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Tapping into Talent: Coupling Education and Innovation Policies for Economic Growth

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Tapping into Talent: Coupling Education and Innovation Policies for Economic Growth*

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Abstract

How do innovation and education policy affect individual career choice and aggregate productivity? This paper analyzes the various layers that connect R&D subsidies and higher education policy to productivity growth. We put the development of scarce talent and career choice at the center of a new endogenous growth framework with individual-level heterogeneity in talent, frictions, and preferences. We link the model to micro-level data from Denmark and uncover a host of facts about the links between talent, higher education, and innovation. We use these facts to calibrate the model and study counterfactual policy exercises. We find that R&D subsidies, while less effective than standard models, can be strengthened when combined with higher education policy that alleviates financial frictions for talented youth. Education and innovation policies not only alleviate different frictions, but also impact innovation at different time horizons. Education policy is also more effective in societies with high income inequality.

Keywords: R&D Policy, Education Policy, Inequality, Innovation, IQ, Endogenous Growth.
JEL Classification: O31, O38, O47, J24.

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1 Introduction

Talent is a scarce resource. There are only a handful of people like Marie Curie and Isaac Newton with the potential to transform the way we live. Similarly, certain occupations have greater potential of generating significant spillovers to society. While some occupations (e.g. the production of goods) can be executed by a wide range of people and have limited spillovers, other occupations, such as inventors in Research and Development (R&D), require talent to raise the shoulders of giants for the next generation. Further, inventive occupations require not just scarce talent but training in order to come up with effective new ideas. These observations give rise to the following important questions: How do societies select which individuals to educate to become inventors? What frictions prevent individuals from investing in human capital to become an inventor? What policies can alleviate the potential talent misallocation? What are the time horizons over which these policies show their full impact? The answers to these questions are crucial to policy debates in innovation and economic growth and are central to addressing certain puzzles in the literature. Our study is an attempt to address these questions theoretically, empirically, and quantitatively.

To answer these questions, we build a novel endogenous growth model that centers on the development of scarce talent. We connect this model to extensive individual level micro-data to explore the connection between talent, human capital, and aggregate innovation. The theoretical framework stresses five main elements: i) talent is necessary for innovation and heterogeneous in quality, ii) it takes time to turn talent into inventive human capital, iii) some talented individuals, such as those born to poor families, may not have enough resources to build inventive human capital, iv) regardless of access to resources, some talented individuals may dislike research, and v) there are limited training slots at universities. The novel elements of this model are motivated by facts in new micro-level data from Denmark.

We turn to the Danish micro-data and uncover ten facts about the connections between talent, human capital, and innovation. We use these ten facts to discipline our theoretical framework and calibrate the model. This enables us to quantify the impact of policies on talent allocation and aggregate innovation. This dataset includes detailed information on individuals, such as their educational attainment, employment and wages, age, IQ, parental background, and parent economic outcomes. We direct our attention to higher education because of its important links to innovation. The IQ and parental background data enable us to link key new elements of the model to the data. In order to speak to innovation and the effects of policies, we match the dataset to external patent data from the European Patent Office (EPO) and policy data from the Danish Ministry of Education. A unique feature of this dataset is detailed information on an innovation and education policy change implemented in 2002, which introduced new R&D subsidies and dramatically increased funding to universities and the level of PhD enrollment. We exploit this variation to better isolate the links between talent, education and innovation. We stress ten facts in our framework made possible by the extensive Danish dataset that shape our analysis:

Fact 1 *Individuals with higher IQ are more likely to obtain a PhD.*

Fact 2 *Individuals with higher parental income are more likely to obtain a PhD.*

Fact 3 *Individuals' IQ is correlated with parental income, but not perfectly.*

Fact 4 *Only a fraction of people with high IQ and high parental income obtain a PhD.*

Fact 5 *PhDs are 20-times more likely to become inventors compared to the average person in society.*

Fact 6 *Conditional on education level, higher IQ people are more likely to innovate.*

Fact 7 *Inventors work in teams and team size is heterogeneous.*

Fact 8 *The probability of innovating as a team leader over an inventor's life-cycle follows an inverted-U shape.*

Fact 9 *An increase in the number of PhD slots is associated with a decline in the average IQ of PhDs.*

Fact 10 *The economy is open in the goods market but closed in the skills market: import and export accounted for 44% and 51% of GDP in 2010, but foreign migrants make up only 10% of inventors in Denmark.*

While the importance of education, IQ and parental background for innovation outcomes have been emphasized in the literature before (Aghion et al., 2017; Akcigit et al., 2017; Bell et al., 2018), the novelty in our study is two-fold. First, we direct our attention to higher education (Facts 1, 2, 4, 5) and evaluate the importance of this factor in the setting of a policy intervention. Starting in 2002, the Danish government required universities to double PhD enrollment in the span of 10 years, with the goal of expanding the pool of scientists and inventors. As highlighted in Fact 9, the increase in PhD enrollment was associated with a decline in average IQ of enrolled PhDs. This empirical finding illustrates constraints the government faces when attempting to expand the pool of researchers and inventors due to the scarcity and heterogeneity of talent, which is a key determinant of innovation outcomes. It also stresses the importance of tapping into other sources of talent in the economy: attracting into research those talented individuals who are financially constrained or work in other sectors.

Second, we organize these facts systemically through the lens of a model and study their theoretical and quantitative implications. In the model, individuals are born with heterogeneous talent, preferences, and financial resources (Facts 1-4). They decide, depending on their talent and preferences, whether they want to get higher education and become a researcher and contribute to aggregate innovation (Fact 5), or enter the production sector. If they are credit constrained, they may not get the education necessary for innovation even if they have sufficient talent and desire. Once in the research sector, individuals join a research team led by a team leader to produce ideas, where the quantity of ideas produced depends on the team leader's talent (Fact 6) and the size of the team (Fact 7). Individuals then continue to learn on the job with the possibility of becoming a team leader (Fact 8). Schools have a fixed amount of PhD slots and will give those slots to the most talented individuals who are not credit constrained and want to get a PhD (Fact 9). A fundamental driving force in the model is that talent is local and scarce (Fact 10).

The model is tractable and delivers certain intuitive results. Higher wages in research make individuals more likely to work in the research sector. Innovation is limited by the availability of educational slots, talent, and the forces that generate a match of talent to education. Talented people might not end up going into research due to either a dislike of research or a lack of resources to afford education. The latter of these two, financial constraints based on parental background, delivers an inefficiency in the allocation of talent. This inefficiency is linked to economic growth through idea production in the research sector. Our model also enables intuitive policy counterfactuals, as it delivers analytical solutions for the balanced growth path impact of innovation and education policies.

The framework is not only well-suited to connect to individual level data on talent and financial frictions, but also to understand the outcomes of policy interventions from the Danish government. From 2002-2013, the Danish government pursued through a program "Innovation Danmark" aggressive policies aimed at promoting innovation and education. We highlight that different policies affect the economy through different channels. For instance, an R&D subsidy boosts profits in the research sector, pulling talented and financially unconstrained individuals into research. A subsidy to education, on the other hand, enables talented and interested, but financially constrained, individuals to enter the research sector. The Danish government also required universities to increase their PhD slots. Our model highlights that this type of policy introduces a trade-off between an increase in the pool of researchers in the economy and a decline in average talent of researchers, because the marginal individual pulled into the research sector is less talented than the existing pool of researchers. This is consistent with the empirical pattern described in Fact 9.

The model thus provides a framework to analyze the empirical facts in the Danish micro-data. Embedding these facts about talent allocation and innovation into an endogenous growth framework not only brings our theory closer to the data, it also rationalizes a puzzle in the literature on the role of innovation policy. In the standard models of endogenous growth, the relationship between the aggregate growth of an economy (g) and the R&D subsidy rate can be approximated by the following formula:[1]

$$\frac{\partial g}{\partial \text{R\&D subsidy rate}} \approx g. \tag{1}$$

Yet, the empirical literature has not been supportive of this strong link between R&D subsidies and economic growth and has given rise to a number of critiques (Goolsbee, 1998; Romer, 2000; Wilson, 2009). Our framework introduces different margins that affect the relationship between R&D subsidies and economic growth, in Equation (1), and makes it function of the following five ingredients, which have an impact in both the short-run and long-run:

$$\frac{\partial g}{\partial \text{R\&D subsidy rate}} = f(\text{talent, time-to-build, parental income, preferences, limited training slots}).$$

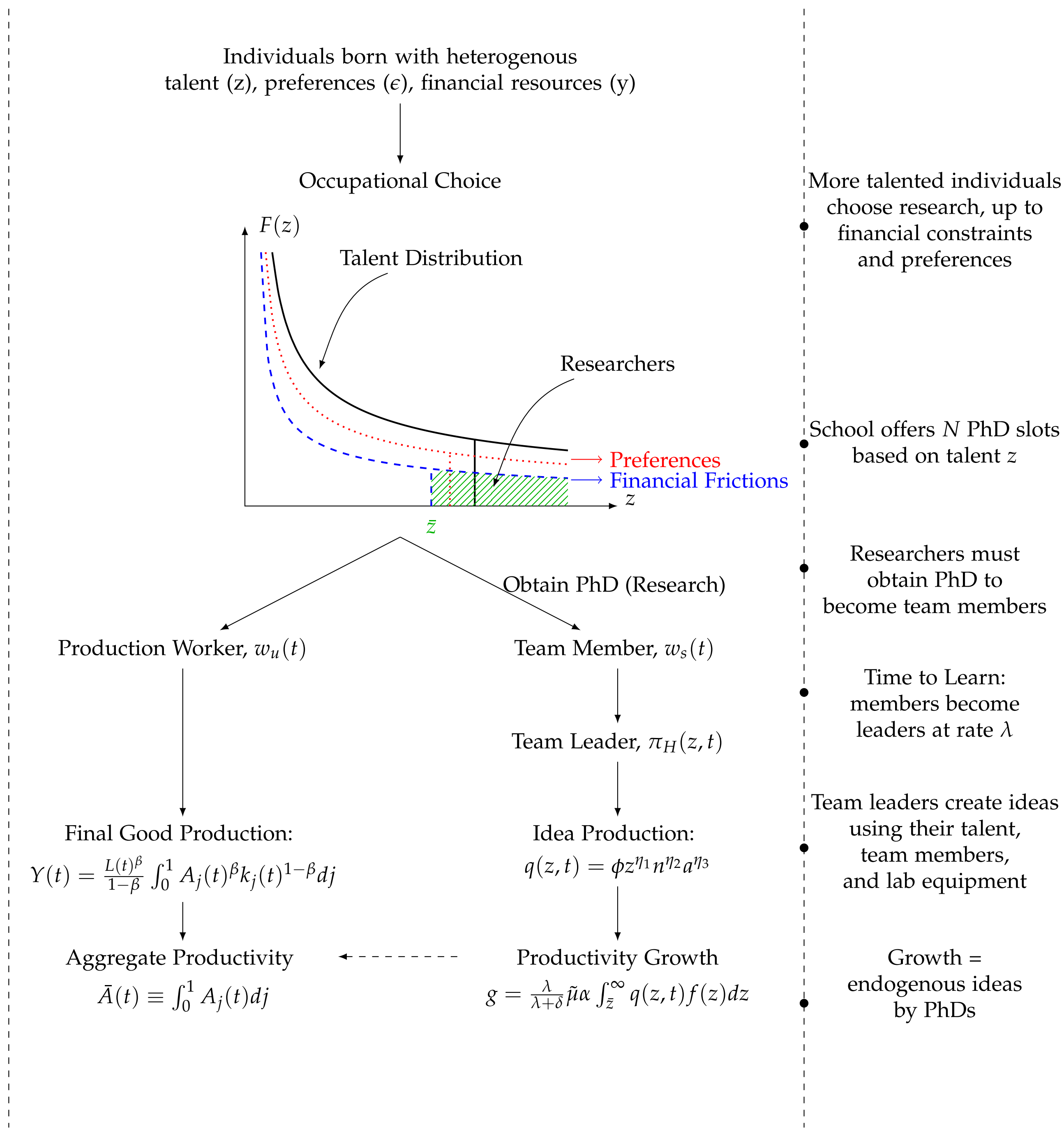
Before proceeding to policy counterfactuals, we validate our model by illustrating its ability to match out of sample moments. First, the model delivers surprising results on selection into higher education depending on parental income. Individuals who enroll in a PhD with wealthier parents have higher IQs than individuals with poorer parents. This result from the model is confirmed in the data. Second, we evaluate the introduction of policies introduced in the Danish education and innovation market. We run these policies through the model and observe the predicted change in the IQ of PhDs. We then compare the model predicted change in IQ to the data and find a close match. After confirming the ability of the model to match out of sample moments, we proceed to innovation and education policy counterfactuals that alter the policy mix. We highlight four main findings.

First, we show that, in steady state, R&D policy has limited effectiveness compared to benchmark models of endogenous growth. R&D policy has even more limited contemporaneous effects. A 10% R&D subsidy only has a contemporaneous effect on the growth rate of about 0.6%, because of the time-to-build component of human capital. Further, the long-run effects of R&D are smaller than standard models due to the fact that the talent pulled into research is restricted by both educational capacity and financial constraints. A 10% R&D subsidy has a long-run effect on the growth rate of 5%, compared to 10% in standard models.

Second, there is a pecking order between education and R&D policies. The optimal policy mix depends on the size of the budget the government has to allocate to research. If the government has 0.5% of

[1] See Appendix C.7 for a derivation of Equation (1).

Figure 1: Summary of the Model



GDP budget to allocate to research, our framework predicts it should allocate all of its resources towards educational subsidies to improve the talent pool by enabling access to education for financially constrained individuals. With a larger budget, the government should mix subsidies to education, subsidies to R&D, and an expansion in the supply of education slots. Given a budget of 1% of GDP, the optimal allocation is between education and R&D subsidies, with 50% of the budget for R&D subsidies and 50% to educational subsidies. Given a budget of 2.5% of GDP, it is optimal to allocate 58% of the budget to R&D subsidies, 26% to education subsidies, and 16% to expanding the educational slots available. The fact that there is an optimal mix highlights the role of the complementarities of the policies in their contribution to economic growth.

Third, our analysis suggests that education policy is more effective in more unequal societies in stimulating innovation. We analyze the responsiveness of policy under different levels of financial frictions. In an environment with lower financial frictions, fewer individuals are unable to pay for schooling; thus, subsidizing the cost of education is less effective in stimulating economic growth. In an economy where no one is constrained, educational subsidies have no effectiveness, as individuals who want to pursue a career in research will do so regardless. In an economy with full frictions, educational subsidies are the most effective policy tool for innovation.

Fourth, we solve for the transitional dynamics of our model and find that it takes time for all of these policies to show their full effect. We find that R&D policy is the most effective policy for innovation in the short-run. On impact, R&D policy stimulates the purchase of R&D inputs other than human capital (e.g. lab equipment) by the existing researchers, which translates into more innovation. Education policy, on the other hand, has limited effectiveness in the short-run due to time-to-build human capital, but is more effective in the medium to long-run. It takes around five years for educational subsidies to surpass R&D policy in terms of its overall growth effect for equivalent expenditure of 0.5% of GDP.

We highlight some overarching themes from these results. First, education policy is an integral component of a policymaker's toolkit for overall innovation. It cannot be thought of separately from its impact on the allocation of talent into research. Second, when observing the aggregate response to any policies, policymakers need to exert patience as each of these policies takes time for their full effects to be realized. Third, the model suggests a framework to think about how inequality and high-skilled immigration interact with these policies. We will revisit these implications in the conclusion.

The rest of the paper proceeds as follows. We complete this section with a review of the literature. Section 2 discusses the institutional background in Denmark and why this is a good setting for our analysis. Section 3 describes the theory, starting with the environment and equilibrium, and moving to theoretical counterfactuals on the introduction of the three policies and the frictions they alleviate. Section 4 introduces the data and the ten facts discussed in the introduction and uses these facts to calibrate the model. We further illustrate the ability of the calibrated model to match out of sample moments. Section 5 performs the quantitative policy counterfactuals. Section 6 discusses robustness checks of these counterfactuals. Section 7 concludes.

Related Literature

This paper primarily builds on and extends the theoretical and empirical literature on innovation and endogenous growth. One of the main departures of our analysis is the focus on individuals instead of firms. In our model, as in the classical endogenous growth models, ideas are the main source of economic growth (Romer, 1990; Aghion and Howitt, 1992), and idea production is heterogeneous in terms of quality, as in Akcigit and Kerr (2018). Alongside this, we stress that the idea creation and transmission process happens through individuals with talent, as in Lucas and Moll (2014), and Perla and Tonetti (2014) and link this talent to objects in the data.

The relationship of scarce talent and aggregate innovation has received lively discussion in recent papers. Aghion et al. (2017), Akcigit et al. (2017), and Bell et al. (2018) find that parental backgrounds influence who becomes an inventor, in Finland, historical US, and modern US, respectively. Through name-matching between modern and historical data, Celik (2015) also finds that a child's likelihood of becoming an inventor correlates with family background. We verify this finding in the case of Denmark in one of our main ten facts. The allocation of talent to innovative occupations has important implications for economic growth, as Waldinger (2016) finds that human capital is much more important than physical capital for innovation in both the short and long-run. We put human capital at the center of this framework, recognizing that human capital must be built from talent.

Understanding the role of human capital in aggregate innovation can help enrich discussions on the interaction between R&D policy and economic growth. Standard growth models of R&D in firms assume the supply of scientists is elastic in steady state (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992; Klette and Kortum, 2004) and predict large effects of R&D subsidies on economic growth. However, there is empirical evidence that the general equilibrium effects of R&D policy are minimal (Goolsbee, 1998; Wilson, 2009). Our analysis enriches this discussion and provides a more micro-founded analysis of the scientists in research. In particular, we introduce a framework that includes matching and financial frictions (e.g., Jovanovic, 2014, Celik, 2015), talent heterogeneity (e.g., Jaimovich and Rebelo, 2017), diffusion and social interaction (Kortum, 1997; Perla et al., 2015), and the delayed process of skill accumulation (e.g. Arrow, 1962; Stokey, 1988; Benhabib et al., 2014; Konig et al., 2016). These new ingredients can bring this theoretical literature closer to empirical studies of the impact of R&D policy.

Empirically, economists have found that R&D subsidies increase firm innovation levels (Bloom et al., 2002; Hall and Van Reenen, 2000). Fiscal policies and public R&D investments also have positive effects in partial equilibrium (Moretti and Wilson, 2014; Azoulay et al., 2018). However, there is also evidence that this is much weaker in general equilibrium. For instance, Wilson (2009) finds that R&D subsidies simply pull innovative activity away from states that do not have subsidies, creating small aggregate effects. Moretti and Wilson (2014) find that R&D subsidies often simply lead to scientist reallocation across states. Dimos and Pugh (2016) review the literature and find mixed effects of public expenditure on R&D spending. Bloom et al. (2019) discuss this literature and suggest a potential combination of policies, which this current paper explores. In particular, we find that innovation policy is much more effective when combined with education policy.

The elasticity of the supply of human capital in research is an important component for explaining the interaction between innovation and R&D policy, as noted by Goolsbee (1998) and Romer (2000). Goolsbee (1998) finds that R&D subsidies mostly transmit to scientist wages. With an inelastic supply of scientists, this price effect does not transmit significantly to overall innovation. Even if human capital is elastic in the long-run, it takes time to build and entering individuals may have different talent than the existing pool. We build a model that formalizes these empirical observations.

In formalizing the connection between education and aggregate innovation, we speak to a literature on the role of universities for innovation. Previous papers have noted the importance of education for innovation (Aghion et al., 2009, 2017; Toivanen and Vaananen, 2016), and the role of education and human capital formation for long-run economic growth (Mincer, 1984; Barro, 2001). Grossman et al. (2017), for instance, find that education and increased levels of schooling are a key component of sustaining balanced growth. Higher education broadly serves two purposes. The first is the production of basic research (Nelson, 1959; Stephan, 1996), which the private sector builds on directly (Ahmadpoor and Jones, 2017). Second, and more central to this paper, is the crucial role higher education plays in training talent. College graduates and PhDs make up a large share of inventors as Aghion et al. (2017) find in Finland, and which we also find in the case of Denmark. However, individuals do not immediately innovate upon entering a PhD program or graduation.

Our model connects to empirical observations on how the human capital of researchers and inventors builds (Jones, 2010; Jones and Weinberg, 2011) and depreciates (Aghion et al., 2020) over the life-cycle. Individuals realize their innovative productivity in teams (Wuchty et al., 2007; Jones, 2009). In each team, individuals take on different roles depending on their human capital (Pearce, 2020), and split the returns to innovation depending on the role within the team (Aghion et al., 2018). In this paper, individuals who serve as team members accumulate skills to become team leaders but also face the possibility of becoming obsolete. This is disciplined by the Danish micro-data with specific attention to how inventive human capital is built from talent.

Given its scarcity, the allocation of talent to specific occupations is central to the production of ideas and growth of an economy. Murphy et al. (1991) note how occupational choice is an important force in economic growth in the context of rent-seeking versus entrepreneurship. Using survey data, Arts and Veugelers (2020) show that individuals with a strong taste for science make better inventors. Rosen (1981) discusses how, in a world without financial frictions or heterogeneous preferences, high ability individuals will sort into careers with the highest returns to talent. Willis and Rosen (1979) show evidence based on this fact and Topel and Ward (1992) study this in the context of early career occupational switches. However, this is complicated by frictions in the allocation of individuals to occupations. Celik (2015) finds that misallocation of talent to non-inventing occupations has a first-order effect on economic growth. Hsieh et al. (2019) find that better occupational allocation for minorities and females over the last 50 years has contributed to aggregate growth, which complements a literature that has addressed this with female labor force participation (Greenwood et al., 2005). If frictions in occupational allocation are not addressed, we find that expansion in the supply of researchers brings in less talented researchers, which will have a smaller impact on growth.

We unite these facts on skill, education, and innovation into an endogenous growth framework with a span of control along the lines of Lucas (1978) and Akcigit et al. (2018). In addition to an otherwise standard span of control model with talent heterogeneity, this paper builds in education, financial frictions, time-to-build human capital, and physical capital for R&D (e.g., lab equipment). These forces enable more realism when it comes to addressing how education and innovation policies interact with talent and human capital in the economy. This realism enables evaluation of the transmission of policies to innovation and economic growth at different horizons. We next turn to the institutional environment in Denmark within which we do our analysis.

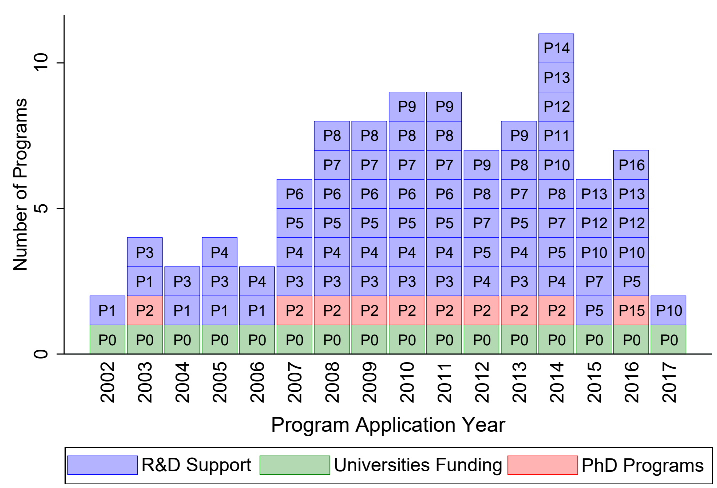
2 Institutional Background

This project is built on Danish micro-data with individuals and firms. We primarily make use of individual identifier data and extensive innovation and education policies in Denmark from Denmark's Statistics Office (DST) and external data sources from 2002-2013. The policies in Denmark and individual outcomes provide a laboratory to understand the effects of specific economic policies targeted towards higher education and R&D. This section provides details on the institutional background for higher education in Denmark and the relevant education and innovation policies. We direct particular attention to PhDs, as they are the main focus of this paper and also an important target in the policies introduced in this time period.

A PhD is the highest level of educational attainment in the Danish education system and the Danish Ministry for Education and Research considers it a key element for supporting scientific capacity in Denmark (Ministry of Education, 2016). A typical PhD program has a duration of three years, and begins after the completion of a 2 years Masters program. There are multiple ways of financing a PhD, including University basic funding, external grants, and funding from research councils or foundations.

The Danish Government has introduced a number of education and innovation policies since 2002. These policies were united in order to build "a comprehensive strategy for the development of Denmark into a leading global growth and knowledge society" (Jensen et al., 2012). On the education side, alongside targets for education attainment at lower levels, the goal was to increase the provision of higher education in order to establish a highly qualified recruitment base of researchers to both the private and public sectors. On the innovation side, the objective of the new R&D programs was to make Danish companies among the most innovative in the world. The "Innovation Danmark" database contains information on several education, research, and innovation programs since 2002. Figure 2 displays the number of active programs in the "Innovation Danmark" database by year. Each color code signifies a program that addresses a different element of the market for idea production – R&D subsidies (blue), educational slots expansion (green), and educational subsidies (red).

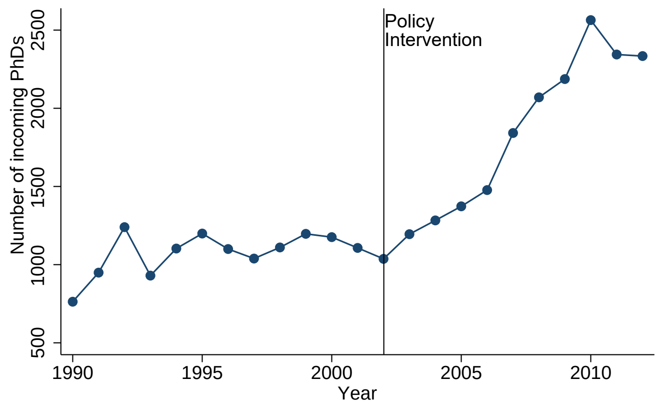
Figure 2: Research and Education Programs by year in the "Innovation Danmark" database



Source: DST, Note: Each box represents an active program in the particular year. The identifiers inside each box represents the program instrument listed in Appendix A. Color codes refer to the role of policies in terms of R&D support to firms (blue), grants for PhD students (red), and subsidies to universities to expand educational slots (green).

As part of the investment in higher education, the government required universities to increase PhD enrollment. This feature of the institutional environment motivates our modeling choice of a fixed number of university slots which can be expanded through government policy.

Figure 3: Increase in PhD Enrollment in Denmark



Source: DST, Note: This graph plots the incoming PhDs by year in Denmark from 1990-2012.

Universities were required to increase the annual intake of PhD students to a target number of 2,400 students, particularly within natural science, technology, medical and health science, and ICT (Ministry of Education, 2016). PhD enrollment, which had been relatively stable at about 1200 individuals in the years prior to 2002, started to gradually increase and reached about 2400 in 2012, as displayed in Figure 3. This was accompanied with increases in funding for universities in the form of educational and research grants.

We will use these programs to discuss both qualitative and quantitative counterfactuals that resemble these policies. In the process, we group the various programs into three main categories: (i) R&D subsidies, (ii) subsidy to the cost of education, (iii) increase in PhD slots. In Section 5, we will discuss quantitative implications of the policies, making use of a calibrated version of our model from Section 4. First, we introduce the model and discuss how the model can be used to explore policy counterfactuals.

3 Model

We start by introducing a tractable endogenous growth model that puts human capital and occupational choice at the center. Individuals are heterogeneous in talent, parental resources, and preference for research. When individuals are born, they choose whether to enter the production sector or research sector as in Figure 1. In order to enter the research sector, they must obtain a PhD. Universities have a fixed number of slots which they give to individuals who (i) have a sufficiently high IQ, (ii) choose to take a career in research, (iii) are able to afford the financial cost of education.[2]

In the research sector, individuals form teams to produce ideas and maximize income. Upon entering the research sector, individuals become members of a research team, until they stochastically accumulate sufficient human capital to become team leaders. The economy is open to trade in the goods sector and capital markets, which implies that the interest rate is exogenous, but the idea production sector is closed

[2]We interpret this cost more broadly than tuition, in that it includes the cost of foregone income that might be required by individuals to support their families.

to trade, as in Grossman and Helpman (1991).

The model is both tractable and connects to the ten empirical facts discussed in the introduction and explored in Section 4. We provide an analytical solution to the steady state of the model and then discuss transitional dynamics from the introduction of subsidies. This framework is amenable to the introduction of various policies which we explore theoretically in Section 3.2. In Section 4, we bring the model closer to the data by matching its quantitative features in order to inform policy counterfactuals.

3.1 Environment and Equilibrium

In this section, we will describe the basic environment and equilibrium. Our focus will be on a balanced growth path equilibrium where all aggregate variables are growing at the same rate g. There is a research market, intermediate goods monopolists who buy ideas from researchers, and a competitive final good production market. In order to discuss the main new features of the framework, we open the model discussion with occupational and educational choice and then turn to a discussion of the research and final goods markets, and close the model with the discussion of the balanced growth path.

Preferences and Career

A unit mass of individuals live in a small open economy. Time is continuous and individuals are born and die at rate δ. Individuals garner hand-to-mouth log utility and attempt to maximize lifetime utility with discount rate ρ. Individuals consume the final good and make educational or career choices based on the expected value of occupations.

Individuals of a certain talent, preferences, and parental background sort into three types of occupations. There is a mass of production workers L_P and two types of skilled researchers – team leaders or head of team, H, and team members, M, such that:

$$L_P + H + M = 1$$

Individuals who obtain a PhD become researchers and will be either a team leader or team member, while production workers work in final good production. We now set up the problem of individuals' educational choice.

Education

Individuals are born with talent, family resources, and career preferences. When they are born, they choose whether or not to apply for a PhD to enter the research sector. Individuals might not get a PhD for three reasons. First, individuals with a low talent or IQ, denoted z, will not be able to get into a PhD program because the school selects those with the highest IQ and has limited slots. Second, individuals who are financially constrained cannot enroll in a PhD program, even if they have high talent. Third, individuals who prefer working outside the research sector will not accept a PhD offer.

The university offers a PhD slot to a share N of individuals born in each cohort. The university attempts to offer the slots to the most talented individuals. In particular, the university admits individuals that will accept the offer with talent z greater than an equilibrium threshold $\bar{z}$ that fills a class of size N. The school is aware of the distribution of preferences, financial constraints, and talent and the cutoff will factor in each of them. Before turning to the school's cutoff, we characterize the individual's problem.

We discuss the talent, financial constraints, and preferences in order. First, $F(z)$ is the c.d.f. that the talent distribution follows, $z \sim F(z)$. We assume a Pareto distribution such that $F(z) = 1 - \frac{z_{min}}{z^{\theta}}$ and we normalize $z_{min} = 1$. Thus, the fraction of individuals of a given cohort above the school's threshold $\bar{z}$ will be $1 - F(\bar{z}) = \bar{z}^{-\theta}$.

Second, whether or not an individual with talent z can afford education, $\tilde{\mu}(z)$, is determined by parental resources. Upon starting a PhD, an individual must pay an upfront cost of education κ. Individuals rely on parental income to cover the cost. We assume that μ individuals have a positive assortative match of IQ to parental income (i.e. more talented individuals are matched to richer parents).[3] For the remaining fraction $(1-\mu)$ of individuals, parental resources are independent of their talent. For these individuals, we assume that income is distributed as a Pareto with shape parameter $\tilde{\theta}$ and that if income were equally distributed, all students could afford education. This is equivalent to assuming that the scale parameter of the income Pareto distribution takes value $\frac{\tilde{\theta}-1}{\tilde{\theta}}\kappa$, and so the probability these individuals can afford education is $\left((\tilde{\theta}-1)/\tilde{\theta}\right)^{\tilde{\theta}}$. Thus, the share of individuals in a given cohort that can afford education is:

$$\tilde{\mu} \equiv \mu + (1-\mu) \times \left(\frac{\tilde{\theta}-1}{\tilde{\theta}}\right)^{\tilde{\theta}} \tag{2}$$

Lastly, even an individual with talent $z > \bar{z}$ and sufficient parental income may not choose to become a researcher. If she has sufficient talent and her parents can afford education, she will compare the alternative of becoming a researcher and a production worker where the preference for being a production worker enters additively as $\ln(\epsilon)$. Thus, the lifetime value function, $V(z, \epsilon, b)$, for an individual born in cohort b with talent z and preference shock ϵ is given by:

$$V(z, \epsilon, b) = \max\{V^{phd}(z, b), V^{worker}(b) + \ln(\epsilon)\}, \tag{3}$$

where $V^{phd}(z, b)$ is the value of becoming a researcher for an individual with talent z and $V^{worker}(b)$ is the value of becoming a production worker. The lifetime value of being a PhD for individuals with talent z in cohort b is given by

$$V^{phd}(z, b) = \int_b^{\infty} e^{-(\delta+\rho)(t-b)} \left[e^{-\lambda(t-b)} \ln(w_s(t)) + (1 - e^{-\lambda(t-b)}) \ln(\pi_H(z, t))\right] dt. \tag{4}$$

Individuals observe the lifetime path of their income as a researcher discounting their lifetime income which they consume with log utility hand-to-mouth. Individuals start off entering the PhD as a team member getting paid a wage for working on a research team w_s. Individuals transition to team leaders at rate λ and as team leaders their income π_H depends on their talent z. The individual value of being a production worker (without including the preference shock) is as follows:

$$V^{worker}(b) = \int_b^{\infty} e^{-(\delta+\rho)(t-b)} \ln(w_u(t)) dt,$$

[3] Formally, for this share μ of individuals, parental resources are proportional to IQ by a constant ζ. For ease of exposition, we present the case where ζ is large enough so that all individuals in this group can afford education. This will be true for the value of ζ that we obtain from our calibration. In Appendix C.6, we derive key model equations for the case where some of the individuals, in the left tail of IQ distribution, with assortatively matched parental resources cannot afford the cost of education.

where w_u indicates the wage earned as a production worker which we derive later in this section.

A given individual compares the value of a research career against a career as a production worker, discounting at $\delta + \rho$. Given wages and profits, forward looking individuals decide whether they want to obtain a PhD. Even if individuals prefer a PhD, they may not obtain it due to frictions or talent. To understand the forces determining the individual decision and school cutoff, we turn to the research production and labor market in order to characterize wages and profits.

Research Production

Once individuals make their career choice, those who pursue an advanced degree (PhD) work in research teams composed of a team leader and team members. Upon enrolling in a PhD program, an individual becomes a team member and earns a skilled wage w_s. While she is a team member, she stochastically transitions to being a team leader with Poisson rate λ. The arrival rate λ captures the time required to build human capital, which includes time for formal education to earn a PhD as well as time spent learning on the job.

At each time t, a team produces a set of ideas. A team leader with talent $z \sim F(z)$ hires n team members and purchases a units of lab equipment at the marginal cost $\bar{A}$. For each total quantity of ideas, a fraction ϕ are implemented successfully.[4] This delivers a net quantity of ideas q:

$$q = \phi z^{\eta_1} n^{\eta_2} a^{\eta_3}.$$

The number of the ideas produced is increasing in the team leader's productivity and in the number of team members. The parameter $\eta_1 \in [0,1]$ denotes the team leader's span of control as in Lucas (1978). The parameters η_2 and η_3 are the team members and lab equipment shares respectively. Given the per unit price p[5], a team leader with talent z chooses the number of members n and lab equipment a to maximize her profits,

$$\pi_H(z) = \max_{n,a \geq 0} \left\{ p\phi z^{\eta_1} n^{\eta_2} a^{\eta_3} - w_s n - \bar{A}a \right\}.$$

We assume $\eta_1 + \eta_2 + \eta_3 = 1$. The profit maximization of the team leader delivers, for a given z, optimal team size n, lab equipment a, quantity of innovation q, and team leader profits π_H:

$$n(z) = z \left(\eta_2 \phi \frac{p}{w_s} \right)^{\frac{\eta_1+\eta_2}{\eta_1}} \left(\eta_3 \phi \frac{p}{\bar{A}} \right)^{\frac{\eta_3}{\eta_1}} \tag{5}$$

$$a(z) = z \left(\eta_2 \phi \frac{p}{w_s} \right)^{\frac{\eta_2}{\eta_1}} \left(\eta_3 \phi \frac{p}{\bar{A}} \right)^{\frac{\eta_1+\eta_3}{\eta_1}} \tag{6}$$

$$q(z) = \phi z \left(\eta_2 \phi \frac{p}{w_s} \right)^{\frac{\eta_2}{\eta_1}} \left(\eta_3 \phi \frac{p}{\bar{A}} \right)^{\frac{\eta_3}{\eta_1}} \tag{7}$$

$$\pi_H(z) = \eta_1 p \phi z \left(\eta_2 \phi \frac{p}{w_s} \right)^{\frac{\eta_2}{\eta_1}} \left(\eta_3 \phi \frac{p}{\bar{A}} \right)^{\frac{\eta_3}{\eta_1}} \tag{8}$$

[4] Another way of interpreting ϕ can be as the common R&D productivity in the economy.

[5] We close the model later in this section and find that the price p is independent of the intermediate good producer to whom a research team sells the idea.

These results deliver the value of being a team leader in the research sector and will inform an individual's occupational choice.

Occupational Choice and Equilibrium in the Research Labor Market

Forward looking individuals observe the research and final good production market and make their career choice as described in Equation (3).[6] This equation governs how the research market influences occupational choice and the school cutoff $\bar{z}$. We find that an individual of talent z is indifferent if:

$$z = \left(\frac{\hat{w}_u}{\hat{w}_s}\right)^{\frac{\delta+\rho}{\lambda}} \frac{\hat{w}_u}{\hat{\pi}} \epsilon^{\chi},$$

where we define $\chi \equiv \frac{(\delta+\rho)(\delta+\rho+\lambda)}{\lambda}$, and $\hat{w}_u$, $\hat{w}_s$, and $\hat{\pi}z$ are the detrended values of being an unskilled worker, skilled worker, and a team leader with talent z respectively.[7] We assume $\epsilon^{\chi} \sim U(0, Ez)$, thus the preference shock scales with individual talent with coefficient E. It follows that the fraction of individuals in a given cohort who prefer the research sector, α, is also time-invariant and independent of z:

$$\alpha \equiv \Pr\left(V^{phd} > V^{worker} + \ln(\epsilon)\right) = \frac{1}{E}\left(\frac{\hat{w}_s}{\hat{w}_u}\right)^{\frac{\delta+\rho}{\lambda}} \frac{\hat{\pi}}{\hat{w}_u}. \tag{9}$$

We return to the fixed set of slots N that the school holds for students, also recalling the three elements that determine entry into PhD. Those with talent $z \geq \bar{z}$, financially unconstrained, $\tilde{\mu}$, and those who prefer working in the research sector, α will enter the PhD:[8]

$$N = \Pr(z \geq \bar{z}) \times \tilde{\mu} \times \alpha. \tag{10}$$

The occupational choice and school cutoff determine the availability of researchers in the economy. In the labor market of researchers, team leaders hire team members and wages and prices equate supply and demand for team members in steady state. Recall that team members M become team leaders H at flow rate λ and individuals exit the economy at rate δ. In steady state, this implies the following flow equation:

$$\lambda M = \delta H.$$

Moreover, recalling that $M + H = N$[9], we obtain that the number of workers in each occupation is:

$$M = \frac{\delta}{\lambda+\delta} N, \tag{11}$$

$$H = \frac{\lambda}{\lambda+\delta} N, \tag{12}$$

$$L_P = 1 - N. \tag{13}$$

[6] Appendix C.1 solves this equation and shows that the solution to the occupational choice problem is time-invariant.

[7] We look for a balanced growth path of the economy where the growth rate of final output g is constant. Then, the expressions for the detrended values $\hat{w}_u$, $\hat{w}_s$, and $\hat{\pi}z$ are such that: $w_u(t) = \hat{w}_u A_0 e^{gt}$; $w_s(t) = \hat{w}_s A_0 e^{gt}$; $\pi_H(z,t) = \hat{\pi} z A_0 e^{gt}$.

[8] Financial constraints and preferences are independent of z.

[9] Notice that, given that population size is constant, along the balanced growth path the share of individuals in a cohort who enroll in a PhD, N, is equal to the share of researchers in the population.

The demand for team members must take into account the distribution of team leaders in the economy. This depends on the arrival rate to becoming a team leader, λ, the school's cutoff for PhDs which we will solve for explicitly, $\bar{z}$, the measure of financial frictions, $\tilde{\mu}$, and the probability an individual with sufficient talent and financial resources chooses a career in research, α. The supply of team members and team leaders depends on the arrival rate to becoming a team leader, λ, and the death rate, δ. The overall equation governing the relationship is the following:

$$\underbrace{\frac{\lambda}{\lambda+\delta}}_{\text{\# team leaders}} \underbrace{\left[\tilde{\mu}\alpha \int_{\bar{z}}^{\infty} n(z) f(z) dz\right]}_{\text{Demand for team members}} = \underbrace{\frac{\delta}{\lambda+\delta} N}_{\text{supply of team members}} . \tag{14}$$

Plugging in N from Equation (10) and $n(z)$ from Equation (5), we solve for the wages of team members as a function of the price of ideas, p:

$$\frac{w_s}{p} = \phi\eta_2 \left[\frac{\lambda\bar{z}}{\delta}\frac{\theta}{\theta-1}\right]^{\frac{\eta_1}{\eta_1+\eta_2}} \left(\eta_3\phi\frac{p}{\bar{A}}\right)^{\frac{\eta_3}{\eta_1+\eta_2}} . \tag{15}$$

We can use these equations to get the expression for equilibrium profits for the team leader z:

$$\pi_H(z) = z\eta_1\eta_3^{\frac{\eta_3}{\eta_1+\eta_2}} \left[\frac{\delta}{\lambda\bar{z}}\frac{\theta-1}{\theta}\right]^{\frac{\eta_2}{\eta_1+\eta_2}} \left(\phi\frac{p}{\bar{A}}\right)^{\frac{1}{\eta_1+\eta_2}} \bar{A}. \tag{16}$$

These equations describe wages and profits in the research sector. We now turn to the final good production to characterize the wages of production workers and then to intermediate goods producers who buy ideas to characterize their prices.

Final Good Production

The final good $Y(t)$ is competitively produced at time t using effective labor L and a continuum of intermediate goods k_j:

$$Y(t) = \frac{1}{1-\beta} L(t)^{\beta} \int_0^1 A_j(t)^{\beta} k_j(t)^{1-\beta} dj,$$

where $A_j(t)$ represents the quality of the intermediate good j at time t. The price of the final good is normalized to 1. The time indices will be suppressed henceforth when it does not cause confusion.

The profit of the final goods producer is equal to their total output minus the prices they pay for intermediate goods $P_j k_j$ and wages paid to effective labor $w_u L$. Each laborer provides ℓ units of labor to final good producers such that $L = \ell L_P$.[10] The standard maximization problem of the final goods producers delivers a demand curve for intermediate inputs k_j as follows:

$$P_j = L^{\beta} A_j^{\beta} k_j^{-\beta}. \tag{17}$$

Each intermediate good is produced by a monopolist that holds the product line j. The marginal cost

[10] We introduce a parameter for labor efficiency units in production, ℓ, to help discipline the connection between wages and profits in the intermediate goods and research sector in the calibration in Section 4.3.

of producing each intermediate good is ψ in terms of the final good and the monopolist maximizes profits subject to the demand curve as follows:

$$\Pi_j = \max_{k_j, P_j} \left\{ P_j k_j - \psi k_j \right\}, \textit{ subject to (17).}$$

The resulting equilibrium profits for the intermediate good producer can be shown to be linear in the quality A_j such that:

$$\Pi_j = \pi_I A_j,$$

where $\pi_I \equiv \beta \left[(1-\beta) / \psi \right]^{\frac{1-\beta}{\beta}} L$. We can then rewrite the demand for each intermediate good as:

$$k_j = \left[(1-\beta) / \psi \right]^{\frac{1}{\beta}} L A_j.$$

Given this equilibrium demand for k_j and the final good producer's optimal choice of labor, the wage of the unskilled worker is then as follows:

$$w_u = \frac{\beta}{1-\beta} \left(\frac{1-\beta}{\psi} \right)^{\frac{1-\beta}{\beta}} \bar{A}, \tag{18}$$

where $\bar{A} \equiv \int_0^1 A_j dj$ is aggregate productivity of the economy. In addition, final good output is:

$$Y = \frac{1}{1-\beta} \left(\frac{1-\beta}{\psi} \right)^{\frac{1-\beta}{\beta}} L\bar{A}. \tag{19}$$

Given their profit $\pi_I A_j$, intermediate goods monopolists have incentives to invest in technology. They can increase their productivity (A_j) through buying ideas from research teams. We now turn to the process of purchasing an idea to characterize prices.

The Market for Ideas

The intermediate goods monopolists buy ideas from research teams. Researchers produce a bundle of ideas q that they sell to the intermediate goods monopolist. This idea bundle increases productivity by a step size $q\bar{A}$, such that a monopolist with technology A_j can move to technology $A_j + q\bar{A}$.

The surplus or change in value from buying an idea is appropriated by the research team. The intermediate goods producers pay unit price p_j for a bundle of ideas q that arrive at rate x.[11] Without loss of generality, we assume that teams are randomly matched to intermediate goods producers.[12] Let us denote the value of owning a product line A_j as $V(A_j)$, which looks as follows:

$$rV\left(A_j\right) = \pi_I A_j + x\left[V\left(A_j + q\bar{A}\right) - V\left(A_j\right) - p_j q\right]. \tag{20}$$

This continuous time Hamilton-Jacobi-Bellman has the following interpretation: the left-hand side equates the safe flow return $rV(A_j)$ to the risky return on the right-hand side. One right-hand side, the first term

[11] The rate x does not need to be pinned down in order to solve for the price of the idea.

[12] The exact market structure in the market for ideas is irrelevant because the value function is linear in A_j, so that the return to an additional unit of productivity is the same across all intermediate good producers and independent of their current level of productivity. See Akcigit et al. (2018).

is the per-period profit flow $\pi_I A_j$; the second term captures the change in firm value due to the increased quality by $q\bar{A}$ minus the unit cost of the idea to the firm, p_j multiplied by the bundle of ideas, q.

Proposition 1 *The equilibrium value function of monopolist j takes the following form*

$$V(A_j) = \frac{\pi_I}{r} A_j,$$

and the unit price of an idea p_j is equal to:

$$p_j = p = \frac{\pi_I}{r} \bar{A}. \tag{21}$$

Proof. Since the research team appropriates all of the surplus from the idea sale, then the surplus to the intermediate goods producer from purchasing the idea is 0, i.e.

$$V\left(A_j + q\bar{A}\right) - V\left(A_j\right) - p_j q = 0 \tag{22}$$

Conjecture the following form of the value function from Equation (20): $V(A_j) = vA_j + \omega\bar{A}$. Substituting the guess into Equation (20), it follows in a straightforward manner that $v = \frac{\pi_I}{r}$ and $\omega = 0$. Substituting this result into Equation (22) delivers the unit price p_j described in Equation (21). ■

The flow gain associated with an idea bundle q is $\pi_I q\bar{A}$ and r encapsulates the discounted sum of this gain. Note the price of an idea is independent of buyer j. The change in aggregate productivity equals the sum of innovation produced by research teams. With the price of ideas characterized closing the model, we can characterize the research pool and growth rate of the economy.

The Research Pool

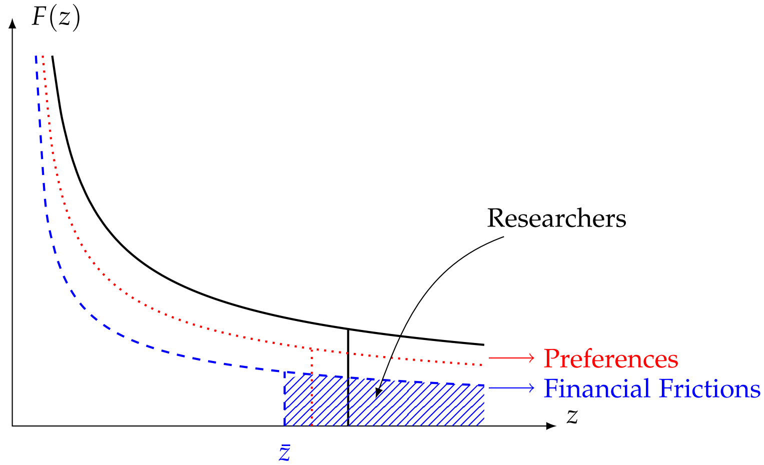
We can now solve for the allocation of talent in the economy. Individuals with talent higher than $\bar{z}$ who have sufficient financial resources and preferences for the research sector will become researchers. The remaining individuals work in the production sector. We can obtain an expression for $\bar{z}$ solving for Equation (10) and plugging in $\tilde{\mu}$ from Equation (2) and α from Equation (9):

$$\bar{z} = \left[\frac{\tilde{\mu}}{NE} \left(\frac{\hat{w}_s}{\hat{w}_u} \right)^{\frac{\delta+\rho}{\lambda}} \frac{\hat{\pi}}{\hat{w}_u} \right]^{\frac{1}{\theta}}. \tag{23}$$

Given prices and wages, Equation (23) delivers an expression for $\bar{z}$. Higher wages (w_s) and team leader profits ($\hat{\pi}$) in research pull in more talent and induce a higher cutoff for the university. One can also observe how preferences and financial frictions can reduce the propensity of individuals to enter the research market. The larger the mean of the preference shock for working outside the research sector (through larger E), the lower the school has to make the cutoff. If there are fewer financially unconstrained people above the cutoff (higher $\tilde{\mu}$), the school would have a higher cutoff for the same number of enrollees. All these equilibrium prices and the fraction of people who can afford and prefer research affects the talent cutoff as a function of the shape of the talent distribution θ.

Figure 4 illustrates the allocation of PhD slots which determine the research pool. The school accepts lower quality students as a result of the financial frictions (low $\tilde{\mu}$) and heterogeneous preferences (high E). If all individuals prefer a career in research and are not financially constrained, the school's cutoff would be at the black vertical line (highest IQ pool). When some high talent people prefer a career in production, due to a distaste for research, this shifts the cutoff to the red vertical dashed line. When, in addition, some high talent people cannot afford education, this shifts the cutoff even further down to the blue vertical dashed line (lowest IQ pool). This latter case generates the misallocation of talent in the society.

Figure 4: The pool of educated researchers



The results in this section clarify the individual decision choice problem and the forces that influence the composition of researchers. We will next turn to the general equilibrium characterization of $\bar{z}$ which links the composition of the research pool to fundamental parameters, which we then use to uncover the balanced growth path.

From Equations (15), (16) and (18) we obtain the values for the detrended variables $\hat{w}_u$, $\hat{w}_s$, and $\hat{\pi}$.[13] Plugging these three variables into Equation (23) allows to solve for the general equilibrium expression for the school's talent cutoff, which contains the elements of the financial frictions $\tilde{\mu}$, preferences E, slots N, as well as the other fundamental parameters in research and final good production. The analytical solution for the cutoff is the following:

$$\bar{z} = \left[\frac{\tilde{\mu}}{NE} \left(\frac{\phi \pi_I}{r} \right)^{\frac{(1+v)}{\eta_1+\eta_2}} \left(\frac{1-\beta}{\beta} \left(\frac{\psi}{1-\beta} \right)^{\frac{1-\beta}{\beta}} \right)^{1+v} \left[\frac{\lambda}{\delta} \frac{\theta}{\theta-1} \right]^{\frac{\eta_1 v - \eta_2}{\eta_1+\eta_2}} \eta_3^{\frac{\eta_3(1+v)}{\eta_1+\eta_2}} \eta_2^{v} \eta_1 \right]^{\frac{\eta_1+\eta_2}{\eta_2(\theta+1)-\eta_1(v-\theta)}}, \tag{24}$$

where $v \equiv (\delta+\rho)/\lambda$. To make the research sector sufficiently desirable, we need to make a mild assumption on the parameter structure. The intuition of the following condition is that individuals get enough compensation for their time spent as a skilled worker. If individuals spend a lot of time as a skilled worker ($(\delta+\rho)/\lambda$ large) then they must be compensated with a large enough share $\frac{\eta_2}{\eta_1}(\theta+1)+\theta$. The condition is as follows:

$$\frac{\delta+\rho}{\lambda} < \frac{\eta_2}{\eta_1}(\theta+1)+\theta.$$

[13]The equilibrium expressions for the detrended variables $\hat{w}_u$, $\hat{w}_s$, and $\hat{\pi}$ are: $\hat{w}_s = \eta_2 \eta_3^{\frac{\eta_3}{\eta_1+\eta_2}} \left[\frac{\lambda \bar{z}}{\delta} \frac{\theta}{\theta-1} \right]^{\frac{\eta_1}{\eta_1+\eta_2}} \left(\phi \frac{\pi_I}{r}\right)^{\frac{1}{\eta_1+\eta_2}}$; $\hat{w}_u = \frac{\beta}{1-\beta} \left(\frac{1-\beta}{\psi} \right)^{\frac{1-\beta}{\beta}}$; $\hat{\pi} = \eta_3^{\frac{\eta_3}{\eta_1+\eta_2}} \eta_1 \left[\frac{\delta}{\lambda \bar{z}} \frac{\theta-1}{\theta} \right]^{\frac{\eta_2}{\eta_1+\eta_2}} \left(\phi \frac{\pi_I}{r}\right)^{\frac{1}{\eta_1+\eta_2}}$.

We verify this in all calibrations and robustness checks. Some remarks are in order on the parameters that govern the school's cutoff $\bar{z}$. First, the cutoff is increasing in the probability an individual can afford a PhD $\tilde{\mu}$, decreasing in the mean of the preference shock (E), and decreasing in the number of slots, N. Second, $\bar{z}$ is more responsive to these three forces the larger η_1 (share to team leader) is. This indicates that as the returns to skills increase, financial constraints induce a larger decline in $\bar{z}$. Having characterized $\bar{z}$, we now move to characterizing the growth rate of the economy as the function of the research market producing ideas.

Balanced Growth Path

The ideas produced by research teams shape the growth rate of aggregate productivity through $\bar{A}(t)$. We focus on growth exclusively coming from this endogenous innovation, but discuss an extension where people outside the research market also produce ideas with an exogenous arrival rate in Section 6.

Proposition 2 *The aggregate growth rate of the economy is given by:*

$$g = \bar{z}^{\frac{\eta_1}{\eta_1+\eta_2}} N \frac{\left(\phi\lambda^{\eta_1}\delta^{\eta_2}\right)^{\frac{1}{\eta_1+\eta_2}}}{\lambda+\delta} \left(\eta_3 \frac{\pi_I}{r}\right)^{\frac{\eta_3}{\eta_1+\eta_2}} \left(\frac{\theta}{\theta-1}\right)^{\frac{\eta_1}{\eta_1+\eta_2}}. \tag{25}$$

Proof. See Appendix C.2. ■

Equation (25) delivers growth as a function of fundamental parameters and $\bar{z}$. We note that g is increasing in the quantity of researchers N and the quality of researchers which itself is a function of $\bar{z}$. It is increasing in ϕ, which is the fraction of effective ideas in the economy, and the price of ideas. The parameters $\lambda, \delta, \eta_1, \eta_2, \eta_3$ and θ and financial frictions μ and distaste for research E all interact with g through both the production of ideas conditional on the talent pool and its effect on the talent pool, $\bar{z}$.

Recall from Equation (19) that aggregate final good output is linear in productivity. Thus, if aggregate productivity grows at rate g, final good output also grows at rate g. We now summarize the characteristics of the balanced growth path (BGP) of the economy, on which the economy's growth rate is constant and the cutoff $\bar{z}$ is time-invariant.

Definition 1 ***Balanced Growth Path.*** *A balanced growth path consists of a constant growth rate g, paths for the skilled wage w_s, unskilled wage w_u, team leaders' profits $\pi_H(z)$, team sizes $n(z)$, new ideas $q(z)$, price of ideas p, individual allocation across occupations H, M, L_P and cutoff $\bar{z}$ such that:*

1. *Team size $n(z)$, lab equipment $a(z)$, idea qualities $q(z)$, and profits $\pi_H(z)$, are given by Equations (5)-(8).*
2. *The allocations of team members M, team leaders H, and production workers L_P, are given by Equations (11)-(13);*
3. *The path for the skilled wage clears the market for researchers and satisfies Equation (14);*
4. *The path for the unskilled wage solves Equation (18);*
5. *The price of ideas is given by Equation (21);*
6. *The school's cutoff is given by Equation (24);*
7. *Aggregate productivity $\bar{A}$ and aggregate output Y grow at rate g as in Equation (25).*

This concludes the characterization of the balanced growth path equilibrium. Next, we will discuss the implications of innovation and education policies.

3.2 Policy Intervention: Innovation and Education Policies

The balanced growth path equilibrium provides a framework to address the steady state response of the economy to the introduction of policies.[14] In Section 5, we discuss transitional dynamics as the economy responds in the short-run from its initial steady state. We compare three types of interventions: R&D subsidies, educational financing subsidies, and expanding university slots. Each subsidy affects the economy through different channels. An R&D subsidy both stimulates innovation through expanding lab equipment conditional on the existing talent pool and increases the returns to the research sector, pulling in talented and financially unconstrained individuals into the research sector who would otherwise choose production work. A subsidy to education enables talented but financially constrained individuals to enter the research sector. Increasing the number of PhD slots expands the pool of researchers that create new ideas in the economy. The government finances these policies with a proportional tax τ on intermediate firms profits.

We now introduce each of the three types of policies (R&D subsidies, education subsidies, and university slots expansion) and analyze how they affect the equilibrium growth rate of the economy. We will then discuss the quantitative effect of these subsidies in Section 5.

3.2.1 R&D subsidy

The government can subsidize research effort. In our case, this amounts to subsidizing the price of ideas that are being purchased by firms from research teams. The subsidy to the price of the idea will induce two effects. First, given the existing pool of talent, team leaders will have an incentive to produce more ideas by increasing the purchases of lab equipment. Second, it will increase the return to being a researcher through both increasing profits and the salaries of skilled workers, hence making research more attractive. This will induce a more talented pool of researchers who initially did not prefer research. However, financially constrained individuals will still be unable to afford education and won't be affected by the subsidy. The group of people who initially preferred the production sector and then decide on the margin to enter research are indicated by the shaded green region in Figure 5.

More formally, the R&D subsidy increases the school's cutoff as follows:[15]

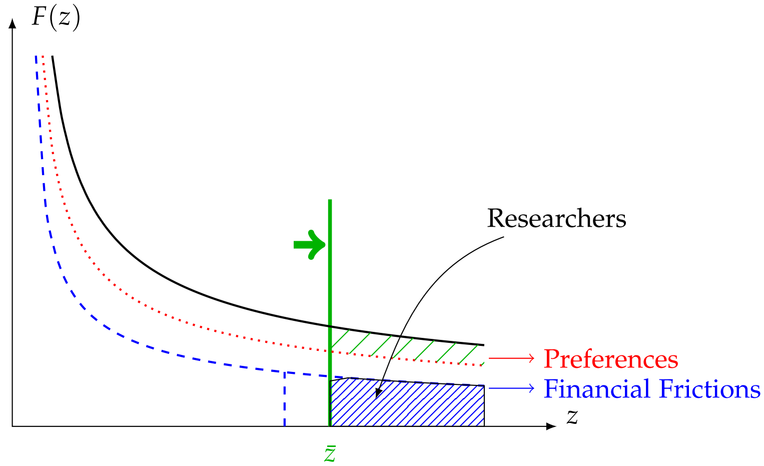
$$\bar{z} = \left[\frac{\tilde{\mu}}{NE} \left(\frac{(1+s)(1-\tau)\phi\pi_I}{r} \right)^{\frac{(1+v)}{\eta_1+\eta_2}} \left(\frac{1-\beta}{\beta} \left(\frac{\psi}{1-\beta} \right)^{\frac{1-\beta}{\beta}} \right)^{1+v} \left[\frac{\lambda}{\delta} \frac{\theta}{\theta-1} \right]^{\frac{\eta_1 v - \eta_2}{\eta_1+\eta_2}} \eta_3^{\frac{\eta_3(1+v)}{\eta_1+\eta_2}} \eta_2^{v} \eta_1 \right]^{\frac{\eta_1+\eta_2}{\eta_2(\theta+1)-\eta_1(v-\theta)}}, \tag{26}$$

where, recall $\pi_I = [(1-\beta)/\psi]^{\frac{1-\beta}{\beta}} \ell(1-N)$. Then the growth rate is as follows:

$$g = \bar{z}^{\frac{\eta_1}{\eta_1+\eta_2}} N \frac{(\phi \lambda^{\eta_1} \delta^{\eta_2})^{\frac{1}{\eta_1+\eta_2}}}{\lambda+\delta} \left(\eta_3 \frac{(1+s)(1-\tau)\pi_I}{r} \right)^{\frac{\eta_3}{\eta_1+\eta_2}} \left(\frac{\theta}{\theta-1} \right)^{\frac{\eta_1}{\eta_1+\eta_2}}.$$

[14] We thus continue without the t index in this analysis.
[15] Please see Appendix C.3 for the formal derivation.

Figure 5: R&D subsidies hit talented people who prefer non-research fields



In our model, R&D subsidy has two effects, as can be seen in the above expressions. First, it has the usual direct effect through the R&D effort in terms of final goods. When the subsidy rate goes up, the growth rate g goes up as a function of the relative importance of lab equipment in the research production function. In the extreme case, as $\eta_3 \to 0$, the subsidy has no direct effect on the growth rate. Second, and more interestingly, there is a new channel in our framework where subsidies to R&D increases the threshold $\bar{z}$ and brings in more talented people by increasing the financial returns to being a researcher. This can be seen in Equation (26).

3.2.2 Subsidy to Cost of Education

The government can introduce a subsidy s_κ to the cost of education such that students pay $(1 - s_k)\kappa$ to enroll in a PhD program. This policy has a direct impact on the fraction of people who can afford education. This means that the probability that an individual can afford education is:

$$\tilde{\mu}_\kappa = \mu + (1-\mu)\left(\frac{\tilde{\theta}-1}{\tilde{\theta}(1-s_\kappa)}\right)^{\tilde{\theta}}.$$

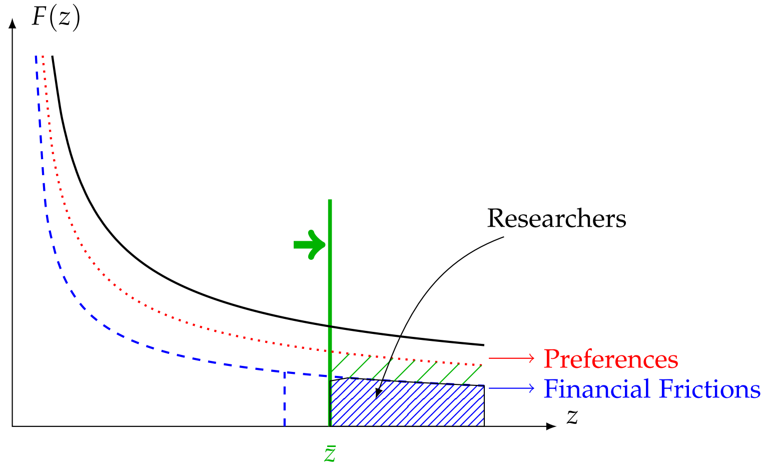
This subsidy allows some talented individuals who are financially constrained to pay for education and enter the research sector, thus increasing the average quality of researchers. This is illustrated by the now unconstrained individuals in the shaded green area in Figure 6.

As a result, the schools' cutoff increases as follows:

$$\bar{z} = \left[\frac{\tilde{\mu}_\kappa}{NE}\left(\frac{(1-\tau)\phi\pi_I}{r}\right)^{\frac{(1+v)}{\eta_1+\eta_2}}\left(\frac{1-\beta}{\beta}\left(\frac{\psi}{1-\beta}\right)^{\frac{1-\beta}{\beta}}\right)^{1+v}\left[\frac{\lambda}{\delta}\frac{\theta}{\theta-1}\right]^{\frac{\eta_1 v-\eta_2}{\eta_1+\eta_2}}(\eta_3)^{\frac{\eta_3(1+v)}{\eta_1+\eta_2}}\eta_2^v\eta_1\right]^{\frac{\eta_1+\eta_2}{\eta_2(\theta+1)-\eta_1(v-\theta)}}.$$

We again note that $\pi_I = [(1-\beta)/\psi]^{\frac{1-\beta}{\beta}}\ell(1-N)$. The increase in the average quality of researchers has

Figure 6: Educational; subsidies hit talented people who are financially constrained



affects the growth rate through $\bar{z}$:

$$g = \bar{z}^{\frac{\eta_1}{\eta_1+\eta_2}} N \frac{(\phi \lambda^{\eta_1} \delta^{\eta_2})^{\frac{1}{\eta_1+\eta_2}}}{\lambda+\delta} \left(\eta_3 \frac{(1-\tau)\pi_I}{r} \right)^{\frac{\eta_3}{\eta_1+\eta_2}} \left(\frac{\theta}{\theta-1} \right)^{\frac{\eta_1}{\eta_1+\eta_2}}.$$

We note that with educational subsidies there is no direct effect of the subsidy on the growth rate. The entire effect of the subsidy comes through $\tilde{\mu}_\kappa$, which relaxes the financial constraint as it increases $\bar{z}$. The increase in the quality of researchers impacts the growth rate through $\bar{z}$. Further, there is also the effect of the increased tax rate on final goods producers that lowers the returns to innovation. The analysis of the two subsidies held the number of researchers fixed; we now turn to an expansion in the supply of researchers.

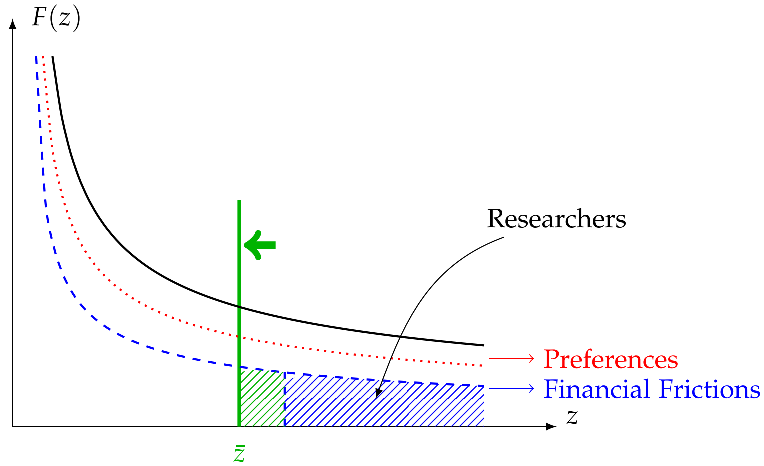
3.2.3 Increasing the number of PhD slots

Another policy tool discussed in the Ministry of Education (2016) report is the expansion of university slots. When the university increases the share of educational slots per cohort from N to N_s, more people flow into research, but the quality of the marginal researcher declines. We return to the expression for the university's cutoff with the new number of slots N_s:

$$\bar{z} = \left[\frac{\tilde{\mu}}{N_s E} \left(\frac{\phi(1-\tau)\beta\left[(1-\beta)/\psi\right]^{\frac{1-\beta}{\beta}} \ell(1-N_s)}{r} \right)^{\frac{(1+v)}{\eta_1+\eta_2}} \left(\frac{1-\beta}{\beta} \left(\frac{\psi}{1-\beta} \right)^{\frac{1-\beta}{\beta}} \right)^{1+v} \left[\frac{\lambda}{\delta} \frac{\theta}{\theta-1} \right]^{\frac{\eta_1 v - \eta_2}{\eta_1+\eta_2}} \eta_3^{\frac{\eta_3(1+v)}{\eta_1+\eta_2}} \eta_2^{v} \eta_1 \right]^{\frac{\eta_1+\eta_2}{\eta_1(\theta-v)+\eta_2(\theta+1)}}.$$

Increasing the number of slots will tend to induce individuals with less talent to enter into research through two direct effects which are amplified through general equilibrium price mechanisms. First, as there are more available slots, the school admits less talented individuals to fill them as captured by the N_s in the denominator. Second, there is a market size effect from the production side as illustrated through the price of an idea in Equation (21). More researchers mean fewer production workers and thus a smaller market size for innovation as seen in the numerator $(1-N_s)$. These effects and the tax τ for funding the slots expansion also operate through the equilibrium prices that amplify the decline in marginal talent. This is filtered through the shape of the innovation production function and the talent distribution through the power term $\frac{\eta_1+\eta_2}{\eta_1(\theta-v)+\eta_2(\theta+1)}$. This decline in average inventors' quality is illustrated in Figure 7.

Figure 7: Increasing educational slots gets more researchers but lower quality



This leaves the following equation for the growth rate of the economy.

$$g = N_s \bar{z}^{\frac{\eta_1}{\eta_1+\eta_2}} \frac{(\phi \lambda^{\eta_1} \delta^{\eta_2})^{\frac{1}{\eta_1+\eta_2}}}{\lambda+\delta} \left(\eta_3 \phi \frac{(1-\tau)\beta \left[(1-\beta)/\psi\right]^{\frac{1-\beta}{\beta}} \ell(1-N_s)}{r} \right)^{\frac{\eta_3}{\eta_1+\eta_2}} \left(\frac{\theta}{\theta-1}\right)^{\frac{\eta_1}{\eta_1+\eta_2}}.$$

When the number of slots increases, the growth rate is affected through four channels: First, growth is affected by the increase in the number of people producing ideas, N_s; second, by a reduction in their average quality, $\bar{z}$; third, there is a reduction in lab equipment investment due to the reduced market size effect, $(1-N_s)^{\frac{\eta_3}{\eta_1+\eta_2}}$; finally, growth responds to the tax τ to finance the increased slots. All these interesting channels will have implications for policy design which we will study quantitatively in Section 5. We first turn to the description of the data and estimation.

4 Data and Calibration

This section describes the data, the empirical facts we use to calibrate the model, and the calibration. We find a close match of model to data and then further illustrate the ability of the calibrated model to hit non-targeted moments in the data. These non-targeted moments leverage policy experiments in Denmark that motivate our own model policy counterfactuals. We will use our calibrated model parameters to study policy counterfactuals in Section 5.

4.1 Data

The empirical and quantitative analysis in this project is built on detailed micro-level data from the Denmark Statistical Office (DST). For data on individuals, we rely on the Integrated Database for Labor Market

Research (IDA). Each individual in Denmark is assigned a unique identifier. We observe individuals on an annual basis in the IDA. From different subsets of this dataset, we can leverage information on individuals' highest completed education, background family characteristics (e.g. parental income), employment status, occupation, and income. In addition, the firm-linked (FIDA) dataset connects individuals to their place of employment (a unique employer identifier) each year. This comes with firm-level data such as sales, profits, and employees.

In addition to the individual level data in IDA, we make use of additional internal data on an individual's academic pursuits. The PHD dataset contains detailed information on individuals who enrolled in a PhD program. PHD contains information on most students' subject of PhD, date of enrollment, and date of graduation. Further, this data describes the funding source of their education and can be linked to individuals' socioeconomic background.

We combine these datasets with IQ data provided from the Danish military test, Borge Prien's Prove, which is required for conscripts at age 18.[16] This test data goes back to 1995. With tests mostly taken at age 18, this provides data on most males entering the workforce or college after 1995. This data informs our measure of talent. In total, we have approximately 500,000 males with IQ data.

Lastly, we turn our focus to innovation – making use of both policy and outcomes of innovative output. DST provides details on project funding related to R&D as the Danish government expanded their support for R&D starting in 2002. We observe both funding through the broad "Innovation Danmark" program and specific R&D subsidies. The innovation program also has information on funding to education, which will be a key component of our policy discussion.

We combine DST data with innovation data made available through patents at the European Trademark Office (EPO). We use a disambiguation algorithm provided by the DST and we are able to match about 75% of inventors on patents from Denmark to the individual information in the IDA dataset. We use these patents as our primary measure of innovation.

The extensive data enables a bridge between model ingredients and data-specific counterparts: the role of IQ in the education, the innovative rate and occupation of PhDs, the team structure of innovation, and potential policies. Detailed parental data enables a study of the determinants of who becomes a PhD based on their background. Further, due to a host of policy tools the Danish government utilized over the main sample period, we have variation related to policy instruments that can provide quantitative tests for the model. This provides a basis for the out of sample moments from the introduction of policies.

The intersection of the datasets cover 2001-2013, but all background data is available from 1980-2013 (i.e., age, education, sex, country of origin). We will focus primarily on the 2001-2013 period. This leaves us with approximately 32 million individual matched employer-employee observations, 10,000 inventors, and 37,000 unique PhDs.

4.2 Stylized Facts

The novel element of our framework is putting individuals and their career choice as central to a model of innovation-led growth. One of the main advantages of our micro-data from Denmark is that we are able to observe the backgrounds of individuals, their career choice, innovations, and a key proxy for ability, IQ. In order to discipline our model, we need to document the relationship between these forces, primarily

[16] Studies have indicated this data is a reliable measure of cognitive ability in similar sense to IQ, e.g. Hartmann and Teasdale (2005). Teasdale (2009) reviews the literature.

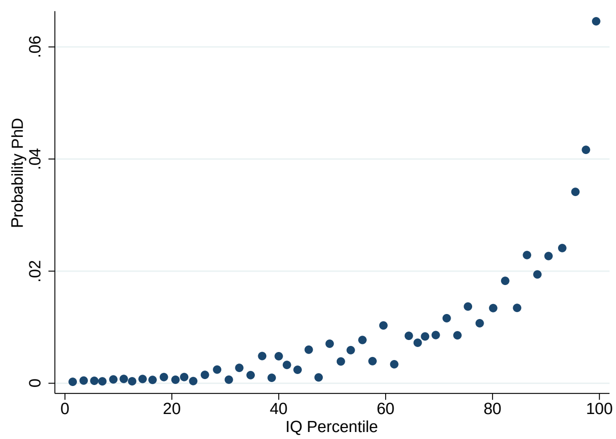
informed by ten empirical facts discussed in this section.

In the model, what leads a newborn person to contribute to aggregate innovation is the following steps. First, an individual is born with ability that will influence their educational choice. Second, since education is costly, parental socioeconomic background will influence the ability to afford education. Third, parental income has an impact on their child's ability through intergenerational talent transmission. Fourth, high ability kids born to rich parents might end up not doing a PhD due to preferences. Fifth, when an individual undertakes a PhD they become a researcher, and join a team to innovate. Sixth, ability influences the innovative capacity of researchers. Seventh, individuals work in teams with heterogeneous scales. Eighth, young researchers learn how to become a team leader over time. Ninth, when the number of educational slots increases, the number of researchers increases but the average quality of researchers decreases. Tenth, this is due to the fact that even though the country is a small open economy, its skill pool is locally determined. Given this unique structure of our model, we are going to discipline each of these mechanisms by looking at the relevant correlations in Danish micro-data in order. We use these facts to provide target moments for the estimation of the model parameters.

Fact 1: *Individuals with higher IQ are more likely to obtain a PhD.*

Figure 8 looks at the relationship between ability, proxied by IQ, and the likelihood of doing a PhD.

Figure 8: Probability of Obtaining a PhD and IQ Percentile (Fact 1)



Source: DST, Note: Fraction of individuals with a PhD by IQ percentile bin.

In this figure, we take an individual's percentile IQ within their same age group on the x-axis and plot the probability an individual enrolls in a PhD program at any time in their life on the y-axis. The relationship between IQ and the probability of becoming a PhD is convex; for each increase in an IQ percentile an individual is increasingly more likely to become a PhD. While the lowest IQ percentile has essentially zero probability of enrolling in a PhD, the median percentile has about 0.5%, and the top percentile about 6-7% probability. Hence, we conclude that more talented people are more likely to enroll in a PhD.

Fact 2: *Individuals with higher parental income are more likely to obtain a PhD.*

Our model predicted talented individuals born in poor families might be unable to afford higher education. To discipline this prediction in the data, we relate the socioeconomic background of parents to the child's likelihood of obtaining a PhD. Here, we focus on father's income percentile in order to avoid time trends in female labor force participation in Denmark. We provide a robustness check in Section 6 that takes the income of both parents. In Figure 9, we take the father's age-adjusted income percentile in the year 2000 for potential PhD enrollees from 1995-2012. We plot the income percentile on the x-axis and the probability of becoming a PhD on the y-axis.

Figure 9: Probability Child Obtains a PhD and Father's income percentile (Fact 2)

Source: DST, Note: Income percentile is age-adjusted percentile in year 2000.

Figure 9 shows that children of higher income fathers are more likely to obtain a PhD. Through the lens of our model, this interesting fact could be driven by two forces. First, this could be due to financial frictions faced by individuals born to poorer fathers. Second, talent transmission across generations could also be responsible for this relationship. In order to tease out these two margins, the next fact focuses on the link between parental income and child's IQ.

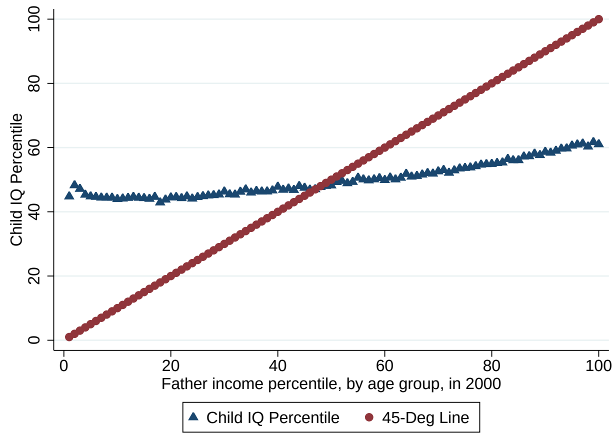
Fact 3: *Individuals' IQ is correlated with parental income, but not perfectly.*

How is talent related to parental income? To answer this question, Figure 10 plots a child's IQ on parental income. We find a positive relationship between child IQ and parental income as can be seen in the blue triangles in this figure. However, the correlation between parental income and IQ is 0.18, which indicates imperfect sorting of talented children to high income parents. The red 45-degree line would indicate perfect sorting in this manner. This suggests that some talented individuals may face financial barriers to higher education enrollment if they are matched to poorer parents.

Fact 4: *Only a fraction of people with high IQ and high parental income obtain a PhD.*

Another important margin in our model for the determination of PhDs was the preference for research. Some talented individuals who are not financially constrained may choose not to become a researcher. Empirically, this would imply that some fraction of unconstrained people will not enter a PhD program. When we consider children of the richest 5% of fathers and the most talented percentile, we still observe that about 90% do not enroll in a PhD program. Through the lens of the model, this suggests some distaste for research that drives individuals who could obtain a PhD into the production sector.

Figure 10: Average individual IQ percentile by father's income percentile (Fact 3)

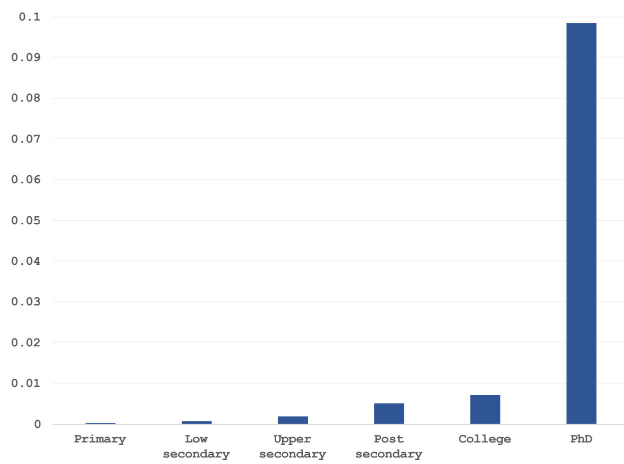


Source: DST, Note: Child IQ × Father's income percentile within age group in 2000 (and 45 degree line).

Fact 5: *PhDs are 20-times more likely to become inventors compared to the average person in society.*

How is education related to innovation? Figure 11 plots the likelihood of becoming an inventor by the six education categories in Denmark. In this figure, we take an individual's highest level of educational

Figure 11: Probability of Being an Inventor by Education Level (Fact 5)



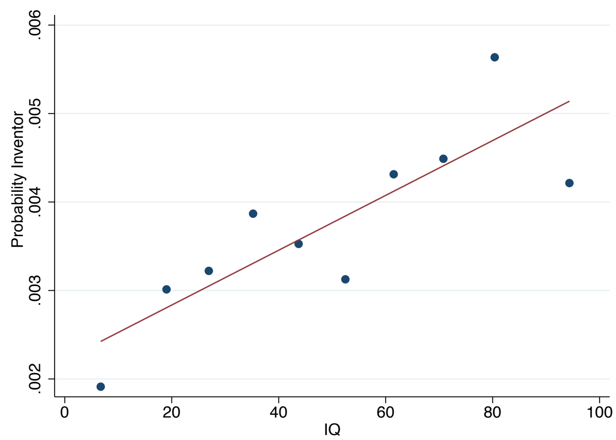
Source: DST, Note: Fraction of individuals who file at least one patent by education level.

attainment (x-axis) and plot the probability of being an inventor (having at least one patent, y-axis) for the member of each cohort. Note that the probability a PhD has a patent is 9.8% while the probability a college graduate has a patent is 0.7%. When we compare PhDs to the population with no higher education, we find an even larger difference in the probability of being an inventor, approximately 30 times the general population. We observe that the link between education and becoming an inventor is monotonically increasing. Most interestingly, individuals with PhDs are disproportionately more likely to become inventors. College graduates are also more likely to be inventors than those without a college degree. This fact illustrates the tight link between higher education and innovation, motivating the focus on PhDs in our analysis.

Fact 6: *Conditional on education, higher IQ people are also more likely to innovate.*

Does talent matter for innovation separately from its impact on acquiring education? To explore this interaction, we take the components of innovation and IQ that are uncorrelated with education. We run a regression of IQ on education level and take the residual IQ from this regression, and do the same with the probability of becoming an inventor. Figure 12 plots the residualized inventor probability on residualized inventor IQ. It shows that people with higher IQs are more likely to become inventors even when we condition on education. Figure 12 illustrates that talent has a direct effect on innovation in addition to its indirect effect through education.

Figure 12: Probability of Becoming an Inventor Conditional on Education, by IQ (Fact 6)



Source: DST, Note: This figure takes on the y-axis the output from the residual of a regression of the probability of being an inventor on the education level of the individual. On the x-axis, we plot the residuals of a regression of individual IQ on their highest educational level to get the residualized response of probability of being an inventor to IQ.

Fact 7: *Inventors work in teams and the team size is heterogeneous.*

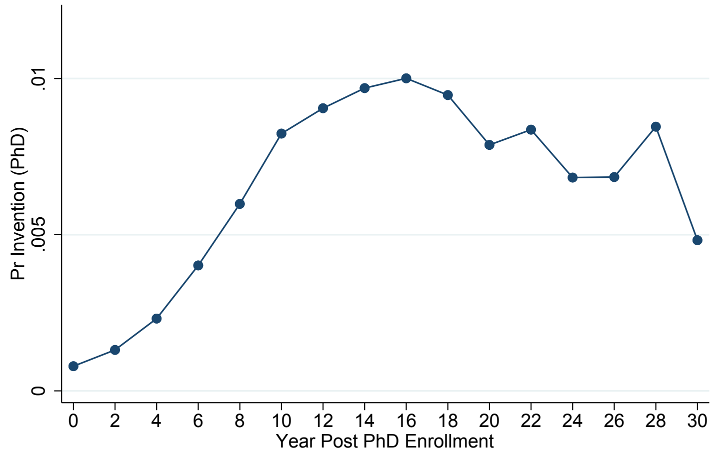
We define team size as the number of inventors listed on a patent file. In our data, the average team has size 2.5 with variance 3.6. Our modeling choice has team size and team leader talent as an important component of idea production. We do not observe team leaders directly by patent, but we identify team leaders as the oldest PhD inventor on a patent. [17]

Fact 8: *The probability of innovating as a team leader over an inventor's life-cycle follows an inverted-U shape.*

How important is the time-to-build element of human capital? In Figure 13, we plot the share of PhDs who apply for a patent as a team leader (as defined in Fact 7) by year after PhD enrollment. Figure 13 shows that the probability of team-leading innovation follows an inverted-U process: the probability of filing a patent over the cohort life-cycle is increasing until its peak at around 16 years and then declines. This observation is important because it highlights how policy interventions aimed at increasing the talent pool of the economy may not immediately take effect, as it takes time for researchers to gain experience and reach their peak productivity.

[17] As a robustness check, we replicate the empirical facts with an alternative definition of team leader as the PhD with the largest experience in each patent, where we measure experience as the stock of patents, and we obtain similar findings.

Figure 13: Fraction of PhDs applying with ≥ 1 patent, year post PhD enrollment (Fact 8)

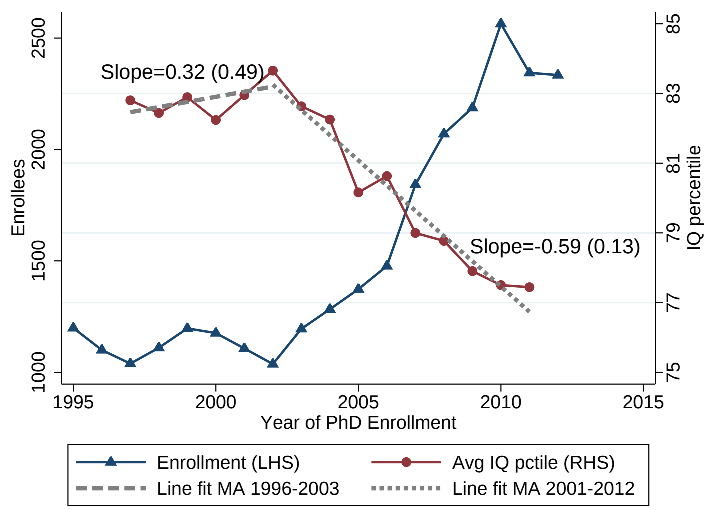


Source: DST, Note: Probability of obtaining a patent year post PhD enrollment (by 2-year intervals).

Fact 9: *An increase in the number of PhD slots is associated with a decline in the average IQ of PhDs.*

Starting in 2002, the Danish Government required the universities to increase the number of PhD slots, as part of a larger initiative to support education and innovation in Denmark (see Section 2 for further institutional details). Figure 14 shows that as the number of slots for PhDs increases, the average IQ of the enrolling students falls. This indicates that there is heterogeneous quality of enrollees and expanding slots may draw in a marginal researcher less talented than the average researcher from the existing pool. Thus, even though policy can increase the supply of researchers, there is a trade-off between expanding the pool of PhDs and the average talent of PhDs in the economy.

Figure 14: PhD Enrollment and Average PhD Enrollee IQ (Fact 9)



Source: DST, Note: This figure plots the enrollment (RHS y-axis) and IQ of enrollees with a three-year moving average (LHS y-axis) from 1995-2015.

Fact 10: *The economy is open in the goods market but closed in the skills market: import and export accounted* *for 44% and 51% of GDP in 2010, but foreign migrants make up only 10% of inventors.*

Fact 10 motivates our treatment of Denmark as a small open economy. In Denmark, exports and imports accounted for about 51% and 44% of GDP, respectively, in 2010 (OECD, 2010), while foreign inventors account for only about 10% of total inventors in our data. This is in contrast with economies that are relatively more closed to trade in the goods market but more open to trade in the skills market, such as the United States, where imports and exports were about 12% and 16% of GDP in 2010 (OECD, 2010), but foreign inventors accounted for 22% of total inventors in the period 2001-2010 (Miguelez, 2016).

4.3 Calibration Technique

The ten facts illustrate the links between human capital, innovation, and forces in the market for ideas that impact who becomes an inventor. In order to quantify the importance of these forces and perform policy counterfactuals, we calibrate our model to the ten facts. We perform a simulated method of moments (SMM) matching exercise to back out the parameters. We provide a description of the quantitative moments we target in the data and the calibrated parameters in Tables 1 and 2, respectively. We proceed to discuss non-targeted moments that illustrate the ability of the model to perform well out of sample in Section 4.4.

Our model has 17 parameters: $\{\rho, r, \beta, \mu, N, \lambda, \delta, \eta_1, \eta_2, \eta_3, \theta, \tilde{\theta}, \phi, \psi, \ell, E, \zeta\}$. We assume parental income has the same shape parameter as talent $\tilde{\theta} = \theta$, and note that $\eta_3 = 1 - \eta_1 - \eta_2$. This leaves us with 15 parameters to be identified. We refer to the literature for three parameters $\{\rho, r, \beta\}$ and directly match one with the data (N). For the 11 remaining parameters, we select 11 informative moments from our facts in Section 4.2, M^E. We then utilize simulated method of moments (SMM) to jointly calibrate the 11 parameters.

We minimize the distance between model-simulated moments, $M(\Theta)$, and their empirical counterparts, M^E, by searching over the parameter space Θ, using a simulated annealing algorithm, as follows:

$$\min_{\Theta} \sum_{i=1}^{11} (M_i^E - M_i(\Theta))^2.$$

4.3.1 External Calibration

The production side of our model is very similar to the existing literature. The key departure in our framework is how an individual's life-cycle and career choice relates to aggregate innovation, as illustrated in the ten steps above. Therefore, on the production side we follow the literature closely and set $\beta = 0.106$ (Akcigit and Kerr, 2018), set $r = \rho$, and set the discount factor to 97% ($\rho = 0.03$).

4.3.2 Internal Calibration

We match the number of enrollees (N) to its empirical counterpart (M1), which we take as our first moment. For the remaining 11 parameters $\{\lambda, \delta, \eta_1, \eta_3, \theta, \phi, \psi, \ell, E, \mu, \zeta\}$, we target 11 moments (M2-M12) jointly. Even though the parameters are identified together, below we provide a heuristic discussion of the parameters that each moment informs.[18]

[18] In Appendix C.5, we discuss derivations of the equations underlying these moments.

M1 *PhD Enrollees per Year*: The number of enrollees per year at universities will inform us on the amount of slots available for universities to let into research positions, N. In our data, PhDs account for 1% of the adult working population before the major policy interventions.

M2 *Mean Percentile IQ of PhDs*: In the model, individuals may not obtain a PhD either because they are constrained (governed by μ) or they have a distaste for research (governed by dispersion parameter E). This moment informs us on E. In the data, it is measured as the mean IQ percentile of PhD enrollees prior to policy interventions in 2002, and we obtain a value of 0.83.

M3 *Assortative Matching between IQ and Parental Income*: In the model, talented individuals may not be born to high-income parents. This correlation is governed by μ. In the data, we take the correlation between IQ and father's income to inform this parameter.

M4 *Dispersion of Parental Income Distribution*: Theoretically, there is a group of individuals who cannot afford education. A key parameter that determines this fraction is ζ, which relates assortative matching to income inequality. In the data, we take the ratio of the mean to standard deviation of the distribution of paternal income and find a value of 1.18.

M5 *Mean Team Size*: In our model, the mean team size is determined by the death rate δ and team-leader arrival rate λ as follows:

$$\frac{\delta}{\lambda} + 1.$$

Hence, we target the mean team size to further inform us on these two parameters. In the DST data, we find the mean number of people on a patent to be 2.25.

M6 *Variance of Team Size*: In our model, the variance of team size is determined as follows:

$$\frac{\theta}{\theta - 2}\bar{z}^{2-\theta}.$$

In the data, we obtain a team size variance of 3.13. This target primarily informs us on the shape parameter θ.

M7 *Peak Year of Innovation for a Team Leader post-PhD Enrollment*: Our model will generate an inverted-U relationship between the probability of team-leading innovation and the experience of a researcher. Targeting the time t of the peak in Figure 13 will help pin down the value for the arrival rate to becoming a team leader, λ, and the death rate, δ. We target the number of years after PhD enrollment at which individuals reach the peak probability of innovation.

M8 *Probability of Innovation at Peak*: We return to the inverted-U relationship between the probability of team-leading innovation and researcher experience. By targeting the height of the peak value in Figure 13, we pin down the value for the innovative rate of team-leading researchers, ϕ. This is taken as the probability a team leader innovates in the peak year post-PhD.

M9 *Post-PhD/PhD Income:* In the model, team leaders and team members receive different returns from research production. The average ratio of their two incomes is given by:

$$\frac{\eta_1}{\eta_2}\frac{\delta}{\lambda}.$$

This moment informs us mainly about the share of team leaders η_1 and team members η_2 in idea production. In the data, we measure it as the ratio of average income for PhD graduates to average income of individuals without a PhD in the same age group.

M10 *Skill Premium of PhDs*: We define skill premium as the ratio of the team member income to outside workers income. In the data, it is measured as the ratio of average income of PhD students to average income of individuals without a PhD in the same age and talent group. This informs us on the cost of raising capital in the intermediate goods sector ψ which influences the unskilled wage.

M11 *Profits to Wages*: We define the profits to wage as the ratio of the total of intermediate goods profits to the wages paid out to production workers. M10 and M11 deliver the cost of raising capital and effective labor units in the final goods market, ψ and ℓ. In the data, it is measured as the ratio of profits of operating firms (with at least one employee) to total wages.

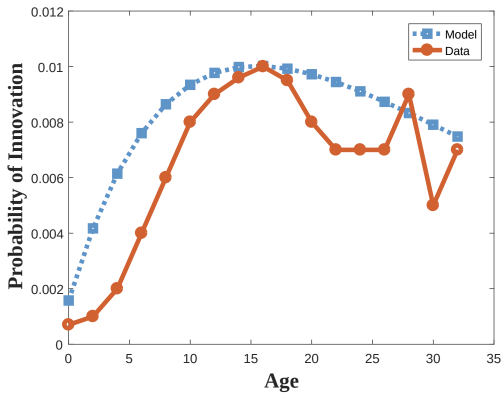
M12 *Growth Rate of the Economy*: The aggregate growth rate of the economy is determined by the frequency of innovations and the composition of inventor teams. Along with the income ratio of team leaders to members, this informs us mainly on the shares in the idea production function, η_1 and η_2. We target a growth rate of 1.5%.

Table 1 summarizes the values of these moments in the data and the corresponding model generated values. As can be seen, our model is generating a close match to the data. Even though we only targeted two specific data-points for the life-cycle and the team size distribution each, Figure 15 shows that the model provides a good fit for the overall patterns.

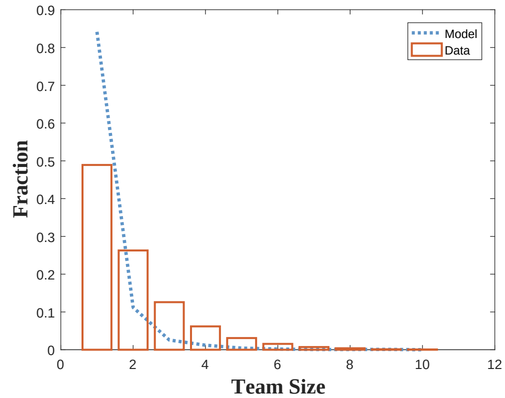
Table 1: Moments

Moment	**Data**	**Model**
PhD Share of the Labor Force (N)	0.01	0.01
Mean Percentile IQ of PhDs (Fact 1+4)	0.83	0.83
Corr(Parent Income, Child IQ) (μ, Facts 2+3)	0.18	0.18
Mean/SD of Parent Income (Facts 2+3)	1.18	1.22
Mean Team Size (Fact 7)	2.25	2.23
Variance of Team Size (Fact 7)	3.13	3.19
Peak Year Innovation Post PhD (Fact 8)	16	16
Pr Innovation at Peak (Fact 8)	0.01	0.01
PhD Graduate/Student Income	1.66	1.66
Skill Premium of PhDs	0.74	0.74
Profits to Wages	0.073	0.022
Growth rate	0.015	0.015

Figure 15: Specific Moments, Red Solid: Data ; Blue Dotted: Model



(a) Innovation over the Life-Cycle



(b) Team Size Distribution

Table 2: Parameter Values

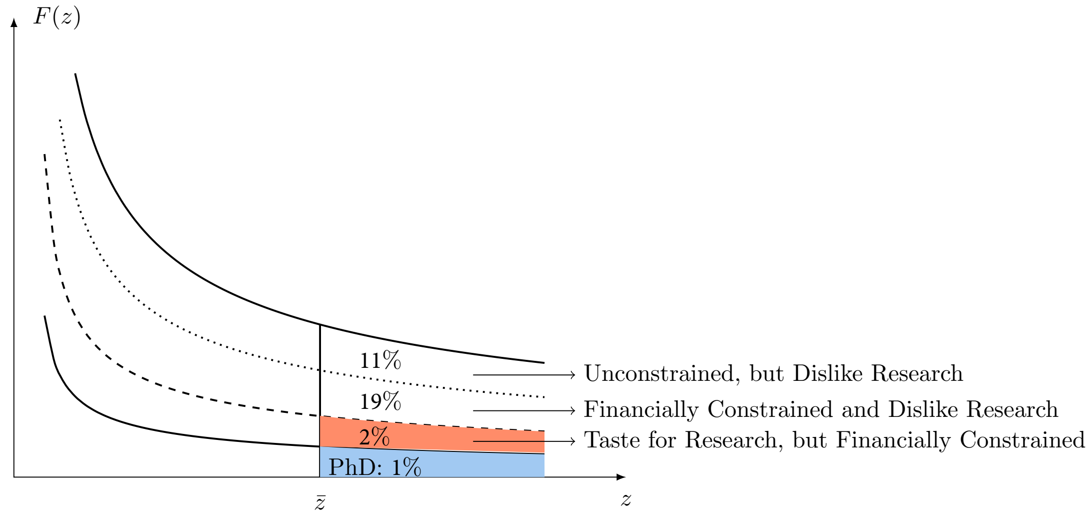
Parameter	Description	Value
	— Panel A. External Calibration —	
ρ	Discount rate	0.030
r	Interest rate	0.030
β	Labor share output	0.106
	— Panel B. Internal Calibration —	
N	PhD share of the labor force	0.010
λ	Rate at which researcher becomes a team leader	0.036
δ	Death rate	0.044
η_1	Skilled labor share idea production	0.511
η_2	Worker share idea production	0.379
η_3	Lab equipment share idea production	0.110
θ	Pareto shape	2.633
ϕ	Innovation rate	0.046
ψ	Cost of raising capital	0.011
ℓ	Labor efficiency in production	0.024
E	Preference shock parameter	3.299
μ	Fraction assortative match IQ - parental income	0.116
ζ	Income/IQ ratio for assortatively matched	7.534

Notes: All parameters are estimated jointly.

The resulting parameter values are reported in Table 2. Here, we discuss parameters of interest. The input shares η_1, η_2, η_3 indicate that the human capital component of research, which consists of team leader ability and number of researchers, is significantly more important (around 46% and 41% respectively) than the physical capital component ($\eta_3 = 11\%$). Three other interesting parameters worth discussing are

μ, θ, and ζ. These parameters jointly imply that around 1/3rd of the potential PhD population ($\tilde{\mu}$) are unconstrained in the sense that they can afford the time spent in a PhD. We would like to reemphasize that the cost of schooling is more broad than tuition cost, in that it includes living expenses and the cost of foregone income that might be required by individuals to support their families.

Figure 16: The allocation of talent in the economy



As illustrated in Figure 16, these parameters predict that the fraction of people who are talented and unconstrained but dislike research are 11% of the population. More interestingly, the pool of people who are talented and willing to do research but are not able to afford education is twice as large as the current pool of researchers (2% versus 1%). This group is misallocated in society. We revisit this in the policy counterfactuals in Section 5. Before turning to policy counterfactuals, we observe how the model hits non-targeted moments in the data.

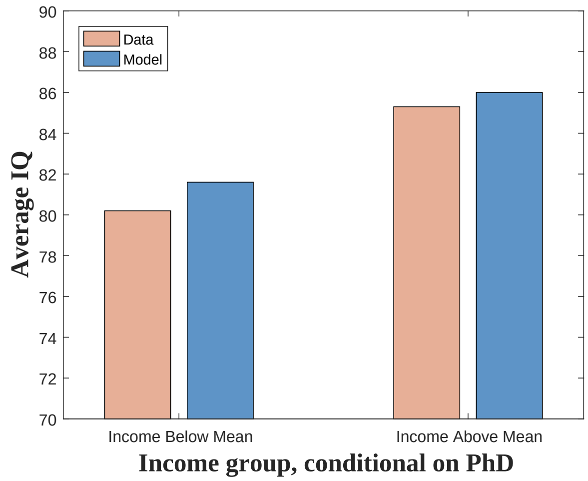
4.4 Non-Targeted Moments

In order to assess the out-of-sample validity of the model, we explore the implications of our results for non-targeted moments in the data. We perform two main out-of-sample matching exercises. First, we consider the relationship between parental income and IQ for individuals who obtain a PhD. For matching the model, we only relied on the pairwise correspondence between parental income, PhD, and IQ, but not the interaction of these three components. Second, we use our model to simulate the observed policy interventions in 2002 and compare the implied results for the talent of PhD enrollees to what we observe in the data.

Relationship between Paternal Income and IQ for PhDs

While we know that those who enroll in a PhD tend to have higher parental income and higher IQ, we have not discussed the relationship between IQ and parental income among PhD enrollees. In this exercise, we use our simulated model to trace out the different IQs of the relatively wealthier and poorer individuals that enter a PhD program. The model implies that PhD enrollees with wealthier parents have higher talent. The result in the model comes from the fact that, on average, individuals with higher IQs have a higher preference shock which proxies for greater opportunities outside the research sector. We find empirical support for this particular implication as shown in Figure 17.

Figure 17: IQ by Income Group, Data and Model Predicted



This figure illustrates model-simulated data alongside the IQ from two income groups of PhDs. We calculate the mean of parental resources for PhD enrollees, and we plot average IQ for PhD students with parental resources below the mean and above the mean. Both in the model and in the data, average IQ increases in parental income conditional on being a PhD enrollee. We now turn to our second out-of-sample exercise.

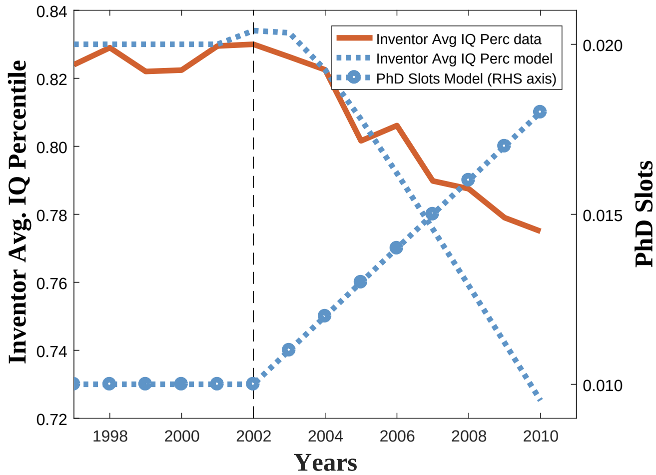
Government Policy Intervention

Here we explore the responsiveness of PhD enrollee quality to a host of policy interventions. We did not directly use any Danish government policy interventions to calibrate the model, yet the major break in our sample provides us an opportunity to test the power of our calibrated framework. Thus, we use the outcome of the policy interventions introduced in 2002 as an out-of-sample test of our model.

The Danish government introduced a number of education and innovation policies starting in 2002 with the goal of fostering innovation and technological progress. The "Innovation Danmark" database contains information on education and innovation programs, including the amount of funding and grants. We group the interventions in the three types of policies discussed in our model: R&D subsidies, subsidies to the cost of education, and increases in PhD slots. We estimate the expenditure for each type of policy from the data. Then, we feed the estimated policy rates into our model and we compare the predicted outcome for average IQ of PhD enrollees predicted by our model to the empirically observed outcome. The procedure to estimate policy rates from the data is outlined in Appendix E.1.

Figure 18 displays the increase in PhD enrollees and the corresponding change in IQ after the policy intervention in both the data and model. Because we calibrate our model to the average IQ of PhDs before the policy intervention, both the data and the model predict the average IQ of enrollees at the 83rd percentile of the IQ distribution in 2002, prior to the introduction of these subsidies and slot expansion.

Figure 18: Policy Intervention and IQ of PhD enrollees: data vs. model.



The introduction of the policies and the expansion of slots push in opposite directions on mean IQ: subsidies to R&D and the cost of education lead to an increase in average IQ, while the expansion of PhD slots leads to a decline in average IQ. Our model overall matches the declining pattern in average IQ of PhD enrollees observed in the data. The quantitative fit is very close between 2002-2008, while it departs slightly from the data counterpart in the last years of the sample.

The two out-of-sample tests show that our model not only does well with the targeted moments but also is able to reproduce both specific facts in the data and the policy intervention which was implemented in 2002. We are now ready to study both the positive and normative quantitative implications of our model.

5 Policy Experiments

Our calibration enables us to study the impact of policies in the short and long-run and speak to optimal policy combinations. In this section, we use our model to perform a number of counterfactual policy exercises to quantify the strength of different policies in shifting the growth rate. We explore the quantitative impact of each policy individually and then introduce them simultaneously in order to evaluate their interaction and the optimal policy mix. We then discuss how the effectiveness of each policy depends on underlying parameters in the economy.

This paper directs attention to the consequences of making a realistic description of human capital central to an innovation-led growth model and what this suggests for longstanding debates in the literature on R&D policies. Section 5.1 looks at the comparison of steady states under different subsidies and puts our new mechanisms into perspective by comparing our findings to the literature. Section 5.2 illustrates the complementarity of education and innovation policies – directing particular focus to the growth-maximizing policy mix for different budgets. Section 5.3 focuses on how the optimal policy depends on the inequality of parental income in the economy. Section 5.4 discusses the transitional dynamics with the introduction of each policy to illustrate the intertemporal tradeoffs and understand what a policymaker should expect on the horizon of the impact.

5.1 Steady State Impact of Policies and Comparison to Existing Literature

We begin by describing the impact of our policies individually in steady state. In particular, we want to focus on revenue equivalent policies and their effects on the quality of PhDs, total number of PhDs, and the overall growth rate. There are three main policies of interest: R&D subsidies, educational subsidies, and expansion of educational slots. Each policy on its own increases innovation but through different channels. One important point to note is that an expansion in the number of PhD slots will increase the number of researchers, but induce a decline in their average talent, while R&D and educational subsidies, without expansion of the pool of researchers, increase the talent of the average researcher through different margins. These different margins were illustrated in Figures 5, 6, and 7 in Section 3.2. Table 3 compares these three policies by showing the overall growth rate effect of a 10% R&D subsidy equivalence spent on R&D, education subsidies, and slot expansion, respectively.[19]

Table 3: Alternative Policy Interventions (10% R&D Subsidy Equivalence)

	%ΔInnovation	Avg. PhD IQ percentile
Baseline	0%	83
R&D subsidy	5%	87
Educational subsidy	20%	93
University slots	2%	78

A number of important observations are in order. First, a 10% subsidy rate to R&D and this policy increases the baseline endogenous innovation by 5%. Note that unlike the standard growth models (Romer, 1990; Aghion and Howitt, 1992), this intervention keeps the number of researchers fixed (e.g. no scale effect), and the effect comes mostly through the quality composition of the inventor pool. The rest of the effect comes from the additional use of physical capital for researchers with a given subsidy. The change in inventor talent happens due to an increase in the compensation of researchers as a result of the subsidized R&D. The returns to being a researcher increases, which pulls in individuals who can afford a career in research, but on the margin prefer working in production.

Second, when we use the same amount of resources to subsidize education, the impact on endogenous innovation is much larger (20% versus 5%) than the R&D subsidy. This is due to the fact that educational subsidies alleviate financial frictions by bringing in high ability but constrained individuals into research. There is evidence in the literature that there are many constrained individuals who have the ability for a research career (Aghion et al., 2017; Akcigit et al., 2017; Bell et al., 2018), and frictions that prevent individuals from entering the sector could have large effects on innovation (Celik, 2015).

Third, when we use the same amount of resources to expand the PhD slots, we observe only a 2% increase in endogenous innovation. Given this subsidy level, an expansion in slots is about half as effective as an R&D subsidy. Note that unlike standard models that often assume homogeneous skills of inventors, this framework with talent heterogeneity will have different implications for increasing the supply of researchers. When researcher supply is increased, schools lower the cutoff and thus the marginal researcher will not have the talent to contribute to the same degree as the upper tail to aggregate innovation. This would create a non-linear relationship between policy and aggregate response (Jaimovich and Rebelo, 2017). This finding also squares facts related to an increasing pool of researchers not creating a significant

[19] This equals approximately 0.5% GDP in our framework. We use this as our baseline expenditure, but we present our results for a wider range of budgets in Appendix E.2 and we explore how changing the budget changes the optimal allocation in Section 5.2. This share is approximately the amount spent on R&D subsidies and education subsidies in Denmark from 2000-2014.

impact on aggregate innovation (Jones, 1995; Bloom et al., 2017).

Lastly, we want to relate the growth implications of these policies in our framework. To put our numbers into perspective, it would be fruitful to start with a discussion of standard endogenous growth models. By assuming a homogeneous population that can do both production and R&D, standard frameworks assume a very elastic margin of effective researchers. These models have been criticized for giving very high responses of growth to policy which are not observed empirically (e.g., Goolsbee, 1998; Romer, 2000; Wilson, 2009, among others). Guided by the Danish micro-data, we have shown that becoming an inventor requires talent, a preference for research, training, and financial resources. We now want to highlight the role of these new ingredients in comparison to the benchmark model.

Recall that the policy impact on growth of the standard model is approximated by the following equation:

$$\underbrace{\partial g}_{0.0015} \approx \underbrace{g}_{0.015} \times \underbrace{\partial \text{R\&D subsidy rate}}_{0.1}.$$

This expression implies that a 10% R&D subsidy (or approximately 0.5% GDP equivalence) should increase the benchmark economy that grows at 1.5% by 0.15 p.p., as reported in the second row of Panel A in Table 4. This amounts to a 10% ($= 1.65/1.5 - 1$) increase in the growth rate. Our model, on the other hand, generates about one half of this impact and increases growth to 1.58%.

Table 4: Comparison of New Features to Standard Model

	Growth rate p.p	% Change in Growth
Benchmark Model w/o subsidy	1.50	
Standard models	1.65	10%
Current model in steady state	1.58	5.4%

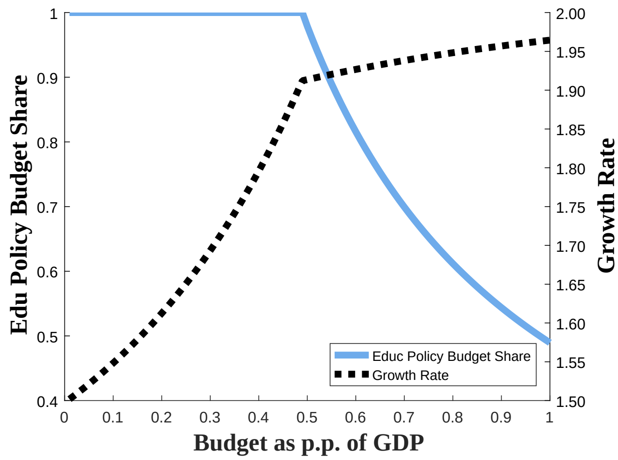
In our model there are two reasons why talented individuals may not enter a PhD: financial frictions and preferences. Each channel dampens the response of the growth rate to R&D subsidies compared to standard models. For instance, if all high-talent individuals are constrained, R&D subsidies have minimal effects because these individuals do not have the ability to respond to the increased wage in the research sector.

5.2 Policy Complementarities and Growth-Maximizing Policy Mix

Motivated by the different margins that each policy hits, we now investigate how these policies interact. First, we consider the growth-maximizing mix of R&D and education subsidies for the existing level of slots, $N = 0.01$. More specifically, the solid blue line in Figure 19 plots the optimal share of education subsidy (with the remaining share going to R&D policy) for any given budget. In the same graph, the dotted black line plots the resulting growth rate.

The results show a pecking order of the two policies. The policymaker prioritizes education subsidies by allocating 100% of the policy budget to education when the budget is low ($\leq$ 0.5% of GDP). This is due to the fact that the policymaker first finds it most effective to reduce financial frictions to strengthen the talent pool. When the level of budget is larger than 0.5% of GDP, the policymaker relies on a mix between education and R&D policies. For instance, when the government allocates 1% of GDP to maximize innovation, it is optimal to split it evenly between R&D and education subsidies. For larger budgets, the

Figure 19: Growth Maximizing Policy Mix at baseline slots level



share of R&D subsidy increases monotonically.

We now investigate what the growth-maximizing policy mix would be if the government could also expand the number of slots available in universities. Figure 20 shows the maximum growth rate attainable at different levels of slots for a budget of 2.5% of GDP. [20]. The x-axis shows the number of slots available, N. The vertical bars display the share of budget spending going to each policy for each level of slots. In particular, the bars represent (i) R&D subsidy (dark blue bars) (ii) subsidizing the cost of education (light blue bars), and (iii) slot creation (grey bars). The grey bars compute the share of the budget going to slots creation corresponding to the level of slots given on the x-axis. The other bars show the optimal allocation of the remaining budget between R&D subsidies and education subsidies at the given slots level. The solid red curve displays the growth rate at each level of slots for the aforementioned policy mix.

Figure 20: Growth-maximizing policy mix at budget=2.5 pp GDP

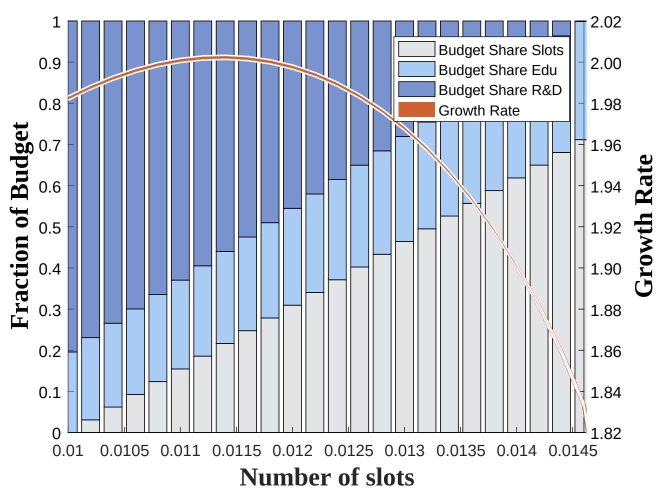


Figure 20 indicates that the growth-maximizing mix given 2.5% of GDP corresponds to roughly spend-

[20]For small budgets, the growth-maximizing policy mix is such that none of the budget should be allocated to expanding slots (see Appendix E.3).

ing 59% on subsidizing R&D, 26% of the budget on subsidizing the cost of education, and 15% of the budget on the creation of new slots. We revisit the role of the dynamics of policy complementarities in Section 5.4.

5.3 Inequality and Education Policy

In this section, we use our quantitative model to explore the link between the distribution of parental income and the impact of education policy. In our model, all potential students can afford education if parental resources are evenly distributed. Yet, the extent of inequality reduces the fraction of people who can afford education, generating a potential source of misallocation. The measure linking inequality to the access to education is given by $\tilde{\mu}$ as defined Equation (2). As $\tilde{\mu}$ tends to 1, the fraction of financially constrained individuals tends to 0.

Figure 21: Effectiveness of Education Policy at Different Levels of Financial Frictions

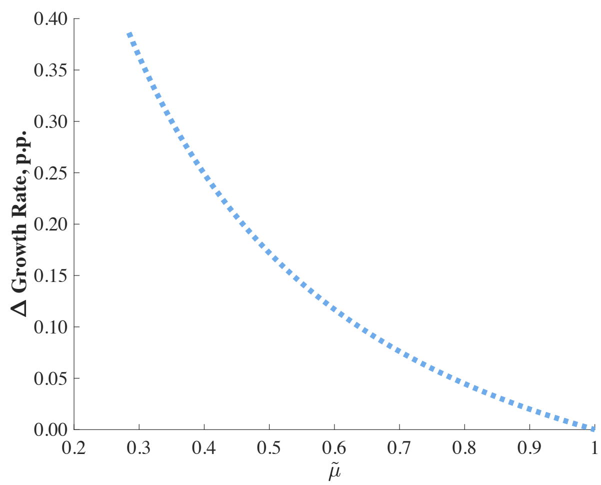


Figure 21 analyzes the effectiveness of education policy depending on the level of financial frictions in the economy. The x-axis represents the fraction of financially unconstrained individuals, $\tilde{\mu}$. We then compute the growth rate in response to a 0.5 p.p. GDP subsidy to the cost of education. On the y-axis, we plot the difference between the growth rate with the subsidy and the baseline growth rate of the economy without subsidy. The figure shows that as the fraction of financially unconstrained individuals increases, the effectiveness of subsidizing the cost of education declines. This is because the subsidy to the cost of education targets talented individuals who would like to obtain higher education, but cannot afford it.

The overall takeaway from this result is that as the society becomes more unequal and more families are financially constrained, government intervention to subsidize education becomes more desirable to preserve the innovative capacity of the economy.

5.4 Transitional Dynamics

We next turn to the dynamic evolution of the economy in response to each policy. A key component of the dynamics is that human capital takes significantly longer to build than physical capital and will thus induce longer delays in the transmission of policies to aggregate innovation. Starting from the initial steady state, the introduction of policies affects the occupational choice decisions of new generations. However, it takes time for the new generations of researchers to acquire human capital to become team leaders and to replace researchers from old generations.

Figure 22: Transitional dynamics response to 0.5% of GDP expenditure

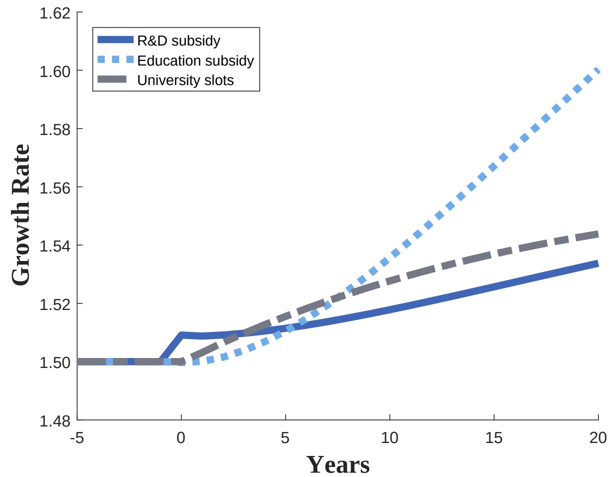


Figure 22 illustrates the evolution of the economy in response to policies.[21] At time 0, we introduce three distinct budget equivalent policies using 0.5% of GDP into the economy that will remain in place permanently. Recall that this budget corresponds to a 10% subsidy rate, which approximately the rate of R&D support in Denmark. We compare these policies over the following 20-year period to observe the corresponding evolution of the growth rate. The solid dark blue line corresponds to an R&D subsidy, the dotted light blue line corresponds to an educational subsidy, and the dashed grey line corresponds to an expansion in PhD slots.

A couple of remarks are in order. First, it takes time for each policy to show its full impact. All of these policies take more than ten years to get halfway to the new steady state from Table 3. Second, the policies that look more effective in the short-run are different from the policies that look effective in the medium to long-run. For instance, R&D subsidies generate immediate growth effects through increased use of lab equipment, while educational slots expansion takes some time and surpasses R&D after six years. Educational subsidies, on the other hand, take the longest to transmit to the growth rate, but gradually become the most effective policy tool in the long-run (see Table 3) through increasing the quality of researchers. The talented new enrollees eventually become team leaders and then transmit their skill to aggregate innovation. The talent increase has general equilibrium effects as the skilled wage increases over time, making research more attractive and bringing in talented individuals who on the margin prefer production.

An important takeaway from these transitional dynamics is that it takes time to build a high quality talent pool. Thus, the impact of policies targeting innovation and productivity through human capital has long time lags. For instance, it takes educational subsidies 30 years to reach the halfway point to the new steady state. Hence, empirical innovation policy evaluations based on data with short time-spans could

[21] We solve the transition dynamics numerically by guessing a vector of cutoffs $\{\bar{z}(t)\}_t$ along the transition and verifying whether the implied wages and profits are consistent with individuals' decisions. The algorithm we use to numerically solve for the equilibrium of the economy along the transition is discussed in Appendix D.

potentially lead to wrong conclusions about their effectiveness if the lagged nature of these policies are not taken into account.

6 Discussion and Robustness

In this section, we revisit our main findings and discuss their robustness. We focus on five variations on the main framework and reference the results in Table 5. Our finding that R&D policy is less effective than in standard models is confirmed in all variations in Panel A of this table which one can compare to benchmark models. Benchmark models imply a steady state increase of growth to the corresponding subsidy by 15%, whereas in our robustness checks it is never larger than 7%. Panel A also indicates how R&D policy is more effective in the short-run, while education tends to be more effective in the long-run. The fact that policymakers should mix policy to make R&D more effective can be seen in Panel B, where usually policymakers should use exclusively education subsidies (for small budgets) or mix policies (for larger budgets). Our other results (the interaction with parental income inequality and the different margins each policy hits), are robust as these are theoretical and hence do not depend on the calibration of the model.

For each robustness check, we report the impact of each of the three policies in both the short- and long-run. In particular, we report the responsiveness of the growth rate to budget equivalent policies of 0.5% of GDP at both one year, ten years, and in steady state. This can be found in Panel A of Table 5. In addition, we report the role of policy complementarities at a budget of both 0.5% of GDP and 2.5% of GDP in Panel B of Table 5. For reference, we repeat the benchmark results in Panel A(i) and B(i).[22] Overall, we find that our main results related to the magnitude of policies compared to benchmark models and at different time horizons are quite robust to the different alternatives that we consider.

6.1 Alternative Estimates of Parental Income: Including Maternal Income

In our benchmark specification, we focus on paternal income as a measure of parental resources in order to avoid time trends in female labor force participation. Here, we measure parental income as the sum of father's and mother's income. This delivers an estimate of the correlation between parental resources and IQ of 0.212, instead of 0.175 in our benchmark calibration. We recalibrate the model accordingly as described in Appendix E.4. Recall that this target moment informs the parameter μ, which captures financial frictions. A higher value of μ implies that fewer individuals are financially constrained. Panel A(ii) denotes the steady state response in the percentage change of the growth rate with this new calibration. Education subsidies are still significantly more effective (Δ17.86% in growth rate) than R&D subsidies (Δ5.54% in growth rate), yet the effectiveness of education policy declines due to better assortative matching. This general pattern of education subsidy effectiveness is illustrated in Figure 21. The time-to-build components still confirm R&D policy is more effective after one year (0.58) than educational subsidies (0.01) and slot expansion (0.23). Lastly, Panel B(i) illustrates that for a small budget of 0.5% of GDP, the government should allocate all of it to education. For a large budget of 2.5% of GDP, the optimal policy mix of 58%, 26%, and 16% for R&D subsidies, education subsidies, and slots increases respectively, looks similar to the benchmark.

[22] Additionally, for the cases that require a re-calibration of the model, we report in Appendix E.4 the implied parameter estimates and match between model and data moments.

Table 5: Robustness: Policy Response and Optimal Mix

Panel A: % Increase in Growth Rate ($\partial g/g$) from 0.5% GDP Policy			
	R&D Subsidy	Education Subsidy	Increase in Slots
(i) Benchmark Calibration			
After 1 Year	0.60	0.01	0.23
After 10 Years	1.19	2.38	1.62
Steady State	5.44	20.12	1.91
(ii) Parental Income as Sum of Father's and Mother's Income			
After 1 Year	0.58	0.01	0.23
After 10 Years	1.17	2.16	1.64
Steady State	5.54	17.86	1.74
(iii) Cost of Education Includes Room and Board Expenses			
After 1 Year	0.59	0.01	0.23
After 10 Years	1.19	0.63	1.62
Steady State	5.44	3.64	1.91
(iv) Alternative Cost of PhD Slots			
After 1 Year	0.59	0.01	0.47
After 10 Years	1.19	2.38	4.31
Steady State	5.44	20.12	9.72
(v) Addition of Exogenous Growth Component			
After 1 Year	0.29	0.01	0.06
After 10 Years	0.54	0.55	0.44
Steady State	2.67	5.33	1.33
(vi) Exogenous Growth Component Responds to Policy			
After 1 Year	0.29	0.01	0.06
After 10 Years	0.51	0.49	0.43
Steady State	2.67	4.67	1.33
Panel B: Respective Share of Policy under Optimal Mix			
	R&D Subsidy	Education Subsidy	Increase in Slots
(i) Benchmark Calibration			
0.5% GDP Budget	0	100	0
2.5% GDP Budget	59	26	15
(ii) Parental Income as Sum of Father's and Mother's Income			
0.5% GDP Budget	0	100	0
2.5% GDP Budget	58	26	16
(iii) Cost of Education Includes Room and Board Expenses			
0.5% GDP Budget	100	0	0
2.5% GDP Budget	51	49	0
(iv) Alternative Cost of PhD Slots			
0.5% GDP Budget	0	100	0
2.5% GDP Budget	43	22	35
(v) Non-PhD Innovation			
0.5% GDP Budget	0	100	0
2.5% GDP Budget	45	39	16
(vi) Non-PhD Innovation Responds to Policy			
0.5% GDP Budget	0	100	0
2.5% GDP Budget	48	12	40

6.2 Alternative Estimates of the Cost of Education

Our benchmark measure for the cost of education is given by the estimated foregone income during three years of a PhD, which we estimate to be about 0.3 million DKK, or 2.4% of a PhD's lifetime discounted income. Here, we are going to take a more aggressive approach and also include the cost of undergraduate studies as part of the bundle of education costs. The incorporation of broader costs by including the college expenses takes an extreme on the cost of a PhD, which enables us to illustrate the mechanics of the model when education policy is very costly.

In particular, the alternative estimate includes expenditures that parents have to bear to enable their kids to prepare for a PhD, such as the cost of room and board during their years in college and masters (tuition for college is free for domestic students). Based on information provided by international study associations[23], we assess the cost of room and board for college students to be approximately 0.32M DKK over five years. Including this cost alongside the 0.3M DKK cost for a PhD, we obtain our alternative estimate of cost of education to be about 5% of a PhD's lifetime income, roughly two times as large as our initial estimate.

The effect of higher cost of education policy is described in Panel A(*iii*). After one year , education policy shows similar effectiveness to the benchmark as in Panel A(*i*). However, after 10 years and in steady state, we can observe that as education becomes very costly, education policy for the same budget has a smaller overall effect. In Panel B(*iii*), we see that optimal policy with 2.5% GDP mixes again with significant reliance (49%) on educational subsidies.

6.3 Alternative Estimates of the Cost of PhD Slots

Our benchmark measure for the cost to the government of creating a PhD slot is given by cost of training a PhD reported by the Danish Ministry of Education (2016), which we estimate to be about 6% of a PhD's lifetime discounted income. Here, we consider an alternative estimate based on information on the tuition fees charged to PhD students. Although domestic students do not pay tuition fees in Denmark, some foreign students have to pay tuition. International study associations[24] report tuition fees for foreign students to be between 0.2-0.5M DKK. According to our calculations, this amounts to about 3% of a PhD's lifetime income. Panel A(*iv*) of Table 5 shows that the effectiveness of increasing PhD slots increases when the cost of PhD slots is estimated to be lower. In fact, the increase in growth rate after 1 year from the expansion of PhD slots increases from 0.23% in the benchmark calibration to 0.47%, after ten years it increases from 1.62% in the benchmark to 4.31%, and in steady state it increases from 1.91% to 9.72%. Panel B(*iv*) shows that the optimal policy mix still includes R&D policy, but more educational budget is allocated to slots and education subsidies.

6.4 Non-PhD Innovation

In our baseline model, we assume that all innovation is coming by PhDs. However, in our data, PhDs account for about a fourth of patents, while remaining patents come from non-PhDs. One concern is that we might overestimate the effect of policies in our benchmark, because in reality they only affect a fraction

[23]https://www.studyportals.com/ is a spin-off from two large international study associations, with the goal of helping students make an informed choice about study programs. It provides a host of information about University programs in many countries, including cost of tuition, cost of living, scholarship and grants, visa requirements.

[24]Ibidem.

of the inventors in the economy. In order to capture innovations created by individuals without higher education, in this section we extend our model to incorporate an exogenous arrival of ideas proportional to the economy-wide talent pool at rate ξ. The total amount of exogenous innovation is thus equal to $\xi \int_{\infty}^{\infty} q(z)f(z)dz = \xi \frac{\theta}{\theta-1}$. The resulting growth rate expression is:

$$g = \underbrace{\phi \frac{\lambda}{\lambda+\delta} \frac{\tilde{\mu}}{E} \left(\left(\frac{\hat{w}_s}{\hat{w}_u} \right)^{\frac{\rho+\delta}{\lambda}} \frac{\hat{\pi}}{\hat{w}_u} \right) \int_{\bar{z}}^{\infty} q(z)f(z)dz}_{\text{endogenous component}} + \underbrace{\xi \frac{\theta}{\theta-1}}_{\text{exogenous component}},$$

where the additional last term reflects the exogenous growth component. We then re-calibrate the model, as described in Appendix E.4, so that endogenous innovation due to PhD accounts for 23% of total innovation, as we observe in patent data. In this setting, only the endogenous component of the growth rate is sensitive to policy. Panel A(v) of Table 5 shows that policy effectiveness is lower when we limit the share of susceptible population. Nonetheless, our key results hold in these settings. R&D policy is less effective than in standard models: an R&D subsidy of magnitude corresponding to a 10% R&D subsidy leads to an increase in the growth rate by 0.29% on impact and about 2.67% in steady state, compared to about 15% in standard models. R&D policy is the most effective in the short run, but education policy is more effective in the long run. Finally, Panel B(v) of Table 5 shows that also the pecking order of policies discussed in our benchmark model still holds in this setting.

6.5 Non-PhD Innovation Responds to Policy

Our baseline model, we assume that all growth comes from PhDs. In the previous exercise in Section 6.4, we discuss an exogenous idea arrival to account for the fact that individuals without PhDs create ideas as well. One concern is that innovation and education policy might change the production of ideas outside the research sector, as a result of driving more numerous and more talented individuals into research. To address this concern, in this section we restrict the arrival rate of exogenous ideas to be proportional to the talent pool outside the research sector. In other words, we assume that PhDs get ideas only through research, and that the exogenous idea arrival is proportional to talent of the production workers. Therefore, as talented individuals are driven into research, the quantity of ideas which exogenously arrive to individuals in the production sector will decline. Formally, this growth rate takes the form:

$$g = \phi \frac{\lambda}{\lambda+\delta} \frac{\tilde{\mu}}{E} \left(\left(\frac{\hat{w}_s}{\hat{w}_u} \right)^{\frac{\rho+\delta}{\lambda}} \frac{\hat{\pi}}{\hat{w}_u} \right) \int_{\bar{z}}^{\infty} q(z)f(z)dz + \xi \left(\int_{1}^{\bar{z}} z dF(z) + (1-\tilde{\mu}\alpha) \int_{\bar{z}}^{\infty} z dF(z) \right)$$

We recalibrate the model according to this new structure, as described in Appendix E.4. In Section 6.4, the part of economic growth due to exogenous ideas arrival does not respond to policy and accounts for 77% (1.157 p.p./ 1.5 p.p.) of the baseline growth. With the current re-calibration, in steady state, the introduction of a 10% R&D subsidy leads to a decline to the exogenous growth component of 0.3% (to 1.153 p.p. GDP); a subsidy of equivalent expenditure to cost of education reduces the exogenous growth component by 0.7% (to 1.149 p.p. GDP), and a subsidy of equivalent expenditure aimed at increasing PhD slots reduces the exogenous growth component by 0.1% (to 1.156 p.p. GDP). An inspection of the overall growth effect in Panel A(vi) of Table 5 shows that the decline in exogenous growth is reflected in overall growth, particularly in the case of steady state response to an education subsidy, where the increase in growth declines from 5.33% to 4.67% from Panel A(*v*) to Panel A(*vi*). Thus, if the growth component due to innovation in the production sector is allowed to respond to policy, the effect of innovation and education policy on the growth rate is mitigated. However, inspection of Panel A(*vi*) indicates that the policy rankings are similar and the overall effect of policy on innovation is only slightly muted. Further, the optimal mix of policies is similar to the benchmark model, as seen in Panel B(*vi*).

7 Conclusion

This paper puts the development of scarce talent and career choice at the center of an endogenous growth framework and uses this framework to understand the effects of education policies, innovation policies, and their interaction. Individuals decide their career path as a function of their talent, preferences, financial constraints, and the time it takes to build human capital. These choices eventually transmit to aggregate innovation as talent builds into human capital and contributes to idea production. We discipline these micro-level decisions and outcomes with rich micro-level datasets from Denmark. Our estimated model not only matches a host of facts in the data, but replicates the response of the talent pool to policy interventions in the 2000s. We then use this framework to study policy counterfactuals of education and innovation policies and square longstanding puzzles in the literature on R&D policy. The framework delivers a number of important messages for understanding and designing optimal policies.

Our analysis delivers four main findings. First, we find that the introduction of a subsidy to innovation is less effective than in standard models due to a host of forces, such as time-to-build human capital and financial frictions, which squares the empirical evidence with these models. However, the impact of R&D subsidies can be strengthened when combined with higher education policy that sorts talented but credit-constrained individuals into research. Second, education and innovation policies are tapping into a different part of the talent distribution depending on the types of frictions individuals face. Third, R&D and education policies impact innovation at different horizons, which makes the optimal policy design a function of the time horizon of the policymaker. Finally, the optimal policy will depend on the amount of parental income inequality in the society. In highly unequal societies, education policy is likely to be significantly more effective than R&D policy because education policy hits credit constrained individuals with talent. In completely equal societies, R&D subsidies are more likely to be effective because financial frictions in obtaining education will be limited, but R&D subsidies can increase the available capital for researchers and induce those who would have otherwise not worked in research to enter the research sector.

We discuss here two exciting extensions for the individual-based endogenous growth agenda in this paper. First, in light of the increasing inequality observed around the world, it would be interesting to apply the current framework to societies where income inequality shows much more extreme dynamics relative to Denmark. Second, our results highlight that domestic talent is scarce and induce a country to run into diminishing returns when the country only relies on a domestic talent pool. One way to ameliorate this problem could be to tap into international talent, drawing implications for immigration policy. These are very fruitful extensions that await further research.

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Appendix

A Denmark Innovation and Education Programs

In this section, we provide a brief description of the various research and education programs included in the "Innovation Danmark" database referenced in Figure 2.

Table A1: Grants and Subsidies from Innovation Programs

	Research Subsidies	
P1 P7	FP6 / FP7	VI & VII European Community Framework Program for Research, Technological Development and Demonstration.
P3	Innovation Consortia	Joint project between firms and knowledge institutions to develop and mature research-based knowledge.
P4 P8	Innovation Pilot / Innovation Coupon	Promote cooperation between SME's and institutions to increase SME's innovation capacity.
P5	Innovation Agents	Innovation Agents offer knowledge and guidance to SMEs for technological innovation and business development.
P6	Open Funds	Public-Private cooperation for other projects.
P9	SPIR	Strategic Platform for Innovation and Research
P10	Horizon 2020	Part of EU Framework Program for Research & Innovation.
P11	High Tech Fund	Supports strategic R&D initiatives in high-tech.
P12	InnoBooster	Knowledge-based innovation projects for SMEs, start-ups and companies established by researchers.
P13	Int'l Cooperation	Projects in cooperation with international partners.
P14	Community P'ship	Innovation-oriented problem-solving for manufacturing.
P16	Grand Solutions	High risk projects with high value creation.

	Education Subsidies	
P0	Universities Funding	Educational and Research grants for Universities.
P2 P15	Business PhD / Business Postdoc	Business-oriented research project conducted in collaboration between a private company and a university.

B Measuring the Cost of Education

It is necessary for the budget equivalence of our policies to compute the cost of education in Denmark. In the literature, it is common to assess the cost of higher education as the sum of tuition fees and foregone income. However, in Denmark, tuition is free for domestic students, so we focus on assessing foregone income for PhD students. We compare the income of PhD students in our data to the income of comparable individuals of the same age working in the private or public sector. We assess the cost of foregone income during the years of PhD education to be approximately 100K DKK per year. The typical duration of a PhD program in Denmark is three years. Thus, we obtain that foregone income is approximately 300K DKK. This is consistent with the findings of a report from the Ministry of Education and Research (Ministry of Education (2016)), which finds that in the first years during and after completion of the PhD program, income of PhD graduates is lower than comparable MSc graduates in both the public and the private sector, but that the trend inverts over time. Given the average discounted income of PhD graduates in our data, we obtain that the cost of education is roughly 2.4% of a PhD's lifetime income. In Section 6, we check the robustness of our main results to different assumptions on the cost of education.

Additionally, we need to assess the cost to the government in creating a PhD slot. A report from the Danish Ministry of Education and Research (Ministry of Education, 2016) estimates the cost of PhD training, including administrative costs, teaching material, etc., to be approximately between 0.7-1.1M DKK. Given the average discounted income of PhD graduates in our data, we then assess the cost of creating a PhD slot to be approximately 6% of a PhD's lifetime income. In Section 6, we check the robustness of our main results to different assumptions on the cost of a PhD slot.

C Theoretical Derivations

This section discusses primarily model derivations from the text. In particular, we focus on the main theoretical model derivations from Appendix C.1-C.6, and details on the derivation for Equation (1) in Appendix C.7.

C.1 Derivation of talent threshold $\bar{z}$ in partial equilibrium

Recall that individuals choose between becoming a researcher or a production worker:

$$V(z,\epsilon,b) = \max\{V^{phd}(z,b), V^{worker}(b) + \ln(\epsilon)\}$$

where $V^{phd}(z)$ is the value of becoming a researcher for an individual with talent z and V^{worker} is the value of becoming a production worker. We can rewrite this maximization problem as:

$$\max\left\{\int_b^\infty e^{-(\delta+\rho)(t-b)}\left[e^{-\lambda(t-b)}\ln(w_s(t)) + (1-e^{-\lambda(t-b)})\ln(\pi_H(z,t))\right]dt, \int_b^\infty e^{-(\delta+\rho)(t-b)}\left[(\ln w_u(t))\right]dt + \ln(\epsilon)\right\}.$$

An individual will be indifferent between becoming a researcher or a production worker if:

$$V^{phd}(z,b) = V^{worker}(b) + \ln(\epsilon).$$

We can write the value of being a PhD or being an unskilled worker as:

$$\int_b^\infty e^{-(\delta+\rho)(t-b)}\left[\left(e^{-\lambda(t-b)}\ln(\hat{w}_s) + (1-e^{-\lambda(t-b)})\ln(\hat{\pi}z)\right) + g(t-b)\right]dt = \int_b^\infty e^{-(\delta+\rho)(t-b)}\left[\ln\hat{w}_u + g(t-b)\right]dt + \ln(\epsilon).$$

Notice that the growth rate cancels out on both sides of the expression. Solving this equation yields:

$$\frac{\ln(\hat{w}_s) - \ln(\hat{\pi}z)}{\delta+\rho+\lambda} + \frac{\ln(\hat{\pi}z)}{\delta+\rho} = \frac{\ln(\hat{w}_u)}{\delta+\rho} + \ln(\epsilon)$$

Notice that the resulting expression is time-invariant. Rearranging this expression, we obtain:

$$\ln(z) = \frac{\delta+\rho+\lambda}{\lambda}\ln(\hat{w}_u) - \frac{\delta+\rho}{\lambda}\ln(\hat{w}_s) - \ln(\hat{\pi}) + \frac{(\delta+\rho)(\delta+\rho+\lambda)}{\lambda}\ln(\epsilon)$$

Finally, exponentiating both sides of this equation we obtain:

$$z = \left(\frac{\hat{w}_u}{\hat{w}_s}\right)^{\frac{\delta+\rho}{\lambda}} \frac{\hat{w}_u}{\hat{\pi}} \epsilon^{\chi},$$

where $\chi \equiv \frac{(\delta+\rho)(\delta+\rho+\lambda)}{\lambda}$. We assume $\epsilon^{\chi} \sim U(0, Ez)$. It follows that α is also time-invariant and independent of z:

$$\begin{aligned}
\alpha &\equiv \Pr\left(V^{phd} > V^{worker} + \ln(\epsilon)\right) \\
&= \Pr\left(z \geq \left(\frac{\hat{w}_u}{\hat{w}_s}\right)^{\frac{\delta+\rho}{\lambda}} \frac{\hat{w}_u}{\hat{\pi}} \epsilon^{\chi}\right) \\
&= \Pr\left(\epsilon^{\chi} \leq z \left(\frac{\hat{w}_u}{\hat{w}_s}\right)^{-\frac{\delta+\rho}{\lambda}} \frac{\hat{\pi}}{\hat{w}_u}\right) \\
&= \frac{z}{Ez} \left(\frac{\hat{w}_s}{\hat{w}_u}\right)^{\frac{\delta+\rho}{\lambda}} \frac{\hat{\pi}}{\hat{w}_u}.
\end{aligned}$$

C.2 Proof of Proposition 2.

Proof.

We can write the change in average productivity $\bar{A}$ in small time interval, Δt, as $\frac{\bar{A}(t+\Delta t)-\bar{A}(t)}{\bar{A}(t)\Delta t}$ and obtain the growth rate of productivity on the balanced growth path by letting Δt go to zero, i.e.:

$$\begin{aligned}
g &= \lim_{\Delta t \to 0} \frac{\bar{A}(t+\Delta t) - \bar{A}(t)}{\bar{A}(t)\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{\int_0^1 \left(A_j + \left(\frac{\lambda}{\lambda+\delta}\tilde{\mu}\alpha\phi \int_{\bar{z}}^{\infty} q(z)dF(z)\right)\bar{A}(t)\right)dj - \bar{A}(t)}{\bar{A}(t)\Delta t} \\
&= \frac{\lambda}{\lambda+\delta}\tilde{\mu}\alpha\phi \int_{\bar{z}}^{\infty} q(z)dF(z)
\end{aligned}$$

Plugging in $\tilde{\mu}$ from Equation (2), α from Equation (9), $q(z)$ from Equation (7) and our functional form for F we obtain:

$$g = \bar{z}^{\frac{\eta_1}{\eta_1+\eta_2}} N \frac{(\phi\lambda^{\eta_1}\delta^{\eta_2})^{\frac{1}{\eta_1+\eta_2}}}{\lambda+\delta} \left(\eta_3 \frac{\pi_I}{r}\right)^{\frac{\eta_3}{\eta_1+\eta_2}} \left(\frac{\theta}{\theta-1}\right)^{\frac{\eta_1}{\eta_1+\eta_2}},$$

delivering the formula in Equation (25). ■

C.3 Details on $\bar{z}$ Derivation with Taxes and Subsidies

Recall the partial equilibrium value for the cutoff $\bar{z}$:

$$\bar{z} = \left[\frac{\tilde{\mu}}{NE}\left(\frac{\hat{w}_s}{\hat{w}_u}\right)^{\frac{\delta+\rho}{\lambda}} \frac{\hat{\pi}}{\hat{w}_u}\right]^{\frac{1}{\theta}}$$

R&D subsidies s and taxes to fund subsidies τ alter the $\hat{w}_s$ and $\hat{\pi}$.[25]

$$\bar{z}^{1-\frac{\frac{\delta+\rho}{\lambda}\eta_1-\eta_2}{\theta(\eta_1+\eta_2)}} = \left[\frac{\tilde{\mu}}{NE} \left(\frac{\eta_2 \eta_3^{\frac{\eta_3}{\eta_1+\eta_2}} \left[\frac{\lambda}{\delta}\frac{\theta}{\theta-1}\right]^{\frac{\eta_1}{\eta_1+\eta_2}} \left(\phi \frac{(1+s)(1-\tau)\pi_I}{r}\right)^{\frac{1}{\eta_1+\eta_2}}}{\frac{\beta}{1-\beta}\left(\frac{1-\beta}{\psi}\right)^{\frac{1-\beta}{\beta}}} \right)^{\frac{\delta+\rho}{\lambda}} \frac{\eta_3^{\frac{\eta_3}{\eta_1+\eta_2}} \eta_1 \left[\frac{\delta}{\lambda}\frac{\theta-1}{\theta}\right]^{\frac{\eta_2}{\eta_1+\eta_2}} \left(\phi \frac{(1+s)(1-\tau)\pi_I}{r}\right)^{\frac{1}{\eta_1+\eta_2}}}{\frac{\beta}{1-\beta}\left(\frac{1-\beta}{\psi}\right)^{\frac{1-\beta}{\beta}}} \right]^{\frac{1}{\theta}}$$

Simplfying both sides leads to Equation (26).

C.4 Details on Steady State Subsidy Equivalence Across Policies

In order to compare different policies, it is crucial to understand how to compare the equivalent revenue cost of each policy. In this section, we provide more details on government policies described in Section 3.2. The government can introduce three types of interventions: R&D subsidies, educational financing subsidies, and expanding university slots. The government finances these policies with a proportional tax τ on intermediate firms profits. We assume that the government balances the budget in every period. Then the problem of the intermediate good producer becomes:

$$\Pi_j = \max_{k_j} \left\{ (L^\beta A_j^\beta k_j^{1-\beta} - \psi k_j)(1-\tau) \right\}.$$

Hence the flow profits for the intermediate good producers in the presence of a tax become

$$\Pi_j = (1-\tau)\pi_I A_j,$$

where $\pi_I \equiv \beta \left[(1-\beta)/\psi\right]^{\frac{1-\beta}{\beta}} L$.

Thus the total tax revenue to the government is:

$$T = \tau \pi_I \bar{A}.$$

We will now derive the tax rates that balance the government budget in every period for each policy instrument.

R&D Subsidy. The government introduces a subsidy to the price of ideas purchased by firms from research teams. Thus the price received by research teams under the subsidy, $p_{\text{R\&D}}$, is :

$$p_{\text{R\&D}} = (1+s)(1-\tau)p = (1+s)(1-\tau)\frac{\pi_I}{r}\bar{A}.$$

Note that this formula also displays the pass-through of the tax τ imposed on monopolists to the research teams. The solution of the model follows the same steps described in the main text, with the only difference that the unit price of ideas now reflects the subsidy and the tax. This procedure leads to the expression for the threshold $\bar{z}$ and growth rate g described in Section 3.2.1. The total cost of an R&D subsidy to the government is given by the unit price of ideas multiplied by the amount of ideas produced:

[25] Recall, $\hat{w}_s = \eta_2 \eta_3^{\frac{\eta_3}{\eta_1+\eta_2}} \left[\frac{\lambda \bar{z}}{\delta}\frac{\theta}{\theta-1}\right]^{\frac{\eta_1}{\eta_1+\eta_2}} (\phi \frac{\pi_I}{r})^{\frac{1}{\eta_1+\eta_2}}$; $\hat{w}_u = \frac{\beta}{1-\beta}\left(\frac{1-\beta}{\psi}\right)^{\frac{1-\beta}{\beta}}$; $\hat{\pi} = \eta_3^{\frac{\eta_3}{\eta_1+\eta_2}} \eta_1 \left[\frac{\delta}{\lambda \bar{z}}\frac{\theta-1}{\theta}\right]^{\frac{\eta_2}{\eta_1+\eta_2}} (\phi \frac{\pi_I}{r})^{\frac{1}{\eta_1+\eta_2}}$.

$$S_{\text{R\&D}} = s(1-\tau)p\phi\frac{\lambda}{\lambda+\delta}\tilde{\mu}\alpha\int_{\bar{z}}^{\infty}q(z)dF(z)$$

For a given value of the subsidy s, the value of τ that balances the budget solves the equation $S_{\text{R\&D}} = T$, which is equivalent to:

$$\tau = s(1+s)^{\frac{\eta_3}{\eta_1+\eta_2}}\left(\phi\frac{(1-\tau)}{r}\right)^{\frac{1}{\eta_1+\eta_2}} N\frac{(\lambda^{\eta_1}\delta^{\eta_2})^{\frac{1}{\eta_1+\eta_2}}}{\lambda+\delta}(\eta_3\pi_I)^{\frac{\eta_3}{\eta_1+\eta_2}}\left(\frac{\theta}{\theta-1}\bar{z}\right)^{\frac{\eta_1}{\eta_1+\eta_2}}$$

Subsidy to Cost of Education. Recall that we defined κ to be the cost of education and here we assume that the government pays a fraction s^{κ} of each PhD's education cost. For ease of comparison of the cost parameters to the data counterpart int the calibration section, we express this cost as a fraction of expected lifetime income for a PhD student, $I^{PhD} \times \bar{A}$. Thus, we let the cost of education be $\kappa = \kappa^{e} \times I^{PhD} \times \bar{A}$. Thus the total cost of the subsidy is:

$$S_{\text{edu}} = s^{\kappa}\kappa^{e}I^{PhD}\bar{A}N$$

and the τ that balances the budget is:

$$\tau = \frac{s^{\kappa}\kappa^{e}I^{PhD}N}{\pi_I}.$$

Increasing the Number of PhD slots. In order to solve for the balanced budget equation of the government, we need to make assumptions on the cost to the government of increasing the slots. Again, for ease of exposition, we translate this cost in terms of fraction of expected lifetime income for a PhD student, $I^{PhD} \times \bar{A}(t)$, and let the cost of a PhD slot be $\kappa^{s} \times I^{PhD} \times \bar{A}(t)$. Thus, the total cost of the subsidy is:

$$S_{\text{slots}} = \kappa I^{PhD}\bar{A}(N^{s}-N).$$

For a given value of the subsidy s, the value of τ that balances the budget solves the equation $S_{\text{slots}} = T$, which is equivalent to:

$$\tau = \frac{\kappa^{s}I^{PhD}(N^{s}-N)}{\pi_I}.$$

In order to carry out meaningful comparisons on the impact of these subsidies on economic growth, our quantitative exercise solves for the subsidy rates equivalent to the same expenditure for the government. We then report the magnitude of the government intervention in terms of total government expenditure as a percentage of GDP for each policy. The concept of GDP in our model is given by final good output, as expressed in Equation (19). Thus we solve for subsidy and tax rates that result in equivalent government expenditures as percentage of GDP as follows:

$$\frac{S_{\text{R\&D}}}{Y} = \frac{S_{\text{edu}}}{Y} = \frac{S_{\text{slots}}}{Y}.$$

C.5 Derivation of Equations Used for Calibration

In Section 4.3 we presented a number of expressions that represent the model counterpart of data moments that we target for calibration. In this section, we present the derivations of those expressions.

The mean of team size in our model can be derived as follows:

$$\mathbf{E}(n) = \frac{\int_{\bar{z}}^{\infty} n(z) f(z) dz}{1 - F(\bar{z})} = \frac{\delta}{\lambda} \frac{\bar{z}^{1-\theta}}{\bar{z} \times \bar{z}^{-\theta}} = \frac{\delta}{\lambda}.$$

We then add one to the mean of team size to also capture the team leader as part of the team.
Similarly, we can derive the variance of team size:

$$Var(n) = \frac{\int_{\bar{z}}^{\infty} (n(z) - \mathbf{E}(n))^2 f(z) dz}{1 - F(\bar{z})} = \frac{\theta}{\theta - 2} \bar{z}^{2-\theta}.$$

We can additionally derive the ratio of Post-PhD income to PhD income as the ratio of average profits of team leaders to skilled wage of team members. First we obtain the ratio of team leader profit to skilled wage for an individual with talent z:

$$\frac{\pi_H(z,t)}{w_s(t)} = \frac{\eta_1}{\eta_2} \frac{\delta}{\lambda} \left[\frac{z}{\bar{z}} \frac{\theta - 1}{\theta} \right].$$

Then we compute the average profits of team leaders to skilled wage of team members for researchers in the economy:

$$\begin{aligned} \frac{\int_{\bar{z}}^{\infty} \frac{\pi_H(z,t)}{w_s(t)} dz}{1 - F(\bar{z})} &= \frac{\eta_1}{\eta_2} \frac{\delta}{\lambda} \left[\frac{1}{\bar{z}^{1-\theta}} \frac{\theta - 1}{\theta} \right] \int_{\bar{z}}^{\infty} z f(z) dz \\ &= \frac{\eta_1}{\eta_2} \frac{\delta}{\lambda} \end{aligned}$$

Finally, we can obtain the PhD skill-premium as the ratio of skilled wage to unskilled wage:

$$\frac{w_s(t)}{w_u(t)} = \frac{1-\beta}{\beta} \left(\frac{1-\beta}{\psi} \right)^{-\frac{1-\beta}{\beta}} \left(\frac{\lambda}{\delta} \bar{z} \frac{\theta}{\theta - 1} \right)^{\frac{\eta_1}{\eta_1+\eta_2}} \eta_2 \eta_3^{\frac{\eta_3}{\eta_1+\eta+2}} \left(\phi \frac{\pi_I}{r} \right)^{\frac{1}{\eta_1+\eta_2}}.$$

C.6 Model Derivations for the Case where Some Individuals with Assortative Match of Parental Resources and IQ Cannot Afford Education

In the main text, we derive equilibrium equations under the assumption that ζ is large enough so that all individuals with an assortative match between parental resources and IQ can afford education. Here we study the more general case where some assortatively matched individuals cannot afford education. In particular, suppose that parental resources for the group of assortatively matched follow the same Pareto CDF as for the individuals with random match of parental resources and IQ. We will also assume that

$\tilde{\theta} = \theta$. Then, individuals whose parental resources are assortatively matched to IQ can afford education only if they are in the top of the IQ distribution. More precisely, recall that the probability that parents can afford education is $\left(\frac{\theta-1}{\theta}\right)^{\theta}$. Thus, individuals whose parental resources are assortatively matched to IQ can afford education if their IQ is larger than a threshold $\underline{z}$ such that $Pr(z \geq \underline{z}) = \left(\frac{\theta-1}{\theta}\right)^{\theta}$. This implies that $\underline{z} = \frac{\theta}{\theta-1}$. Thus, the fraction of individuals who can afford education in the economy is:

$$(1-\mu)\left(\frac{\theta-1}{\theta}\right)^{\theta} + \mu \int_{\underline{z}}^{\infty} dF(z) = \left(\frac{\theta-1}{\theta}\right)^{\theta}$$

Following the same steps outlined in the main text, we obtain that the partial equilibrium expression for $\bar{z}$ is:

$$N = \begin{cases} \alpha \tilde{\mu} \bar{z}^{-\theta} & \text{if } \bar{z} \geq \underline{z} \\ \alpha \left((1-\mu)\left(\frac{\theta-1}{\theta}\right)^{\theta} \bar{z}^{-\theta} + \mu \underline{z}^{-\theta} \right) & \text{if } \bar{z} < \underline{z} \end{cases}$$

where:

$$\alpha = \frac{1}{E}\left(\frac{\hat{w}_s}{\hat{w}_u}\right)^{\frac{\delta+\rho}{\lambda}} \frac{\hat{\pi}}{\hat{w}_u}.$$

The labor market clearing equations for team members becomes:

$$\frac{\lambda}{\lambda+\delta}\alpha\left((1-\mu)\left(\frac{\theta-1}{\theta}\right)^{\theta}\int_{\bar{z}}^{\infty} n(z)f(z)dz + \mu\int_{\max\{\underline{z},\bar{z}\}}^{\infty} n(z)f(z)dz\right) = \frac{\delta N}{\lambda+\delta}$$

If $\bar{z} \geq \underline{z}$ we default to the case we already solved in the main text. Thus, we focus on the case $\bar{z} < \underline{z}$. The resulting relationship between wages and prices is:

$$\frac{w_s}{p} = \eta_2\phi\left[\frac{\lambda}{\delta}\frac{\theta}{\theta-1}\frac{\left((1-\mu)(\bar{z})^{1-\theta} + \mu\left(\frac{\theta}{\theta-1}\right)\right)}{(1-\mu)\bar{z}^{-\theta}+\mu}\right]^{\frac{\eta_1}{\eta_1+\eta_2}}\left(\eta_3\phi\frac{p}{\bar{A}}\right)^{\frac{\eta_3}{\eta_1+\eta_2}}$$

while the expression for profits of the team leader is:

$$\pi_H(z) = z\phi p\eta_1\left[\frac{\lambda}{\delta}\frac{\theta}{\theta-1}\frac{\left((1-\mu)(\bar{z})^{1-\theta} + \mu\left(\frac{\theta}{\theta-1}\right)\right)}{(1-\mu)\bar{z}^{-\theta}+\mu}\right]^{\frac{-\eta_2}{\eta_1+\eta_2}}\left(\eta_3\phi\frac{p}{\bar{A}}\right)^{\frac{\eta_3}{\eta_1+\eta_2}}.$$

Substituting these expressions into the formula for α we obtain :

$$\alpha = \frac{1}{E}\eta_1\eta_2^{\tilde{\rho}}\left(\phi\left(\eta_3\frac{\pi_I}{r}\right)^{\eta_3}\right)^{\frac{\tilde{\rho}+1}{\eta_1+\eta_2}}\left(\frac{1-\beta}{r}\right)^{\tilde{\rho}+1}\left[\frac{\lambda}{\delta}\frac{\theta}{\theta-1}\frac{\left((1-\mu)(\bar{z})^{1-\theta} + \mu\left(\frac{\theta}{\theta-1}\right)\right)}{(1-\mu)\bar{z}^{-\theta}+\mu}\right]^{\frac{\eta_1\tilde{\rho}-\eta_2}{\eta_1+\eta_2}}$$

where $\tilde{\rho} \equiv \frac{\delta+\rho}{\lambda}$. Recall that $N = \alpha\left(\frac{\theta-1}{\theta}\right)^{\theta}\left((1-\mu)\bar{z}^{-\theta}+\mu\right)$. Plugging in α into this expression we obtain:

$$N = \left(\frac{\theta-1}{\theta}\right)^{\theta} \frac{\eta_1 \eta_2^{\tilde{\rho}}}{E} \left(\phi \left(\eta_3 \frac{\pi_I}{r}\right)^{\eta_3}\right)^{\frac{\tilde{\rho}+1}{\eta_1+\eta_2}} \left(\frac{1-\beta}{r}\right)^{\tilde{\rho}+1} \left[\frac{\lambda}{\delta}\frac{\theta}{\theta-1}\left((1-\mu)\left(\bar{z}\right)^{1-\theta} + \mu\left(\frac{\theta}{\theta-1}\right)\right)\right]^{\frac{\eta_1\tilde{\rho}-\eta_2}{\eta_1+\eta_2}} \left((1-\mu)\bar{z}^{-\theta}+\mu\right)^{1-\frac{\eta_1\tilde{\rho}-\eta_2}{\eta_1+\eta_2}}$$

Solving this expression delivers the general equilibrium value of $\bar{z}$ for the case $\bar{z} < \underline{z}$.

C.7 Derivation of Equation (1)

We describe a simple model of Schumpeterian endogenous growth that delivers the benchmark Equation (1) for the effect of R&D subsidies on the rate of economic growth in standard endogenous growth models.

Assume that time is discrete and, in each period, there is a fixed number L of individuals that live only for one period. Final good Y_t is produced competitively according to the production function:

$$Y_t = x_t^{\alpha}\left(A_t L_t\right)^{1-\alpha}$$

where A_t is productivity and x_t is the amount of the intermediate good used in final good production. The price of the final good is normalized to 1. The intermediate product is produced by a monopolist using the final good as the only input with one-for-one technology. The monopolist maximizes expected profits subject to the demand:

$$\max_{p_t, x_t} p_t x_t - x_t$$
$$\text{s.t. } p_t = \alpha x_t^{\alpha-1}\left(A_t L_t\right)^{1-\alpha}$$

We obtain that intermediate profits are:

$$\Pi_t = \pi L A_t$$

where $\pi \equiv (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}$. In each period a potential entrant attempts to innovate and create a more productive version of the intermediate product. If innovation at time t is successful, productivity of the intermediate good increases up to $A_t = \gamma A_{t-1}$, where $\gamma > 1$. The probability that innovation is successful depends on the amount of resources spent on R&D. The cost of doing R&D and generating a successful innovation with probability μ is:

$$R_t(\mu) = \left(\frac{\mu}{\lambda}\right)^{\frac{1}{\sigma}} A_t$$

where $\sigma \in (0,1)$ and $\lambda > 0$ is R&D productivity. The maximization problem for a potential entrant is :

$$\max_{\mu} \mu \pi L A_t - \left(\frac{\mu}{\lambda}\right)^{1/\sigma} A_t$$

which delivers the innovation rate:

$$\mu^* = (\sigma \pi L)^{\frac{\sigma}{1-\sigma}} \lambda^{\frac{1}{1-\sigma}}$$

The expected productivity growth rate can be expressed as:

$$g = E\left(\frac{A_t - A_{t-1}}{A_{t-1}}\right) = \mu^*(\gamma - 1).$$

Now suppose that the government introduces a subsidy s to R&D. Then the problem of the potential entrant becomes

$$\max_{\mu} \mu \pi L A_t - (1-s)\left(\frac{\mu}{\lambda}\right)^{1/\sigma} A_t$$

and innovation rate becomes:

$$\mu^s = \frac{\mu^*}{(1-s)^{\frac{\sigma}{1-\sigma}}}$$

Thus we can obtain that the expected growth rate upon the introduction of the subsidy becomes

$$g^s = \frac{\eta}{(1-s)^{\frac{\sigma}{1-\sigma}}}$$

where $\eta = (\sigma \pi L)^{\frac{\sigma}{1-\sigma}} \lambda^{\frac{1}{1-\sigma}} (1-\gamma)$. Taking log on both sides we obtain

$$\ln(g^s) = \ln(\eta) - \frac{\sigma}{1-\sigma}\ln(1-s) \approx \ln(\eta) + \frac{\sigma}{1-\sigma}s$$

It follows that :

$$\frac{\partial \ln(g^s)}{\partial s} = \frac{g'(s)}{g(s)} \approx \frac{\sigma}{1-\sigma}$$

For the value of the parameter of R&D productivity $\sigma = 0.5$, we obtain the expression from Equation (1):

$$\frac{g'(s)}{g(s)} \approx 1.$$

D Details on Transitional Dynamics

In this section, we present the algorithm used to solve for transitional dynamics of the equilibrium of the economy upon the introduction of a permanent subsidy. We solve the transition dynamics numerically by guessing a vector of cutoffs $\{\bar{z}(t)\}_t$ along the transition and verifying whether the implied wages are consistent with individuals' decisions. We then apply the following steps to numerically solve for transitional dynamics.

1. Guess a vector of cutoffs $\{\bar{z}(t)\}_t$ along the transition
2. Obtain a vector of taste-probabilities $\{\alpha(t)\}_t$ consistent with the guess for cutoffs, i.e. taste-probabilities must satisfy:

$$N(t) = Pr(z \geq \bar{z}(t)) \times \tilde{\mu}(t) \times \alpha(t)$$

3. Solve for labor market clearing along the transition at every point in time τ given thresholds and probabilities:

$$\frac{\lambda}{\lambda+\delta}\left[\int_0^{\tau} H(t,\tau)\tilde{\mu}(t)\alpha(t)\int_{\bar{z}(t)}^{\infty} n(z,\tau)f(z)dzdt\right]=\frac{\delta N}{\lambda+\delta}$$

   where $H(t,\tau)$ indicates the fraction of team leaders at time τ from cohort t. Then solve recursively for $\frac{p(\tau)}{w_s(\tau)}$ along the transition:

$$\frac{p(\tau)}{w_s(\tau)}=\frac{1}{\eta_2\phi}\left(\frac{\frac{\delta}{\lambda}\frac{\theta-1}{\theta}N}{\left(\eta_3\phi\frac{\pi_I(1+s)(1-\tau)}{r}\right)^{\frac{\eta_3}{\eta_1}}\left[\int_0^t H(t,\tau)\alpha(t)\tilde{\mu}(t)\bar{z}(t)^{1-\theta}\right]}\right)^{\frac{\eta_1}{\eta_1+\eta_2}}$$

   from this we also obtain that profits for team leaders along the transition are given by:

$$\pi(\tau)=\phi\eta_1 p(\tau)\left(\eta_2\phi\frac{p(\tau)}{w_s(\tau)}\right)^{\frac{\eta_2}{\eta_1}}\left(\eta_3\phi\frac{(1+s)(1-\tau)\pi_I}{r}\right)^{\frac{\eta_3}{\eta_1}}$$

4. Obtain the implied taste-probabilities given the wages and profits. An individual born at time b will be indifferent between becoming a researcher or a production worker if:

$$V^{phd}(z,b)=V^{worker}(b)+\ln(\epsilon)(b)$$

$$\int_b^{\infty} e^{-(\delta+\rho)(t-b)}\left[e^{-\lambda(t-b)}\ln w_s(t)+(1-e^{-\lambda(t-b)})ln(\pi_{H,t}z)\right]dt=\int_b^{\infty} e^{-(\delta+\rho)(t-b)}\left[(\ln w_u(t))\right]dt+\ln(\epsilon)$$

   Normalize wages and profits by $p(t)$ and rearrange to obtain:

$$\int_b^{\infty} e^{-(\delta+\rho)(t-b)}\left[e^{-\lambda(t-b)}\ln\frac{w_s(t)}{p(t)}+(1-e^{-\lambda(t-b)})\ln(\frac{\pi_H(t)}{p(t)})\right]dt+\frac{\ln(z)}{\chi}=\int_b^{\infty} e^{-(\delta+\rho)\mathbf{t}}\left[(\ln\frac{w_u(t)}{p(t)})\right]dt+\ln(\epsilon)$$

   We are interested in computing the probability:

$$Pr(\ln(\epsilon)\le V^{phd}(z)-V^{worker})=$$
$$=Pr(\epsilon^{\chi}\le e^{\chi\left(V^{phd}(z)-V^{worker}\right)})$$

   Recall the following error dist: $\epsilon^{\chi}\sim U(0,Ez)$, where $\chi\equiv\frac{(\delta+\rho)(\delta+\rho+\lambda)}{\lambda}$. then:

$$Pr(\epsilon^{\chi}\le e^{\chi\left(\int_b^{\infty} e^{-(\delta+\rho)(t-b)}\left[e^{-\lambda(t-b)}\ln\frac{w_s(t)}{p(t)}+(1-e^{-\lambda t})ln(\frac{\pi_{H,t}}{p(t)})\right]dt+\frac{\ln(z)}{\chi}-\int_b^{\infty} e^{-(\delta+\rho)(t-b)}\left[(\ln\frac{w_{u,t}}{p(t)})\right]dt\right)})=$$
$$Pr(\epsilon^{\chi}\le ze^{\chi\left(\int_b^{\infty} e^{-(\delta+\rho)(t-b)}\left[e^{-\lambda(t-b)}\ln\frac{w_{s,t}}{p(t)}+(1-e^{-\lambda t})ln(\frac{\pi_{H,t}}{p(t)})\right]dt-\int_b^{\infty} e^{-(\delta+\rho)(t-b)}\left[(\ln\frac{w_{u,t}}{p(t)})\right]dt\right)})=$$
$$=\frac{e^{\chi\left(\int_b^{\infty} e^{-(\delta+\rho)(t-b)}\left[e^{-\lambda t}\ln\frac{w_{s,t}}{p(t)}+(1-e^{-\lambda(t-b)})ln(\frac{\pi_{H,t}}{p(t)})\right]dt-\int_b^{\infty} e^{-(\delta+\rho)(t-b)}\left[(\ln\frac{w_{u,t}}{p(t)})\right]dt\right)}}{E}$$

5. Verify whether the taste-probabilities obtained in point 4 are consistent with the taste-probabilities

implied by the guess. If not, repeat point 1-4 until reaching convergence.

6. Once step 5 converges, compute the growth rate along the transition given the obtained values of the cut-off $\{\bar{z}(t)\}_t$.

E Quantitative Appendix

In this section, we discuss the policy counterfactuals in the quantitative section of the model. We first address the inputs for subsidies which we use in Section 4.4, then discuss in greater detail the steady state policy comparisons.

E.1 Policy Values for Comparison Non-Targeted Moments

Starting in 2002, the Danish government introduced a number of education and innovation policies. We use the outcome of policy interventions as an out-of-sample test of our model. We group the policies into R&D subsidies, education subsidies, and slot expansion.

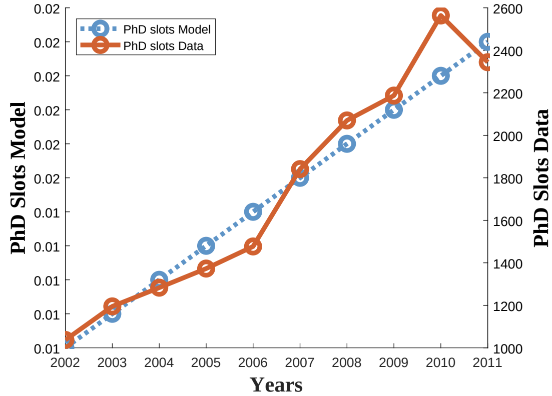
We use the following procedure to estimate policy rates from the data. To compute the R&D subsidy rate, we proceed in multiple steps. First, we compute the expenditures of the government for grants under the "Innovation Danmark" programs for the period 2002-2010. For most grants, we observe the exact amount received by firms in the program. Second, for those grants for which the expenditure is missing, we impute the median value of grants under the same program. Third, to apportion expenditures over each year, we use the year in which the program rolled out and, for those programs that last more than one year, we spread the expenditures proportionally for each year of duration. Fourth, we add up the expenditures for all grants year by year and obtain yearly government expenditures for R&D subsidies. Fifth, to convert the expenditures into subsidy rates, we divide the total expenditure in subsidies in a given year by the total business R&D expenditures reported by DST. We then compute the average subsidy rate between 2002 and 2010 and obtain a value of 1.22%.

We follow a similar procedure to compute the educational subsidy rates. We consider grants within "Innovation Danmark" aimed at PhD students ("Business PhD" grant and "Business Postdoc" grant, see Appendix A for further details). After repeating the first four steps outlined above, we then convert year expenditures into subsidy rates by dividing the total expenditure in education subsidies in a given year by the total cost of education for enrolled PhD students computed according to our measure of cost. Although we do not need to make assumptions on the cost of education to solve the model, estimating the costs are necessary to understand the budgetary costs of each policy. In Appendix B, we explain in detail how educational costs are backed out. Finally, we compute the average subsidy rate between 2002 and 2010 and we obtain a value of 12.5%.

For the increase in PhD slots, we compute the percentage increase in PhD enrollment from our data. PhD enrollment doubled from 2002-2012; however, the increase was not sudden, but rather gradual over the ten years. Thus, we feed a corresponding gradual increase in slots to our model, as displayed in figure E1.

We then feed the estimated time-series of policy rates to our model and we solve for the equilibrium of the economy along the transitional dynamics since the introduction of the subsidy. Note that, in order to solve for the transitional dynamics, we need to know the value of subsidy rates in every time period

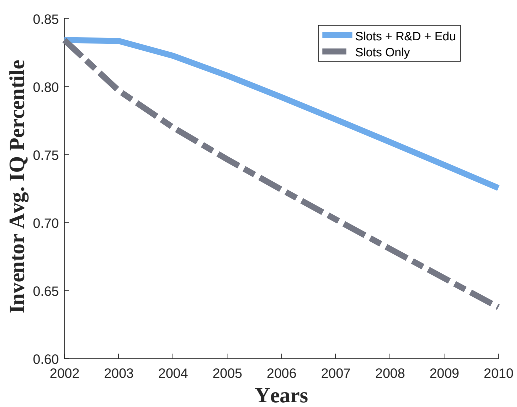
Figure E1: Increase in PhD slots for Non-Target Moment Exercise from the Data



until the economy reaches the new steady state. We assume that, for the time periods after our data ends, the government continues to operate the R&D and education subsidy at a rate equal to the average rate that we observe between 2002-2010 and that the slots are maintained at the new level observed in 2011.

The 2002 government policy intervention discussed in Section 4.4 used the three policies from this paper simultaneously. Figure 18 illustrated how our model replicates fairly well the decline in average IQ of the pool of individuals who enrolled in a PhD even though it did not directly target this decline. Here, we use the transitional dynamics to decompose the contribution of each policy to the decline in the IQs of enrolling PhDs. We use the same policy rates estimated from the data as for Figure 18, and we compute the counterfactual change in average IQ of PhD students under different choices of policy mix, as displayed in Figure E2.

Figure E2: Policy Intervention: Decomposition of the Contribution of each policy

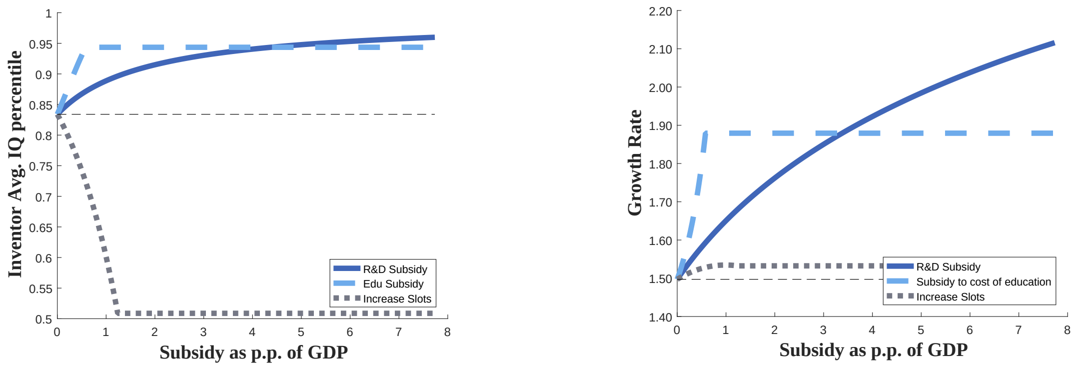


By 2007, the combination of R&D subsidies, education subsidies, and the increases in PhD slots chosen by the government led to a decline in IQ of PhD students from the 83rd to the 78th percentile of the IQ distribution . If the government had only increased the PhD slots, our model predicts IQ would have declined much more strongly, all the way to the 70th percentile of the distribution. Thus, the government intervention expanded the pool of talent in the economy while mitigating the decline in average talent by using a mix of complementary policies that target different frictions to the allocation of talent. The fact that there was a significant decline in average IQ, however, shows that the economy already had well-allocated individuals to research prior to the policy intervention.

E.2 Steady State Results in terms of p.p of GDP

In Figure E3, we display the steady state impact of the introduction of policies by expressing the magnitude of policies in terms of p.p. of GDP. In Appendix C.4, we provide details on the computation of the revenue equivalence across policies. We observe that, as explained in Section 5.1, for smaller budgets, a subsidy

Figure E3: Comparing subsidies at different budget levels.



(a) inventor quality and subsidy

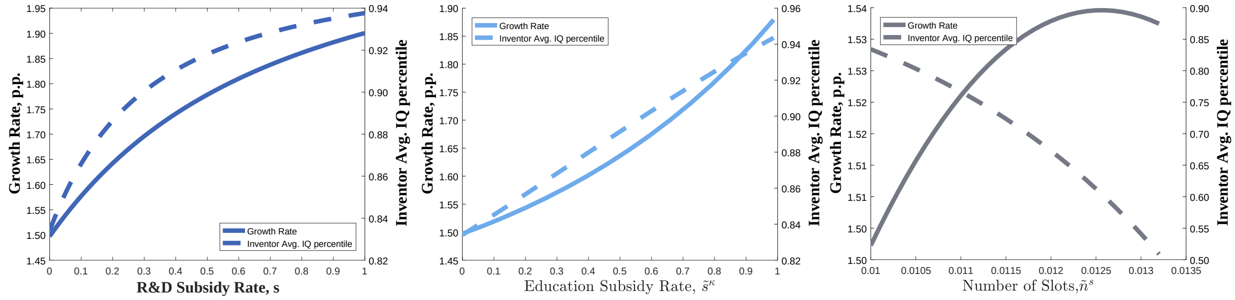
(b) growth and subsidy

to the cost of education is the most effective in stimulating growth in steady state. However, for large enough budgets, the effectiveness of education subsidies levels off. This happens when the education subsidy has effectively enabled all individuals to afford the cost of education: subsidizing education any further is ineffective because there are no more financially constrained individuals. The effectiveness of expanding slots also levels off for large enough budgets. This happens when the slot expansion is so large that there are more open slots in the economy than individuals who are financially unconstrained and have a taste for research and thus willing to fill them. Finally, the effectiveness of R&D does not level off, but it is reduced when subsidies are so large to convince anyone in the economy to prefer research over the production sector. Even at that point, R&D subsidies are still somewhat effective, because they can stimulate the purchase of lab equipment.

Beyond comparing the subsidies overall, here we present the effect of subsidies when computed as a subsidy rate. Figure E4 shows the response of growth rate (primary axis) and inventor quality (secondary axis) for different policies.

In line with the predictions from the theoretical analysis of the policies in Section 3.2, we find that increasing education subsidy and R&D subsidy rates and increasing the number of slots leads to an increase in the growth rate. However, the effect of different policies on the composition of the inventor pool varies. Both the subsidy to the cost of education and the R&D subsidy result in higher quality of average inventors, but they operate through different channels: the former affects talented but financially constrained individuals, while the latter affects talented individuals with a distaste for research. Increasing the number of slots results in an expansion of the number of researchers, but at the cost of a declining average talent of the research pool.

Figure E4: Steady State Policy Comparisons



(a) R&D Subsidies

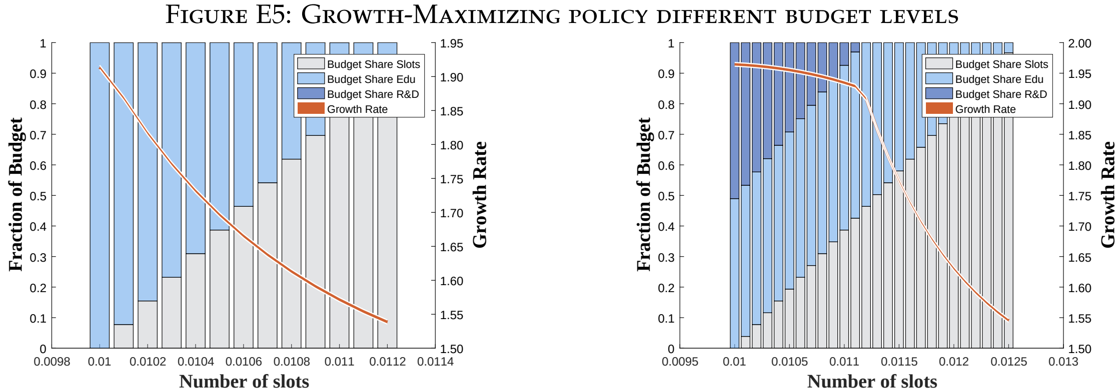
(b) Educational Subsidies

(c) University Slots

E.3 Growth-maximizing policy

Section 5.2 discussed the growth-maximizing policy mix for a budget of 2.5% of GDP. Here, we discuss the growth-maximizing policy mix for smaller budgets. Figure 19 shows that for small budgets, when slots are fixed at the baseline level of 0.01, it is optimal to spend the entire budget on subsidies to the cost of education. This result still holds when we allow for the number of slots to be increased as well. In Figure E5a we show the growth-maximizing policy mix at a budget of 0.5% of GDP. Growth is maximized when spending the entire budget on subsidies to education. For intermediate budget levels (larger than 0.5% but smaller than approximately 2% of GDP), it is optimal to mix subsidies to the cost of education and to R&D, but still none of the budget is allocated to expanding PhD slots. For example, Figure E5b shows the growth-maximizing policy mix at a budget of 1% of GDP. Growth is maximized when splitting the budget equally between subsidies to the cost of education and to R&D. Finally, for budgets larger than 2% of GDP, it is growth maximizing to use all three policies instruments (expansion of PhD slots, subsidy to education and R&D), as exemplified in Figure 20 .

Figure E5: Growth-Maximizing policy different budget levels



(a) Budget = 0.5 pp GDP

(b) Budget = 1 pp GDP

E.4 Robustness: Calibration Parameters Estimates and Targeted Moments Fit

In this section, we present the estimated parameters and target moments for the robustness exercises outlined in Section 6.

E.4.1 Alternative Estimates of Parental Income Including Maternal Income

Here we use the sum of paternal and maternal income as a measure of parental income. This delivers a higher assortative matching parameter. The recalibration can be seen in Tables E2 and E3.

Table E2: Moments

Moment	Data	Model
PhD Share of the Labor Force (N)	0.01	0.01
Mean Percentile IQ of PhDs (Fact 1+4)	0.83	0.85
Corr(Parent Income, Child IQ) (μ, Facts 2+3)	0.21	0.22
Mean/SD of Parent Income (Facts 2+3)	1.20	1.22
Mean Team Size (Fact 7)	2.25	2.23
Variance of Team Size (Fact 7)	3.13	3.12
Peak Year Innovation Post PhD (Fact 8)	16	16
Pr Innovation at Peak (Fact 8)	0.01	0.01
PhD Graduate/Student Income	1.66	1.66
Skill Premium of PhDs	0.75	0.64
Profits to Wages	0.07	0.02
Growth rate	0.015	0.015

Table E3: Parameter Values

Parameter	Description	Value
	— Panel A. External Calibration —	
ρ	Discount rate	0.030
r	Interest rate	0.030
β	Labor share output	0.106
	— Panel B. Internal Calibration —	
N	PhD share of the labor force	0.010
λ	Rate at which researcher becomes a team leader	0.036
δ	Death rate	0.044
η_1	Skilled labor share idea production	0.511
η_2	Worker share idea production	0.379
η_3	Lab equipment share idea production	0.110
θ	Pareto shape	2.633
ϕ	Innovation rate	0.046
ψ	Cost of raising capital	0.011
ℓ	Labor efficiency in production	0.024
E	Preference shock parameter	3.299
μ	Fraction assortative match IQ - parental income	0.15
ζ	Income/IQ ratio for assortatively matched	7.534

Notes: All parameters are estimated jointly.

E.4.2 Non-PhD Innovation

In this section we introduce an exogenous ideas arrival rate, ξ, which accounts for innovation coming from individuals without higher education. To calibrate the additional parameter, we add one target moment, which is the share of patents applications from PhDs, which is 23% in our data. The resulting moments and calibrated parameters are displayed in Tables E4 and E5.

Table E4: Moments

Moment	Data	Model
PhD Share of the Labor Force (N)	0.01	0.01
Mean Percentile IQ of PhDs (Fact 1+4)	0.83	0.83
Corr(Parent Income, Child IQ) (μ, Facts 2+3)	0.18	0.19
Mean/SD of Parent Income (Facts 2+3)	1.20	1.23
PhD Patent Share (Fact 5)	0.23	0.23
Mean Team Size (Fact 7)	2.47	2.50
Variance of Team Size (Fact 7)	3.62	3.60
Peak Year Innovation Post PhD (Fact 8)	16	16
Pr Innovation at Peak (Fact 8)	0.01	0.01
PhD Graduate/Student Income	1.66	1.69
Skill Premium of PhDs	0.75	0.74
Profits to Wages	0.10	0.11
Growth rate	0.015	0.015

Table E5: Parameter Values

Parameter	Description	Value
	— Panel A. External Calibration —	
ρ	Discount rate	0.030
r	Interest rate	0.030
β	Labor share output	0.106
	— Panel B. Internal Calibration —	
N	PhD share of the labor force	0.010
λ	Rate at which researcher becomes a team leader	0.031
δ	Death rate	0.047
η_1	Skilled labor share idea production	0.459
η_2	Worker share idea production	0.409
η_3	Lab equipment share idea production	0.132
θ	Pareto shape	2.561
ϕ	Innovation rate	0.053
ψ	Cost of raising capital	0.086
ℓ	Labor efficiency in production	0.107
E	Preference shock parameter	2.863
μ	Fraction assortative match IQ - parental income	0.111
ζ	Income/IQ ratio for assortatively matched	0.016
ξ	Exogenous idea arrival	0.007

Notes: All parameters are estimated jointly.

E.4.3 Non-PhD Innovation Responds to Policy

Here we recalibrate the model where the exogenous stock of ideas responds to a change in the talent pool. The idea here is that when individuals are pulled away from non-inventing occupations, they may reduce their "inventiveness" in those occupations. Thus, the overall targeted growth rate takes this into account in Tables E6 and E7.

Table E6: Moments

Moment	Data	Model
PhD Share of the Labor Force (N)	0.01	0.01
Mean Percentile IQ of PhDs (Fact 1+4)	0.83	0.83
Corr(Parent Income, Child IQ) (μ, Facts 2+3)	0.18	0.19
Mean/SD of Parent Income (Facts 2+3)	1.20	1.23
PhD Patent Share (Fact 5)	0.23	0.23
Mean Team Size (Fact 7)	2.47	2.50
Variance of Team Size (Fact 7)	3.62	3.60
Peak Year Innovation Post PhD (Fact 8)	16	16
Pr Innovation at Peak (Fact 8)	0.01	0.01
PhD Graduate/Student Income	1.66	1.69
Skill Premium of PhDs	0.75	0.74
Profits to Wages	0.10	0.11
Growth rate	0.015	0.015

Table E7: Parameter Values

Parameter	Description	Value
	— Panel A. External Calibration —	
ρ	Discount rate	0.030
r	Interest rate	0.030
β	Labor share output	0.106
	— Panel B. Internal Calibration —	
N	PhD share of the labor force	0.010
λ	Rate at which researcher becomes a team leader	0.031
δ	Death rate	0.047
η_1	Skilled labor share idea production	0.459
η_2	Worker share idea production	0.409
η_3	Lab equipment share idea production	0.132
θ	Pareto shape	2.561
ϕ	Innovation rate	0.053
ψ	Cost of raising capital	0.086
ℓ	Labor efficiency in production	0.107
E	Preference shock parameter	2.863
μ	Fraction assortative match IQ - parental income	0.111
ζ	Income/IQ ratio for assortatively matched	0.016
ξ	Exogenous idea arrival	0.0072

Notes: All parameters are estimated jointly.