Macroeconomic Models for 
Monetary Policies: A Critical 
Review from a Finance Perspective

Winston W. Dou †, Andrew W. Lo ‡ and Ameya Muley §

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†Department of Financial Economics, MIT Sloan School of Management, 100 Main St, E62–615, Cambridge, MA, 02139, USA, wdou@mit.edu.
‡Department of Financial Economics, MIT Sloan School of Management, and Chief Investment Strategist, AlphaSimplex Group, LLC. 100 Main Street, E62–618, Cambridge, MA 02142–1347, (617) 253–0920 (voice), alo@mit.edu.
§Department of Economics, MIT, 50 Ames Street, Cambridge, MA, 02139, USA, muley@mit.edu.
Abstract
This paper surveys critically the macroeconomic models for monetary policy at central banks from a finance perspective. The complex yet crucial role played by financial systems in monetary policy and macroeconomic forecasting had been largely overlooked in the formal tools used by monetary authorities. We review the history of the monetary policy modeling and set forth a simple and canonical framework that incorporates the key up-to-date theoretical advances for the reader unfamiliar with the literature. As this is done, challenges for the existing models and quantitative methodologies are brought out. The topics discussed are: (1) government balance sheet and unconventional monetary policies, (2) heterogeneity and reallocation, (3) macroeconomic impact of sizable and nonlinear risk premium dynamics, (4) time-varying uncertainty, (5) financial sector and systemic risks, (6) imperfect product market and markups, and (7) solution, estimation and evaluation methods for dynamic quantitative structural models. At last, we review the current core monetary models employed by the major central banks.

Keywords: Leverage; Liquidity; Risk Management; Financial Regulation; Financial Institutions; Financial Crisis

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1 Introduction

The recent financial crisis and the Great Recession of 2007 - 2009 revealed serious gaps in our ability to define, measure, and manage financial sector activities that pose risks to the macroeconomy as a whole. Current macroeconomic models for monetary policy typically lack the analytical specificity to account for important financial sector influences on the aggregate economy. A new generation of enhanced models and advanced empirical and quantitative methodologies are demanded by policymakers and researchers to better study the impact of shocks that are initially large or build endogenously over time. The modeling advances of the past several decades are the foundation on which the new generation of models will be built, rather than a mistaken detour, as some have declared. However, developing a new generation of models is a long-term venture. As a starting point, we begin with a review, as explicit and comprehensive as possible, of these macroeconomic models and their empirical methods. Through this review, we hope to clearly sort out the important main challenges faced by existing macroeconomic models for monetary policy analysis, and comprehensively summarize the recent advances in new modeling and quantitative techniques. The primary goal of this paper is to provide insight and explicit guidance into the next step of developing new generation of models for monetary policy decisions.

There has been a remarkable evolution of macroeconomic models used for monetary policy at major central banks around the world, in aspects such as model formulation, solution methods, estimation approaches, and importantly, communication of results between central banks. Central banks have developed many different classes and variants of macroeconomic models in the hopes of producing a reliable and comprehensive analysis of monetary policy. Early types of models included quantitative macroeconomic models\(^1\), reduced-form statistical models, structural vector autoregressive models, and large-scale macroeconometric models, a hybrid form combining the long-run structural relationships implied by a partial equilibrium treatment of theory (e.g., the decision rule for aggregate consumption) and reduced-form short-run

\(^1\)For example, the Wharton econometric model and the Brookings model.
relationships employing error-correcting equations.

Over the past 20 years in particular, there have been significant advances in the specification and estimation for New Keynesian Dynamic Stochastic General Equilibrium (New Keynesian DSGE) models. Significant progress has been made to advance policymaking models from the older static and qualitative New Keynesian style of modeling to the New Keynesian DSGE paradigm. The New Keynesian DSGE model is designed to capture real world data within a tightly structured and self-consistent macroeconomic model. The New Keynesian DSGE model has explicitly theoretical foundations, allowing it to circumvent the Sims Critique (see Sims, 1980) and the Lucas Critique (see Lucas, 1976), and therefore it can provide more reliable monetary policy analysis than earlier models. A consensus baseline New Keynesian DSGE model has emerged, one that is heavily influenced by estimated impulse response functions based on Structural Vector Autoregression (SVAR) models. In particular, a baseline New Keynesian DSGE model has recently been shown by Christiano et al. (2005) to successfully account for the effects of a monetary policy shock with nominal and real rigidities. Similarly, Smets and Wouters (2003, 2007) show that a baseline New Keynesian DSGE model can track and forecast time series as well as, if not better than, a Bayesian vector autoregressive (BVAR) model. New Keynesian DSGE models have been developed at many central banks, becoming a crucial part of many of their core models.\(^2\) Sbordone et al. (2010) have emphasized that an advantage of New Keynesian DSGE models is that they share core assumptions about the behavior of agents, making them scalable to relevant details to address the policy question at hand. For example, Smets and Wouters (2007) introduced wage stickiness and investment frictions into their model, Gertler et al. (2008) incorporated labor market search and wage bargaining, and Bernanke et al. (1999), Chari et al. (1995) and Christiano et al. (2008) studied the interaction between the financial sector and macroeconomic activity.

\(^2\)Examples includes the Bank of Canada (QPM, ToTEM), the Bank of England (MTMM, BEQM), the Central Bank of Chile (MAS), the Central Reserve Bank of Peru (MEGA-D), the European Central Bank (NAWM, CMR), the Norges Bank (NEMO), the Sveriges Riksbank (RAMSES), the US Federal Reserve (SIGMA, EDO), the Central Bank of Brazil, the Central Bank of Spain, the Reserve Bank of New Zealand, the Bank of Finland, and IMF (GEM, GFM and GIMF). In particular, the Bank of Canada, the Bank of England, the Central Bank of Chile, the Central European Bank, the Norges Bank, the Sveriges Riksbank, and the U.S. Federal Reserve have incorporated New Keynesian DSGE models into their core models.
Interestingly, DSGE models with richer structures have been included in core models of several central banks. We shall discuss the current generation of New Keynesian DSGE models in more detail in Section 3. In particular, we conduct a standard policy exercise based on a simple canonical New Keynesian DSGE model in Section 3.2.2. This simple model incorporates financial intermediation (see, e.g. Gertler and Karadi, 2011) and time-varying rare disaster risk (see, e.g. Gourio, 2012) into the key New Keynesian foundation (see, e.g. Christiano et al., 2005; Smets and Wouters, 2003).

For illustrative purposes, we will conduct experiments on the effects of three shocks in the economy: (1) a financial sector shock impairing the ability of banks to borrow and hold productive assets, (2) a technology shock that impairs the quality of the physical capital, and (3) a risk shock. We will also show how the economy would respond to various policy responses to those shocks.

The devastating aftermath of the financial crisis and the Great Recession has prompted a rethink of monetary policy and central banking. Central bank monetary policy models face new challenges. Many macroeconomists (and in fact, many of the world’s leading thinkers) have called for a new generation of DSGE models. The first and foremost critique of the current state of the art of New Keynesian DSGE models is that these models lack an appropriate financial sector with a realistic interbank market, and as a result, the models fail to fully account for an important source of aggregate fluctuations, such as systemic risk. Second, the linkage between the endogenous risk premium and macroeconomic activity is crucial for policymakers to understand the transmission mechanism of monetary policy, especially in financially stressed periods. In models that lack a coherent endogenous risk premium, policy experiments become unreliable in stressed periods, and the model cannot provide a consistent framework for conducting experimental stress tests regarding financial stability or macroprudential policy. Third, heterogeneity among the players in the economy is essential to our understanding of inefficient allocations and flows between agents. These inefficiencies have an extremely important effect on the equilibrium state of the economy. Without reasonable heterogeneity among agents in models, there is no way to infer the distributional effects of monetary policy.

Finally, and perhaps most importantly in terms of government policy, a new
generation of models is in strong demand to provide policymakers a unified and coherent framework for both conventional and unconventional monetary policies. For example, at the onset of the financial crisis, the zero lower bound went from a remote possibility to reality with frightening speed. This led central banks to quickly develop unconventional measures to provide stimulus, including credit easing, quantitative easing and extraordinary forward guidance. These unconventional measures demanded a proper platform to be analyzed. Furthermore, these unconventional monetary policies have blurred the boundary between monetary policy and fiscal policy. Through these policies, central banks gave preference to some debtors over others (e.g. industrial companies, mortgage banks, governments), and some sectors over others (e.g. export versus domestic). In turn, the distributional effects of monetary policy were much stronger than in normal times. As a result, these measures are sometimes referred to as quasi-fiscal policy. As Sims emphasized, a reliable monetary policy experiment cannot ignore the effect of ongoing fiscal policy. In order to implement unconventional measures during the crisis, central banks put much more risk onto government balance sheets than ever before, which had the potential to lead to substantial losses. Thus the government balance sheets in these models should be forward-looking, and its risk characteristics are crucial to the success of the model. In Section 4, we discuss these critical features to be incorporated into New Keynesian DSGE models in detail.

There are also serious methodological and technical challenges that prevent reasonable policy experiments from being conducted with New Keynesian DSGE models as well. First, advanced nonlinear solution methods and estimation approaches are necessary to guarantee that key nonlinear dynamics in the financial market and the macroeconomy are eventually captured in quantitative analysis. Second, data availability and risk measurement are always a central challenge in macroeconomic modeling, but especially so in the wake of the global financial crisis and the subsequent global economic recession. Brunnermeier et al. (2012) pointed out that our current measurement systems are outmoded, leaving regulators, academics, and risk managers in a dangerous position. Assessing systemic risk requires viewing data on the financial sector through the lens of a macroeconomic model. However, macroeconomics in particular frames questions and builds models based on available data, and we have
so far lacked the data to construct macro-financial models. New infrastructure for
detailed micro-level financial data collecting is necessary and critical for further risk
measurement development and model construction. In fact, the Office of Financial
Research (OFR) at U.S. Department of the Treasury already has this mandate, and
the first steps toward a new, comprehensive, financial data collection system are
already underway. More details are discussed in Section 4.

To conclude, we review the core models employed by central banks to forecast the
economic activities and to analyze their monetary policies. We shall focus on the US
Federal Reserve (FRB/US, FRB/Global, SIGMA, EDO), the European Central Bank
(AWM, NAWM, CMR), the Bank of England (MTMM, BEQM, COMPASS), and the
Bank of Canada (QPM, ToTEM).

2 The Goals and Mechanisms of Monetary Policy

2.1 The Goals and Responsibilities of Monetary Policy

In order to understand the existing monetary policy models employed by central banks,
one needs to understand the primary goals of monetary policy, and the reasons why it
is inevitable that monetary policy analysis will largely be conducted with the use of
mathematical models. According to a 1977 amendment to the Federal Reserve Act,
the U.S. Federal Reserve’s monetary policy has three basic objectives. They include
promoting “maximum” sustainable output and employment, promoting a moderate
long-term interest rate, and promoting “stable” prices\(^3\). These three basic goals of
monetary policy are shared by most major central banks. For example, the Treaty
on the Functioning of the European Union also promotes the primary objective of
supporting stable prices, while at the same time achieving “optimal” employment and
growth.

Price stability is an economic environment that avoids both prolonged inflation and
deflation. In such an environment, households and firms can make financial decisions

\(^{3}\)The terms “price stability” and “inflation stability” are often used synonymously, and we shall
do the same in this review.
without worrying about where prices are headed. Moreover, price stability is all the Fed can achieve in the long run, i.e. its effects are long-run monetary neutral. Periods of high inflation rarely follow a steady course, since the inflation and the volatility of inflation tend to move together. It has been argued that price stability contributes to high levels of employment and economic growth. First, price stability improves the transparency of the price mechanism. Under conditions of price stability, agents can recognize changes in relative prices between different goods and assets without being confused by changes in the overall price level. This allows them to make well-informed consumption and investment decisions, and to allocate resources more efficiently. Second, price stability reduces inflation risk premia in interest rates, the compensation creditors ask for the risks associated with holding nominal assets. This reduces real interest rates and increases incentives to invest. Third, price stability deters unproductive financial activities that hedge against the negative impact of inflation or deflation. Fourth, price stability reduces the distortions caused by inflation or deflation, which may exacerbate the distortionary impact of tax and social security systems on economic behavior. Finally, price stability can help prevent an arbitrary redistribution of wealth and income as a result of unexpected inflation or deflation, and therefore contribute to financial stability.

However, one major lesson the recent financial crisis has taught us is that price stability does not guarantee financial stability. It has previously been argued that a monetary policy directed at maintaining aggregate price level stability would lessen both the incidence and severity of financial instability, most famously in the Schwarz Hypothesis (see e.g. Schwartz, 1988; Schwarz, 1995). While agreeing that low and stable inflation promotes financial stability, we stress from the evidence of the 2007-2009 financial crisis and Great Recession that price stability and financial stability are largely independent, in the sense that large financial imbalances can and do build up during the periods of stable aggregate price(see e.g. Borio and Lowe, 2002). More precisely, price stability is sometimes associated with excessive credit growth and emerging asset bubbles, which may ultimately compromise the goal of price stability. Furthermore,

\footnote{The ECB’s Governing Council has announced a quantitative definition of price stability: “Price stability is defined as a year-on-year increase in the Harmonized Index of Consumer Prices (HICP) for the euro area of below 2%. “}
price stability can encourage excessive optimism, which may lead to overestimates of future growth in income and asset prices, creating a self-reinforcing asset and credit boom, for example, as emphasized by the volatility paradox in Brunnermeier and Sannikov (2014). Most importantly, it has been argued that systemic risk within the financial sector is invisible to measurements based on prices and cash instruments (e.g., balance sheet and income statement items), because it builds up in the background before being triggered by agent response to macroeconomic shocks, only materializing in a crisis (see, e.g., Brunnermeier et al., 2012).

There has been a long debate on whether the central bank is the natural guarantor of the stability of the financial system. In this paper, we emphasize the natural responsibility of the central bank in this matter for the following reasons. First, the central bank is the only provider of the legal means of payment, and therefore it is the only provider of immediate liquidity during a financial crisis. Second, a natural role of the central bank is to ensure the smooth functioning of the national payment system. As such, it is centrally positioned to monitor and combat systemic risk, defined here as the risk of collapse of the entire financial system. Under conditions of high systemic risk, adverse shocks to a few individual banks could create problems at other banks, in particular, those which make up the core of the national payment system. In other words, should systemic risk become a systemic reality, problems at a few individual banks would cascade through the interconnections of the national payment system, likely leading to a downturn in the real economy as a whole. Third, the financial system is the transmission mechanism through which monetary policy has its effect on the real economy. The status of the financial system is critical for the central bank to have any desirable impact and achieve its monetary objectives. For this reason alone, central banks have a natural interest in maintaining a sound financial system. Finally, financial stability also plays an important role in guaranteeing price stability, already a basic role of the central bank. The central bank must avoid the creation of moral hazard in order to effectively achieve financial stability, a difficult challenge. Here, financial innovations may help the development of new tools for the central bank to accomplish the dual objectives of promoting financial stability and avoiding the creation of moral hazard.
It is also one of a central bank’s main responsibilities to maintain a sound central bank balance sheet. Central bank balance sheets have proved crucial in designing and understanding policies pursued in the wake of financial crises in recent years. In particular, large-scale asset purchase programs became the primary tools in efforts to prevent any renewal of the financial meltdown as the effective zero lower bound for interest rates was reached. With short-term interest rates near zero, and the effectiveness of conventional monetary policies constrained as a result of a liquidity trap, these policies sought to provide additional monetary stimulus by lowering the long-term interest rate on government bonds. A loss of confidence in banks and in many financial products in the advanced economies disrupted global financial markets. This occurred in large part because the normal operations of financial markets became impaired, blocking the transmission of lower policy rates to the real economy. Central banks countered this by buying unconventional assets on a large scale. They started by short-term lending, or by buying short-term assets, but progressively moved towards buying long-term paper. At present, the aggregate size of central bank balance sheets in the advanced countries is nearly $8 trillion, the equivalent of more than 20% of GDP. In some cases, balance sheets are still growing.

It is hard to imagine that a central bank would be able to handle another crisis of similar severity given a balance sheet and its associated risks at the current levels of many advanced economies. A country is surely better off if the central bank has the full financial strength needed to carry out its functions. A lack of capacity to conduct effective monetary policy puts the soundness of the whole economy on the hook, with a potentially huge adverse effect. Moreover, the massive size of central bank balance sheets has the possibility to cause huge risks to the real economy through more direct channels, including inflation, financial instability, distortions in financial markets, and conflicts with government debt managers. Finally, the sizable buildup of the asset side of central bank balance sheets also requires a comparable increase in domestic liabilities. However, since these liabilities are the assets of banks and other financial institutions, the process of domestic financial intermediation has been altered, with potentially serious consequences.
2.2 The Mechanisms and Tools of Monetary Policy

It is necessary to understand the particular channels of how monetary policies operate and affect the real economy in order to correctly evaluate the success of those policies. Because monetary authorities cannot directly control the employment and growth of an economy, central banks have to target some measurements to affect the key variables in an indirect way. For example, the Federal Open Market Committee (FOMC) at the U.S. Federal Reserve controls interest rates and the money supply via open market operations, discount loans, and reserve requirements. In particular, the Federal Reserve usually buys and sells short-term government bonds by open market operations from and to banks and other financial institutions to achieve the targeted short-term Treasury interest rates. When basic monetary policies have limited effect on the economy (e.g. during a liquidity trap), unconventional monetary policies, such as quantitative easing, credit easing and forward guidance, can work as the last line of defense against financial vulnerability and economic downturn. For example, quantitative easing has been used extensively during the recent financial crisis and the Great Recession. In quantitative easing, the central banks purchase a predetermined amount of bonds or other assets from financial institutions, without reference to the interest rate. The goal and economic effect of quantitative easing is to increase the monetary base (i.e. reserve money) rather than decrease the interest rate or necessarily increase the money supply (because banks can keep cash provided by the central bank in liquidity reserve).

The scope of monetary policy is limited in terms of what variables the central banks can directly control, to what extent, and for how long the impact of monetary policy will last. In the short run, a change in money market interest rates induced by the central bank will trigger a number of mechanisms and actions in different economic sectors (e.g. financial institutions, firms and households), as well as by different agents within each sector. Ultimately, the heterogeneous reactions of different agents in multiple sectors will together influence economic variables such as output or prices. The process of how the shock in monetary policy leads to changes in aggregate economic variables, including inflation, output, employment, consumption
and investment, via financial institutions, firms and households, is known as the
monetary policy transmission mechanism. The transmission mechanisms are usually
highly complex. While the broad features of monetary transmission channels have
been studied by researchers and are well understood, there is no consensus on the
detailed functioning of the monetary policy transmission mechanism. Although theory
has suggested a wide range of transmission channels, economic practice has emphasized
the interest rate channel, the inflation expectations channel, the balance sheet channel,
the bank credit channel (i.e. the bank lending channel), the exchange rate channel,
and the asset price channel. The accurate projection of a monetary policy shock, which
is key to reliable policy experiments, depends on the specifications of the model, the
accuracy of the solution methodology, and the estimation approach. This is exactly
the reason why central banks have needed to develop complex macroeconomic models,
accompanied by advanced solution/estimation methods, for monetary policy analysis.

Monetary policy impulses coming from the central bank are usually transmitted
through the financial system through the banks. There is a tight relationship between
financial intermediaries such as commercial banks and the monetary authority in the
general context of financial markets like the money market and the foreign exchange
market. Normally, the central bank can control short-term interest rates relatively
efficiently, because it has the ability to manage liquidity in the market. Although
monetary policy impulses can pass quite quickly through an advanced economy with
a sound financial system, these impulses are usually transmitted rather imperfectly
and only with a time lag, depending on the structural characteristics of the economy
and the soundness of the financial system of the nation. Figure 1 illustrates the main
transmission channels of monetary policy impulses.

We first consider the interest rate channel. A change in the official short-run
interest rates directly affects money market interest rates, but it only indirectly affects
lending and deposit rates, which are set by commercial banks to their debtors and
depositors, respectively. While the central bank can control short-term interest rates,
the real economy is mainly affected by the medium- and long-term deposit and lending
rates charged by these commercial banks to their customers. These rates depend
not only on the interest rate set by the monetary authority, but also on a number
Figure 1: Main Transmission Channels of Monetary Policy Impulse. The source is European Central Bank.
of other determinants, such as inflation expectations and the risk premium of other channels, since they are of utmost importance for investment, consumption, and savings decisions. The conventional interest rate channel is characterized by the proposition that lower nominal short-term interest rates lead to lower real interest rates, because prices are sticky. Therefore, lower interest rates promote investment and consumption, but discourage savings, while higher interest rates stimulate savings and lower consumption and investment in the short run. As a result, changes in the interest rate affect the aggregate demand in the economy. This can be seen explicitly from the dynamic IS curve. In the short run, aggregate supply has only a limited ability to adjust to the new level of demand. However, in the long run, aggregate supply gradually adjusts its response to shocks in fundamental economic factors such as production capacity, labor force, and technology. Monetary policy has almost no influence on the long-run aggregate supply. Thus, in the short-to-medium run, monetary policy can influence only the difference between the actual level of economic activity and the one that is sustainable over the long run, the potential GDP. This difference is called the output or GDP gap.

We next consider the inflation expectations channel. The New Keynesian Phillips curve demonstrates that the real GDP gap, together with inflation expectations, is a key determinant for price inflation dynamics. Moreover, to the extent that agent expectations are model-consistent, inflation dynamics will reinforce inflation expectations. In particular, a demand for consumption or investment goods in excess of the supply will put pressure on their marginal cost of production. Faced with an increase in production costs, some firms might decide to reduce their profit margins because they have to leave the final sale price unchanged. However, in the medium term, if the production costs rise systematically, firms will gradually transfer these costs onto the final price, which will eventually lead to a rise in the price of consumption goods, thus generating realized inflation. Conversely, an aggregate demand deficit will exert opposite effects. Therefore, in a rational expectation framework, the rising force of inflation will boost the agents’ inflation expectations. Inflation expectations are heavily affected by the perceptions of economic agents regarding the central bank’s commitment towards achieving its primary objectives. Anchoring inflation
expectations can be one of the most powerful and efficient channels of monetary policy transmission, provided that it is transparent, and its actions are regarded as credible. However, it is still not clear how conventional monetary policies effectively anchor the desired inflation expectations. As pointed out by Blanchard (2009),

“[...] although we very much want to believe that monetary policy can anchor inflation expectations, I am not sure we actually understand whether and how it can actually achieve it.”

The balance sheet channel is deeply associated with the external finance premium, which is defined as the wedge between the cost of capital internally available to firms, and the cost of raising capital externally by issuing equity or borrowing from corporate debt markets. External financing is more expensive than internal financing, and the external finance premium will exist positively so long as external financing is not fully collateralized. Fully collateralized financing implies that even under the worst-case scenario, the payoff of the project is at least sufficient to guarantee full loan repayment. If the net worth on a firm’s balance sheet can be used as collateral for external borrowing, the external finance premium should be inversely related to the firm’s net worth. An increase in interest rates will tighten the balance sheet channel for firms. An increasing interest rate increases the interest payments on outstanding or floating-rate debt, and decreases the value of the firm’s collateral through decreased asset prices. This will lead to a high external financial premium. Meanwhile, an increasing interest rate reduces the demand for a firm’s products, which reduces the firm’s revenue, while its short-run fixed costs do not adjust. The reduction in cash inflow erodes the firm’s net worth, and hence increases the firm’s external finance premium over time. The balance sheet channel is potentially dangerous since it could amplify and propagate small fluctuations via a pecuniary externality or an adverse feedback loop, as is emphasized in Kiyotaki and Moore (1997). The balance sheet channel is also critical for households in determining the aggregate demand for durable goods and houses.

The bank lending channel has arguably been the most important channel in monetary policy during the recent financial crisis and the Great Recession. The bank credit channel is essentially the balance sheet channel as applied to the operations of
lending institutions. The first study of the bank lending channel in monetary policy was by Bernanke (1983). Since then, there has been an extensive academic literature on the topic; see, for example, Bernanke and Blinder (1988), Kashyap and Stein (1994), and Bernanke and Gertler (1995), among others. The uniqueness of the bank credit channel for monetary policy transmission is mainly a result of the special role of the financial sector in the economy relative to other sectors. While there is some evidence that small firms may be especially dependent on banks for financing, there are conflicting opinions on whether bank lending is directly affected by monetary policy actions. As emphasized in Morris and Sellon (1995), for monetary policy to operate through a credit channel, not only must there be bank dependent borrowers, but monetary policy must also directly affect banks’ willingness and capacity to lend. Monetary policies may change the supply of loanable funds available to banks, and consequently the total amount of credit they can extend to borrowers, including both firms and households. It has been argued that the most direct way monetary policy is able to affect the willingness and capacity of bank lending is to control the supply of bank reserves. For instance, a drop in the supply of bank reserves will force banks to shrink their balance sheets, and hence cut risky corporate and household lending. A lesson from the recent crisis and recession is that disruptions in the financial system could generate large losses and affect the liquidity and solvency of both banks and borrowers.

We next consider the exchange rate channel. Since the central bank has relatively efficient control over short-term interest rates, the central bank can also influence the exchange rate, which reflects the willingness of economic agents to hold domestic currency over holding foreign currency. However, the exchange rate is also the outcome of other influences, including the risk aversion of foreign investors, domestic and external balance sheets, political factors, and so on. Monetary policy has some capacity to influence these factors. The exchange rate channel operates most directly through the relative price of domestic goods versus foreign goods. In particular, if the exchange rate falls (that is, if the domestic currency depreciates), an exporter would profit from converting the price in foreign currency of goods sold overseas back to domestic currency. In contrast, if the domestic currency rises (or appreciates), an
importer would profit when selling goods purchased abroad on the domestic market. As a result, monetary policy impulses which initially have an effect on the exchange rate can be transmitted, although with a lag, to real economic activity through the so-called net exports (or trade) channel. Real economic activity can also be affected through a combination of interest rates and exchange rates, via the so-called wealth and balance sheet channel. Exchange rate depreciation lowers the incentive to borrow in foreign currency. At the same time, depreciation reduces the disposable income that is left after servicing the regular payments on a foreign currency loan, since economic agents with revenues denominated in domestic currency would have to pay a greater amount following a depreciation of the currency. On the other hand, domestic currency appreciation will have the opposite effect, lowering the costs associated with loans denominated in foreign currency.

Finally, we consider the asset price channel. It is argued that monetary policy can affect agent investment and consumption decisions through stock prices, risky bond prices, and real estate prices, through what is called the asset price channel. For example, lower interest rates will cause more capital to flow into stocks and consequently raise the stock prices, leading to higher investment via two main channels. First, higher stock prices generate higher Tobin’s Q, and hence entice higher investment. Second, higher stock prices make it easier for firms to obtain outside equity financing, and as a result, increase their investment.

The basic mechanisms described above are the standard benchmark mechanisms now incorporated into the core macroeconomic models of the major central banks. However, in Section 4, we shall discuss some important transmission mechanisms that are ignored by these models, for example, endogenous risk premium dynamics, government balance sheet forward-looking decisions and risk characteristics, and interactions with fiscal policy.
3 Overview of Monetary Policy Models in Central Banks

3.1 A Brief History

According to Galí and Gertler (2007), economists and policymakers began to be skeptical about large-scale macroeconometric modeling during the 1970s for two related reasons. First, some existing models, like the Wharton econometric model and the Brookings model, failed to forecast the stagflation of the 1970s. These traditional large-scale macroeconometric models were originated by (see, e.g. Klein, 1985, 1991) and have been in use for decades. Second, leading macroeconomists leveled harsh criticisms about their underlying framework. Lucas (1976) and Sargent (1981), for example, argued that the absence of an optimization-based approach to the development of the structural equations meant that the estimated model coefficients were likely not invariant to shifts in policy regimes, or to other types of structural changes. Similarly, Sims (1980) argued that the absence of convincing identification assumptions to sort out the vast simultaneity among macroeconomic variables meant that one could have little confidence that the parameter estimates would be stable across different policy regimes. More precisely, Sims (1980) argued that large-scale macroeconometric models may fit the data well, but that they will provide misleading answers due to non-credible identification restrictions.

Despite the criticisms by Lucas (1976) and Sims (1980), many central banks continued to use large-scale macroeconometric models and reduced-form statistical models in the 1980s and 1990s to produce forecasts of the economy that presumed no structural change. However, they did so knowing that these models could not be used with any degree of confidence to predict the outcome of policy changes. Over the past two decades, quantitative microfounded macroeconomic frameworks for monetary policy evaluations have made their debut. The building blocks for the development of this new framework were two independent literatures that emerged in response to the downfall of traditional macroeconometric modeling: New Keynesian theory and real business cycle (RBC) theory. The New Keynesian paradigm arose in the
1980s as an attempt to provide microfoundations for key Keynesian concepts such as the inefficiency of aggregate fluctuations, nominal price stickiness, and the non-neutrality of money (see, e.g. Mankiw and Romer, 1991). The models of this literature, however, were typically static and designed mainly for qualitative as opposed to quantitative analysis. By contrast, real business cycle theory, which was developed concurrently, demonstrated how it was possible to build quantitative macroeconomic models exclusively from the “bottom up” – that is, from explicit optimizing behavior at the individual level (see, e.g. Prescott, 1986). The RBC models, abstracted from monetary and financial factors, could not address the issues related to monetary policy. The new frameworks reflect a natural synthesis of the New Keynesian and the real business cycle approaches. A variety of labels have been used for this new framework. For example, Goodfriend and King (1997) employs the term “New Neoclassical Synthesis”, while Woodford (2003) uses “NeoWicksellian” and Clarida et al. (1999) uses “New Keynesian”. Usually, however, these type of models are called New Keynesian Dynamic Stochastic General Equilibrium models. The key innovation of the New Keynesian DSGE model for monetary policy evaluation is to incorporate nominal stickiness and the resulting monetary non-neutrality into a fully specified dynamic general equilibrium framework.

Central banks use a wide range of macroeconomic models and tools for forecasting and monetary policy analysis, including large-scale macroeconometric models, reduced-form statistical models, structural autoregressive models, and New Keynesian DSGE models. The characteristics of the various models are summarized in Table 1. Large-scale macroeconometric models constrain purely data-driven models in such a way that the long-run dynamic behavior of the variables converges to the theoretical long-run steady state. In econometric terms, the macroeconometric models developed and used by central banks are essentially large-scale restricted vector error-correction models (VECM). This approach puts less emphasis on theory, insofar as short-run dynamics are largely data-driven and long-run relations implied by theory still have to be confirmed by empirical work. For instance, the modeler would not insist that the model has a balanced growth equilibrium, but instead would test whether the cointegrating relation implied by this was present in the data. Examples of this type
of macroeconometric model include the Bank of England’s earlier MTMM model, the Bank of Canada’s QPM model, the Federal Reserve’s MPS and FRB/US model, and the European Central Bank’s AWM model.

Although the large-scale macroeconometric model still plays an active role at major monetary authorities such as the Federal Reserve, there has been a steady shift towards models that place greater emphasis on theoretical consistency. For example, the Bank of Canada’s shifted its principal model from the QPM model to the ToTEM model in late 2005, the European Central Bank replaced its AWM model with the NAWM model, and the Federal Reserve started to build various DSGE models such as SIGMA and ODE. This vintage of new macroeconometric models uses a calibrated theoretical model to pin down a set of steady-state attractors to describe an error-correcting relationship. Dynamics are driven by assuming that there are adjustment costs between current and long-run levels for variables on a partial equilibrium basis. Higher orders of adjustment costs introduce a role for forward-looking expectations. The full model is a mixture of structural relations implied by a partial equilibrium treatment of theory, such as the decision rule for aggregate consumption, and some reduced-form relations, such as their trade equations, which employs error-correcting relationships.) Finally, structural VAR models were first introduced by Sims (1980) as an alternative to traditional large-scale macroeconometric models.

One benefit of having multiple models is the opportunity to examine the robustness of policy strategies across models with quite different foundations. According to Tovar (2009) and Chung et al. (2010), central bankers emphasize that in their experience, model-based policy analysis is enhanced by considering multiple models, and indeed, they often learn as much when models disagree as when they agree.

In the following subsection, we shall focus on recent advances in the development of New Keynesian DSGE models, which now serve as core models and warhorses at several major central banks. Recent efforts on the academic side include the incorporation of financial frictions (i.e., the financial accelerator channel), financial intermediation (i.e., the bank funding channel), nontrivial fiscal policies, and the government/central bank balance sheet in order to analyze unconventional monetary policies. We shall first lay out a canonical simple New Keynesian DSGE model, of the
Table 1: Macroeconometric Models, SVAR, and New Keynesian DSGE Models

<table>
<thead>
<tr>
<th>Example</th>
<th>Macroeconometric</th>
<th>SVAR</th>
<th>DSGE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FRB/US, FRB/Global, AWM, MTMM, QPM</td>
<td>Linear Approx. to DSGE Models</td>
<td>SIGMA, ODE, CMR ToTEM, NAWM</td>
</tr>
<tr>
<td>Dynamic</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Long-run Relations</td>
<td>Based on Steady State Equilibrium in Theory</td>
<td>Based on Theory and Restrictions</td>
<td>Based Explicitly on Individual Optimization in a Coherent Manner</td>
</tr>
<tr>
<td>Short-run Dynamics</td>
<td>Based on Ad-hoc Adjustment Dynamics</td>
<td>Based on Theory and Restrictions</td>
<td>Based Explicitly on Individual Optimization in a Coherent Manner</td>
</tr>
<tr>
<td>Sims Critique i.e. Reliable Structural Exogenous Shocks?</td>
<td>Partly</td>
<td>Yes (ideally)</td>
<td>Yes (ideally)</td>
</tr>
<tr>
<td>Lucas Critique i.e. Reliable Policy Analysis?</td>
<td>Partly</td>
<td>Yes (ideally)</td>
<td>Yes (ideally)</td>
</tr>
<tr>
<td>Policy Experiment i.e. Impulse Response Analysis</td>
<td>Yes (less credible)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Forecast?</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Estimated ?</td>
<td>Estimation &amp; Calibration</td>
<td>Yes</td>
<td>Estimation &amp; Calibration</td>
</tr>
<tr>
<td>Nonlinearity?</td>
<td>Maybe</td>
<td>No</td>
<td>Maybe</td>
</tr>
</tbody>
</table>

Type which has been the core component of all central bank DSGE models. Following this, we shall discuss extending the model by adding an imperfect credit market and financial intermediation.

3.2 New Keynesian DSGE Models

An increasing number of central banks have started to use New Keynesian DSGE models as their core models. These central banks and policy institutions include the US Federal Reserve, the European Central Bank, the Bank of England, the Bank of Canada, the Bank of New Zealand, and the International Monetary Fund. DSGE models were originally built on the neoclassical growth model, with stochastic ingredients added from real business cycle models, and real/nominal frictions such
as the cost of capital adjustment, nominal wage and price rigidity, and monopolistic competition. An excellent introduction to the basics of New Keynesian DSGE models can be found in Galí (2008) and Woodford (2003). The standard approach used by the central banks is start with the basic New Keynesian DSGE model, and keep adding additional relevant ingredients, such as

(i) exogenous shocks, including preference shocks, marginal efficiency shocks, global shocks, risk premium shocks, fiscal policy shocks, etc.

(ii) frictions in the financial market, including collateral constraints, information-based frictions, moral hazard-based frictions, and limited commitment. See, for example, Kiley and Sim (2011b,a), Brunnermeier and Pedersen (2009), Chari et al. (1995), Bernanke et al. (1999). There are other recent papers exploring the policy implications of both nominal rigidities and credit frictions, including collateral-based borrowing constraints (see, e.g. Iacoviello, 2005) and limited access to financial markets (see, e.g. Galí et al., 2004, 2007).

(iii) financially constrained (occasionally) intermediaries. See, for example, Gertler and Kiyotaki (2010), He and Krishnamurthy (2008), Christiano et al. (2010), and Adrian and Shin (2010a,b); Adrian et al. (2010).

3.2.1 Why and How to Use DSGE Models

The DSGE model has become increasingly attractive to the central banks over the past two decades. The reasons why this has happened also naturally provide answers to how we may use DSGE models. We list three of the most important here.

First, its explicit account of the role of expectations and its identification of deep structural parameters makes the DSGE analysis less subject to the Lucas Critique, and more suitable for policy analysis and counterfactual experiments. DSGE models emphasize the important role of expectations in assessing alternative policy actions. The DSGE model is able to relate the reduced-form parameters to deeper structural parameters, which makes the use of the model for policy analysis less subject to the Lucas critique, as those structural parameters are less likely to change in response
to changes in policy regime. Therefore, the DSGE model provides a solid organizing framework for understanding and analyzing the economy and policy impacts.

Second, impulse-response analysis allows the DSGE model to identify and decompose economic and policy structural shocks on the quantitative level. A reasonable identification of structural shocks greatly improves the reliability of policy analysis and counterfactual experiments, making the analysis less subject to the Sims critique. The nature of the DSGE model’s structure, not only in terms of its parameters, but also in the way exogenous shocks drive the economy according to the model, makes it possible to tell coherent stories and structure forecasts around it.

Third, the DSGE model’s capacity to link model implications to time-series and cross-sectional data makes it particularly useful to discover deep structural parameters. Recent advances in the construction, simulation and estimation of DSGE models have made it possible to combine a rigorous microeconomic derivation of the behavioral equations of macroeconomic models with an empirically plausible calibration, an estimation which fits the main features of a macroeconomic time series. Beginning with a series of seminal papers, including Mehra and Prescott (1985) and Hansen and Singleton (1982, 1983), it has been shown that asset pricing data are extremely useful in understanding the deep structural parameters of DSGE models. In addition, these structural parameters can be calibrated/estimated using off-model information, especially when time series are short. In terms of the accuracy of the estimation of structural parameters, the DSGE model reduces the risk of overfitting by helping identify parameters and shocks hitting the economy.

### 3.2.2 A Simple Canonical New Keynesian DSGE Model with Financial Intermediation and Risk Premia

In this subsection, we review a canonical New Keynesian DSGE model with financial intermediation, which allows for unconventional monetary policy effects (see, e.g. Gertler and Karadi, 2011), and also time-varying rare disaster risk and its premia (see, e.g. Gourio, 2012). The New Keynesian component of the model is a simplified version based on Christiano et al. (2005) and Smets and Wouters (2003). A simple
New Keynesian DSGE model featuring monopolistic competitive firms and rigid nominal prices without endogenous capital accumulation can be found in Galí (2008). Kollman (1997) and Erceg et al. (2000) both introduced nominal sticky wages which are adjusted according to the Calvo rule (see Calvo, 1983). In terms of endogenous capital accumulation, we follow Christiano et al. (2005), which incorporates endogenous capital accumulation with an adjustment cost characterized by the relative level of investment, rather than the investment-capital stock ratio, as commonly assumed in the real business cycle literature. We adopt their external habit formation in consumption, which helps generate persistence in the consumption process in the data. We also incorporate “capital quality shocks” similar to Smets and Wouters (2003) and Christiano et al. (2005) in a New Keynesian setting and Gourio (2012) in asset pricing setting, inspired by Greenwood et al. (1988) and King and Rebelo (1999) in the real business cycle literature. The financial intermediation component in our model is based on Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). Our model is a simplified version of Christiano et al. (2010) and Christiano et al. (2014), which are state-of-the-art New Keynesian DSGE models with nontrivial financial intermediation. Our model is meant for illustrative purposes, and we have left out many exogenous shocks that would be studied in a full-scale DSGE model.

Our model creates a simplified but recognizable version of the real economy. In this model, households maximize their individual utility function with consumption and labor over an infinite horizon. The utility function is characterized by external habit formation. The habits depend on lagged aggregate consumption that is unaffected by any single household’s decision. Abel (1990) calls this the “catching up with the Joneses” effect. For simplicity, we assume households face flexible nominal wages. The members of each household are divided into bankers and workers. Bankers and workers can both supply labor. However, only bankers own capital in the economy, and rent capital to the intermediate goods firm to extract rents. In this way, bankers decide how much capital to accumulate given the capital adjustment costs. The representative intermediate goods firm produces one kind of intermediate good, which can be used for investing and creating capital goods, or sold wholesale to retailers who simply convert the intermediate goods into differentiated goods for consumption.
The retailers produce differentiated goods, giving them some monopoly power over goods prices, with a downward sloping demand for goods from households. The intermediate goods firm decides on labor and capital inputs, and at the same time, the retailers re-optimize goods prices according to the Calvo rule. Eventually, under the assumption that the bankers and the workers can fully insure their idiosyncratic risks in consumption, the representative households have the full claim on the dividends paid out by the intermediate goods firm and the retailers.

It should be noted that, in the generation of New Keynesian DSGE models built on Christiano et al. (2005) and Smets and Wouters (2003), the financial and credit markets play no role in determining asset prices except for the term structure of real interest rates and expectations of future payouts, and have no impact on the real economy in the canonical New Keynesian DSGE models. An equivalent statement is that these models adopt the assumptions underlying the Modigliani and Miller (1958) Theorem, which implies that financial structure is both indeterminate and irrelevant to real economic outcomes. In order to quantitatively study how credit market imperfections influence the transmission of monetary policy, Bernanke et al. (1999) incorporated a countercyclical credit-market friction which is endogenously generated from first principles (i.e. using agent optimization) into an otherwise standard New Keynesian DSGE model. This countercyclical credit-market friction, first emphasized in Kiyotaki and Moore (1997), is shown to amplify and propagate productivity shocks. In more recent work, Christiano et al. (2010) extend the simple model in Bernanke et al. (1999) in many dimensions, including financially-constrained intermediations, but the key financial and credit market imperfections are not far removed from the financial accelerator mechanism in Bernanke et al. (1999). The crucial feature of constrained financial intermediation has been characterized by Gertler and Karadi (2011) in a simple but transparent model, which we build upon.

Our basic desire is to understand and illustrate the effects of three shocks in this model economy: (1) a financial sector shock impairing the ability of banks to borrow and hold productive assets, (2) a technology shock that impairs the quality of the physical capital, and (3) a risk shock. We also desire to show how the economy would respond to various policy responses to these shocks.
**Households**

We begin with a description of households in our simple canonical New Keynesian DSGE model. There is a continuum of households of unit mass. The members of each household are either workers or bankers. Though there are two groups of agents, and certain portfolio constraints among them, we assume the representative framework following Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) by assuming that the full set of Arrow-Debreu securities are available to the members within each household (but not across households), so that the idiosyncratic consumption risks can be fully insured, and the agents in two groups have identical preferences. At any time, a fraction $f$ of the members of the household are bankers. Bankers live for a finite number of periods with probability 1. At any time, a fraction $1 - \theta$ of randomly selected existing bankers exit and become workers, and return their net worth to their household. At the same time, an equal number of workers become bankers within each household, so the proportion of workers and bankers remains fixed. The new bankers receive some start-up funds from their household, which we describe below. The “perpetual youth” assumption in our model is purely technical in our model, with the purpose of guaranteeing the survivorship of both groups of agents.

The preferences of the household are given by

$$
E_t \left[ \sum_{\tau=0}^{\infty} \beta^\tau \left( \frac{(C_{t+\tau} - hC_{t+\tau-1})^{1-\gamma}}{1-\gamma} - \frac{\chi}{1+\varphi} R_{t+\tau}^{1+\varphi} \right) \right],
$$

where $C_t$ is the consumption and $L_t$ is the labor supply at time $t$. $\varphi$ is the Frisch elasticity of labor supply and it is positive. The subject discount rate $\beta \in (0,1)$ and habit parameter $h \in (0,1)$. Also, we assume that $\chi > 0$.

Both bankers and workers within each household can hold nominally risk-free debt issued by the government, and can deposit its cash with a financial intermediary that pays a nominally risk-free rate. Assuming that both assets are perfect substitutes, we denote by $R_{t+1}$, the real gross interest rate paid by either of these assets. It should be noted that the gross real interest rate $R_{t+1}$ is possibly random up to the information
set at time $t$, because the debt contract is written on the nominal term and the inflation $\Pi_{t+1}$ is random up to the information set at time $t$. Let $B_{t+1}$ denote the quantity of this debt held by the household at the end of period $t$. The household then faces a state-by-state budget constraint

$$C_t = W_t L_t + \Pi_t + T_t + R_t B_t - B_{t+1},$$

where $W_t$ is the real wage, $\Pi_t$ are the profits from the various firms the household owns (which we describe below), and $T_t$ are the real lump-sum taxes. The first-order conditions to the household’s utility maximization problem include the intra-temporal Euler equation for working hours

$$\Lambda_t = \chi \frac{L^e_t}{W_t},$$

(1)

and the Inter-temporal Euler equation for risk-free bond holding

$$1 = \mathbb{E}_t \left[ \beta \frac{\Lambda_{t+1}}{\Lambda_t} R_{t+1} \right],$$

(2)

where

$$\Lambda_t \equiv (C_t - hC_{t-1})^{-\gamma} - \beta h \mathbb{E}_t (C_{t+1} - hC_t)^{-\gamma}$$

(3)

Financial Intermediaries

Here we describe how financial intermediaries are implemented within our model. Financial intermediaries borrow funds from households at a risk-free nominal rate, and pooling this with their own net worth or wealth, they invest in the equity of the representative intermediate goods firm. We describe the intermediary using real variables in what follows. The balance sheet of intermediary $j$ at the end of time $t$ is
given by

\[ Q_t S_{j,t} = N_{j,t} + B_{j,t+1}, \]  

where \( Q_t \) is the price of the intermediate goods firm’s equity, \( S_{j,t} \) is the quantity of equity held by the intermediary, \( N_{j,t} \) is the net worth and \( B_{j,t+1} \) are the deposits raised from households. The intermediary earns a gross return \( R_{k,t+1} \) from the equity investment at time \( t+1 \), and must pay the gross interest \( R_{t+1} \) on the deposit. The net worth of the intermediary, therefore, evolves as

\[ N_{j,t+1} = (R_{k,t+1} - R_{t+1})Q_t S_{j,t} + R_{t+1}N_{j,t}. \]

The intermediaries face a constraint on raising deposits from households. They cannot raise deposits beyond a certain level, which is determined endogenously in the equilibrium. We shall describe this constraint in more detail below. Since the bankers own the intermediaries, we use the bankers’ stochastic discount factor (SDF), which coincides with the SDF of the representative agent, \( \beta^t \Lambda_{t+\tau}/\Lambda_t \) to compute the value of assets to the intermediary. The presence of the borrowing constraints implies

\[ \mathbb{E}_t \left[ \beta^\tau \frac{\Lambda_{t+\tau+1}}{\Lambda_t} (R_{k,t+\tau+1} - R_{t+\tau+1}) \right] \geq 0, \quad \forall \tau \geq 0, \]  

with equality if and only if the intermediary faces no borrowing constraint.

Since the intermediary ceases being a banker each period with probability \( 1 - \theta \), the value of the intermediary \( j \), or of its terminal wealth is given by

\[ V_{j,t} = \max_{\{S_{j,t+\tau},B_{j,t+\tau+1}\}_{\tau \geq 0}} \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} (1 - \theta)^{\tau} \theta^\tau \beta^\tau \frac{\Lambda_{t+\tau+1}}{\Lambda_t} N_{j,t+\tau+1} \right] \]

\[ = \max_{\{S_{j,t+\tau},B_{j,t+\tau+1}\}_{\tau \geq 0}} \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} (1 - \theta)^{\tau} \theta^\tau \beta^\tau \frac{\Lambda_{t+\tau+1}}{\Lambda_t} [(R_{k,t+\tau+1} - R_{t+\tau+1})Q_{t+\tau} S_{j,t+\tau} + R_{t+\tau+1}N_{j,t+\tau+1}] \right]. \]
The borrowing constraint on financial intermediaries is necessary to guarantee the existence of an equilibrium. This is because the discounted risk premium is positive in every period, i.e.

\[
\beta^T \Lambda_{t+\tau}(R_{k,t+\tau+1} - R_{t+\tau+1}) > 0, \quad \forall \ t, \tau.
\]

(6)

Thus, the value-maximizing financial intermediary would lever up infinitely by borrowing from the household. In such a case, the economy is not well defined since no equilibrium exists. In order to motivate the borrowing constraint faced by financial intermediaries, we introduce a simple moral hazard/costly enforcement problem. We assume that the banker can choose to liquidate the financial intermediation and divert the fraction \( \lambda_t \) of available funds from the value of the financial intermediation.

The borrowing constraint is modeled as follows. At any time \( t \), the manager of the intermediary can divert a fraction \( \lambda_t \) of the intermediary’s assets to his household for his own benefit. The logarithm of \( \lambda_t \) follows an AR(1) process

\[
\log \lambda_t = (1 - \rho_\lambda) \log \lambda_{ss} + \rho_\lambda \log \lambda_{t-1} + \sigma_\lambda \epsilon_{\lambda,t},
\]

(7)

where \( \lambda_{ss} \) is the long term mean or the quantity \( 1 - \lambda_{ss} \) measures the steady-state pledgeability of the intermediary’s asset. If the value of the intermediary falls below \( \lambda_t Q_t s_{j,t} \), the intermediary will simply divert the assets, and the households will get a zero gross return from their deposits. In order for the households to have an incentive to deposit cash with the intermediary, the following condition must hold:

\[
V_{j,t} \geq \lambda_t Q_t s_{j,t}.
\]

We conjecture that the value of the intermediary is linear in its net worth and the value of the assets it holds:

\[
V_{j,t} = \nu_t Q_t s_{j,t} + \eta_t N_{j,t}.
\]
We see that the incentive constraint binds only if $0 < \nu_t < \lambda_t$, otherwise the marginal value to the intermediary of increasing the assets is larger than the marginal value of diverting them, and the intermediary has an incentive to increase its assets. As in the equilibrium in Gertler and Karadi (2011), we assume that the incentive constraint always binds in the local region of the long run mean $\lambda_{ss}$ of $\lambda_t$. When the constraint binds, we have the condition

$$\nu_t Q_t S_{j,t} + \eta_t N_{j,t} = \lambda_t Q_t S_{j,t}, \text{ or}$$

$$Q_t S_{j,t} = \frac{\eta_t}{\lambda_t - \nu_t} N_{j,t} = \phi_t N_{j,t}. \quad (9)$$

Using the definition of $\phi_t$, we can rewrite the evolution of the intermediary’s net worth as

$$N_{j,t+1} = N_{j,t} \left[ (R_{k,t+1} - R_{t+1}) \phi_t + R_{t+1} \right]. \quad (10)$$

This is the standard wealth or net worth law of motion with leverage, where $\phi_t$ can be viewed as the share of net worth invested in risky asset (i.e. equity).

We verify the guess to the solution of the value function of the intermediary when the incentive constraint binds, and obtain

$$\nu_t = \mathbb{E}_t \left[ (1 - \theta) \beta \frac{\Lambda_{t+1}}{\Lambda_t} (R_{k,t+1} - R_{t+1}) + \theta \beta \frac{\Lambda_{t+1}}{\Lambda_t} \phi_{t+1} \nu_{t+1} ((R_{k,t+1} - R_{t+1}) \phi_t + R_{t+1}) \right] \quad (11)$$

and

$$\eta_t = \mathbb{E}_t \left[ 1 - \theta + \theta \beta \frac{\Lambda_{t+1}}{\Lambda_t} \eta_{t+1} ((R_{k,t+1} - R_{t+1}) \phi_t + R_{t+1}) \right]. \quad (12)$$

Since $Q_t S_{j,t} = \phi_t N_{j,t}$, and since $\phi_t$ does not depend on intermediary-specific factors,
we can aggregate over the equation to get

\[ Q_t S_t = \phi_t N_t, \quad (13) \]

where \( S_t \) is the aggregate investment in the equity and \( N_t \) is the aggregate wealth of the intermediaries.

Finally, we determine the evolution of the aggregate net worth of the intermediaries. The aggregate net worth is the sum of the net worth of the existing intermediaries \( N_{n,t} \) and the net worth of the new entrants \( N_{e,t} \):

\[ N_t = N_{e,t} + N_{n,t}. \quad (14) \]

Since a fraction \( \theta \) of the bankers from \( t-1 \) survive up to \( t \), we have

\[ N_{e,t} = \theta N_{t-1} \left[ (R_{k,t} - R_t) \phi_{t-1} + R_t \right]. \quad (15) \]

The new entrants receive funds from the households to “start up”. As in Gertler and Karadi (2011), we assume that each entering intermediary receives a fraction \( \frac{\omega}{1-\theta} \) of the value of the final period assets of the exiting intermediaries, which is \( (1-\theta)Q_t S_{t-1} \). This gives

\[ N_{n,t} = \omega Q_t S_{t-1}. \]

Thus, we have the evolution of the aggregate net worth

\[ N_t = \theta N_{t-1} \left[ (R_{k,t} - R_t) \phi_{t-1} + R_t \right] + \omega Q_t S_{t-1}. \quad (16) \]
The Intermediate Goods Firm

Here we discuss the role of the intermediate goods firm in our model. The financial intermediaries invest in the equity of the intermediate goods firm. We assume one such representative firm that produces the intermediate good used by the retail firms to produce differentiated goods. This firm has no wealth of its own. It uses the proceeds of the investment by the intermediaries to purchase capital from the capital producing firm for the next period. The number of shares issued by the intermediate goods firm is equal to the number of units of capital purchased, so that

$$Q_t S_t = Q_t K_{t+1}.$$  \hspace{1cm} (17)

The intermediate firm faces no informational or incentive problems. It purchases capital and hires labor to produce the intermediate good using the production function

$$Y_t^I = A_t (U_t \xi_t K_t)^{\alpha} L_t^{1-\alpha},$$  \hspace{1cm} (18)

where $A_t$ is the TFP shock, $\xi_t$ is the quality of capital, $K_t$ is the capital stock determined at period $t - 1$, and $U_t$ is the utilization rate of capital chosen in period $t$ right before the production of intermediate goods. The utilization rate will be chosen endogenously because the capital depreciation rate depends on $U_t$ inversely. In particular, we assume the depreciation rate is

$$\delta(U_t) \equiv \delta_{ss} U_t^{\varphi},$$  \hspace{1cm} (19)

where $\delta_{ss}$ is the steady-state depreciation rate with the steady-state utilization rate to be assumed as 1, and $\varphi$ is the elasticity of marginal depreciation with respect to the utilization rate.

Denote by $P_{m,t}$ the price of the output of the intermediate good firm. The first-order
condition with respect to labor gives

\[ P_{m,t}(1 - \alpha) \frac{Y^I_t}{L_t} = W_t. \]  

(20)

Moreover, the first-order condition with respect to capital utilization rate \( U_t \) is

\[ P_m \alpha \frac{Y^I_t}{U_t} = \delta'(U_t) \xi_t K_t. \]  

(21)

Since the intermediate firm has no wealth of its own and makes zero economic profits due to constant returns to scale, it simply pays out the profits (inclusive of the liquidation of the leftover capital stock) to the financial intermediaries, giving

\[ R_{k,t+1} = \frac{P_{m,t+1} Y^I_{t+1} - W_{t+1} L_{t+1} + \left[ Q_{t+1} - \delta(U_{t+1}) \right] \xi_{t+1} K_{t+1}}{Q_t K_{t+1}}. \]

where \( \delta(U_{t+1}) \) is the depreciation rate in period \( t + 1 \). Plugging in the first-order condition for labor, we get

\[ R_{k,t+1} = \frac{P_{m,t+1} \alpha Y^I_{t+1} \xi_{t+1} K_{t+1} + Q_{t+1} - \delta(U_{t+1})}{Q_t} \xi_{t+1}. \]  

(22)

Disasters and Disaster Probability

Here we discuss the implementation of time-varying disaster risk in our model. In the disaster process, the economy switches between normal times \((z_t = 0)\) and disaster times \((z_t = 1)\). The Markov transition matrix for this process is

\[
\begin{bmatrix}
1 - p_t & p_t \\
1 - q & q
\end{bmatrix}
\]  

(23)
where $p_t$ is the conditional probability that the economy switches from normal times to disaster times, and $q$ is the conditional probability that the economy remains in disaster times.

We model the productivity variables as

$$A_t = \begin{cases} A_t^0, & z_t = 0 \\ A_t^1, & z_t = 1 \end{cases}, \quad \text{and} \quad \xi_t = \begin{cases} \xi_t^0, & z_t = 0 \\ \xi_t^1, & z_t = 1 \end{cases}. \quad (24)$$

The TFP shocks evolve exogenously as

$$\log \xi_t = \rho_\xi \log \xi_{t-1} + \sigma_\xi \epsilon_{\xi,t} - 1_{(z=1)} \zeta_{\xi,t}, \quad \text{for } z = 0, 1. \quad (25)$$

and the capital quality shocks evolve exogenously as

$$\log A_t = \rho_A \log A_{t-1} + \sigma_A \epsilon_{A,t} - 1_{(z=1)} \zeta_{A,t}, \quad \text{for } z = 0, 1. \quad (26)$$

where $\epsilon_{\xi,t}$ and $\epsilon_{A,t}$ are i.i.d. standard normal disturbances and $\zeta_{\xi,t}$ and $\zeta_{A,t}$ are the disaster size variable, which follows i.i.d. normal $N(\mu_{\zeta,\xi} - \frac{1}{2} \sigma_{\zeta,\xi}^2, \sigma_{\zeta,\xi}^2)$ and $N(\mu_{\zeta,A} - \frac{1}{2} \sigma_{\zeta,A}^2, \sigma_{\zeta,A}^2)$, respectively.

Finally, in line with Gourio (2012), which is in contrast to Barro et al. (2013), we allow the probability of disaster occurrence $p_t$ to be time-varying. More precisely, we assume that

$$\log(p_t) = \rho_p \log(p_{t-1}) + (1 - \rho_p) \log(p_{ss}) + \sigma_p \epsilon_{p,t}, \quad (27)$$

with $\epsilon_{p,t}$ follows i.i.d. $N(0, 1)$.

**Capital Producing Firms**

Here we discuss the implementation of capital producing firms in our model. The representative capital producing firm purchases capital from the intermediate goods
firm at the end of every period. It refurbishes worn-out capital at unit cost. It also produces new capital, which it sells at a price $Q_t$. Producing this new capital is subject to adjustment costs

$$f \left( \frac{I_{n,t} + I_{ss}}{I_{n,t-1} + I_{ss}} \right) (I_{n,t} + I_{ss}),$$

where $I_{n,t} = I_t - \delta \xi_t K_t$ is the new capital created, $I_t$ is the gross investment, and $I_{ss}$ is the steady state of gross investment. The function $f$ is such that $f(1) = f'(1) = 0$ and $f''(1) > 0$. Effectively, we assume the quadratic adjustment cost function

$$f(x) \equiv \frac{\vartheta}{2} (x - 1)^2, \quad \text{with } \vartheta > 0. \quad (28)$$

We assume that the costs of producing new capital is subject to “investment cost shocks” $z_t$. Thus, the discounted value of the profits of the capital goods producer is

$$\max_{\{I_{n,\tau}\}_{\tau \geq 0}} \sum_{\tau = t}^{\infty} \mathbb{E}_t \beta^{\tau-t} \Lambda_t \left[ Q_{\tau} I_{n,\tau} - z_{\tau} I_{n,\tau} - z_{\tau} f \left( \frac{I_{n,\tau} + I_{ss}}{I_{n,\tau-1} + I_{ss}} \right) (I_{n,\tau} + I_{ss}) \right], \quad (29)$$

which gives the first-order condition (Q-relationship)

$$Q_t = z_t \left\{ 1 + f(\cdot) + \frac{I_{n,t} + I_{ss}}{I_{n,t-1} + I_{ss}} f'(\cdot) - \mathbb{E}_t \left[ \beta \frac{\Lambda_{t+1}}{\Lambda_t} \left( \frac{I_{n,t+1} + I_{ss}}{I_{n,t} + I_{ss}} \right)^2 f'(\cdot) \right] \right\}. \quad (30)$$

The investment shock evolves as

$$\log z_t = \rho_z \log z_{t-1} + \sigma_z \epsilon_{z,t}. \quad (31)$$

Finally, capital evolves as

$$K_{t+1} = I_{n,t} + \xi_t K_t = I_t + [1 - \delta(U_t)] \xi_t K_t. \quad (32)$$
Retail Firms

Here we discuss the implementation of retail firms in our model. There is a unit mass continuum of retail firms. Retail firms use the (single) intermediate good to produce differentiated goods. Each firm $f$ can use one unit of the intermediate good to produce one unit of the differentiated good. If $Y_{f,t}$ denotes the final output of the retail firm, the final output composite good is the CES aggregator

$$Y_t = \left[ \int_0^1 Y_{f,t}^{(\varepsilon_t-1)/\varepsilon_t} df \right]^{\varepsilon_t/(\varepsilon_t-1)}, \tag{33}$$

where $\varepsilon_t$ is the elasticity of substitution. The steady state of the elasticity of substitution is $\varepsilon_{ss}$, and the elasticity evolves as

$$\log \varepsilon_t = (1 - \rho_\varepsilon) \log \varepsilon_{ss} + \rho_\varepsilon \log \varepsilon_{t-1} + \sigma_\varepsilon \varepsilon_t. \tag{34}$$

If $P_{f,t}$ is the price that each retail firm charges for its good, then cost minimization for the users of the final good gives

$$Y_{f,t} = \left( \frac{P_{f,t}}{P_t} \right)^{-\varepsilon_t} Y_t, \tag{35}$$

where $P_t$ is the ideal price index

$$P_t = \left[ \int_0^1 P_{f,t}^{1-\varepsilon_t} df \right]^{1/(1-\varepsilon_t)}. \tag{36}$$

The retail firms are monopolistically competitive, and face a downward sloping demand for their goods. At each time $t$ a random fraction $1 - \zeta$ of the retail firms can reset their price. A firm which can reset its price sets it to maximize the discounted value

34
of its future profits as long as it is stuck with that price, i.e.

$$\max_{P_t^*} \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{\Lambda_{t+\tau}}{\Lambda_t} \left( \frac{P_t^*}{P_{t+\tau}} Y_{f,t+\tau} - P_{m,t+\tau} Y_{f,t+\tau} \right) \right] \equiv (37)$$

The first-order condition is

$$\mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{\Lambda_{t+\tau}}{\Lambda_t} \left( P_t \left( \frac{P_t^*}{P_{t+\tau}} \right)^{-\varepsilon_t} - P_{m,t+\tau} \left( \frac{P_t^*}{P_{t+\tau}} \right)^{-\varepsilon_t} \right) Y_{t+\tau} \right] = 0.$$

Defining

$$J_t = \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{\Lambda_{t+\tau}}{\Lambda_t} \left( \frac{P_t}{P_{t+\tau}} \right)^{1-\varepsilon_t} Y_{t+\tau} \right],$$

$$H_t = \mathbb{E}_t \left[ \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{\Lambda_{t+\tau}}{\Lambda_t} \frac{\varepsilon_t}{\varepsilon_t - 1} P_{m,t+\tau} \left( \frac{P_t}{P_{t+\tau}} \right)^{-\varepsilon_t} Y_{t+\tau} \right],$$

we have

$$\frac{P_t^*}{P_t} = \frac{H_t}{J_t}.$$

At any time $t$ a fraction $\varsigma$ of the retail firms will be unable to change their prices and their aggregate price index will be simply $P_{t-1}$, which means that the price indexation parameter $\varsigma_p$ is zero. In general, those firms which cannot adjust their prices at $t$ would have price $\Pi_{t-1}^\varsigma P_{t-1}$. A fraction $1 - \varsigma$ will be able to reset the prices, and they will all reset their price to $P_t^*$. Thus, by the law of large numbers, we
have $P_t^{1-\varepsilon t} = \varsigma P_{t-1}^{1-\varepsilon t} + (1 - \varsigma)P_{t-1}^{1-\varepsilon t}$. Rearranging, and substituting for $P_t^*$ from the first-order condition, we get

$$
\frac{1 - \varsigma \Pi_t^{\varepsilon t-1}}{1 - \varsigma} = \left( \frac{H_t}{J_t} \right)^{1-\varepsilon t},
$$

where $\Pi_{t+1} = P_{t+1}/P_t$ is the inflation. We can rewrite $J_t$ and $H_t$ recursively as

$$
J_t = Y_t + \varsigma \beta \mathbb{E}_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \Pi_t^{\varepsilon t-1} J_{t+1} \right],
$$

$$
H_t = \frac{\varepsilon_t}{\varepsilon_t - 1} P_{m,t} Y_t + \varsigma \beta \mathbb{E}_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \Pi_t^{\varepsilon t} H_{t+1} \right].
$$

The relationship between the intermediate goods output and the final goods output is

$$
Y_t^I = \int_0^1 Y_{f,t}df = \frac{Y_t}{P_t^{\varepsilon t}} \int_0^1 P_{f,t}^{-\varepsilon t}df = \frac{Y_t}{P_t^{\varepsilon t}} \left[ (1 - \varsigma)(P_t^*)^{-\varepsilon t} + \varsigma P_{t-1}^{-\varepsilon t} \right] = Y_t \left[ (1 - \varsigma)\frac{1}{P_t^{\varepsilon t}} (1 - \varsigma \Pi_t^{\varepsilon t-1}) + \varsigma \Pi_t^{\varepsilon t} \right].
$$

**Government Policies**

Here we discuss the implementation of government policies in our New Keynesian DSGE model. First, we specify the interest rate policy. The central bank sets the short-term nominal risk-free interest rate $i_t$ at time $t$. This gives the expression for the real interest rate (possibly random, due to inflation)

$$
R_{t+1} = \frac{1 + i_t}{\Pi_{t+1}}.
$$
The central bank uses a Taylor rule to set the interest rate:

\[ i_t = (1 - \rho_i) [i_{ss} + \kappa_\pi \log \Pi_t + \kappa_y (\log Y_t - \log Y^*_t)] + \rho_i i_{t-1} + m_t, \quad (42) \]

where \( Y^*_t \) is the natural level of output that would hold in a flexible price equilibrium, \( i_{ss} \) is the steady-state nominal interest rate, the smoothing parameter \( \rho_i \) lies between zero and one, and \( \kappa_\pi \) and \( \kappa_y \) are constants greater than 1. Here, \( m_t \) is an exogenous shock to monetary policy. It evolves as

\[ m_t = \rho_m m_{t-1} + \sigma_m \epsilon_{m,t}. \quad (43) \]

Now we specify the credit policy. The central bank is also willing to buy the shares of the intermediate goods firm to facilitate lending. It buys a fraction \( \psi_t \) of the total outstanding shares of the intermediate goods firm, so that

\[ Q_t S_t = \phi_t N_t + \psi_t Q_t S_t, \quad (44) \]

where \( \phi_t \) is the leverage ratio for the privately-held asset, i.e. \( Q_t S_{p,t} \equiv \phi_t N_t \) and the government-held asset is \( Q_t S_{g,t} = \psi_t Q_t S_t \).

We define the total leverage ratio \( \phi_{c,t} \) as follows

\[ Q_t S_t = \phi_{c,t} N_t. \quad (45) \]

The leverage ratio \( \phi_{c,t} \) is the leverage ratio for total intermediated funds, public as well as private, which has the following relation with the private leverage ratio \( \phi_t \) and the intensity of government credit intervention,

\[ \phi_{c,t} = \frac{\phi_t}{1 - \psi_t}. \quad (46) \]
The central bank issues government bonds $B_{gt} = \psi_t Q_t S_t$ to fund the purchase of these shares. From this activity, the central bank thus earns an amount $(R_{k,t+1} - R_{t+1}) B_{g,t}$ every period.

In particular, we assume that at the onset of a crisis, which is defined loosely to mean a period when the credit spread rises sharply, the central bank injects credit in response to movements in credit spreads, according to the following rule for $\psi_t$:

$$\psi_t = \psi_{ss} + \nu \mathbb{E}_t \left[ (\log R_{k,t+1} - \log R_{t+1}) - (\log R_{k,ss} - \log R_{ss}) \right],$$

(47)

where $\psi_{ss}$ is the steady-state fraction of intermediation, $\log R_{k,ss} - \log R_{ss}$ is the steady-state risk premium, and the sensitivity parameter $\nu$ is positive. According to the rule (47), the central bank expands credit as the credit spread increase relative to the steady-state credit spread.

From (46), it is clear that, when the private leverage ratio $\phi_t$ is kept fixed, the expanding credit policy increases the total leverage of financial intermediaries, i.e. $\phi_{c,t}$ rises.

**Government Budget Constraint**

Here we discuss the implementation of governmental budget constraints in our model. The government spends a fraction $g_t$ of output $Y_t$ in period $t$. That is,

$$G_t = g_t Y_t.$$  

(48)

It also funds the central bank’s purchase of shares by issuing purchasing bonds worth $\psi_t Q_t K_{t+1}$. Its revenues include the taxes $T_t$ and the central bank’s income from intermediation $\psi_{t-1} Q_{t-1} K_t (R_{k,t} - R_t)$. Thus, the government budget constraint is

$$G_t + \psi_t Q_t K_{t+1} = T_t + \psi_{t-1} Q_{t-1} K_t (R_{k,t} - R_t).$$

(49)
To simplify our illustration, we assume that government expenditures are exogenously fixed at a constant fraction of output. We denote steady-state government spending by \( g_{ss} \). Government spending evolves as

\[
\log g_t = (1 - \rho_g) \log g_{ss} + \rho_g \log g_{t-1} + \sigma_g \epsilon_{g,t}.
\] (50)

Since the taxation \( T_t \) effectively takes up any slack that shows up on the government balance sheet, and given the existence of representative agents in the economy, the intertemporal budget constraint of the representative household and the intertemporal budget constraint of the government can be combined together with taxes left out. Intuitively, then, by Walras’ Law, both budget constraints are redundant in determining the equilibria. However, this is very different from saying that the size and composition of the government balance sheet are irrelevant for pinning down the equilibrium under efficient financial market conditions, as was proposed by Wallace (1981). This is simply because not all investors can purchase arbitrary amount of the same assets at the same market prices as the government in this model. Put more precisely, unlike private financial intermediation, the government intermediation is not balance sheet constrained.

**Resource Constraint**

The resource constraint for the final good in our model is given by

\[
Y_t = C_t + I_t + f \left( \frac{I_t - \delta(U_t) \xi_t K_t + I_{ss}}{I_{t-1} - \delta \xi_{t-1} K_{t-1} + I_{ss}} \right) (I_t - \delta(U_t) \xi_t K_t + I_{ss}) + G_t + \psi_t Q_t K_{t+1}
\]

3.2.3 Calibration Analysis

We use our model to understand the response of the economy to various shocks. We use a calibrated version of the model, basing our parameter choices mainly on those in Gertler and Karadi (2011) and Gourio (2012), and the estimated dynamic parameters in Smets and Wouters (2007). In particular, we set the steady-state government
expenditure to be $g_{ss} = 20\%$ and the steady-state government credit intervention to be $\psi_{ss} = 0$. These values are close to the average government expenditure and investment in the United States over the time period 1934 to 2010. We note that credit intervention by the U.S. government has historically been negligible, only becoming substantial after the recent crisis. Nevertheless, we include such intervention in our analysis to understand the effects of modern policy responses. The parameter values are summarized in Table 2 and 3.

**Experiment I: Capital Quality Shock**

In our first experiment, we examine the effect of a capital quality shock on our model economy. We argue that an initial adverse disturbance of capital quality can approximately capture a decline in the quality of intermediary assets, leading to a severe decline in the net worth of the financial intermediaries. There are two major effects of the quality shock, one exogenous, and one endogenous. The first effect is the exogenous impact of the destruction of capital on output and asset values. The second effect is endogenous. The balance sheet of intermediaries is weakened by the decline of asset values, and hence intermediaries reduce their demand for investment goods, which suppresses the price of capital $Q$. The endogenous feedback effect of decline in $Q$ is to further weaken the balance sheet of the financial intermediaries.

We choose the shock size to be a 5% deviation from the steady-state $\xi$ level (i.e. $\sigma_\xi = 0.05$). Conditional on occurring, the shock obeys an AR(1) decaying path with a persistent parameter $\rho_\xi = 0.66$. The shock path is displayed in the top-left corner of Figure 2. The real economy’s response to this capital quality shock are shown in Figure 2. The capital quality shock triggers dramatic drops in output, investment, labor, and capital stock. Also, the intermediate good price and capital goods price decline due to the weakened demand for investment. Interestingly, however, we can see that credit policies do not show any significant power to combat the capital quality shock, which can be seen even more clearly in Figure 3. In Figure 3, we find that the change in credit policy lasts for a long period, although the total amount of credit intervention is low overall. The credit policy does bring down the leverage and the
Table 2: Static Parameter Calibration (Quarterly)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Household preference</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount rate</td>
<td>( \beta )</td>
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<td>Relative risk aversion</td>
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<td>Habit parameter</td>
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<td>Inverse Frisch elasticity of labor supply</td>
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<td><strong>Financial intermediaries</strong></td>
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<tr>
<td>Steady-state fraction of divertible capital</td>
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</tr>
<tr>
<td>Proportional transfer to new bankers</td>
<td>( \omega )</td>
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</tr>
<tr>
<td>Survival rate of bankers</td>
<td>( \theta )</td>
<td>0.972</td>
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<tr>
<td><strong>Intermediate good firms</strong></td>
<td></td>
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</tr>
<tr>
<td>Steady-state disaster probability</td>
<td>( p_{ss} )</td>
<td>0.72%</td>
</tr>
<tr>
<td>Probability of going back to normal state</td>
<td>( q )</td>
<td>91.4%</td>
</tr>
<tr>
<td>Average disaster size in ( \xi )</td>
<td>( \mu_{\xi,\xi} )</td>
<td>0.9%</td>
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<tr>
<td>Average disaster size in ( A )</td>
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<td>( \alpha )</td>
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<td>Elasticity of marginal depreciation with respect to utilization rate</td>
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<td>Depreciation rate</td>
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<td><strong>Capital producing firms</strong></td>
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<tr>
<td>Adjustment cost coefficient</td>
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<tr>
<td><strong>Retail firms</strong></td>
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<td>Elasticity of substitution</td>
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<td>Steady state government share of capital</td>
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Table 3: Dynamic Parameter Calibration (Quarterly)

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<thead>
<tr>
<th>Parameter</th>
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<tr>
<td>Capital Quality</td>
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<tr>
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<tr>
<td>Volatility</td>
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<td>Volatility</td>
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<td>Persistence</td>
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<td>Volatility</td>
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<td>Monetary Policy</td>
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<td>Persistence</td>
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<tr>
<td>Volatility</td>
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<tr>
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risk premium for the intermediaries, but only by a moderate amount.

**Experiment II: Margin Shock**

In our second experiment, we examine the effect of a margin shock on our model economy. The crisis scenario of a sudden collapse of funding can be roughly captured by an adverse shock in $\lambda_t$, meaning the deterioration of the pledgeability of the financial intermediaries’ assets. This can also be interpreted as a margin shock, as is emphasized by Geanakoplos (2001, 2009), among others.

The adverse margin shock affects asset value only through the endogenous channel, by deteriorating the capacity of the intermediaries to take on leverage. Like the capital quality shock, it also triggers a feedback effect via the balance sheet of intermediaries.

We choose the shock size to be one standard deviation from the steady-state $\lambda$ level (i.e. $\sigma_\lambda = 0.20$). We use such a large shock to get the impulse response to the real variables to be roughly of the size they were during the recent financial crisis. Conditional on occurring, the shock obeys an AR(1) decaying path $a$ with persistent parameter $\rho_\xi = 0.66$, the same as in experiment I. The shock path is displayed in the top-left corner of Figure 4. The real economy’s responses to this margin shock are shown in Figure 4. The margin shock causes severe but very temporary drops in output, investment, and employment. However, it only generates a small decline in capital stock, different from the response of capital quality shock. Also, the prices of the intermediate good and capital goods decline due to the weakened demand for investment. In contrast to the case of capital quality shock, credit policies are very efficient in alleviating the adverse impact of the margin shock, which can be seen even more clearly in Figure 5. In Figure 5, we find that the credit policy only lasts for about 10 quarters, with a level similar to that of the policy response to the 5% capital quality shock. From Figure 5, it is obvious that the credit policy is a powerful tool to maintain the stability of the financial system when a sudden funding crisis occurs.
Figure 2: Real quantities' response to capital quality shock: 5% deviation from steady state. The solid curve is for the case of zero credit policy intervention ($\nu = 0$). The dashed curve is for the case of moderate credit policy intervention ($\nu = 10$). The dash-dotted curve is for the case of intensive credit policy intervention ($\nu = 100$). The nominal interest rate has a zero lower bound.
Figure 3: Financial variables' response to capital quality shock: 5% deviation from steady state. The solid curve is for the case of zero credit policy intervention ($\nu = 0$). The dashed curve is for the case of moderate credit policy intervention ($\nu = 10$). The dash-dotted curve is for the case of intensive credit policy intervention ($\nu = 100$). The nominal interest rate has a zero lower bound.
Figure 4: Real quantities’ response to intermediary margin shock: one standard deviation from steady state. The solid curve is for the case of zero credit policy intervention ($\nu = 0$). The dashed curve is for the case of moderate credit policy intervention ($\nu = 10$). The dash-dotted curve is for the case of intensive credit policy intervention ($\nu = 100$). The nominal interest rate has a zero lower bound.
Figure 5: Financial variables’ response to intermediary margin shock: one standard deviation from steady state. The solid curve is for the case of zero credit policy intervention ($\nu = 0$). The dashed curve is for the case of moderate credit policy intervention ($\nu = 10$). The dash-dotted curve is for the case of intensive credit policy intervention ($\nu = 100$). The nominal interest rate has a zero lower bound.
Experiment III: Risk Shock

In our third experiment, we examine the effect of a disaster risk shock on our model economy. There is no exogenous direct adverse effect on the fundamentals of the economy when the disaster probability rises high. Agents in the economy adjust their decisions on quantities and asset prices endogenously, purely through the expectations channel. This channel seems particularly relevant to the recent Great Recession. Essentially, our risk shock captures the same effect as the uncertainty shocks or second moment shocks in Bloom (2009), Gilchrist et al. (2010), and Christiano et al. (2014).

We design a large shock in $p_t$ as in Gourio (2012), where the disaster probability $p_t$ starts from 6% (i.e. $\sigma_p = 8.33$) in the first period, and then decays in an AR(1) manner with a persistent parameter $\rho_p = 0.95$. The shock path is displayed in the top-left corner of Figure 6. The real economy’s response to this risk shock is shown in Figure 6. The disaster shock causes severe but very temporary drops in output, investment, and labor, followed by a large “overshooting” in the medium run. The overshooting effect of uncertainty shocks in the medium run has been highlighted in Bloom (2009). This is an interesting feature of risk and uncertainty shocks compared to other shocks, including the capital quality shock and the margin shock examples analyzed above. As with the margin shock, the disaster shock only generates a small decline in capital stock, rather different from the response to the capital quality shock. However, it causes a “disaster” in consumption which takes about 40 quarters to recover. Moreover, the intermediate good price and the capital goods price decline due to the weakened demand for investment. As in the case of the margin shock, and in contrast to the case of capital quality shock, credit policies are very efficient in smoothing out the potentially huge adverse impact of the risk shock. In Figure 7, we find that only a very high level of credit policy intervention lasts for a long time. It is obvious that credit policy is an equally powerful tool to maintain the stability of the financial system in the face of increased risk.
Figure 6: Real quantities’ response to risk shock: 1 standard deviation from steady state. The solid curve is for the case of zero credit policy intervention ($\nu = 0$). The dashed curve is for the case of moderate credit policy intervention ($\nu = 10$). The dash-dotted curve is for the case of intensive credit policy intervention ($\nu = 100$). The nominal interest rate has a zero lower bound.
Figure 7: Financial variables’ response to risk shock: 1 standard deviation from steady state. The solid curve is for the case of zero credit policy intervention ($\nu = 0$). The dashed curve is for the case of moderate credit policy intervention ($\nu = 10$). The dash-dotted curve is for the case of intensive credit policy intervention ($\nu = 100$). The nominal interest rate has a zero lower bound.
3.2.4 Variance Decomposition

In order to better understand the relative importance of the various shocks in our model, we perform a forecast error variance decomposition analysis. We use the calibration in Table 3 for the size and persistence of the shocks. The calibrations for the monetary policy shock, the government spending shock, the price markup shock, the TFP shock and the investment shock are taken from the estimated values in Smets and Wouters (2007). The calibrations for the disaster shock are taken from Gourio (2012). The calibrations for the capital quality shock are taken from Gertler and Karadi (2011). We pick the calibrations for the margin shock based on the description in the impulse response analysis. We assume no credit intervention policies in the analysis.

The variance decomposition at a horizon of 100 quarters for the macroeconomic and financial variables is shown in Figure 8. We find that the capital quality and TFP shocks play a significant role in determining all variables in the long run. This is because these shocks directly affect the productive capacity of the economy. The disaster shock is important for determining the long-run output, investment and level of capital. The monetary policy shock is important for determining the long-run real interest rate, but its effect on inflation in the long run is dominated by other shocks, particularly the TFP shock. The margin shock plays a significant role in determining
the risk premium in the long run. Apart from the investment shock, which has a lingering effect on $Q$ in the long run, other shocks, namely the markup and spending shocks, are relatively insignificant in the long run.

The variance decomposition for output at different horizons is shown in Figure 9. We find that the TFP shock is most important in determining this variable, and its importance increases over time. In the short run, the capital quality and spending shocks are also important, but their effect dissipates in the long run. The persistent disaster shock plays a small but important role in determining output in both the short and long runs.

The variance decomposition for consumption at different horizons is shown in Figure 10. Consumption is mainly driven by the TFP shock and the capital quality shock. The relative importance of these shocks remains stable over time. All the other shocks are insignificant in determining consumption.

The variance decomposition for inflation at different horizons is shown in Figure 11. In the short run, inflation is mostly driven by the monetary policy shock and the disaster shock. The relative importance of the persistent disaster shock peaks in the medium run, as the economy goes into a deleveraging and deflationary spiral. In the
Figure 10: Forecast Variance Decomposition for Consumption at a horizon of 1, 2, 4, 10, 40 and 100 quarters.

Figure 11: Forecast Variance Decomposition for Inflation at a horizon of 1, 2, 4, 10, 40 and 100 quarters.
long run, however, the TFP and capital quality shock assume their generally observed importance.

The variance decomposition for the real interest rate at different horizons is shown in Figure 12. As expected, the monetary policy shock is the key driver of the real interest rate in the short run and the long run. In the short run, the disaster shock also significantly drives the real interest rate, as inflation plummets. The TFP and the capital quality shocks become significant in the long run.

The variance decomposition for the risk premium at different horizons is shown in Figure 13. The most striking characteristic of this figure is the importance of the margin shock in the short run. When the financial intermediary sector is hit by a pledgeability shock, the net worth and the ability to borrow of the intermediary plummets due to the net-worth amplification channel. The low supply of investible risk-free assets causes the interest rate to crash. The inability of the financial intermediaries to finance the intermediate goods producers also lowers the price of capital, which increases the expected return on capital, further increasing the risk premium. The effect of the margin shock on the risk premium dissipates very slowly over time.
3.3 Estimation Analysis

In this section, we perform an estimation exercise to demonstrate the difficulty faced by current estimation methods in correctly applying the data to the model. Since the static parameters of Table 2 are reasonably well understood and estimated, we fix them in place, and only estimate the dynamic parameters for exogenous shocks using a Bayesian method (see, e.g. Smets and Wouters, 2007). To make sure that $\lambda_{ss}$ and $p_{ss}$ are between zero and one, we use the transformations

$$
\lambda_{ss} = \frac{1}{1 + \exp(\lambda_{ss})}, \text{ and}
$$

$$
p_{ss} = \frac{1}{1 + \exp(p_{ss})},
$$

Figure 13: Forecast Variance Decomposition for the Risk Premium at a horizon of 1, 2, 4, 10, 40 and 100 quarters.
We use quarterly data on the data variables

\[
\begin{pmatrix}
    \log(Y_t) - \log(Y_{t-1}) \\
    \log(C_t) - \log(C_{t-1}) \\
    \log(I_t) - \log(I_{t-1}) \\
    \log(W_t) - \log(W_{t-1}) \\
    \log(L_t) - \log(L_{t-1}) \\
    \log(\Pi_t) \\
    i_t \\
    R_{kt}
\end{pmatrix}
\]

from 1948Q2 to 2013Q4 for our estimation. Formally, we calibrate the parameters Θ in Table 2 and estimate the dynamic parameters Ψ. We use the Metropolis-Hasting algorithm to numerically compute the posterior distribution

\[
f(\Psi|\mathbf{Y}_t, \Theta) \propto g(\mathbf{Y}_t|\Psi, \Theta) \cdot p(\Psi),
\]

where \(f(\Psi|\mathbf{Y}_t, \Theta)\) is the posterior distribution of the parameters, \(g(\mathbf{Y}_t|\Psi, \Theta)\) is the likelihood function or the conditional distribution of the observables given the parameters, and \(p(\Psi)\) is the prior distribution of the parameters. We use the Kalman filter to compute the likelihood function. We simulate the posterior using a sample of 4000 draws after dropping 45% of the draws. We report the priors and the estimated mean and the 90% HPD interval in Table 4.

After comparing the seminal work of Smets and Wouters (2007), we include the stock return time series into the scope of our empirical analysis for the DSGE model. The estimation results are summarized in Table 4. Interestingly, it is evident that the estimated shock sizes are unreasonably huge, even after imposing tight priors on the volatilities of the shocks. This is because the likelihood function of the model used in the Bayesian estimation is based on the first-order approximation around the deterministic steady state. The first-order approximated likelihood function brutally suppresses the intentionally significant nonlinear structure of the model design. The nonlinear structural components are critical to capture the wild fluctuations in risk.
premia and their important equilibrium feedback in the real economy. The Bayesian estimation based on first-order approximated likelihood functions performs successfully in DSGE models without financial intermediaries or risk premia data in Smets and Wouters (2007). However, it simply fails in DSGE models with financial frictions, trying to capture the volatile dynamics of risk premia in the data by ignoring the nonlinear features in the model.

When we include the risk aversion and habit parameters in the estimation, the results are similar, with very large estimated values of the shock sizes.

4 Challenges for the Next Generation of New Keynesian DSGE Models

There are a number of model features and quantitative methodologies that are crucial to our understanding of the financial market and the macroeconomy that the standard New Keynesian DSGE models of the current generation (such as the simple canonical example in the previous section) simply do not incorporate. The recent crisis and recession have put many of these missing pieces into the spotlight. It is evident that these missing pieces have a first-order impact on the economy as whole, and have profoundly affected how governments have conducted their policies. In this section, we discuss these major missing components and methodological challenges. We hope to shed some light on the path along which researchers may advance current New Keynesian DSGE models to the next generation, one which will be more useful to monetary authorities. The issues of the current generation of New Keynesian DSGE models and the challenges of future improvements to these models are fundamentally and deeply interconnected. Therefore, in order to truly improve these models in one dimension, we may need to simultaneously tackle all the others to some degree.

4.1 The Irrelevance of the Government Balance Sheet

Classic monetary macroeconomic theory, as used in modern macroeconomic models, taught in graduate school textbooks, and employed by major central banks all over
<table>
<thead>
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<th>Parameter</th>
<th>Description</th>
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<th>Posteriors</th>
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<tr>
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<td>$\sigma_z$</td>
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<td>Inv Gam</td>
<td>0.05</td>
</tr>
</tbody>
</table>
the world, starts from the simple national income accounting identity

\[ Y = C + I + G + X, \]

where \( Y \) is the aggregate output of the economy, \( C \) is the aggregate household consumption, \( I \) is the aggregate investment, \( G \) is the government spending, and \( X \) is the net export. The only role played by government in this model is through government spending, whose dynamics are specified exogenously. In other words, the effects of the government balance sheet and any intertemporal budget constraint on government are totally abstracted out of the analysis. This omission is not just some reduced-form modeling trick to simplify the analysis of monetary policy. In fact, the omission of the government balance sheet is completely justifiable in terms of both legislative practice and fundamental economic principles.

In legislative practice, monetary policy decisions by law are independent of government, i.e. the fiscal anchor is independent of the monetary anchor, although the monetary anchor and the fiscal anchor inevitably have interactions. These monetary-fiscal interactions mainly include: (1) interest rate changes, leading to changes in the “interest expense” item in the government budget, thereby leading to changes in the growth rate of government debt, which of course depends on whether taxes and expenditures react to the original changes in interest rate, and if so, by how much; (2) central banks holding earning assets (usually bonds) to back the currency they issue (which does not earn interest), giving the banks a stream of revenue (so-called “seigniorage”), which they generally turn over to the treasury (i.e. the government); (3) increased inflation reducing the real burden of the stream of future payments specified in long-term government bonds. As emphasized by Sims (2008), monetary independence could be sustained on a fair level because, up to 2007, there had been little risk on the Federal Reserve’s balance sheet. Its liabilities were mainly currency outstanding and reserve balances, and its assets were mainly short-term U.S. government debt. More precisely, before 2007, there was little risk on the U.S. Fed’s balance sheet because (1) while exchange rate movements or inflation can change the value of the dollar, since assets and liabilities were all in dollars, there was no effect on net
worth; (2) changes in long-term interest rates can change the market value of long bonds, but since the assets were mainly short term, this effect had a minor effect; (3) the U.S. government was extremely unlikely to default outright on its nominal bonds, in part because under conditions where this might be an attractive possibility, inflation to reduce the value of the debt would be easier and more efficient. Therefore, it is fair to assume that the government balance sheet plays a very limited role in U.S. Federal Reserve’s monetary policy decisions before 2007.

The justification from fundamental economic principles is more involved. The efficiency of the financial market is the key. More precisely, the financial market needs to be efficient enough so that the following assumptions are satisfied:

(1) assets are valued only for their pecuniary returns. This means that assets only fail to be perfect substitutes from the standpoint of investors due to their different risk characteristics, but not due to any other reasons.

(2) all investors can purchase arbitrary amounts of the same assets at the same market prices as the government.

(3) the government conducts a Ricardian fiscal policy, indicating that the government budget constraint must be satisfied for all realizations of the price level (see, e.g. Woodford, 1995). In the presence of multiple equilibria, a non-Ricardian spending or tax policy can trim the set of monetary policy-derived equilibria, as we discuss in Section 4.1.1.

Under these assumptions, the government balance sheet has no impact on the equilibrium of the economy, and hence neither does the open-market purchase of securities by the government. Thus, monetary policy models need only assume a government printing press which creates additional “money” at a greater or lesser rate, which is then put in the hands of private parties, perhaps by dropping it from helicopters. These assumptions lie at the heart of the classic monetarist view: the amount of

However, the story for Europe is very different. The European Central Bank (ECB)’s assets and liabilities are denominated in different currencies, because they have large non-euro reserves. Many other major economies in the world also face the same situation. In addition, it is unique to the ECB that there is no single fiscal counterpart to pressure them over seigniorage or interest expense.
monetary liabilities by the central bank matters for macroeconomic equilibrium, but it does not matter at all what kinds of assets might back those liabilities on the other side of the balance sheet, or how the base money gets to be in circulation.

The irrelevance or neutrality of the government balance sheet in determining market equilibrium is essentially the theoretical macroeconomic analog to the Modigliani-Miller Theorem in corporate finance, as noted in the seminal work by Wallace (1981). In that paper, the author emphasized that this result of irrelevancy implies that both the size and the composition of the central bank or government balance sheet should be irrelevant for market equilibrium in a world with frictionless financial markets (or more precisely, a world in which the above postulates hold). Similar to Wallace (1981), Eggertsson and Woodford (2003) derive a neutrality result in a New Keynesian model. In their framework, which assumes Ricardian fiscal policies, the portfolio of assets held by the central bank is irrelevant towards determining the set of equilibrium output and price levels. This does not, however, mean that monetary policy is irrelevant in such a world, as is sometimes thought; it simply means that monetary policy cannot be implemented through open-market operations. Control of the short-term nominal interest rate by the central bank remains possible in a frictionless environment. The central bank is still free to determine the nominal interest rate on overnight balances at the central bank. This interest rate must then be linked in equilibrium to other short-term interest rates, through arbitrage relations; and hence the central bank can determine the level of short-term nominal interest rates in general. Moreover, the central bank’s adjustment of nominal interest rates matters for the economy as a whole. Even in an endowment economy with flexible prices for all goods, the central bank’s interest rate policy can determine the evolution of the general level of prices in the economy. In a production economy with sticky prices and/or wages, it can have important real effects as well. However, even in this classic model, the effectiveness of short-term nominal interest rate policies depends heavily on the absence of arbitrage in the financial market, a condition which can be significantly violated.

The irrelevance result can be easily understood in a representative agent setting, although the result does not depend on the representative agent assumption. In representative-household theory, the market price of any asset should be determined
by the present value of the random returns to which it is a claim, where the present value is calculated using an asset pricing kernel (a stochastic discount factor) derived from the representative household’s marginal utility of income in different future states of the world. Insofar as a mere reshuffling of assets between the central bank and the private sector should not change the real quantity of resources available for consumption in each state of the world, the representative household’s marginal utility of income in different states of the world should not change. Hence the pricing kernel should not change, and neither should the market price of one unit of a given asset, assuming that the risky returns to which the asset represents a claim have not changed. More intuitively, if the central bank takes more risky securities onto its own balance sheet, and allows the representative household to hold only securities that pay as much in the event of a crash as in other states, this does not make the risk disappear from the economy. The central bank’s earnings on its portfolio will be lower in the crash state as a result of the asset exchange, and this will mean lower earnings distributed to the treasury, which will in turn mean that higher taxes will have to be collected by the government from the private sector in that state; so the representative household’s after-tax income will be just as dependent on the risk as before. This explains why the asset pricing kernel does not change, and why asset prices are unaffected by open market operations.

A similar result can also be derived when there are heterogenous agents in the economy. If the central bank buys more of asset X by selling shares of asset Y, private investors should wish purchase more of asset Y and divest themselves of asset X by exactly the amounts that undo the effects of the central bank’s trades. The reason that they optimally choose to do this is in order to hedge the additional tax/transfer income risk that they take on as a result of the change in the central bank’s portfolio. If share $\theta_h$ of the returns on the central bank’s portfolio are distributed to household $h$, where the \{\theta_h\} are a set of weights that sum to 1, then household $h$ should choose a trade that cancels exactly fraction $\theta_h$ of the central bank’s trade, in order to afford exactly the same state-contingent consumption stream as before. Summing over all households, the private sector chooses trades that in aggregate precisely cancel the central bank’s trade. The result holds even if different households have very different
attitudes toward risk, different time profiles of income, different types of non-tradeable income risk that they need to hedge, and so on, and also regardless of how large or small the set of marketed securities may be. One can easily introduce heterogeneity of the kind that is often invoked as an explanation of time-varying risk premia without implying that any “portfolio balance” effects of central bank transactions should exist.

In fact, the portfolio balance effect is contrary to the proposition that the balance sheet size and composition are irrelevant. The portfolio balance effect of central bank transactions means that if the central bank holds less of certain assets and more of others, then the private sector is forced to hold more of the former and less of the latter as a requirement for equilibrium, and a change in the relative prices of the assets will almost always be required to induce the private parties to change the portfolios that they prefer. Therefore, portfolio balance effects imply that open market purchases of securities by the central bank must inevitably affect the market prices of those securities and hence other prices and quantities as well.

However, the recent financial crisis and the Great Recession taught us that all the assumptions that guarantee the irrelevance of the government balance sheet can be violated. First, we can see in Table 5 that from October 2007 to October 2008 the size, composition and risk characteristics of the U.S. Federal Reserve’s balance sheet changed dramatically. By October 22, 2008, its assets were no longer mainly government bonds. Through the open market purchase programs, the Fed had built up a new balance sheet with assets mainly consisting of risky loans from private sector. These assets could potentially have suffered substantial capital loss not offset by reductions in the liabilities. On the liability side, we see that more than 25% of its liabilities were in the form of special deposits from the U.S. Treasury. This made the Federal Reserve’s independence fragile, and the government balance sheet began to play a potentially important role in monetary policy.

From Figure 14, we can see that the component of the Fed’s liabilities constituted by reserves held by depository institutions changed in an especially remarkable way: by the fall of 2008, reserves were more than 100 times larger than they had been only a few months earlier. This explosive growth led some commentators to suggest that the main instrument of US monetary policy had changed from an interest rate policy to
Table 5: The U.S. Fed Balance Sheet – Assets and Liabilities

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<tr>
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<td>29,137</td>
<td>29,137</td>
</tr>
<tr>
<td>Float</td>
<td>-2,476</td>
<td>-1,048</td>
<td>-558</td>
</tr>
<tr>
<td>Central Bank Liquidity Swaps</td>
<td>33,315</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Other Federal Reserve Assets</td>
<td>90,476</td>
<td>522,906</td>
<td>481,050</td>
</tr>
<tr>
<td><strong>Gold Stock</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>11,041</td>
<td>11,041</td>
<td>0</td>
</tr>
<tr>
<td><strong>Special Drawing Rights Certificate Account</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5,200</td>
<td>2,200</td>
<td>0</td>
</tr>
<tr>
<td><strong>Treasury Currency Outstanding</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>42,605</td>
<td>38,773</td>
<td>389</td>
</tr>
<tr>
<td><strong>Liability Items</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Currency in Circulation</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>913,756</td>
<td>854,517</td>
<td>41,706</td>
</tr>
<tr>
<td><strong>Reverse Repurchase Agreements</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Foreign Official and International Accounts</td>
<td>65,737</td>
<td>98,110</td>
<td>61,384</td>
</tr>
<tr>
<td>Dealers</td>
<td>65,737</td>
<td>73,110</td>
<td>36,384</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>25,000</td>
<td>25,000</td>
</tr>
<tr>
<td><strong>Treasury Cash Holdings</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>284</td>
<td>276</td>
<td>-46</td>
</tr>
<tr>
<td><strong>Deposits with FR Banks</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U.S. Treasury, General Account</td>
<td>86,496</td>
<td>554,927</td>
<td>542,431</td>
</tr>
<tr>
<td>U.S. Treasury, Supplementary Financial Account</td>
<td>43,241</td>
<td>23,166</td>
<td>18,120</td>
</tr>
<tr>
<td>Foreign Official</td>
<td>29,992</td>
<td>524,771</td>
<td>524,771</td>
</tr>
<tr>
<td>Service-Related</td>
<td>2,297</td>
<td>254</td>
<td>155</td>
</tr>
<tr>
<td>Required Clearing Balances</td>
<td>3,237</td>
<td>6,138</td>
<td>-441</td>
</tr>
<tr>
<td>Adjustments to Compensate for Float</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Other</td>
<td>7,730</td>
<td>598</td>
<td>289</td>
</tr>
<tr>
<td><strong>Other Liabilities and Capital</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>61,537</td>
<td>46,213</td>
<td>4,273</td>
</tr>
</tbody>
</table>
one often described as “quantitative easing”. It seems that quantitative easing became the only important monetary policy decision once the overnight rate (the federal funds rate) reached the zero lower bound, as it effectively has in the U.S. since December 2008. From Figure 15, the past two years have also seen dramatic developments in regard to the composition of the asset side of the Fed’s balance sheet. Whereas prior to the fall of 2007, the Fed had largely held Treasury securities on its balance sheet, other kinds of assets have rapidly grown in importance. A variety of new “liquidity facilities”, new programs under which the Fed essentially became a direct lender to certain sectors of the economy, targeted purchases of certain kinds of assets, including more than a trillion dollars’ worth of mortgage-backed securities. Decisions about the management of these programs have occupied much of the attention of policymakers during the recent period.

Moreover, financial market frictions apparently affected the transition dynamics of monetary policy in a nontrivial way. Indeed, there is some evidence suggesting that at least some of the Fed’s special credit facilities, and similar programs of the other central banks, have affected asset prices. As a simple example, Figure 16 shows the behavior of the spreads between yields on various categories of commercial paper and the one-month overnight interest-rate swap rate (essentially, a market forecast of the average federal funds rate over that horizon), over the period just before and after the introduction of the Fed’s Commercial Paper Funding Facility at the beginning of October 2008. (The darkest solid line shows the quantity of purchases of commercial paper by the Fed, which spikes up sharply at the introduction of the new facility.) The reason for the introduction of the new facility had been a significant disruption of the commercial paper market, indicated by the explosion of spreads in September 2008 for all four of the types of commercial paper shown in the figure. The figure also shows that spreads for three classes of paper (all except the A2/P2 paper) came back down again immediately with the introduction of the new facility, these three series being precisely the ones that qualified for purchases under the CPFF. In contrast, the spread for the A2/P2 paper remained high for several more months, though this spread as well returned to more normal levels eventually, with the general improvement of financial conditions. Spreads did not decline in the case of paper not eligible for
Figure 14: Liabilities of U.S. Federal Reserve. (Source: Federal Reserve Board)
Figure 15: Assets of U.S. Federal Reserve. (Source: Federal Reserve Board)
purchase by the new facility, suggests that targeted asset purchases by the Fed did change the market prices of the assets.

During the recent crisis, conventional monetary policy measures, such as targeting a short-term nominal interest rate, had negligible effect given the zero lower bound. As a result, unconventional policy measures have become of great importance. Unconventional measures which use the central bank balance sheet as an instrument include: (1) changes in the supply of bank reserves beyond those required to achieve an interest rate target, (2) changes in the assets acquired by central banks (e.g. “quantitative easing” and “credit easing”), and (3) changes in the interest rate paid on reserves. In order to analyze these unconventional monetary policies, we need to extend the standard New Keynesian DSGE model to allow a role for the government balance sheet in equilibrium determination, and we need to consider the connections between these alternative monetary policy measures and traditional interest rate policy. For example, Crdia and Woodford (2011) extended the standard New Keynesian DSGE model by allowing a transactions role for central bank liabilities, and heterogeneous households to guarantee that the government balance sheet has a nontrivial effect on determining equilibrium. This allows Crdia and Woodford (2011) to provide a framework to analyze unconventional monetary policy measures. In addition, Gertler and Karadi (2011) develop a quantitative monetary DSGE model with financial intermediaries that face endogenously determined balance sheet constraints to evaluate the effects of the central bank using unconventional monetary policy to combat a simulated financial crisis.

One important channel through which the balance sheet takes effect is the intertemporal budget constraint

\[
\text{real value of net government liabilities at period } t = \sum_{k=t}^{\infty} \mathbb{E}_t \left[ \frac{\Lambda_{t+k}}{\Lambda_t} \left( s_k + \frac{i_k M_k}{1 + i_k P_k} \right) \right]
\]

where \( \Lambda_t \) is state price density in period \( t \), \( M_k \) is the money supply, \( P_k \) is the price level, and \( s_k \) is the real primary government budget surplus, which is the difference between revenues from taxes, real investments, the premium from insurance/guarantees, the
Figure 16: Spreads between yields on four different classes of commercial paper and the 1-month OIS rate, together with the value of paper acquired by the Fed under its Commercial Paper Funding Facility. (Source: Federal Reserve Board.)
assets held by the central bank and the treasury, etc. on the one hand, and the costs of capital for interest expenses on government debt, real investments, insurance payments, etc. on the other. Lucas (2012) reviews the theoretical and practical rationale for treating market risk as a cost to governments, presenting an interpretive review of the growing literature that applies the concepts and tools of modern finance to evaluating the costs of government policies and projects. Lucas (2012) stresses that governments typically understate their cost of capital because they identify it with their borrowing costs, rather than with a rate of return commensurate with the risk of a project. A consequence is that the official cost estimates for many government investment and financial activities are significantly understated. However, in a few cases risk adjustment lowers estimated cost relative to official estimates. Lucas (2012) emphasizes that when the financial market is incomplete, the choice of appropriate state price density $\Lambda_k$ becomes critical, and in practice rather tricky. In such a complex case, different cash flows could require different stochastic discount factors. However, there is still a debate on whether the government balance sheet is constrained.

4.1.1 Fiscal Theory of the Price Level

Many specifications of monetary policy by themselves fail to determine a unique equilibrium in their inflation dynamics. These multiple equilibria arise when monetary policy is the sole focus of these models, the government budget constraint is ignored, and it is assumed that fiscal policy is completely accommodating to monetary policy. However, as pointed out by Leeper (1991), Sims (1994), and Woodford (1994, 1995), an active fiscal policy will be able to, in the words of Cochrane (2011), trim the set of equilibria, and achieve not just a determinate solution for inflation, but also for the price level. We elaborate this point below using a simple model from Cochrane (2011).

We consider a deterministic, perfect foresight model with the representative consumer maximizing the utility

$$\sum_{t=0}^{\infty} \beta^t u(C_t).$$

(51)
Every period, the consumer can buy (or sell) one-period bonds which pay the nominally risk-free interest rate \(i_t\). Denote by \(B_t\) the number of such bonds bought by the consumer at time \(t\). The consumer maximizes utility subject to the present value budget constraint

\[
\sum_{t=0}^{\infty} Q_{0,t} P_t C_t = B_{-1} + \sum_{t=0}^{\infty} Q_{0,t} P_t (Y_t - T_t).
\]

Here, \(Y_t\) is the exogenous output, which we assume to be a constant \(Y\), \(C_t\) is the consumption, \(P_t\) is the price level, \(T_t\) are the (real) lump-sum taxes. The government has no expenditures. For \(t < s\), the nominal discount rate

\[
Q_{t,s} = \Pi_{j=0}^{s-t-1} \left( \frac{1}{1 + i_{t+j}} \right),
\]

where \(i_t\) is the nominal interest rate set by the central bank.

The stochastic discount factor used by the consumer to value nominal claims is

\[
\frac{\Lambda_{t+1}}{\Lambda_t} = \beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t}{P_{t+1}}.
\]

Since \(C_t = Y\) for all \(t\), we have

\[
\frac{\Lambda_{t+1}}{\Lambda_t} = \beta \frac{1}{\Pi_{t+1}},
\]

where \(\Pi_{t+1} = P_{t+1}/P_t\) is the inflation rate. The consumer Euler equation for the risk-free bond is

\[
1 = \frac{\Lambda_{t+1}}{\Lambda_t} (1 + i_t),
\]
which gives the Fisher relation, by defining the real interest rate as $1 + r = 1/\beta$,

$$1 + i_t = (1 + r)\Pi_{t+1}. \quad (53)$$

Substituting the Fisher relation into the budget constraint (52), we get

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \frac{T_{t+j}}{(1 + r)^j}. \quad (54)$$

The last equation represents the government budget constraint, which must hold by Walras' law. It says that the real value of government debt must be equal to the net present value of its tax revenues.

We now have three sequences of variables $\{i_t, B_t, T_t\}$ which can be controlled by the government. However, only two of these can be independent because the government budget constraint

$$(1 + i_{t-1})B_{t-1} = B_t + P_tT_t$$

must hold period-by-period. Suppose then that the government controls $\{i_t, T_t\}$, with the monetary authority setting $\{i_t\}$ and the fiscal authority setting $\{T_t\}$.

Let us see now whether the monetary policy $\{i_t\}$ can, by itself, determine the price level. Suppose this policy is given by

$$1 + i_t = (1 + r)\Phi(\Pi_t). \quad (55)$$

Combining Equations (53) and (55), we get the difference equation for inflation

$$\Pi_{t+1} = \Phi(\Pi_t).$$

Clearly, in general, without an initial or terminal condition for $\Pi_t$, the monetary
policy by itself will not determine the inflation rate, let alone the price level. A Taylor rule implies a locally deterministic equilibrium if we ignore equilibria with explosive inflation dynamics. Figure 17 shows this point. If \( \Pi^* \) is the desired optimum level of inflation, a Taylor rule should have \( \Phi'(\Pi^*) > 1 \). However, this is an unstable equilibrium. As pointed out by Cochrane (2011), the only reason that the inflation \( \Pi^* \) will hold is that for any other starting \( \Pi_0 \neq \Pi^* \), the monetary authority would threaten to “blow up” the economy by creating explosive inflation. Cochrane (2011) points out that “ruling out” such explosive equilibria has no basis either in economic theory (since it is only inflation, or the nominal side, that is blowing up, not the real side, such as the asymptotic value of the debt), or in economic history, which has many recorded instances of hyperinflation. Thus, such explosive equilibria are also valid solutions in addition to \( \Pi^* \). Moreover, since \( i_t \geq 0 \), we must have a stable equilibrium \( \Pi_L \) where \( \Phi'(\Pi_L) < 1 \), and to which any path of inflation starting from \( \Pi_0 < \Pi^* \) must converge. This adds to the multiplicity of equilibria.

All such equilibria, however are valid only because we have ignored the government budget constraint (54), and assumed a “Ricardian” fiscal policy. A Ricardian fiscal policy means that the budget constraint must hold for all price levels, and taxes must adjust to accommodate the price level obtained from monetary policy dynamics.
An “active” (in the sense of Leeper, 1991) or non-Ricardian (Woodford, 1994) fiscal policy would force exogenous values of \( \{T_t\} \) such that only one price level given by the budget constraint (54) can hold as an equilibrium. The policy is called non-Ricardian because it does not satisfy the government budget constraint for all given realizations of the price level. In other words, the government budget constraint is satisfied on the equilibrium path, but not necessarily off it. Non-Ricardian policies trim the set of equilibria, or in this case, select one equilibrium, from the many that are implied by the monetary policy. With reference to Figure 17, the non-Ricardian fiscal policy would select a starting \( \Pi_0 \), following which inflation would follow a deterministic path to either \( \Pi_L \) or \( \Pi^* \).

To avoid explosive inflation dynamics (rather than ruling them out ad hoc), the following combination of active fiscal policy and passive monetary policy is very effective. The active fiscal policy takes the form of non-Ricardian policies as above, while the passive monetary policy in this case means that the interest rate should react less than one-for-one to inflation, or \( \Phi'(\Pi^*) < 1 \). This would ensure stability and convergence to \( \Pi^* \) of any equilibrium path, the start \( \Pi_0 \) of which would be determined by the active fiscal policy. See Figure 18.

The multiplicity of equilibria is a common feature in New Keynesian models. The
key to determining a unique equilibrium, when monetary policy is unable to determine the equilibrium alone, is the specification of additional policies to the monetary policy. Optimal policy prescriptions in zero-lower-bound situations, such as Werning (2011), must rely heavily on the Taylor rule to select the right equilibrium path. The Taylor rule, however, generates uniqueness only locally, not globally, as seen in Figure 17, and that after ruling out ad hoc explosive inflation dynamics. Quantitative easing, if we interpret it as an active policy for long-term interest rates, can help select the appropriate non-deflationary equilibrium in such cases. The crux of the ability to set long-term interest rates independently of the short-term rates can only be seen in a non-linearized model with uncertainty. Changing the long-term interest rate independently of the short-term rate implies reweighting the future short term rates by a different stochastic discount factor (Muley, 2013).

4.2 Heterogeneity Irrelevance

In the core monetary models employed by major central banks, the existence of a representative agent is assumed, as seen in the simple canonical model we illustrated in Section 3.2.2. It has been widely recognized that important features of macroeconomic data are difficult or impossible to explain within the representative agent framework. These include the cyclical behavior of the factor shares of national income, the cyclical behavior of the large risk premium (see, e.g. Basak and Cuoco, 1998; Guvenen, 2009), the cyclical behavior of cross-sectional stock returns (see, e.g. Fama and French, 1992), the cyclical behavior of the distribution of income and wealth and leverage of households, and the cyclical behavior of the distribution of leverage, asset and cash holdings in firms. The data also show strong cyclical patterns of capital, labor and credit reallocation among firms, as well as cyclical behavior of in the bankruptcy rate of firms, entries and exits of firms, and mergers and acquisitions among firms (see, e.g. Eisfeldt and Rampini, 2006, 2008). These behaviors all should have significant equilibrium effects on aggregate quantities. One important feature in the data is that households do not mainly save by themselves to finance their consumption; instead, one side of households finances the other side’s consumption (see, e.g. Guerrieri and
Theoretically, the representative agent can be justified by the assumption of complete markets. Complete market conditions imply perfect insurance for agents in the economy, or that insurance in the economy is costless. However, in reality, the profits of insurance companies in the U.S. and costly transactions and portfolio constraints demonstrate the violation, or at least the poor approximation, of the complete market assumption. Also, under assumptions like a time-separable utility function and the stationarity of the economy, dynamically complete market models predict the capital structure of firms and the asset portfolio of households to be constant, even when the investment opportunity varies over time and there are heterogeneous agents (see e.g. Judd et al., 2003). This is largely contradicted by the real data. Moreover, Sims (2006) argues that there is no overall aggregate capital and no aggregate consumption good, and that the real economy has a rich array of financial markets, which models have so far not included in a widespread or successful manner. Aggregate identities are problematic when we realize that different agents in the economy deal with uncertainty and risk differently.

There are three main reasons for incorporating rich heterogeneity into macroeconomic models. First, it is crucial not to ignore the significant equilibrium effects of the distribution of wealth, income, leverage, cash holdings, etc. on the aggregate quantities and the transitional dynamics of various monetary policies (see e.g. Guerrieri and Lorenzoni, 2012). Moreover, from first principles, the cross-sectional distribution serves as an infinite-dimensional important endogenous state variable, and hence it has a strong impact on equilibrium. Second, heterogeneity makes it possible to analyze the equilibrium effect of extensive margins on the aggregate quantities and the transitional dynamics of monetary policies. Third, the heterogeneity of agents provides a framework to assess the welfare properties of different monetary policy measures, fiscal policies, and other government policies, including unemployment insurance and social security programs.

The key feature of models with heterogeneous agents that makes them different from representative agent models is that the set of of possible trades available for agents is restricted. The trading restrictions are usually modeled either in the form of
an incomplete set of Arrow-Debreu securities, or in the form of portfolio constraints, or in the form of trading frictions. This prevents various aggregation results from holding (see e.g. Deaton, 1992). For example, Constantinides and Duffie (1996) shows that the heterogeneous agent model with incomplete markets can be represented as several homogeneous agent models, as long as the shocks in the model are such that agents do not gain anything by trading, even in the presence of those markets. Therefore, computing the equilibrium requires keeping track of the distribution of agents. In each period of time, the state of the economy is characterized by exogenous state variables driven by exogenous shocks, and by endogenous state variables whose law of motions are endogenously determined in the economy. The endogenous state variables usually include the distribution of agents. The difference between these distributions and distributions from ordinary endogenous state variables, such as capital stock, is that they are usually infinitely-dimensional mathematical objects. The equilibrium prices and quantities are functions of the (potentially infinitely-dimensional) endogenous and exogenous state variables. The law of motion for endogenous state variables is usually called the transition map. Because the law of motion for endogenous state variables is an equilibrium output, it is intrinsically difficult to solve the transition map for a infinite-dimensional state variable. The aggregate level of prices and quantities is not enough to characterize the state of the economy or to predict future endogenous state variables. It is the key feature distinguishing the economy of heterogeneous agents and incomplete markets from the representative agent economy with complete markets:

“No information available in period $t$ apart from the level of consumption, $c_t$, helps predict future consumption, $c_{t+1}$, in the sense of affecting the expected value of marginal utility. In particular, income or wealth in period $t$ or earlier [is] irrelevant, once $c_t$ is known.” Hall (1978, corollary 1, page 974).

The heterogeneous agent model requires to solve the policy functions and the transition map simultaneously. Mathematically, solving the equilibrium amounts to solving a fixed-point problem for an infinitely-dimensional object. The standard numerical methods include discretization of the state space, parameterization of distributions, backward and recursive methods, and so on. To see the extra computational complexity caused by an endogenous transition map more clearly, we note that solving
the problem of the agent for a given law of motion of the endogenous state variables (i.e. the transition map) is not enough. The correct transition map has to be found at the same time. This requires a double-layer iteration algorithm. To circumvent the complexity of the double-layer iteration algorithm, or to avoid iterations on the transition map, one needs to prevent the distribution of agents from affecting relative prices. Preventing this dramatically simplifies the computations. One example is Aiyagari (1994) which focuses on the steady state of an economy without aggregate fluctuations. In similar fashion, Imrohoruglu (1989) utilizes a storage technology that pins down the rate of return of savings exogenously, while Díaz-Giménez (1990) assumes that the government commits itself to a specific inflation rate policy that does not depend on the asset distribution.

4.2.1 Heterogeneity and New Keynesian Models

Heterogeneous agent models have lately been incorporated into the New Keynesian DSGE framework to study the effects of monetary policy. Algan and Ragot (2010) show the importance of a new precautionary savings motive in an incomplete market model in which the traditional redistributive effects of inflation are also introduced. The paper also shows that the long-run neutrality of inflation on capital accumulation obtained in complete market models does not hold under household binding credit constraints. They demonstrate that there is a quantitative rationale for the observed hump-shaped relationship between inflation and capital accumulation. Borrowing-constrained households are not able to adjust their money holdings differently compared to unconstrained households, since they cannot rebalance their financial portfolio when fluctuations in inflation become large. Inflation therefore increases capital accumulation due to the precautionary saving motive under heterogeneity.

It is necessary to understand heterogeneity to better study the redistribution effects of monetary policy. Gornemann et al. (2012) show that heterogeneous workers vary in their employment status due to search and matching frictions in the labor market, their potential labor income, and their amount of savings. This New Keynesian model quantitatively assesses who stands to gain or to lose from unanticipated monetary
accommodation, and who benefits the most from systematic monetary stabilization policy. This paper finds substantial redistribution effects from monetary policy shocks. A contractionary monetary policy has opposing effects on the wealthiest 5 percent versus the rest of the population. The top 5 percent enjoy increases in income and welfare, while the remaining 95 percent suffer under a contractionary monetary policy shock. Consequently, the negative effect of a contractionary monetary policy shock to social welfare is larger if heterogeneity is taken into account.

4.2.2 Heterogeneity is Not New in Macroeconomics

The effects of heterogeneity have long been studied in macroeconomics, leading to its serious adoption in New Keynesian DSGE models. Imrohoruglu (1989) is perhaps the first published paper to compute the equilibrium of a model with heterogeneous agents, and to calibrate it to match key U.S. observations. Imrohoruglu (1989) considers different institutional market arrangements under three different environments, and it also evaluates the welfare difference across institutional market arrangements. Similar welfare differences indicate that the existence of liquidity constraints in an economy is trivial for welfare considerations. Díaz-Giménez (1990) explores the business cycle implications of alternative insurance technologies using a similar methodology to Imrohoruglu (1989), which could be easily adjusted to study the welfare effects of monetary and fiscal policy. Díaz-Giménez (1990) compares perfect insurance and monetary arrangements with pervasive liquidity constraints, finding that the welfare costs of monetary arrangement were 1.25 percent of output in zero-inflation economies. Hansen and Imrohoroglu (1992) find that the optimal level of unemployment insurance is very low, even when there is a very small amount of moral hazard.

Huggett (1993) explains the puzzle of very low risk-free interest rates in the postwar period in the United States by assessing the importance of the role played by the lack of insurance. Huggett (1993) does not have aggregate uncertainty, and assumes an economy that agents, subjected to idiosyncratic labor market shocks of the same type as in Imrohoruglu (1989), can lend and borrow up to certain limits, at a rate that is endogenously determined by nontrivial market-clearing conditions, which are
necessary to solve for the equilibrium of this economy.

Aiyagari (1994) describes two features in this paper: the first, an endowment economy that has no possibilities to save as a whole; the second, the level of aggregate savings affecting the society’s ability to produce goods. Aiyagari (1994) incorporates these two by using the standard neoclassical growth model with production. In order to measure the size of the role of precautionary savings, especially those motivated by self-insurance against idiosyncratic risk, Aiyagari (1994) has to deviate from the endowment economy setting from Huggett (1993). Aiyagari (1994) finds that with moderate and empirically plausible parameter values, uninsured idiosyncratic risk accounts for a 3 percent increase in the aggregate savings rate.

Krusell and Smith (1998) propose an important method for solving models with heterogeneity and aggregate uncertainty. When there are aggregate shocks in the model, the entire wealth distribution is an endogenous state variable, but its distribution can be approximated by its first few moments. The authors find that this approximate aggregation is reasonable, and to forecast future prices and quantities, it is enough to use the mean wealth instead of the entire cross-sectional wealth distribution. The distribution of wealth is unimportant to aggregate quantities such as aggregate consumption, when most agents have the same marginal propensity to consume after aggregate shocks. Most agents achieve good self-insurance in the model, which is equivalent to saying that the consumption policy functions are roughly linear. Aggregate capital by design is three times larger than output, and therefore most agents are rich enough to almost completely smooth out shocks. Only very poor agents, who account for a small fraction of aggregate consumption, do not have self-insurance. Krusell and Smith (1998) conducted an experiment to compare the model under complete market and incomplete market conditions, and found that heterogeneity has little effect on the model’s business cycle properties.

4.2.3 Liquidity and Heterogeneous Firms

It is well known that efficient trade and the reallocation of resources among different agents and sectors have a crucial impact on the macroeconomic performance and
transitional dynamics of monetary policy (see, e.g. Walsh, 2012). However, the data show that resource mobility is far from frictionless, and the intensity of resource reallocation has strong cyclical patterns (see, e.g. Eisfeldt and Rampini, 2006). The imperfect nature of resource mobility plays a surprisingly small role in most policy models in major central banks. In those core New Keynesian DSGE models which are central bank warhorses, for example, it is costly for firms to adjust their selling prices, but those same firms can hire and fire workers without cost, and both workers and capital can frictionlessly shift from one firm to another.

Theoretically, ignoring the potential costs associated with shifting real economic resources is consistent with a standard economy with one sector of homogeneous firms and representative households. However, real world economies consist of multiple sectors and heterogeneous agents, and the data shows that different sectors of the economy and different firms and households behave very differently over the course of the business cycle. For example, durable goods producing sectors are more cyclically sensitive than service sectors. Economic fluctuations may be associated with shifts in relative prices across sectors, or with persistent shifts in relative demand that may require labor and capital to shift from contracting to expanding sectors of the economy, and from low productivity firms to high productivity firms. These shifts require resources to transfer, yet differences in labor skills or in the type of capital employed in different occupations or sectors may make sectoral reallocations costly. The costs that arise because resources are not fully mobile may have consequences for policies on aggregate demand. Monetary policy shocks will definitely alter the transitional dynamics of the demand shock. For example, Walsh (2012) concludes that resource mobility matters for both the transitional dynamics of monetary policy shocks and the goals of monetary policy. Resource mobility affects the transmission mechanism that links monetary policy instruments to inflation and the real economy, thereby affecting the tradeoffs faced by the policy authority, and it affects the way policymakers weigh their objectives.

One important type of resource reallocation is capital reallocation. In Eisfeldt and Rampini (2006), the authors define the ease of capital reallocation between firms as capital liquidity, and show that the amount of capital reallocation between U.S. firms
is procyclical. In contrast, the benefits to capital reallocation appear countercyclical. The benefits to capital reallocation are approximated by the dispersion among the productivity of firms. This is intuitive because smart capital should flow out of low productivity firms into high productivity firms. They document that capital mobility is far from frictionless, and that mobility is particularly difficult in bad economic times. In order to quantify the amount of cost for capital reallocation, they calibrated a simple model economy in which capital reallocation is subject to a standard adjustment cost function, and impute the cost of reallocation. They find that reallocation costs need to be substantially countercyclical to be consistent with the observed joint cyclical properties of reallocation and productivity dispersion. In Eisfeldt and Rampini (2008), the authors provide one possible microfounded explanation for this endogenous inefficient capital reallocation. The authors argue that when managers have private information about the productivity of assets under their control and receive private benefits, substantial bonuses are required to induce less productive managers to declare that capital should be reallocated. Capital is less productively deployed in downturns because agency costs make reallocation more costly.

Another important type of resource reallocation is labor reallocation. Work by Davis et al. (1998, henceforth DHS) has been central to the surge of interest in this area. Their empirical analysis is based on data for manufacturing plants covering the period from the early 1970s to the mid-1980s. After defining employment increases at new and growing plants as job creation, and decreases at dying and shrinking plants as job destruction, they pointed out a number of empirical regularities. One striking feature is that the data is marked by a high rate of job creation and destruction. On average, close to one out of ten manufacturing jobs disappeared in a given year, while the rate of new job creation is slightly lower. These changes are quite persistent: a year later, nearly seven out of ten newly created jobs were still in existence, and about eight in ten lost jobs were still lost. In addition, job creation and destruction tended to be concentrated at plants that experienced large changes in employment (those associated with plant shutdowns and startups, for instance). Another finding is that job destruction varied more noticeably over the cycle than job creation. The data show that job destruction tended to increase sharply during a recession and then fall
back, while job creation did not move as much.

Some questions have been raised about these results. For instance, some economists have cautioned against relying on data for a single sector of the economy, especially manufacturing, where employment has been shrinking so noticeably. Furthermore, the data cover a relatively limited span (the 1970s and the 1980s), and it is possible that the recessions of this period differ fundamentally from previous (or subsequent) recessions in terms of restructuring and reallocation. Though the issue is not settled yet, some of the DHS findings have been replicated elsewhere. For instance, Blanchard and Diamond (1990) rely mainly on data from the Current Population Survey, which is not restricted to manufacturing alone, and they confirm the finding about the relative volatility of job creation and destruction. For example, they find that “[...]booms are times of low job destruction rather than high job creation” (Blanchard and Diamond, 1990, page 87); similar patterns have been discovered in data for foreign countries as well.

In a recent working paper by Kuehn et al. (2012), the authors argue that frictions in the labor market are important for understanding the equity premium in the financial market. The authors embed the Diamond-Mortensen-Pissarides search framework into a DSGE model with recursive preferences. The model produces realistic equity premium and stock market volatility, as well as a low and stable interest rate. In particular, the show that in their model the job flows and matching friction can help generate disasters in employment, output, consumption à la Rietz (1988) and Barro (2009). Moreover, when incorporated into otherwise standard real business cycle models, it has been shown to improve significantly their empirical performance. More importantly, it allows one to analyze the cyclical behavior of unemployment, job vacancies, and job flows, important phenomena which general equilibrium models based on Walrasian labor markets are not designed to address. For example, see Merz (1995), which tries to explain some cyclical behavior in the US labor market by introducing a two-sided search in the labor market as an economic mechanism propagating technological shocks into a standard business cycle model; Andolfatto (1996), which shows that the labor market search is a quantitatively important propagation mechanism in generating business cycles; den Haan et al. (2000), which
stresses the economic importance of the interaction between the capital adjustment cost and the labor destruction rate in propagating technology shocks; Gertler and Trigari (2009), which extends period-by-period Nash bargaining to staggered multiperiod wage contracts, and shows that it can account for the volatile behavior of labor market activities, and Hall (2005), which generates endogenous wage stickiness under a matching framework, and shows that sticky wages in turn make labor market activities realistically sensitive to aggregate shocks.

Given the significant equilibrium effects of job market reallocation, it is reasonable for us to speculate that the job market mobility should have an important impact on the transitional dynamics of monetary policy shocks. In fact, there is an extensive literature that focuses on the positive implications of labor market friction in New Keynesian models, i.e. how search and matching frictions affect the empirical performance of the New Keynesian model and the transitional dynamics of monetary policy. See, for example, Cheron and Langot (2000), Walsh (2005), Trigari (2006), Moyen and Sahuc (2005), Christoffel and Linzert (2005), and Krause and Lubik (2007), among many others. In Thomas (2008), the author analyzes the optimal monetary policy under the New Keynesian framework with search and matching frictions. Monetary policy shocks should affect job market flows in a nontrivial way.

Finally, the most important resource reallocation is the reallocation of credit or funding among firms or agents. The reallocation of funding is crucial, partly because it can possibly explain capital reallocation and labor reallocation, as is discussed in Eisfeldt and Rampini (2006). In Bernanke et al. (1999) and Kiyotaki and Moore (1997), essentially the representative firm or the productive agent is impatient enough such that the firm or agent does not save very much, and does not escape its financial constraints. Consequently, the models have two salient features: first, the firm or the agent saves by itself, and uses the savings to invest later; and second, the economy as a whole is financially constrained in the steady state. However, these two implications are both inconsistent with the data. On the contrary, the data suggest that only a fraction of firms are occasionally bound by financial constraints, and that firms also finance each other’s investment. Fund reallocation among firms is one of the key functions of the financial sector.
In Chari et al. (2008), the authors stress two facts that have been underappreciated. First, non-financial corporations in the aggregate can pay their capital expenditures entirely from their retained earnings and dividends without borrowing from banks or households. Second, in the aggregate, increases in non-financial corporate debt are roughly matched by increases in their share repurchases. More precisely, Figure 19 shows that in the aggregate, without any funds from the rest of the economy, the cash available to these firms from their operations can easily pay for their investment expenditures. Figure 20 shows that equity repurchases are roughly matched by funds raised through credit market instruments. The data suggest that in the aggregate, firms raise debt to buy back their shares, and not to finance investment.

However, it is totally misleading to conclude from the aggregate data that the financial crisis is irrelevant to macroeconomic recession. Among many others, Shourideh and Zetlin-Jones (2012) emphasize the role that financial markets play in reallocating funds from cash-rich, low productivity firms to cash-poor, high productivity firms. In their calibrated model, they found that a shock to the collateral constraints, generating a one standard deviation decline in the debt-to-asset ratio, leads to a 0.5% decline in aggregate output on impact, roughly comparable to the effect of a one standard deviation shock to aggregate productivity in a standard real business cycle model. They find that disturbances in financial markets are a promising source of business cycle fluctuations when non-financial linkages across firms are sufficiently strong.

### 4.3 Risk Premium Dynamics

Does the risk premium matter for macroeconomic dynamics and the transitional dynamics of monetary policy? The key features for risk premium dynamics include high levels of volatility, nonlinearity, and countercyclicality. However, as explained in Jermann (1998), Lettau and Uhlig (2000), and Kaltenbrunner and Lochstoer (2010), it is often difficult to generate endogenously a large and time-varying market price of risk in a production economy.

Rouwenhorst (1995) shows that the standard real business cycle model fails to explain the equity premium because of consumption smoothing. Using models with
Figure 19: Retained Earnings, Dividends, and Capital Expenditure.
Figure 20: New Debt and Net Repurchases of Equity.
internal habit preferences, Jermann (1998) and Boldrin et al. (2001) use capital adjustment costs and cross-sector immobility, respectively, to restrict consumption smoothing to explain the equity premium. However, both models struggle with excessively high interest rate volatilities. Using recursive preferences to curb interest rate volatility, Tallarini (2000) and Kaltenbrunner and Lochstoer (2010) show that baseline production economies without labor market frictions can explain the Sharpe ratio, but still fail to match the equity premium and the stock market volatility. Uhlig (2007) shows that wage rigidity helps explain the Sharpe ratio and the interest rate volatility in an external habit model, but in this model the equity premium and the stock market volatility are close to zero. Gourio (2011) shows that operating leverage derived from labor contracting helps explain the cross-section of expected returns, but it does not study aggregate asset prices. Favilukis and Lin (2012) quantify the role of infrequent wage renegotiations in an equilibrium asset pricing model with long-run productivity risk and labor adjustment costs. They argue that, in standard models, highly procyclical and volatile wages are a hedge against adverse shocks of productivity for the shareholder. The residual – profit or dividends – becomes unrealistically smooth, as do returns. Smoother wages act like operating leverage, making profits more risky. Bad times and unproductive firms are especially risky because committed wage payments are high relative to output. Instead of specifying the wage rule exogenously, Kuehn et al. (2012) differs from the prior studies by using the search framework to derive equilibrium wages. Because dividends equal output minus wages minus total vacancy costs (in an analogous manner to investment), providing a microfoundation for equilibrium wages makes the dividends truly endogenous in a production economy.

Gourio (2012) shows that an increase in disaster risk leads to a decline in employment, output, investment, stock prices, and interest rates, and an increase in the expected return on risky assets. The model matches the data well on quantities, asset prices, and particularly the relationship between quantities and prices, suggesting that variation in aggregate risk plays a significant role in some business cycles. More precisely, the mechanism is that an increase in the disaster probability affects the economy by lowering expectations, and by increasing risk. Because investors are risk averse, this higher risk leads to higher risk premia, with significant implications for both business cycles and asset prices: stock prices fall, employment and output contract, and investment especially declines. Demand for precautionary savings increases, leading the yield on less risky assets to fall, while expected excess returns on risky securities increase. These dynamics occur in the absence of any change in total factor productivity.

Risk premia are important in understanding many macroeconomic questions, for instance, why investment is often low despite low riskless interest rates. Here, the relevant user cost of capital may well be high if the riskless interest rate is low precisely because of high disaster risk. This will directly affect the transitional dynamics of monetary policies.

Introducing time-varying risk requires solving a model using nonlinear methods, i.e. going beyond the first-order approximation and considering higher order terms in the Taylor expansion. Researchers disagree on the importance of these higher order terms, and a fairly common view is that they are irrelevant for macroeconomic quantities. Lucas (2003), in his presidential address, summarizes this point of view:

“Tallarini uses preferences of the Epstein-Zin type, with an intertemporal substitution elasticity of one, to construct a real business cycle model of the U.S. economy. He finds an astonishing separation of quantity and asset price determination: The behavior of aggregate quantities depends hardly at all on attitudes toward risk, so the coefficient of risk aversion is left free to account for the equity premium perfectly.”

Gourio (2012) shows, however, that when the risk is large and varies over time, risk aversion affects macroeconomic dynamics in a significant way. In a similar spirit, but
using a two-country open economy setting, Dou and Verdelhan (2014) show that the time-varying risks generate rich joint volatile dynamics of international asset prices and capital flows.

The following are some particular examples of the potential importance of the time-varying risk premium on macroeconomic dynamics and the transitional dynamics of monetary policy. Gilchrist and Zakrajsek (2012) show that the default premium, rather than the default probability, is the informative variable about macroeconomic conditions. Gilchrist et al. (2010) show that an uncertainty shock can boost the default premium strongly, without increasing the default probability of firms significantly. The extremely high default risk premium prevent firms from investing optimally, even when the risk-free rate is low. The term premium is crucial to accurately characterize the aggregate demand relationship (the IS curve). According to Galí and Gertler (2007), the aggregate demand depends on the gap between the long-term interest rate and its natural correspondence in a model economy with flexible prices. The relationship between long-term interest rate and the short-term interest rate is captured by the term premium, which depends on risk premium of investors. According to Galí and Gertler (2007), the IS curve is also characterized by the relationship between aggregate demand and marginal $q$. The dependence between marginal $q$ and the short-term interest rate also largely captured by financial friction and the risk premium. However, to the best of our knowledge, generating a realistic term premium is still a challenging task in a model production economy. Reasonable risk premia, including currency risk premium and sovereign risk premium, are crucial to understand international financial linkages, capital flow dynamics, and so on. In today’s world, these international financial dynamics have a definite nontrivial impact on the implications of monetary policy.

Finally, as emphasized in Section 3.2.1, the availability of rich financial data makes DSGE models particularly useful in learning deep structural parameters. However, the absence of reasonable risk premium dynamics in DSGE models wastes the information embedded in asset prices.
4.4 Uncertainty

The uncertainty shock has been shown to have adverse effect on macroeconomic quantities and even be able to drive business cycles. For example, the Federal Open Market Committee minutes repeatedly emphasize uncertainty as a key factor driving the 2001 and 2007-2009 recessions, while Stock and Watson (2012) state

“The main contributions to the decline in output and employment during the [2007-2009] recession are estimated to come from financial and uncertainty shocks.”

In addition, in recent studies (see, e.g. Christiano et al., 2010; Negro and Schorfheide, 2012), economists have empirically found that the two most important shocks that are driving aggregate fluctuations are the “financial disruption” and the “heightened uncertainty”. However, as emphasized by Hansen (2012), it is crucial to have a better understanding of the sources of financial and uncertainty shock in macroeconomic models, and their endogenous interactions.

4.4.1 Impact of Uncertainty Shocks

Since the recent financial crisis and the Great Recession, policy authorities and academic researchers have been enjoying a vigorous debate on the impact of uncertainty shocks on the joint dynamics of macroeconomic quantities and asset prices. Policy authorities, including the Federal Reserve Board and the European Central Bank\(^6\), have claimed that uncertainty has had an adverse effect on their economy, and have built uncertainty shocks into their core DSGE models as a main driver of the aggregate fluctuations (see, e.g. Christiano et al., 2010, 2014). Moreover, there is an extensive academic literature showing the adverse effect of uncertainty, e.g., Bloom (2009); Bloom et al. (2013), Gilchrist et al. (2010), and Basu and Bundick (2011), among others.

However, there are two major concerns that challenge the above argument. First, the causal relationship between fluctuations in uncertainty and fluctuations in the

\(^{6}\)For example, Federal Reserve Bank of Dallas President Richard Fisher gave a formal speech titled “Uncertainty matters. A lot.” emphasizing the uncertainty might worsen the Great Recession and on-going recovery, at the 2013 Causes and Macroeconomic Consequences of Uncertainty Conference.
economy is far from clear to policymakers and researchers. Although the correlation between fluctuations in uncertainty and the economy is evident, it is still undetermined whether the heightened uncertainty partially caused the Great Recession, and whether it should be blamed for prolonging the recovery process out of the Great Recession. This is due both to the lack of crystal-clear empirical evidence, and the lack of comprehensive theoretical studies on the equilibrium feedback effect between fluctuations in uncertainty and the economy, as suggested by Bloom (2013), a review paper on uncertainty. Second, it has been argued that uncertainty could have a positive effect on investment and the stock market. Pastor and Veronesi (2006) use a simple calibrated stock valuation model with uncertainty to show that the fundamental value of a firm increases with uncertainty about its average future profitability, and this uncertainty was extremely high in the late 1990s. Bar-Ilan and Strange (1996) showed that in a high uncertainty environment, the benefits from investment, including the growth opportunity caused by investment lags and abandoned project options, can dominate the cost of investment, the loss of the real option value of waiting. As a result, high uncertainty can sometimes promote investment.

In fact, there is a rich if contradictory literature on the relationship between uncertainty and macroeconomic quantities including consumption and investment. Different theories emphasize different channels, some showing a positive relationship and others showing a negative relationship. As a whole, the impact of uncertainty is still ambiguous. The basic channels under consideration include the real option channel (i.e. the option to wait), the risk premium channel, the precautionary savings channel, the growth opportunity channel, the Oi-Hartman-Abel-Caballero channel, and the learning-by-doing channel.

The first channel under consideration, the real option channel, appears to be the most direct channel through which uncertainty can potentially affect a firm’s investment and hiring decisions. The idea is that the sizable adjustment cost in investment and hiring (see, e.g. Ramey and Shapiro, 2001; Cooper and Haltiwanger, 2006) and its irreversibility (see, e.g. Pindyck, 1991; Kogan, 2001) together make

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7There is a long literature on the real option effect, including Bernanke (1983), Brennan and Schwartz (1985), McDonald and Siegel (1986), and Dixit and Pindyck (1994).
the investment decision effectively a decision on exercising call options. This real option can be viewed as a option to wait, and the opportunity cost of delay is the foregone income from the project, which is unaffected by uncertainty. This asymmetric effect of uncertainty on the benefits and costs of waiting captures the essence of the real option effect. This is referred to as the “bad news principle” by Bernanke (1983). However, the real option effect can be alleviated or even overturned when environmental variables shift. For example, when projects have a liquid reallocation market (i.e., reversible), the real option effect is negligible. Another, more relevant, example is when firms are financially constrained. As demonstrated in Bolton et al. (2013), for financially constrained firms, the uncertainty shock can have both a positive and a negative effect on a firm’s investment and financing decisions.

The idea behind the second channel, the risk premium channel, is that uncertainty reduces aggregate investment, hiring, and growth through a sharp increase in risk premium. The risk premium channel plays a key role in linking asset pricing to the interaction between uncertainty and investment/hiring, an idea which has been missing in the uncertainty literature, although with a few exceptions, including Gilchrist et al. (2010), Arellano et al. (2011) and Christiano et al. (2014). The key idea is that in an economy with corporate debt and costly default, higher uncertainty raises the default probability for those firms that are already near default boundaries, and hence the cost of debt financing increases. This in turn reduces investment, and increases the default probabilities for firms originally further from the default boundaries, and accordingly diminishes hiring, which in turn leads to lower consumption of households. This adverse feedback loop causes a ripple effect, dragging the whole economy into recession, while creating sky-high credit spreads. It is clear that if financial intermediaries are strong, and very few firms are close to their financial binding constraints, the risk premium effect on an economic downturn will largely be dampened. This is a nontrivial point in generating rich and realistic endogenous uncertainty dynamics.

The third channel, the precautionary saving channel, focuses on households. It is evident that higher uncertainty depresses household consumption expenditures (see, e.g. Bansal and Yaron, 2004). In a full-closed economy, the motivation to increase precautionary savings will also reduce contemporaneous consumption, and at the same
time increase investment. However, the investment will also drop when uncertainty is high, assuming price rigidity (see, e.g. Basu and Bundick, 2011; Leduc and Liu, 2012).

Growth opportunities, the fourth channel we consider, are the major forces generating a positive association between uncertainty and investment. This idea is usually implemented in two ways in the literature. Following Bar-Ilan and Strange (1996), the first method assumes that there is an investment lag with a time-to-build $h > 0$ and an abandonment option available for each project. The abandonment option means the loss is bounded below in bad states, while the time-to-build feature forces the firm to invest earlier, in order to be able to capture opportunities in the near future. The two components together cause the rational firm to invest sooner in a high uncertainty environment. The second method is to model two capital goods: traditional capital called “trees”, and investment options called “seeds” (see, e.g. Jovanovic, 2009). In a high uncertainty environment, the investment in “seeds” experiences a gradual boom.

Fifth, the Oi-Hartman-Abel-Caballero channel is based on the work by Oi (1961), Hartman (1972), Abel (1983) and Caballero (1991). The key idea of these models is that the adjustment cost of capital makes investment less flexible than labor adjustment. This concept, combined with a constant-return-to-scale technology, makes the marginal product of capital a convex function of output price. It follows from Jensen’s inequality that uncertainty in output price leads to a high marginal product of capital, and hence to a high intensity of investment.

Finally, the learning-by-doing channel assumes that investors or firms have imperfect information about the underlying state of the economy, and that the only way receive extra signals about its true state is by a sequence of investment. It naturally follows that in a high uncertainty environment, firms conduct earlier and more intensive investment to learn the true state (see, e.g. Roberts and Weitzman, 1981; Pindyck, 1993; Pavlova, 2002).

An important and still unanswered question is which channel dominates under which economic conditions. It is possible that the sign and magnitude of the impact of uncertainty shocks on investment and asset prices depend on the soundness of the financial system and the prevailing external financing costs. When financial
intermediaries are strong and the risk premium is low, negative effect channels such as the real option channel and the risk premium channel will have limited impact, because investment options are deep out of the money, and it is hard to trigger a crash in the financial market. In contrast, positive effect channels are given full play in this environment. Therefore, higher uncertainty should lead to earlier and more intensive investment and create a stock market boom. When the financial intermediaries are fragile, however, the real option channel with liquidity hoarding and the risk premium channel will dominate, while positive effect channels will play a very limited role.

4.4.2 Measurements of Uncertainty are Observable Endogenous Variables

It is impossible to measure uncertainty directly. Uncertainty is unobservable, conceptual, and ex ante. After all, it lives in people’s minds, and has no direct material instantiation. Therefore, a range of proxies have been employed to study the impact of the “uncertainty shock”. Aggregate market volatility and aggregate TFP volatility are among the most popular proxies for uncertainty shock in the existing literature (see, e.g. Bloom, 2009; Bansal et al., 2014; Campbell et al., 2012). These aggregate volatility proxies are usually referred to as “macro uncertainty”. In other papers (see, e.g. Bloom, 2009; Gilchrist et al., 2010), the uncertainty shock is approximated by an increase in the cross-sectional dispersion among agents. These dispersion-based measures are referred to as “micro uncertainty”. There are also measures based on survey data. These measures include forecaster disagreement and news mentions of uncertainty. Empirically, they are all believed to be reasonably good proxies because they co-move over time. This co-movement itself is a nontrivial puzzle to solve, and its solution should shed light on providing a better proxy for uncertainty, and therefore a better equilibrium impact of uncertainty shocks.

4.5 The Financial Sector and Systemic Risk

Systemic risk is believed to be a key driver of the recent financial crisis and the Great Recession. Systemic risk is rooted in the financial sector, and through contagion

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8Holding cash can be viewed as a stronger form of a holding option to wait.
it has a strong adverse effect on the whole economy. However, the key players in systemic risk, financial intermediaries, are missing in the New Keynesian DSGE models used by the central banks. When incorporating financial intermediation into macroeconomic models, it is essential to consider two crucial effects, the balance sheet effect for financial intermediaries, and the effect of imperfections in the interbank lending market. First, in order to analyze how shocks in the financial sector spill over to the whole economy, either the nonfinancial corporate balance sheet or the integrated bank-firm balance sheet is not satisfactory. Second, in order to model the endogenous systemic risk, properly modeling of the interbank lending market and the interconnections between financial intermediaries are crucial. Incorporating the financial sector into these models is necessary to allow us to study endogenous systemic risk in the economy. This is critical for analyzing conventional monetary policy, unconventional monetary policy and macroprudential policy, since the major task of the monetary authority is to maintain a healthy financial system in normal times, and restore a distorted financial system in times of crisis.

Most of the recent macroeconomics literature on financial frictions focuses on credit market constraints on nonfinancial corporate borrowers without any real role for financial intermediation. The most recent advances include Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Bernanke et al. (1999). Among them, Bernanke et al. (1999) introduces a financial accelerator into the New Keynesian DSGE framework. However, they all focus on the balance-sheet effect of nonfinancial firms, while ignoring the unique properties of financial intermediaries. The financial accelerator channel from the balance sheet of nonfinancial corporate borrowers is definitely a relevant financial friction, but it is just one aspect of many possible financial frictions.

One of the first papers which tried to incorporate the financial sector into macroeconomic models, and studied the effects of financial intermediary balance sheets and the interbank lending market is Gertler and Kiyotaki (2010). The authors focus on understanding how disruptions in financial intermediation can induce a crisis that affects the real economy. The credit market constraints on financial intermediation are incorporated into a real business cycle framework, modified with habit formation
and flow investment adjustment costs instead of capital adjustment costs. More precisely, the financial intermediaries are assumed to play three unique roles in the economy, as discussed extensively in the literature. First, the financial intermediaries are delegated monitors and specialists. They are conduits that channel funds from households to nonfinancial firms. Second, the financial intermediaries engage in maturity transformation. In the model, they are assumed to issue short-term debts and hold long-term assets. Third, the financial intermediaries facilitate liquidity provision via the interbank lending market. In the model, it is assumed that there is a continuum of “banks” which fund the goods producers and finance their investment from both a wholesale market, i.e., an interbank lending market, and a retail market, where banks hold deposits from households. To simplify this analysis, the authors assume constant returns to scale production, perfect labor mobility, and goods producers without financial constraints. With these assumptions, there is no need to keep track of the distribution of capital held by producers or their net worth. It is also assumed that the banks and the nonfinancial firms are “buddies”, in the sense that there is no financial friction in their funding relationship. In other words, it is essentially assumed that the producers’ balance sheet can be viewed as part of the banks’ balance sheet. To achieve such a simplification, following Gertler and Karadi (2011), the authors assume complete consumption insurance among workers and bankers, and independent and identically distributed random turnovers between workers and bankers. By doing so, this guarantees that a representative household exists to determine aggregate consumption and prices, with no need to track the wealth distribution of households. In this complete market economy, there is unique stochastic discount factor (SDF) while the agents are actively borrowing and lending in equilibrium.

In contrast, He and Krishnamurthy (2013) incorporates the financial intermediary into a standard macro-finance model with segmentation between bankers and workers and reasonable terms of macroeconomic calibration in order to generate extremely nonlinear risk premium dynamics. They model the dynamics of risk premia during crises in asset markets where the marginal investor is a financial intermediary. In this model, intermediaries face an equity capital constraint. Risk premia rise when the constraint binds, reflecting the capital scarcity. The calibrated model matches the
nonlinearity of risk premia during crises and the speed of reversion in risk premia from crisis levels back to pre-crisis levels. They evaluate the effect of three government policies: reducing intermediaries borrowing costs, injecting equity capital, and purchasing distressed assets. Injecting equity capital is particularly effective because it alleviates the equity capital constraint that drives the model’s crisis. However, it is still far from satisfactory for monetary policy decision making, because the model simplifies many important features, and hence there is no way to see how the constrained financial intermediaries would affect the economy as a whole in such a model.

If the theory of He and Krishnamurthy (2013) is correct, the marginal value of wealth for financial intermediaries should therefore provide a more informative stochastic discount factor (SDF) than that of a representative consumer. Empirically, Adrian et al. (2010) use shocks to the leverage of securities broker-dealers to construct an intermediary SDF. Intuitively, deteriorating funding conditions are associated with de-leveraging and a high marginal value of wealth. Their single-factor model prices size, book-to-market, momentum, and bond portfolios with an $R^2$ of 77 percent and an average annual pricing error of 1 percent, performing as well as standard multi-factor benchmarks designed to price these assets. It empirically documents that financial intermediary balance sheets contain strong predictive power for future excess returns on a broad set of equity, corporate, and Treasury bond portfolios. They also show that the same intermediary variables that predict excess returns forecast real economic activity and various measures of inflation. Their findings point to the importance of financing frictions in macroeconomic dynamics and provide quantitative guidance for preemptive macroprudential and monetary policies.

Moreover, Gilchrist and Zakrajsek (2012) empirically relate the predictive power of bond premia for the business cycle to the risk-bearing capacity of the marginal investors in these bonds. These investors act in a more risk-averse way when their capital becomes impaired, which translates to an increase of the bond premium and a reduction in the supply of credit available to potential borrowers.

However, the literature of occasionally binding financial constraints on financial intermediation, including Brummermeier and Sannikov (2014), He and Krishnamurthy (2012) and Danielsson et al. (2011), stresses that precautionary effects can generate
endogenous tightening of margins. This is in contrast to the literature of the financial accelerator, including Bernanke and Gertler (1989), Bernanke et al. (1999), Kiyotaki and Moore (1997), Gertler and Kiyotaki (2010), and Christiano et al. (2010), the financial constraints are always binding. The precautionary effect means that even if the borrowing constraint is not currently binding, an increase in likelihood that it could be binding in the future, possibly due to increased uncertainty, can induce a tightening of margins. While the papers on occasionally-binding financial constraints on financial intermediation provide reasonable asset pricing dynamics with enough nonlinearities, they generally fail to generate enough nonlinearities in macroeconomic quantities. Dewachter and Wouters (2012) add a reduced-form working capital constraint on goods producers, which quantitatively improves the macroeconomic dynamics of the model.

Recently the CMR framework, introduced by Christiano et al. (2010), has been adopted by the European Central Bank. However, the CMR framework is still subject to some major concerns. First, while a crisis in this framework can now originate in the financial sector, rather than through risk in the production sector, the interbank market is still missing in this version of the model. Second, the model poorly accounted for the external financing premium variables (e.g. the credit spread), negating the important advantage of modeling the financial sector separately from the real sectors. Third, the log-linearization method failed to capture key dynamics of the financial sector and the real economy. Fourth, the absence of precautionary effect makes it hard to generate realistic nonlinear dynamics in asset prices.

4.6 Goods Market and Markups

In the baseline New Keynesian DSGE model, the desired markup of price over marginal cost is constant. This is mainly due to two factors: a constant elasticity of substitution among differentiated goods, and the validity of the Modigliani-Miller theorem. As pointed in Blanchard (2009), however, the desired markup appears to be anything but constant\(^9\), and how it varies in response to other factors is still unknown territory.

\(^9\)For example, some evidence comes from findings on the pass-through effects of exchange rate movements. Recent empirical work on the United States, using disaggregated prices, shows that, when
in macroeconomics.

One popular model for the desired markup is the customer market mechanism, in which a firm that lowers its current price not only sells more to its existing customers, but also expands its customer base, leading to higher future sales at any given price. This idea was first introduced by Phelps and Winter (1970), and formalized by Gottfries (1986), Klemperer (1987), Farrell and Shapiro (1988), and Bils (1989), among others. Several strands in the marketing and industrial organization literature provide empirical support for the customer model. For example, Houthakker and Taylor (1966) use 80 detailed items of consumer expenditure, and find that current demand depends positively on existing inventory, suggesting some habit formation. Guadagni and Little (1983) use a multinomial logit model of brand choice, calibrated on 32 weeks of purchases of regular ground coffee by 100 households, showing high statistical significance for the explanatory variables of brand loyalty. Erdem (1996) find that for margarine, peanut butter, yogurt, and liquid detergent, accounting for habit formation improves both in-sample and out-of-sample fit. More recently, Genesove and Mullin (1997) find that the American Sugar Refining Company sharply cuts its price to maintain market share and to deter entry of competitors. Bronnenberg et al. (2009) find an early entry effect on a brand’s current market share and perceived quality across U.S. cities. There is also direct evidence provided by firm-level surveys in several OECD countries (Hall et al. (1997), Aucremanne and Druant (2005), Fabiana et al. (2005), Amirault et al. (2006)), all pointing out that price stickiness is mainly driven by customer relationships. Dou and Ji (2014) build on the idea of the customer market, and analyze how financing decisions interact with strategic pricing when the financial market is imperfect. One major focus of Dou and Ji (2014) is the endogenous relationship between financing and the price-setting behavior of firms. A closely related paper is Chevalier and Scharfstein (1994), which studied the impact of imperfect financial markets on firms’ price-setting decisions. However, the authors use limited supermarket data to test the causal effect of liquidity shocks on goods price.

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Import prices are denominated in dollars at the border, exchange rate movements have minimal effect on the prices for these imports in the United States. Conversely, if import prices are denominated in foreign currency at the border, exchange rate movements lead to nearly one-for-one effects on prices for those imports (see e.g. Gopinath et al., 2010).
Chevalier and Scharfstein (1994) is the first paper that introduces capital market imperfection into a customer market model, in an attempt to interpret countercyclical markups. Their focus is on how liquidity constraints affect the pricing behavior: liquidity constrained firms have an incentive to raise prices in order to boost current profits to meet their liabilities and finance investment. A recent paper by Sim et al. (2013) provides more evidence using product-level price data. They find that during the recent crisis, firms with weak balance sheets increased prices significantly relative to industry averages, whereas firms with strong balance sheets lowered prices. A general equilibrium model with financial market distortions is proposed to rationalize these findings. The idea of Dou and Ji (2014) is related to Chevalier and Scharfstein (1994) and Sim et al. (2013), but they look for more evidence on firms’ pricing and financing behavior in normal times across industries and on the frequency of price resetting over business cycles. Moreover, Dou and Ji (2014) focus more on the interaction between financing and pricing. Their DSGE model provides a unified theory on Tobin’s q, corporate investment, financing, price setting, and asset pricing. In a recent work, Weber (2013) showed that firms that adjust their product prices infrequently earn a cross-sectional return premium of more than 4% a year.

The cyclicality of markups has significant implications for economic fluctuations, since countercyclical markups would tend to dampen fluctuations in economic activity, whereas procyclical markups would amplify fluctuations. To generate procyclical factor prices, Rotemberg and Woodford (1991) propose that markups should be countercyclical, and they provide evidence using aggregate data. The countercyclical markup is also found in Bils (1987) and the supermarket industry (Chevalier and Scharfstein (1994). However, other papers find that markups are procyclical using different industry-level data (Domowitz et al. (1986), Machin and Van Reenen (1993), Ghosal (2000), Nekarda and Ramey (2013)).

Based on the customer market model introduced by Phelps (1998), Gilchrist et al. (2012) investigate the effect of financial conditions on price-setting behavior during the recent financial crisis by assuming a deep habit component in the model. In their model, firms have an incentive to set a low price to invest in market share. In other words, the loss from setting a lower price can be viewed an investment cost for
positive NPV projects (i.e. market shares). When financial distortions are severe, firms forgo these investment opportunities and maintain high prices, because the marginal value for cash dominates the profits from investment in market share. The model with financial distortions implies a substantial attenuation of price dynamics in response to contractionary demand shocks relative to the baseline without financial distortions, which has important policy implications. Empirically, the authors find theory-consistent evidence that, at the peak of the crisis, firms with relatively weak balance sheets increased prices, while firms with strong balance sheets lowered their prices.

4.7 Methodology: Solution, Estimation and Evaluation

The proper methodologies for the solution, estimation and evaluation of New Keynesian DSGE models are critically important in economics, yet extremely hard to do technically. Without proper methods, the credibility of monetary policies based on these models will be dramatically compromised, and their results may be extremely misleading, even if the modeler constructs a perfect model, one which incorporates all the mechanisms discussed in previous sections. In this section, we shall review the methodologies used by central banks, and point out their principal issues.

4.7.1 Solution Methods

Solving the DSGE model with heterogeneous agents in incomplete markets and severe nonlinearity is mathematically equivalent to solving a huge system of nonlinear equations. The nonlinearity and infinite dimensionality of the model makes the problem very difficult for mathematicians and computer scientists. Therefore, given the limited mathematical knowledge of the modeler and the technical constraints of computational power, the modeler needs to make thorny trade-offs between the complexity of the model and the solvability of the model.

Mainly because of these computational difficulties, macroeconomists at central banks prefer to use simpler models and the log-linearization solution method. The DSGE model relies on log-linearization around the steady state. As pointed out in
Tovar (2009), due to the computational burden often associated with the likelihood
evaluation for the solution of the nonlinear expectation equations implied by DSGE
models, the empirical literature has concentrated its attention on the estimation
of first-order linearized DSGE models. First-order approximations have been until
recently the main tool employed for empirically evaluating DSGE models and for
forecasting. However, as Judd (1997) states

“If theoretical physicists insisted on using only closed-form solutions or proofs of
theorems to study their models, they would spend their time examining the hydrogen
atom, universes with one star, and other highly simplified cases and ignore most
interesting applications of physical theories.”

The log-linearization approximation method has several important drawbacks.
First, the solution methodology makes it impossible to model and study systemic risk.
The most recent papers on modeling financial intermediaries, such as Brunnermeier and
Sannikov (2014) and He and Krishnamurthy (2013), show that the nonlinearity of the
amplification effect is a key aspect of systemic risk. Second, first-order approximations
fail to be appropriate for evaluating welfare across policies that do not affect the
steady state of the economy, e.g., when asset prices and the risk premium are taken
into consideration. Log-linearization around a constant steady state is not applicable
to asset pricing, because by construction, it eliminates all risk premiums in the model.
In fact, the risk premium is zero in a first-order approximation, and constant in
the case of a second-order approximation, therefore higher-order approximations are
required. See, for example, Schmitt-Grohé and Uribe (2004) and Kim et al. (2005)
for a discussion on second-order approximations, and also An and Schorfheide (2007).
Third, Fernández-Villaverde et al. (2006) consider log-linearization approximation to
be unsatisfactory as they prove that second-order approximation errors in the solution
of the model have first-order effects on the likelihood function. In other words, the
likelihood implied by the linearized model diverges from the likelihood implied by the
exact model.
4.7.2 Estimation Method

Nowadays, most central banks have adopted Bayesian likelihood estimation methods, instead of the more traditional equation-by-equation estimation used for large macro models. The main reasons are as follows. First, as shown in Canova (2009), the likelihood function of DSGE models is often flat and irregular in a number of parameters. Prior information helps overcoming such identification issues. (However, there are general issues on justifying the correct choice of priors, and it is dangerous to use too strong of priors.) Second, the Bayesian approach can deal explicitly with measurement errors, unobservable state variables, large data sets and different sources of information. Third, the Bayesian approach allows for decision making under uncertainty for policymakers. Fourth, although the Bayesian method is exposed to the “stochastic singular” problem that occurs when the number of variables is more than the number of the shocks, there are some useful techniques to tackle the problem. See, for example, Harrison and Oomen (2010) where “structural shocks” were added into the baseline model to overcome the stochastic singular issue and improve the fitting of the data. This can also be viewed as an example of the models lying between the data-driven and structural models.

The other main reason macroeconomists at central banks resort to log-linearization approximation is to make estimation easy. Since Smets and Wouters (2003), the Bayesian estimation method has become the most popular estimation approach at central banks. However, as is well known, the standard Bayesian method requires full specification of the likelihood function of the model. This seems implausible for complex New Keynesian DSGE models without log-linear approximation. However, advanced Bayesian computing techniques, such as the Approximate Bayesian Computing (ABC) method, can be adopted. This method is able to work with the Dynare software platform, which allows higher-order approximations of the model. A simple example using the ABC method to estimate a dynamic macroeconomic finance model can be found in Chen et al. (2013). Finally, it is important when using these methods to match the impulse response instead of only matching the moments of the model.
4.7.3 Evaluation Method

The traditional method of evaluating DSGE models is to compare the simulated subset of moments with those observed in the data. More cautious researchers have conducted sensitivity analysis to check the fragility of the model. However, they mostly conduct this robustness check in an informal way, by perturbing parameters one by one and measuring the difference in model effects. The choices of the parameters and the magnitude of disturbance are ad hoc. In Chen et al. (2013), the authors point out that, even when the model is stable along each parameter, it could be the case that the model is fragile along a combination of multiple parameters. Zin (2002) points out that a primary goal of characterizing asset market data using a tightly parameterized general equilibrium model is to try to uncover deep structural parameters for policy purposes. However, he emphasizes that it is not an easy task, mainly because the aggregate historical data are usually not enough to provide an informative statistical test on the models or their structural stability. To demonstrate this idea more explicitly, he discusses a simple asset pricing model which cannot be rejected by data if the asset pricing moment restrictions depend on high-order moments (e.g. the fifth moment) of the distribution of the fundamental process (i.e. the endowment process). It is difficult to visualize or even to describe high-order moments of fundamental processes, and hence it is difficult to believe these asset pricing explanations would be deemed structural or useful. However, these high-order moments could be macroeconomic “dark matter”, as in Chen et al. (2013). The solution proposed in Zin (2002) is to augment the statistical tests with subjective non-sample-based judgments about the reasonableness of the assumptions. Chen et al. (2013) significantly improve Zin’s argument by explicitly defining and quantitatively measuring dark matter, while Zin (2002) only demonstrates the idea qualitatively. More importantly, Zin only focuses on the “weak identification” side of dark matter, while missing the more important side to dark matter: that it may cause model implications to become extremely sensitive to parameters. While Zin (2002) and Chen et al. (2013) both stress the insufficiency of current statistical tests for structural model evaluation, Chen et al. (2013) propose an explicit, quantitative, and implementable method focus on “dark matter” or “fragility”
in models, to augment conventional statistical specification tests for model evaluation.

5 Core Monetary Policy Models Used by Central Banks

In this section, we review the core models used by major central banks to analyze monetary policy. As discussed in previous sections, they include both large-scale macroeconometric models and New Keynesian DSGE models. In the following tables, we summarize the key models that the major monetary authorities have used in the past or are currently using.

5.1 The U.S. Federal Reserve (Fed)

5.1.1 Large-scale Macroeconometric Models at the U.S. Fed

The U.S. Federal Reserve Board makes its monetary policy principally using a core model called FRB/US Reifschneider et al. (1997). At the same time, however, it uses a class of “periphery” models, mostly reduced-form econometric models, such as VAR models, and some medium- and small-scale calibrated New Keynesian DSGE models, focusing on a few particular mechanisms in equilibrium. As pointed out by David J. Stockton, the director of Division of Research and Statistics at the Fed, one reason these periphery models are separated from the core model at the Fed is because of the difficulty incorporating them into the core model in a robust way. Caballero (2010) has emphasized that it is very dangerous to add micro-level insights or mechanisms into the core model in a brute force manner. The ad hoc manner of incorporating the Fed’s periphery models with its core’ model exposes it to the danger of being “too brutal”, as stressed by Caballero (2010).

The Federal Reserve’s first generation large-scale macroeconometric model, which still serves as the primary formal model of the U.S. economy for the Federal Open Market Committee, is the MPS model, which was adopted from the late 1960s until the beginning of 1996. An overview of the MPS model can be found in Brayton and
<table>
<thead>
<tr>
<th>Model Full Name</th>
<th>US Fed</th>
<th>ECB</th>
<th>BOC</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIGMA</td>
<td>U.S. Fed multi-country open economy</td>
<td>NAWM</td>
<td>ToTEM</td>
</tr>
<tr>
<td>Managed by</td>
<td>FOMC</td>
<td>Governing Council</td>
<td>Governing Council</td>
</tr>
<tr>
<td>Adjustment Friction</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Financial Sector</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Habit Utility?</td>
<td>Habit</td>
<td>Habit</td>
<td>Habit</td>
</tr>
<tr>
<td>Open Economy?</td>
<td>Yes Multicountry</td>
<td>Yes Small</td>
<td>Yes Small</td>
</tr>
<tr>
<td>Estimation?</td>
<td>Est. &amp; Calibration</td>
<td>Est. &amp; Calibration</td>
<td>Est. &amp; Calibration</td>
</tr>
<tr>
<td>Linearization?</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Housing Market?</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Endo. Risk Premium</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Number of Parameters</td>
<td>About 36</td>
<td>About 66</td>
<td>About 54</td>
</tr>
<tr>
<td>Number of Equations</td>
<td>About 37</td>
<td>About 102</td>
<td>About 72</td>
</tr>
<tr>
<td>Number of Shocks</td>
<td>About 16</td>
<td>About 18</td>
<td>About 7</td>
</tr>
<tr>
<td>Frequency of Data/Updates</td>
<td>Quarterly</td>
<td>Quarterly</td>
<td>Quarterly</td>
</tr>
<tr>
<td>Short-run Fluctuation</td>
<td>Supply/Demand</td>
<td>Supply/Demand</td>
<td>Supply/Demand</td>
</tr>
<tr>
<td>Long-run Steady State</td>
<td>Supply</td>
<td>Supply</td>
<td>Supply</td>
</tr>
<tr>
<td>Expectation Formation</td>
<td>Rational</td>
<td>Rational</td>
<td>Rational</td>
</tr>
<tr>
<td>Microfounded</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Table 7: The Hybrid Open-Economy Models at Central Banks

| BOE Model Full Name | BOE Model Name | BOE Model Description | BOE References | BOE Financial Friction | BOE Utility | BOE Open? | BOE Estimation? | BOE Linearization? | BOE Housing Market? | BOE Endo. Risk Premium | BOE Number of Parameters | BOE Frequency of Data/Updates | BOE Short-run Fluctuation | BOE Long-run Steady State | BOE Expectation Formation | BOE Microfounded |
|---------------------|---------------|-----------------------|----------------|----------------------|------------|--------|------------|----------------|------------------|---------------------|---------------------|----------------------|--------------------------|--------------------------|------------------------|----------------------|-----------------|
| Bank of England Quarterly Model | BEQM | Bank of England Quarterly Model | Harrison et al. (2005) | Yes | Habit | Yes Small | Estimation & Calibration | Yes | No | No | About 147 | Quarterly | Supply/Demand | Supply | Rational | Yes for the core model |
Table 8: The Core Closed-Economy DSGE Models at Central Banks

<table>
<thead>
<tr>
<th></th>
<th>US Fed</th>
<th>ECB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>EDO</td>
<td>CMR</td>
</tr>
<tr>
<td>Model Full Name</td>
<td>Fed’s Estimated, Dynamic,</td>
<td>Christiano-Motto-Rostagno</td>
</tr>
<tr>
<td></td>
<td>Optimization-based model</td>
<td>model</td>
</tr>
<tr>
<td>Managed by Foundation</td>
<td>FOMC</td>
<td>Governing</td>
</tr>
<tr>
<td>Adjustment</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Friction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial Sector</td>
<td>No</td>
<td>Yes financial accelerator</td>
</tr>
<tr>
<td>Financial Friction</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Utility</td>
<td>Habit</td>
<td>Habit</td>
</tr>
<tr>
<td>Open?</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Estimation?</td>
<td>Bayesian</td>
<td>Bayesian</td>
</tr>
<tr>
<td></td>
<td>Calibrate SS</td>
<td>Calibrate SS</td>
</tr>
<tr>
<td>Linearization?</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Housing Market</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Endo.Risk Premium</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Number of Parameters</td>
<td>About 43</td>
<td>About 74</td>
</tr>
<tr>
<td>Number of Equations</td>
<td>About 66</td>
<td>About 49</td>
</tr>
<tr>
<td>Number of Shocks</td>
<td>About 11</td>
<td>About 16</td>
</tr>
<tr>
<td>Frequency of Data/Updates</td>
<td>Quarterly</td>
<td>Quarterly</td>
</tr>
<tr>
<td>Short-run Dynamics</td>
<td>Supply/Demand</td>
<td>Supply/Demand</td>
</tr>
<tr>
<td>Long-run Steady State</td>
<td>Supply</td>
<td>Supply</td>
</tr>
<tr>
<td>Expectation Formation</td>
<td>Rational</td>
<td>Rational</td>
</tr>
<tr>
<td>Microfounded</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
### Table 9: The Core Macroeconometric Models at Central Banks

<table>
<thead>
<tr>
<th></th>
<th>US Fed</th>
<th>US Fed</th>
<th>ECB</th>
<th>BOE</th>
<th>BOC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model</strong></td>
<td>FRB/US</td>
<td>FRB/Global</td>
<td>AWM</td>
<td>MTMM</td>
<td>QPM</td>
</tr>
<tr>
<td><strong>Adjustment</strong></td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Friction</strong></td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td><strong>Financial</strong></td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td><strong>Sector</strong></td>
<td>No</td>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Open?</strong></td>
<td>No</td>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Calibrate for steady state</td>
<td>Calibrate for steady state</td>
<td>Calibrate for steady state</td>
<td>Calibrate for steady state</td>
<td>Calibrate for steady state</td>
</tr>
<tr>
<td><strong>Linearization?</strong></td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Endo.Risk</strong></td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td><strong>Premium</strong></td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td><strong>Number of Core Equations</strong></td>
<td>about 300</td>
<td>about 1700</td>
<td>about 80</td>
<td>about 110</td>
<td>about 155</td>
</tr>
</tbody>
</table>
Mauskopf (1985). This model employed quarterly data and had about 125 stochastic behavioral equations and more than 200 identities.

The MPS model was replaced by the FRB/US model in mid-1996, mainly aiming at improving the expectation formation dynamics and the long-run equilibrium component of the model. The FRB/US model specifies a neoclassical long-run steady state and dynamic behavior designed to address the Lucas critique through considering the influence of expectations and other sources of dynamics. A key feature of the FRB/US model, compared to the MPS model, is that expectations of future economic conditions are explicitly specified in many of its equations. For example, the FRB/US model can show how the anticipation of future events, such as a policy shift, may affect the economy today. The adoption of the rational expectations assumption alleviates the Lucas critique, and is regarded as a major paradigm shift. Rational expectations allow agents in the model to look at policy changes as a contingent plan rather than as a one-time shock. In the FRB/US model, rational expectations are only a baseline assumption, and hence the policy makers can easily add learning on the top of the baseline.

Another key feature of the FRB/US model, compared to the MPS model, is that the FRB/US model incorporates polynomial adjustment frictions. The equations based on these adjustment costs are difficult to interpret because there is little microeconomic justification motivating them. Theoretically, these frictions imply decision rules in an error-correction format. The significant coefficients on lagged changes in the variables suggest adjustment costs that are of an order higher than quadratic (see, e.g. Tinsley, 1993). In Taylor (1997), the author called for new models in which the polynomial adjustment cost functions could be replaced by more microfounded frictions and structures.

The steady state properties of the FRB/US model is close to the steady-state properties of the MPS model. The steady state is characterized using the neoclassical framework. In particular, all markets clear, and the marginal product of each factor of production is equal to its relative price in the long run. The growth of output in the long run depends on the exogenous population growth and the exogenous productivity growth of the production factors, following the assumption of a constant return to
scale production technology. The short-run properties of the model are Keynesian in spirit. For example, the output and employment are mainly determined by the aggregate demand because wages and prices are assumed to be sticky.

The FRB/US model has about 30 stochastic behavioral equations and about 300 identities. The behavioral equations can be categorized into four fundamental building blocks: arbitrage equilibria for financial variables, equilibrium planning for variables not determined in financial markets, dynamic adjustments for activities in nonfinancial sectors, and the formation of expectations. A detailed list of equations and identities is unmanageable for this paper; for more details, please refer to Brayton and Tinsley (1996) and Tinsley (1993). However, the following are some representative examples for the behavioral equations in all categories.

The aggregate consumption equations include the long-run equilibrium relationship equation and short-run dynamic adjustment equation:

\[
c^* = 1.0v + 0.62s_{trans} - 0.15s_{prop} + 0.52s_{stock} + 1.28s_o + 0.13\tilde{x}, \quad (56)
\]

and

\[
\Delta c_t = -0.12(c_{t-1} - c^*_{t-1}) + 0.17\text{lags}_1(\Delta c_{t-i}) + 0.75\text{leads}_\infty(\Delta c^*_{t+i}) + 0.09\Delta y_t. \quad (57)
\]

The equilibrium equation (56) belongs to the category of equilibrium planning for aggregate consumption. Here, \( v \) is the log of wealth \( (V) \), that is, the present value of permanent income. The income consists of three main components: labor income (denoted by \( s_{labor} \) after normalized by \( V \)), transfer income (denoted by \( s_{trans} \) after normalized by \( V \)), and property income (denoted by \( s_{prop} \) after normalized by \( V \)). The variables \( s_{stock} \) and \( s_o \) are normalized by \( V \), and represent the value of corporate equity and other net financial/tangible assets, respectively. \( \tilde{x} \) is the aggregate output gap.

The dynamic adjustment equation (57) is categorized in the group of dynamic adjustments. Here, the lag operator \( \text{lags}_k \) is defined as \( \text{lags}_k(a_t) := \sum_{i=1}^{k} w_i a_{t-i} \) with
\sum_{i=1}^{k} w_i = 1. Furthermore, the lead operator \( \text{leads}_k \) with \( \text{leads}_k(a_t) := \sum_{i=0}^{+\infty} w_i a_{t+i} \) and \( \sum_{i=0}^{k} w_i = 1 \). The superscript “\( \cdot \)” indicates the current market forecasts based on information available in period \( t \). The expectations of future variables are approximated by using small-scale VAR models. Finally, \( y \) is the log total income.

The coefficients of the equations (56) and (57) are estimated using quarterly U.S. data from 1963Q1 and 1995Q4. For more details, please refer to Brayton and Tinsley (1996).

Another example is the non-arbitrage equation for 10-year government bond rates in financial market:

\[
r_{10,t} = 0.46 + 1.0 \text{leads}_{40}(r_e^t) - 0.79 \text{leads}_{40}(\tilde{x}_e^t) + 0.85 \text{lags}_{1}(\mu_{10,t}).
\]  

(58)

Here, \( r_{10,t} \) is the 10-year government bond rate and \( r_t \) is the federal funds rate. The term \( \mu_{10,t} \) represents the term premium of the U.S. Treasury yield curve between the 10-year rate and the 3-month rate. The coefficients are estimated based on U.S. quarterly data from 1965Q1 to 1995Q4. Again, for more details, please refer to Brayton and Tinsley (1996).

In fact, equations for three long-term interest rates and the stock market comprise the core of the financial market sector in the FRB/US model. Unlike nonfinancial behavior, where frictions make it too costly to move immediately to equilibrium values, asset prices are assumed to be in equilibrium continuously. The financial market equations are all exact non-arbitrage conditions.

The Federal Reserve also uses a multi-country large-scale macroeconometric model, the FRB/MCM model. The FRB/MCM model consists of about 1400 equations. The model treats each country in a roughly symmetric manner. The countries modeled include the G-7 economies, Mexico, non-G-7 OECD economies, the newly industrialized economies, OPEC countries, and the rest of the world. The Fed also employs a large-scale macroeconometric model, called the World or FRB/Global model, which merges the non-U.S. parts of the FRB/MCM model and the FRB/US model. A more detailed introduction to the FRB/Global model can be found in Levin et al. (1997).
5.1.2 New Keynesian DSGE models at the Fed

We review two of the major New Keynesian DSGE models constructed and employed at the Federal Reserve for monetary policy analysis and decision making. The first is a multi-country open economy model named SIGMA, and the second is a model of the U.S. economy, the FRB/EDO model, standing for the Federal Reserve Board’s Estimated, Dynamic, Optimization-based model. Both models are extensions of the Christiano et al. (2005) and Smets and Wouters (2007) models whose core ingredients were reviewed in Section 3.2.2. We shall point out key features of the models added on top of the canonical model.

The SIGMA model, compared to the canonical New Keynesian DSGE model with habit persistence in consumption and adjustment costs in investment, incorporates the open economy framework of Obstfeld and Rogoff (1995). Furthermore, the SIGMA model also incorporates international pricing and trading frictions, such as the local currency pricing adjustment cost (see e.g. Betts and Devereux, 1996; Devereux and Engel, 2002) and the cost of adjusting trade flows. Another feature of the SIGMA model is that the agents have incomplete information about the persistence of shocks. More precisely, the agents learn the nature of the shocks using the Kalman filter. This learning mechanism produces gradual responses of the economy to shocks. The third important feature of the SIGMA model is that there are non-Ricardian households who are simply assumed to consume their current after-tax disposable income. The main goal of introducing information frictions and non-Ricardian households is to generate a high persistence in the fiscal multiplier. A review documenting the SIGMA model is in Erceg et al. (2006), in which the authors compared the short-run responses of the SIGMA model to those of the FRB/Global model, and showed that they were quantitatively close.

The FRB/EDO model moves beyond the canonical two categories of private demand, consumption and investment, as found in Section 3.2.2. It divides private consumption into two categories, one of consumer durable goods, and one of consumer non-durable goods and non-housing services. It also separates residential investment from non-residential investment. The model features two final goods sectors, in order
to capture key long-run growth patterns, and to produce different cyclical patterns of different durable expenditures, such as those in consumer durable goods, residential investment, and non-residential investment. One sector produces goods mainly for consumption, and the other sector produces goods that are used for investment or capital accumulation. Like Christiano et al. (2005), this model allows for the variable utilization of capital. The non-residential capital is assumed to be owned by specialists who make decisions on non-residential investment, and hence non-residential capital accumulation. The model incorporates exogenous risk premia shocks trying to capture the financial accelerator effect in Bernanke et al. (1999). For further details on the FRB/EDO models, please refer to its documentation by Chung et al. (2010).

5.2 The European Central Bank (ECB)

5.2.1 Macroeconometric Models at the ECB

The Area-Wide Model (AWM) is a traditional macroeconometric model of the euro area that has been extensively used at the ECB over the past fifteen years. Like the FRB/US model at the Federal Reserve, the AWM model describes the dynamics of the economy through two major components. One is the long-run component, which characterizes the steady state of the economy, and is consistent with neoclassical theory, while the other is the short-run component, which captures the demand-driven short-run dynamics in the data justified by the sluggish adjustment of prices and quantities. As a macroeconometric model, the short-run dynamics are not explicitly derived from an optimization framework, but are instead specified in a more ad hoc form, and estimated on the basis of historical data. Importantly, like other macroeconometric models, the dynamics are “disciplined” by the need to fulfill long-run steady state properties by the use of error-correction terms and appropriate homogeneity properties.

Similar to the FRB/US model, the rest of the world is not explicitly modeled in the AWM framework. More precisely, the AWM model does not include any equations for variables that describe the rest of the world, which is instead treated as exogenous shocks to the model. There are two important drawbacks to the AWM model. First, a number of important channels are ignored. For example, there is no
explicit role of financial and credit markets in shaping the transmission dynamics of monetary policy, because financial quantities and credit variables have no explicit impact on the decisions made by agents in the model. Second, in most equations, expectations are treated implicitly by the inclusion of lagged values of the variables (i.e. adaptive expectations), with the exception of some equations for financial variables (e.g. exchange rates and long-term interest rates). The backward-looking expectation formation method is unrealistic and clearly not satisfactory for policy analysis.

Because of the high level of aggregation in its data, the size of this model is relatively small compared to the FRB/US model. It contains about 84 equations, of which only 15 are estimated behavioral equations. We provide a simple illustration of the equations in the following example. For more detailed documentation, please refer to Fagan et al. (2001).

The aggregate real consumption of households is a function of real GDP, real disposable income and real wealth:

$$\Delta c_t = 0.77\Delta y_t - 0.066 \times [0.74 + c_{t-1} - 0.8(s_{\text{dis},t-1} - \pi_{c,t-1}) - 0.199(s_{\text{wealth},t-1} - \pi_{c,t-1})].$$

Here, $c_t$ is log real consumption, $y_t$ is log real output, $s_{\text{dis},t}$ is nominal households' disposable income, $\pi_{c,t}$ is consumption deflator, and $s_{\text{wealth},t}$ is nominal wealth which is defined as the sum of the capital stock, net foreign assets and public debt. The coefficients in the equation above are estimated based on quarterly data from 1980Q1 to 1997Q4.

### 5.2.2 New Keynesian DSGE Models at the ECB

The European Central Bank (ECB) has developed several DSGE models, which it uses to analyze the economy of the eurozone as a whole. In other words, the model does not analyze individual European countries separately. The models are intended as alternatives to the Area-Wide Model, a more traditional macroeconometric model which the ECB has been using for fifteen years.

According to Smets et al. (2010), the core models at ECB include two different
models. The first is the New Area-Wide Model (NAWM), which is mainly based on Christiano et al. (2005), Smets and Wouters (2003) and Adolfson et al. (2007). Smets and Wouters (2003) originally estimated a closed economy DSGE model of the euro area using Bayesian techniques, while Adolfson et al. (2007) estimated a small-open economy DSGE model of the euro area using Bayesian methods. The second model is the CMR model based on Christiano et al. (2010)\(^{10}\), which basically incorporates the New Keynesian components in Smets and Wouters (2003) and Christiano et al. (2005) with the imperfect credit market mechanism in Bernanke et al. (1999) and Chari et al. (1995).

The NAWM model is similar to the Federal Reserve Board’s calibrated open economy model, SIGMA (see Section 5.1.2). More precisely, besides the long-run neoclassical nature and short-run Keynesian features of nominal stickiness, it incorporates real frictions such as consumption habit persistence and investment adjustment costs. Moreover, it also incorporates frictions relevant in an open-economy model, including local currency pricing, which generates an imperfect exchange rate pass-through in the short run, and costs of adjusting trade flows. Using Bayesian estimation methods, the model is estimated on eighteen key macroeconomic variables, including real GDP, private consumption, total investment, government consumption, exports and imports, a number of deflators, employment and wages, and the short-term nominal interest rate. In addition, data for the nominal effective exchange rate, euro area foreign demand, euro area competitors’ export prices as well as oil prices are used, which are deemed important variables in projections capturing the influence of external developments. Eighteen structural shocks are considered in the estimation. The NAWM model assumes that households are all Ricardian. An important feature (or limitation) of the NAWM model is that it distinguishes between producers of tradable differentiated intermediate goods and producers of three non-tradable final goods. There are three non-tradable final goods, including a private consumption good, a private investment good, and a public consumption good. In addition, there are foreign intermediate goods producers that sell their differentiated goods in domestic markets, and a foreign retail firm that combines the exported domestic intermediate goods.

\(^{10}\)Also see Christiano et al. (2008).
International linkages arise from the trade of intermediate goods and international assets, allowing for imperfect risk sharing and limited exchange-rate pass-through on the import side. For detailed documentation on the NAWM model, please refer to Christoffel et al. (2008).

A distinguishing feature of the CMR model, compared to our canonical New Keynesian DSGE model (see Section 3.2.2), is its incorporation of the financial accelerator channel with imperfect credit market, as emphasized in Bernanke et al. (1999), and the banking system of Chari et al. (1995). In the CMR model, firm investment in physical capital is leveraged, giving rise to the need for external financing. In particular, part of the working capital has to be financed prior to the time revenues from selling current production becomes available. That is, firms need to pay for working capital in advance of production. Another main feature of the model is that the savers and the lenders do not interact directly, but via financial intermediaries. Intermediaries have their own balance sheet with liabilities, mainly different types of deposits, making it possible to construct aggregates such as M1 and M3, and assets, mainly different types of loans. The production of deposits requires resources in terms of capital, labor, and excess reserves. The presence of excess reserves captures the intermediaries’ need for maintaining a liquidity buffer to accommodate unexpected withdrawals. In this model, intermediaries cannot default. Financial contracts are denominated in nominal terms. As borrowers and lenders are ultimately interested in the real value of their claims, shifts in the price level unforeseen at the time the financial contract is signed bring about real effects. This is a way to include the Fisher debt-deflation channel in the model. A detailed illustration and analysis of the CMR model can be found in Christiano et al. (2010).

5.3 The Bank of England (BOE)

5.3.1 Macroeconometric Models at the BOE

The Medium-Term Macroeconometric Model (MTMM, or just MM) is a traditional macroeconometric model of the British economy that has played a central role at the Bank of England. Like the FRB/US model at the Federal Reserve and the AWM
model at the European Central Bank, the MTMM model is built around a number of estimated econometric relationships between important variables, while simultaneously disciplined by long-run properties consistent with economic theory. More precisely, as a macroeconometric model, its short-run dynamics are not explicitly derived from an optimization framework, but are instead specified in a more ad hoc form, estimated on the basis of historical data. Its long-run steady-state properties are imposed in the form of parameter restrictions that are implied by theory.

Like the FRB/US model and the AWM model, the rest of the world is not explicitly modeled in the MTMM model. The MTMM model treats the British economy and the rest of the world in an asymmetric manner, with variables for the rest of the world not appearing in equations as endogenous variables, but only as exogenous shocks in the equations. For example, in the MTMM model, which models Britain as an open economy, aggregate demand can be met from overseas as well as from domestic supply, and domestic supply can be sold overseas to meet foreign demand. So a stylized IS-curve model of aggregate demand can be written as:

\[ c = \gamma_0 + \gamma_1 s^h + \gamma_2 s^w + \gamma_3 r + \gamma_4 x, \]  

(59)

where \( c \) is the real aggregate demand, \( s^h \) is the real domestic income, \( s^w \) is the real income of the rest of the world, \( r \) is the real interest rate, and \( x \) is the real exchange rate. Here, the variable \( s^w \) shows up as an exogenous variable.

The MTMM model has two drawbacks similar to the AWM model. First, a number of important channels are missing, such as imperfect financial and credit markets. Second, expectations of the exchange rate one period ahead are assumed to be formed in a forward-looking manner, which implies that the exchange rate will jump in response to unexpected changes in interest rate differentials or in the long-run exchange rate level. However, other asset prices are not treated in a forward-looking manner, but are assumed to move in ways that are broadly consistent with the long-run growth path of the economy. Inflation expectations are assumed to exhibit a degree of inertia: wage-setters, for example, take time to respond to new information (i.e.
adaptive expectations).

The MTMM model is, to some extent, a restricted vector error-correction Model (VECM). It consists of about 20 key behavioral equations determining endogenous variables, together with about 90 identities defining relationships between variables. We provide a simple illustration of the equations in the following example. For more details, please refer to the Bank of England official documentation.\footnote{“Economic models at the Bank of England” by Bank of England.}

The aggregate households’ consumption is described by

\[
\Delta c_t = -0.036 + 0.19s_{labor,t} + 0.052(s_{nonlabor,t-1} - \pi_{t-1} - c_{t-1}) - 0.068ur_{t-1} \\
+ 0.14(s_{housing,t} - \pi_{t-1} - c_{t-1}) + 0.014(s_{fin,t} - \pi_{t-1} - c_{t-1}) - 0.0016r_t - 0.0017\Delta r_{t-1} \\
- 0.17 \times [c_{t-1} - 0.89s_{labor,t-1} - 0.11(s_{wealth,t-1} - \pi_{t-1} - c_{t-1}) + 0.0028(r_{t-2} - \pi_{t-2})]
\]

where \( \pi_t^e \) is expectation of inflation. In the model, it can be estimated by past inflation rates:

\[
\pi_t^e = 1.1 + 1.2\pi_{t-1} - 0.6\pi_{t-2}. \tag{60}
\]

Here, \( c_t \) is real log aggregate consumption, \( s_{labor,t} \) is real post-tax log labor income, \( s_{nonlabor,t} \) is nominal log non-labor income, \( \pi_{c,t} \) is log total final consumers’ expenditure deflator, \( ur_t \) is the log unemployment rate, \( s_{housing,t} \) is log of the total housing wealth in nominal term, \( s_{fin,t} \) is the nominal log net financial wealth, \( r_t \) is base log interest rate, and \( s_{wealth,t} \) is nominal log total household sector wealth. The coefficients are estimated based on quarterly U.K. data from 1975Q1 to 1998Q1.

### 5.3.2 Hybrid Models at the BOE

The Bank of England, like other major monetary authorities, has developed a macroeconometric model for use in preparing its Monetary Policy Committee quarterly economic projections and inflation reports. Motivated by fears of potential technical
insufficiency and the demand for tractability, the Bank of England has built a model with two distinct layers. Since 2003, the Bank of England Quarterly Model (BEQM) has become the main tool in the suite of models employed by the staff and the Monetary Policy Committee in the construction of the projections contained in its quarterly inflation report. The core layer is a tightly specified theoretical model, containing dynamic decision rules derived from the solution of standard New Keynesian DSGE models. The non-core layer consists of equations that include additional lags and variables to match dynamics that are not modeled formally in the core. These non-core equations also allow the imposition of judgments based on “off-model” information or the judgment of the monetary authorities. The final forecast path can be thought of as a combination of theoretical insight from the structural core model, and the direct application of judgment or ad hoc estimated behavioral dynamics. A detailed illustration of the BEQM is provided in its official documentation by the Bank of England Harrison et al. (2005).

The core of the BEQM is a standard New Keynesian DSGE model for a small-open economy. The model can be used to analyze a wide range of economic issues. Some standard features in its theoretical structure are designed to help match dynamic responses in the data, including potentially consumption habits, labor adjustment costs, capital and investment adjustment costs, inertia in prices and nominal wages, wage and price inflation stickiness, and slow import price pass-through. Because of the size of the core model, it does not fully capture all of the economic channels and dynamic relationships affecting the observed correlations between economic variables. This in part reflects the choice not to include in the core model certain features of the economy which could make the core model too large and complex to be tractable, such as credit market frictions. Moreover, its theoretical assumptions, such as Calvo mechanisms and price adjustment costs, which try to match some aspects of these correlations (for example, the degree of persistence of many nominal variables), are not yet well understood, because they model components that are still “reduced form” on some level.

The non-core layer of the BEQM consists of ad hoc or “data-driven” dynamics on top of its theoretical structure. Incorporating the additional structure in the core
model consistently would make the full model much more complicated and potentially difficult to run. Additionally, there are some effects that seem empirically robust, but are very difficult to model formally. For these reasons, the BEQM tries to embed the additional structures for data coherence, while at the time making the model sufficiently flexible and tractable for forecasting applications. For example, one can think of a neoclassical story about consumption being combined with proxies for credit effects for investment, supplemented by terms for firms preparing for the short run. The only restriction on the structure of ad hoc non-core equations is that the projected path for a given variable should always converge to the long-run equilibrium imposed by the core theory.

The full model is a hybrid combination of core and non-core elements, which matches past movements in the data better than either element on its own, and enables a straightforward application of judgment to the forecast. One interpretation of this hybrid approach is that the final projections are a weighted average of three types of information: a structural story coming from the core model, extra short-run correlations from the non-core model, and judgment applied by the user through the non-core model (the relative weights on these types of information will vary across different parts of the model).

The model has the general format for a non-core equation as follows

\[ A(L)y_t = B(L)y^\text{core}_t + C(L)z_t + \epsilon_t \]  

(61)

where \( A, B \) and \( C \) are polynomials in the lag operator \( L \). The variable \( y \) is the endogenous variable, and the prediction of the variable in the core model is denoted by \( y^\text{core} \). The variable \( z \) represents a vector of selected endogenous and exogenous variables, and \( \epsilon \) is an error term.

In addition, the model has another slightly different form of non-core equation, which simply follows the idea of combining forecasts. For example,

\[ y_{t+1} = \gamma_0 + \gamma_1y^\text{core}_{t+1} + \gamma_2y^{SR}_{t+1}. \]  

(62)
Here $y$ is an endogenous variable, $\hat{y}^{\text{core}}_{t+1}$ is the one-step-ahead forecast generated by the core model, and $\hat{y}^{\text{SR}}_{t+1}$ is a one-step-ahead forecast produced by a statistical “short-term” model.

A key feature of this approach is the strict separation between the core and non-core elements of the model. If the ad hoc elements are introduced into the core, it would risk violating the underlying theoretical assumptions of the core model, and it could also produce an unstable system. One way of viewing this hybrid approach is that the path from the core model is treated as a regressor, along with additional variables and ad hoc dynamics, in the full model equation, as in (61).

Projections from non-core equations feeding back into the core model are not allowed, because this would bring about similar problems of instability and an undermining of the microfoundations of the core theory. Instead, the model uses a “non-feedback” approach, which maintains the distinction between the values from the core and the full forecasting models. This also facilitates the direct application of judgment to the forecast model, so that it is easy to impose desired paths for particular variables.

However, there are several concerns about this hybrid approach. First, the ad hoc component of the model is subject to the Lucas Critique for monetary policy analysis. Second, there is no transparent interpretation for the parameteric form of the dependence of the endogenous variables on their correspondence in the core model. Third, the one-way causal relationship between the projections from the non-core model and the projections from the core model makes the full model theoretically inconsistent.

Following the November 2011 Inflation Report, the forecast process at the Bank of England has been supplanted by the COMPASS platform. The detailed structure of the COMPASS platform is documented in Burgess et al. (2013). The COMPASS platform essentially uses the same idea as the BEQM model, consisting of four components: (1) the Central Organizing Model for Projection Analysis and Scenario Simulation (COMPASS) which is the core theoretical model with microeconomic foundations, (2) the suite of modes alongside the core model, (3) the Model Analysis and Projection
System (MAPS), a MATLAB toolkit built and maintained by economists at the Bank of England, and (4) the Economic Analysis and Simulation Environment (EASE), a new IT user interface.

The core model, COMPASS, is the platform’s main organizing framework for forecast production. COMPASS is an open economy New Keynesian DSGE model, sharing many features with earlier models at other central banks, such as the European Central Bank’s New Area-Wide Model (see, e.g. Christoffel et al., 2008). As a DSGE model, the model is by definition stochastic, in the sense that exogenous random shocks to preferences, technologies and constraints will affect agent decisions. In the absence of shocks, the model settles on a balanced growth path where all variables grow at constant (but possibly different) rates, reflecting exogenous population and technology trends. Shocks push the variables in the model away from the balanced growth path temporarily, with the speed at which they return to the balanced growth path governed by the persistence of the shocks and the strength of the model’s propagation mechanisms, which in turn depend on the specific frictions in the model. COMPASS follows rational expectations as its baseline assumption (i.e. “model-consistent” expectations). The MAPS toolkit can assist analyzing COMPASS using alternative expectation formation assumptions.

Given the recent arguments that the current generation of New Keynesian DSGE models are ill-suited to analyzing the causes and consequences of financial crises, using a model like COMPASS may seem incomplete, particularly given that the model does not include a financial sector. Economists at the Bank of England believe that, at current levels of understanding, the benefits of adding a financial sector to COMPASS would be outweighed by the costs of the added complexity. It is possible that they will come to a different view in the future, as this rapidly developing area advances. Although COMPASS does not include an explicit role for a banking sector, there are several models in its suite that can be used to consider the impact of credit on the economy and explore the effects of an impaired banking sector.

According to Burgess et al. (2013), there are other main economic channels missing from COMPASS beside the financial sector. For example, the COMPASS model does not explicitly account for energy as an input to production or consumption. Changes
in energy prices impact the marginal cost and inflation in a substantially different way from the changes in the prices of other goods and services. Further, the fiscal policy is only modeled in a very simple way. Government spending is assumed not to affect household utility, distortionary taxes play little role, and households behave in a way which guarantees Ricardian equivalence holds. Another example is that there is only a single, short-term interest rate in the COMPASS model. This makes the core model unable to analyze the effects of unconventional policies such as quantitative easing.

However, these missing channels are included in the suite of models. The suite consists of more than fifty separate specific models, covering a wide range of different channels and ways of thinking about the economy, which are not as yet included in the core COMPASS model. Different models can be selected from the suite, depending on what insight is required. The suite provides the means to cross-check the projections in COMPASS, expand the forecast to cover more variables, incorporate potentially critical mechanisms into analysis, and challenge the key judgments in the forecast. The non-core models are incorporated in the core model in an ad hoc manner. The suite includes various extensions of COMPASS to incorporate financial sector channels. For example, one model introduces credit spreads into COMPASS. These drive a wedge between the official policy rate and the effective marginal interest rates faced by households and firms. The household rate enters the consumption equation in both the sticky and flexible price models (in the same way as the risk premium shock), so a rise in credit spreads has a similar effect to a negative demand shock. Meanwhile, a working capital channel is included on the production side. Firms have to borrow to pay for their labor and capital in advance of sales, so a higher credit spread increases their marginal cost. This means that a shock which increases spreads faced by firms leads to higher inflation and a fall in output. The model also allows for a monetary policy response to credit spread shocks.

Finally, the IT infrastructure of the COMPASS platform is a key improvement on the Bank of England’s earlier model, BEQM. This infrastructure consists of two components: a modeling toolbox called MAPS and a user interface called EASE.
5.4 The Bank of Canada (BOC)

5.4.1 Macroeconometric Models at the BOC

The Quarterly Projection Model (QPM) has been one of the core models of the Bank of Canada since September 1993. A detailed documentation of the QPM can be found in Poloz et al. (1994) and the related papers therein. The QPM as a system has two formal components: one, the steady state model based on economic theory at some level of rigor, and the other, a set of short- to medium-run dynamic relationships that provide paths linking the starting conditions to solutions implied by the steady state.

The long-run equilibrium component is called SSQPM. The SSQPM contains several interesting structural features not shared by other steady-state components of macroeconometric models. First, households are modeled using a theoretical device known as “overlapping generations”. Consumers live an uncertain length of time and must plan their consumption and savings over that unknown lifetime. In doing so, they must balance the desire for current consumption with the incentive to save to generate higher consumption levels later in life. The QPM provides solutions for both the desired financial wealth of consumers in the long run, and the consumption/savings paths that will sustain that level. Second, it is an “almost small open economy”. A typical small open economy is characterized by exogenous prices for its exports and borrowing costs. The SSQPM model relaxes the assumption of exogenous exports prices. The idea is that the Canadian economy as a whole has some effect on the price of exports, even though individual firms act in a competitive manner. Such aggregate market power may arise from the fact that Canada is a large exporter of certain goods – wheat, lumber and natural gas, for example. If the supply of these goods increases, the price falls, since the foreign demand curve for these products is not perfectly elastic. While this phenomenon is judged to be important enough to be included in the Canadian model, the effect is assumed to be too small to influence the general level of prices in the rest of the world. Moreover, Canada is assumed to have no influence on the world level of the prices of imported goods. Third, the SSQPM introduces an exogenous risk premium into the firm’s specification. In particular, the risk premium is put into the cost of capital first-order condition as a wedge.
The dynamic structure of the QPM consists of three distinct types of equations. First, there are adjustment dynamics originating from the real and nominal frictions in the economy, including the investment adjustment costs and the labor market contracts. The adjustment features give rise to a gradual response to disturbances. Second, there are separate formation dynamics for expectations. The expectations in QPM are modeled as a mixture of backward- and forward-looking components. The model user can change the relative weights on the two components to generate the sort of stylized facts that are desired. In the core version of QPM, considerable weight is put on the backward-looking portion in order to capture the slow adjustment of expectations apparent in economic data. The forward-looking component is solved conceptually as described above, while the backward-looking portion is usually specified as a simple weighted average of recent historical data. Third, there are automatic policy reactions to disturbances. Accordingly, QPM is specified with inflation control targets and rules of behavior that the monetary authorities will follow, should projected inflation deviate from those targets. Specifically, QPM includes a monetary policy reaction function, according to which a rise in anticipated inflation above target produces a rise in interest rates intended to move inflation back towards its target level over an horizon of six or seven quarters.

In sum, QPM has 27 behavioral equations. There are a total of 329 equations in the model, not counting the satellite structures. There are 155 equations describing expectations; most of the rest are identities. There are only 10 variables for which expectations are required. The large number of expectations equations is needed because the model must keep track of a number of leading terms for each of them.

5.4.2 New Keynesian DSGE models at the BOC

The Terms-of-Trade Economic Model (ToTEM) replaced the Quarterly Projection Model in December 2005 as the Bank’s principal projection and policy analysis model for the Canadian economy. ToTEM is an open-economy, New Keynesian DSGE model. Interestingly, ToTEM contains producers of four distinct finished products: consumption goods and services, investment goods, government goods, and export
goods. ToTEM also contains a separate commodity producing sector. Commodities are either used in the production of finished products, purchased directly by households as a separate consumption good, or exported on world markets. The law of one price is assumed to hold for exported commodities, whereas temporary deviations from the law of one price are permitted for commodities that are purchased domestically.

Recall that QPM only went partway towards incorporating fully rational expectations. Expectations in QPM are a weighted average of model-consistent expectations, or expectations based on forecasts that use the entire structure of the model, and adaptive expectations, which are based only on extrapolations of past values of the variable in question. In traditional macroeconometric models, adaptive expectations are utilized to yield the persistence inherent in the macroeconomic data, including inflation persistence. In ToTEM, a rational expectations DSGE model, expectations can be sticky if monetary policy is viewed as being less than fully credible. A detailed technical description of ToTEM can be found in Fenton and Murchison (2006).
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