Lifetime-Laffer Curves
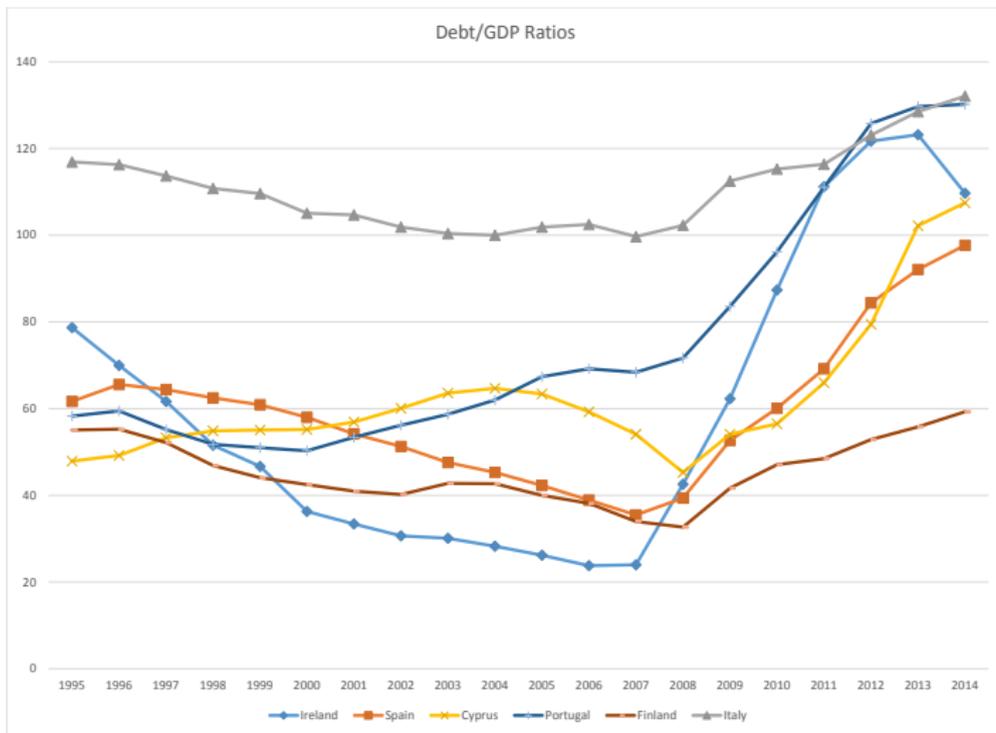# and the Eurozone Crisis

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University of Notre Dame

MFM Group Winter Meetings:
January 29th, 2016

Motivation

- Eurozone crisis: Key features
    1. Sentiments seemed to play role (OMT)
        - Not liquidity Bocola and Dovis (2015)
    2. Borrowing into high spreads → Debt-to-GDP exploded

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Debt/GDP Ratios

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    1. Parsimonious model to generate such crises
        - Multiplicity of financing trajectories
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        - Could be responsible for more than 380 basis points (84.6%) of average spread for Ireland

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    3. Policy prescriptions
    4. Extensions

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- This paper: Impose commitment in terminal periods, but not in initial
    - Work in progress: Commitment in all periods

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- **Budget Condition**

$$d_0 = q_0(b_1 - b_0)$$

$$d_1 = q_1(b_2 - b_1)$$

Model Outline: Lenders

- Default risk in period 2: $g : \mathcal{R} \to [0, 1]$
  - g is increasing, continuous, differentiable, and convex up to $\bar{b} < \infty$ s.t.

    $$g(\bar{b}) = 1$$

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- Implies **No-Arbitrage Condition**

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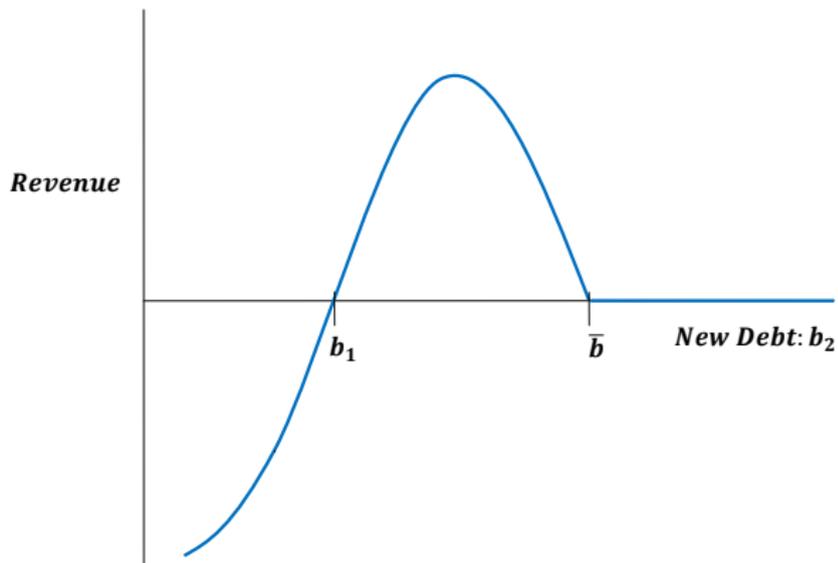
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  Terminal period: *Contemporaneous commitment to debt issuance*

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- Where $\mathbf{D} = d_0 + \frac{d_1}{R}$

Multiplicity via the Lifetime-Laffer Curve

- Let

$$\mathbf{D}^{\star}(b_0) = \max_{b_2} \frac{[1 - g(b_2)]\hat{B}}{R^2}[b_2 - b_0]$$
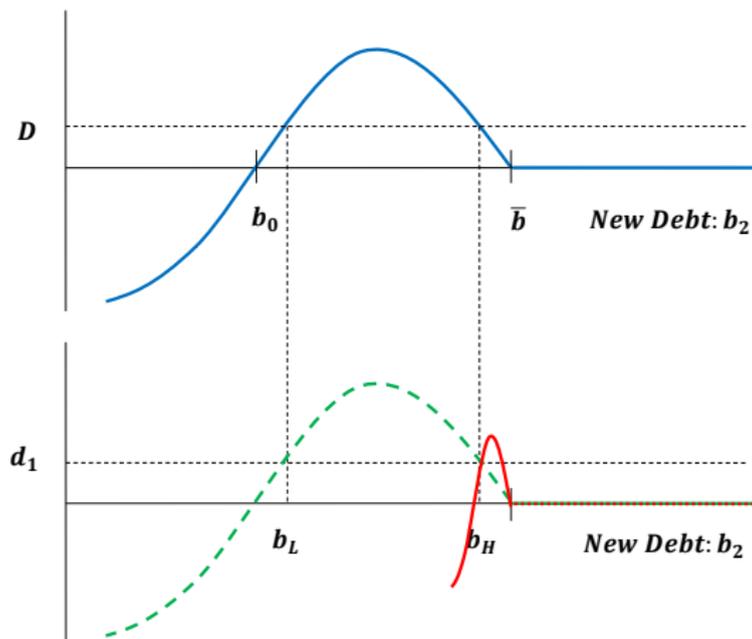
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Proposition

Suppose that $0 < \mathbf{D} < \mathbf{D}^{\star}(b_0)$. Then two solutions exist if and only if the sovereign's primary deficit stream is sufficiently front-loaded.

Call b_2 for each of these solutions b_L and b_H

Graphical Example

The Logic of Front-Loading

Formal **Front-Loading Condition**

$$\frac{d_1}{R} \leq \frac{\hat{B}[1 - g(b_H(\mathbf{D}))]^2}{R^2 g'(b_H(\mathbf{D}))}$$

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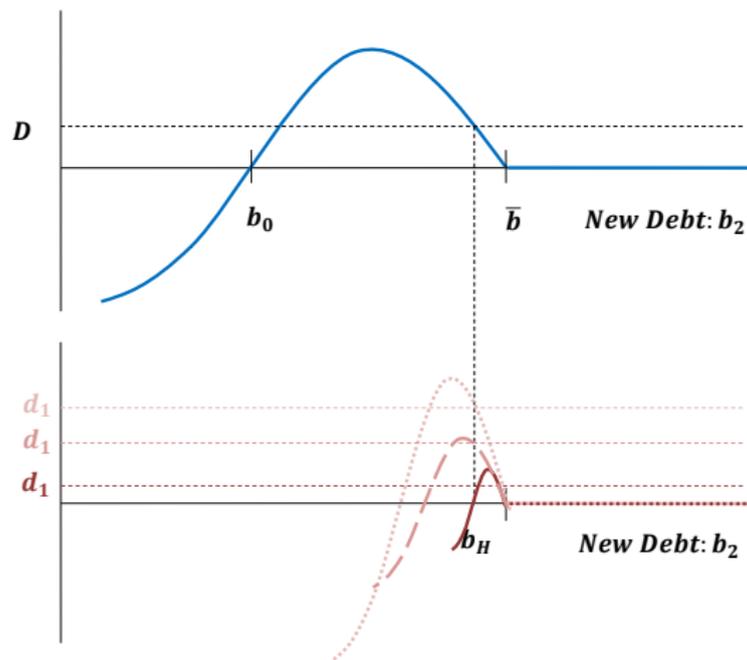## The Logic of Front-Loading

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  - Holds for $b_H \to$ Holds for b_L

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- Delayed austerity *guarantees* existence of two solutions
- Front-loads deficit stream

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*The central bank can costlessly eliminate the high-debt solution by
pledging to provide liquidity at $< q_{0,L}, q_{1,L} >$.*

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- Calibrate T-period model to Irish data: 2008-2013
  - Explains 380 bp of 450 bp spread
  - Very little change in counterfactual B/Y

Extension 2: Deficit Response

- In T-period model, suppose that $d_t(b_T)$
    - Sovereign responds to *expected* debt build-up (or spreads, default prob, etc.)
    - Assume $d_t(\cdot)$ continuous, twice differentiable

Proposition

Under a feasibility condition, a positive economy $<< b_0, \{d_t(\cdot)\}_{t=0}^{T-1} >>$ will have at least two distinct financing trajectories. Further, if each $d_t(\cdot)$ is increasing and convex and a front-loading condition holds, then exactly two solutions exist.

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- Can augment model to include possibility of banking sector bailout
    - Much more action on B/Y

Extension 3: Uncertainty

- Introduce rollover risk in period 1
    - $N < \infty$ potential states with distribution $\{\pi_s\}_{s=1}^{N}$
    - Period 1 deficit is state-dependent: $d_1(s)$
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- Economy is described by $<< b_0, d_0, \{d_1(s)\}_{s=1}^{N} >>$ and the distribution across s
- Solution given by $<< b_1, \{b_2(s)\}_{s=1}^{N} >>$

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Under a feasibility condition and a front-loading condition, a positive economy $<< b_0, d_0, \{d_1(s)\}_{s=1}^{N} >>$ will have at least two distinct solutions. Further, if $<< b_{L,1}, \{b_L(s)\}_{s=1}^{N} >>$ and $<< b_{H,1}, \{b_H(s)\}_{s=1}^{N} >>$ are components of those two distinct solutions and wlog it must be the case that $b_L(s) \leq b_H(s)$ for any $s \in S$ and that only the lesser solution will be numerically stable.

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Generalizes deterministic existence result

Example with Uncertainty: $N = 25$
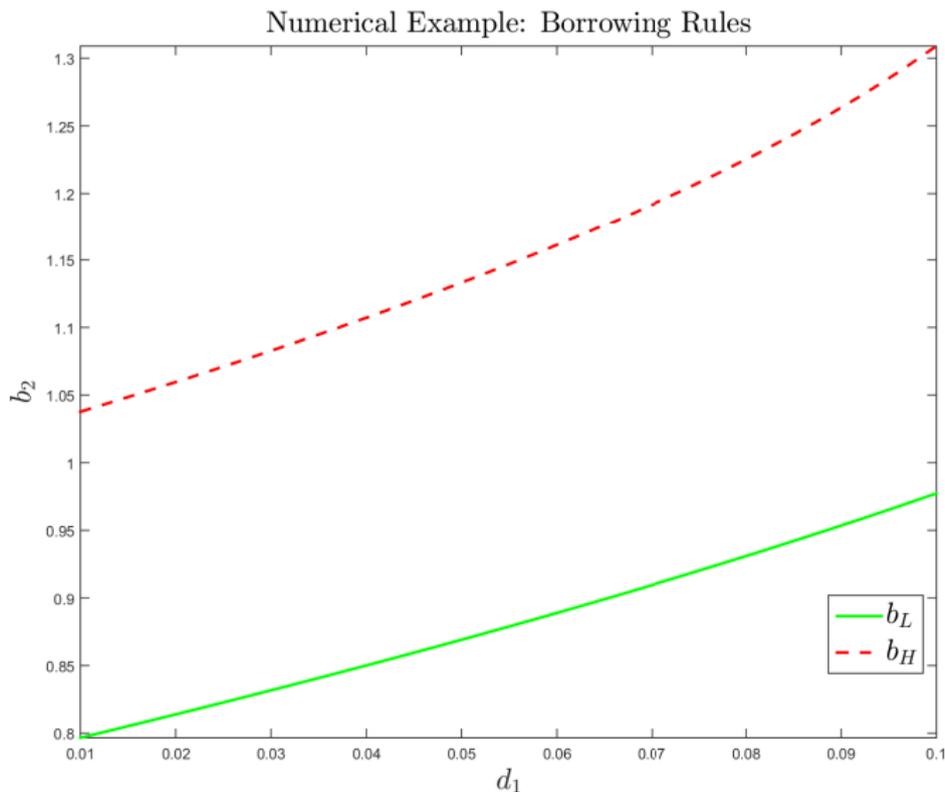
Numerical Example: Borrowing Rules

Conclusion

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    1. Liquidity provision by central bank effective
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- Further work:
    - Infinite-horizon limiting case: Commitment in all periods (in progress)
    - Empirical identification
    - Application to other markets (commercial paper, municipal debt, etc.)