X - \mu s(X) - (1 - \mu)I)\mathbf{1}\{X \geq \hat{X}\}] \\
& \text{s.t.} \quad \varepsilon + \hat{X} - \mu s(\hat{X}) - (1 - \mu)I \geq 0 \\
& q \int_{\hat{X}}^X (s(X) - I)f(X)dX + (1 - q) \int_I^1 (s(X) - I)f(X)dX \geq 0 \\
& \mu \int_{\hat{X}}^1 (s(X) - I)f(X)dX \geq K
\end{aligned}$$

If $\hat{X}(s_{X-N}(\cdot), 1) < I - \frac{\varepsilon}{1-\mu^E}$, then (s_X, μ^E) , which was provided in proposition 4 part (b), the third condition does not bind and the solution is this design. Otherwise, one can show that if $(s(\cdot), \mu)$ constitutes the optimal design, $(s(\cdot), 1)$ is an optimal design as well. Hence, the solution given in part (c) is optimal in that case.

Proof of Proposition 6

a) It is easy to show that lemma A4 and Corollary 7 still hold for the more general case that different securities can be used for insider and outsider investors. These two characterize the equilibrium payoff and information design after paying K in the first round. Therefore, the optimum design only needs to satisfy IR constraint for both insider and outsider investors.

b) We show that for every triple $(b_I(\cdot), b_O(\cdot), \mu)$ that satisfies the constraints in (20), the expected value for the entrepreneur does not exceed V_F^* , where:

$$V_F^* = \mathbb{E}[(\varepsilon + X - I)\mathbf{1}\{\varepsilon + X - I \geq 0\}] - K \quad (44)$$

In the proof of part (c), we introduce a design that implements this value.

Suppose $(b_I(\cdot), b_O(\cdot), \mu)$ satisfies the constraints in (20). Then:

$$\begin{aligned}
& \mathbb{E}[(\varepsilon + X - b_I(X) - (1 - \mu)I)\mathbf{1}\{\varepsilon + X - b_I(X) - (1 - \mu)I \geq 0\}] \\
& = \mathbb{E}[(\varepsilon + X - I)\mathbf{1}\{\varepsilon + X - b_I(X) - (1 - \mu)I \geq 0\}] - \mathbb{E}[(b_I(X) - \mu I)\mathbf{1}\{\varepsilon + X - b_I(X) - (1 - \mu)I \geq 0\}] \\
& \leq \mathbb{E}[(\varepsilon + X - I)\mathbf{1}\{\varepsilon + X - I \geq 0\}] - K = V_F^*
\end{aligned}$$

c) It is easy to show that for a specific value of $\varepsilon > 0$, every optimal design should satisfy $b_I(I - \varepsilon) = \mu I$. As a result, every robust design should satisfy $b_I(X) = \mu I$ for every $X \in [I - \varepsilon, I]$.

Proof of Proposition 7

Proof. The proof for the case of single-security design without supervised learning is provided. The proof for the other case is similar.

Define μ^{VC} as follows:

$$\mu^{VC} \in \operatorname{argmax}_{\mu} \quad \mu \mathbb{E}[(X - I) \mathbf{1}\{X \geq I - \frac{\varepsilon}{1 - \mu}\}]$$

According to Corollary 7, in addition to the result in part (a) of proposition 4, the insider investor solves the following optimization problem:

$$\max_{s(\cdot), \mu \in [0,1]} \mu \mathbb{E}[(s(X) - I) \mathbf{1}\{\varepsilon + X - \mu s(X) - (1 - \mu)I \geq 0\}]$$

In part (b) of proposition 4, it was shown that $\hat{X}(s(\cdot), \mu) \leq \hat{X}(s_X(\cdot), \mu) \leq I$ (where $s_X(X) = X$). Therefore, in the optimal design, $s(X) = X$. Therefore, it is clear that $(s_X(\cdot), \mu^{VC})$ solves the insider's optimization problem. The result follows.

By definition of μ^{VC} , it is clear that $\mu^E \leq \mu^{VC}$. Moreover, note that $\hat{X}(s_X, \mu) = I - \frac{\varepsilon}{1 - \mu}$ and the social welfare associated with an investment function is given by:

$$SW(I(\cdot)) = \mathbb{E}[(\varepsilon + X - I)I(X)]$$

Consequently, a smaller μ implements a more socially efficient outcome. The result follows. \square

Proof of Proposition 8

Note that the investor, after observing signal z , solves the following maximization problem:

$$\max_{\alpha \in [0,1]} r(\alpha) \mathbb{E}[s(X)|z] - \alpha I \tag{45}$$

We also know

$$r(\alpha) \mathbb{E}[s(X)|z] - \alpha I \leq \alpha (\mathbb{E}[s(X)|z] - I) \leq \max\{\mathbb{E}[s(X)|z] - I, 0\}.$$

Hence it is optimal to choose $\alpha \in [0, 1]$. The rest of proof is straightforward following lemma 1.

Proof of Lemma 3

a) It is derived by characterizing the FOC for maximization problem 45.

b) Suppose $\alpha^*(z) > 0$. The FOC implies

$$r'(\alpha^*(z)) \mathbb{E}[s(X)|z] \geq I.$$

Then

$$U^I(z; (\mathcal{Z}, \pi)) = r(\alpha^*(z)) \mathbb{E}[s(X)|z] - \alpha^*(z)I \geq (r(\alpha^*(z)) - \alpha^*(z)r'(\alpha^*(z))) \mathbb{E}[s(X)|z]$$

Note that $r(\cdot)$ is concave in $[0, \alpha^*(z)]$, therefore:

$$r(\alpha^*(z)) - \alpha^*(z)r'(\alpha^*(z)) > r(0) = 0$$

Clearly $\mathbb{E}[s(X)|z] > 0$. It completes the proof.

Proof of Proposition 9

Proof. For simplicity of exposition, we define $\tilde{r}(\cdot) \equiv (r')^{-1}(\cdot)$. It is easy to see that:

$$\begin{aligned} u_{xx} &= 0 \\ u_{xs} &= -\frac{I^2}{s^3} \frac{1}{r''(\tilde{r}(\frac{I}{s}))} \\ u_{xs} &= \frac{I^2}{s^3} \frac{1}{r''(\tilde{r}(\frac{I}{s}))} \left(-1 + \frac{3(x+\varepsilon)}{s}\right) - \frac{I^3(x-s+\varepsilon)}{s^5} \frac{r'''(\tilde{r}(\frac{I}{s}))}{r''(\tilde{r}(\frac{I}{s}))^3} \end{aligned} \quad (46)$$

a) Note that we need at least two signal, as there would be no investment after no-disclosure. Suppose there are two posteriors with expected states (x_1, s_1) and (x_2, s_2) , both of which implement positive investment i.e. $\tilde{r}(\frac{I}{s_i}) > 0$ for $i = 1, 2$. We show that function $u(\cdot, \cdot)$ is concave over the line connecting these two points. Therefore, the entrepreneur can increase its expected utility by pooling these two posteriors.

Suppose $x_1 > x_2$ and $s_1 > s_2$, which is the case according to the MPS (Monotone Partial Signals) condition. The function is concave over the line connecting these two points if:

$$-\frac{I^2}{s^3} \frac{1}{r''(\tilde{r}(\frac{I}{s}))} \left(2\Delta x + \left(-1 + \frac{3x+\varepsilon}{s}\right)\Delta s\right) < 0 \quad (47)$$

for all $(x, s) = \alpha(x_1, s_1) + (1-\alpha)(x_2, s_2)$, for some $\alpha \in [0, 1]$. Note that for every such (x, s) , $\frac{x}{s} > \frac{\Delta x}{\Delta s} > 1$. Therefore, the condition 47 holds for ever point in the interval. It shows the function is concave over the interval, which shows the entrepreneur can benefit from pooling different signals.

b) A debt contract has a linear part, which is convex and has another part that pays the face value to the investor. Since the investment is only sensitive to the amount that the security pays to the investor, the entrepreneur at most needs one signal for all outcomes in the fixed part. Part (a) shows that at most two signals are needed for the rest of the outcomes. Therefore, at most threes signals are enough. Two of them induce an intermediate and an high level of investment.

c) Consider security $s(X) = \sqrt{x}$. It is easy to show that the function $u(\cdot, \cdot)$ is convex over every interval connecting two points of $\{(x, s)|s = s(x), x \geq \bar{x}\}$, for some $\bar{x} \in (0, 1)$. Therefore no pooling over interval $[\bar{x}, 1]$ is optimal. It shows infinite number of signals is required. □