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# Measuring Racial Discrimination in Algorithms

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## Abstract

There is growing concern that the rise of algorithmic decision-making can lead to discrimination against legally protected groups, but measuring such algorithmic discrimination is often hampered by a fundamental selection challenge. We develop new quasi-experimental tools to overcome this challenge and measure algorithmic discrimination in the setting of pretrial bail decisions. We first show that the selection challenge reduces to the challenge of measuring four moments: the mean latent qualification of white and Black individuals and the race-specific covariance between qualification and the algorithm’s treatment recommendation. We then show how these four moments can be estimated by extrapolating quasi-experimental variation across as-good-as-randomly assigned decision-makers. Estimates from New York City show that a sophisticated machine learning algorithm discriminates against Black defendants, even though defendant race and ethnicity are not included in the training data. The algorithm recommends releasing white defendants before trial at an 8 percentage point (11 percent) higher rate than Black defendants with identical potential for pretrial misconduct, with this unwarranted disparity explaining 77 percent of the observed racial disparity in algorithmic recommendations. We find a similar level of algorithmic discrimination with regression-based recommendations, using a model inspired by a widely used pretrial risk assessment tool.

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# 1 Introduction

Algorithms guide an increasingly large number of high-stakes decisions, including criminal risk assessment (Chohlas-Wood, 2020), resume screening (Raghavan and Barocas, 2019), and medical testing (Price II, 2019). Alongside this rise of algorithmic decision-making is a concern that it can entrench or worsen discrimination against legally protected groups (Angwin et al., 2016). This concern has fueled a rich theoretical literature in computer science, where algorithmic discrimination is formalized as the differential treatment of equally qualified individuals (Zafar et al., 2017; Berk et al., 2018). With algorithmic recommendations for pretrial release decisions, for example, a risk assessment tool may be racially discriminatory if it recommends white defendants be released before trial at a higher rate than Black defendants with equal risk of pretrial misconduct.

Bringing the theory of algorithmic discrimination to data, however, is often hampered by a fundamental selection challenge. Data on an individual’s latent qualification for treatment may only be available for a group of individuals who were endogenously selected for treatment by an existing human or algorithmic decision-maker. In the pretrial setting, this challenge arises because pretrial misconduct potential is only observed among the defendants who a judge chooses to release before trial (Kleinberg et al., 2018; Lakkaraju et al., 2017). Such selection can both introduce bias in algorithmic predictions and complicate the measurement of algorithmic discrimination, since unobserved qualification cannot be conditioned on to compare white and Black treatment.

This paper develops new tools to measure racial discrimination in algorithmic predictions by extending methods previously developed in Arnold, Dobbie and Hull (2020). We first show how the fundamental selection problem can be solved by estimating four race-specific parameters: the average qualification rates of white and Black defendants and the race-specific covariances of qualification and algorithmic recommendations. In Arnold, Dobbie and Hull (2020) we show how the first set of mean risk moments can be used to measure racial discrimination in individual judge decisions. Here we extend this logic by showing how the additional race-specific covariances identify racial discrimination in hypothetical algorithmic release recommendations.

We next show how the four key moments can be estimated by extrapolating reduced-form variation across quasi-randomly assigned bail judges. In Arnold, Dobbie and Hull (2020) we use extrapolations of the judge-specific misconduct rates for released white and Black defendants to estimate the mean risk parameters. Here we use similar extrapolations of judge-specific second moments, of misconduct and algorithmic recommendations for released white and Black defendants, to estimate the race-specific covariances. We show how both sets of extrapolations can be conducted flexibly, without

specifying a model of judge decision-making.

We illustrate this approach to measuring algorithmic discrimination in New York City (NYC), home to one of the largest pretrial systems in the country. We find that a sophisticated machine learning algorithm, which does not train directly on defendant race or ethnicity, recommends the release of white defendants at a significantly higher rate than Black defendants with identical pretrial misconduct potential. When calibrated to the average NYC release rate of 73 percent, the algorithm recommends an 8 percentage point (11 percent) higher release rate for white defendants than equally qualified Black defendants. This unwarranted disparity explains 77 percent of the observed racial disparity in release recommendations, grows as the algorithm becomes more lenient, and is driven by discrimination among individuals who would engage in pretrial misconduct if released. We find a similar level of algorithmic discrimination with regression-based recommendations, using a model inspired by a widely used pretrial risk assessment tool.

This paper adds to a recent empirical literature that uses quasi-experimental variation to test for bias and discrimination in the criminal justice system. Arnold, Dobbie and Yang (2018) use the release tendencies of quasi-randomly assigned bail judges to test for racial bias in a conventional linear instrumental variables (IV) framework, while Marx (2018) uses a similar approach to test for racial bias at the margin of police stops. Arnold, Dobbie and Hull (2020) show how quasi-experimental judge assignment can be used to measure a more comprehensive measure of racial discrimination, which includes racial bias, statistical discrimination, and discrimination on seemingly non-race characteristics. Other recent work in this literature includes Rose (2020) and Feigenberg and Miller (2020).<sup>1</sup>

Our paper also adds to theoretical and empirical work on algorithmic fairness in both computer science and economics. We propose a measure of algorithmic discrimination that is closely related to the idea of “conditional procedure accuracy equality” or “equalized odds” in the computer science tradition (Zafar et al., 2017; Berk et al., 2018), and we show how our approach can be used to quantify alternative unfairness measures such as “equality of opportunity” (Hardt, Price and Srebro, 2016) and “sufficiency” (Zafar et al., 2017). An important empirical consideration in this literature is the “selective labels problem” (Kleinberg et al., 2018; Lakkaraju et al., 2017), which may induce racial bias in algorithmic predictions. We show how this problem, which may also confound the measurement of algorithmic discrimination, can be overcome in the pretrial bail context. Our paper also relates to a recent literature on how algorithmic recommendations interact with human decision-makers; examples

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<sup>1</sup>Rose (2020) shows that a policy reform that sharply reduced prison punishments for technical probation violations nearly eliminated the racial disparity in incarceration without significantly increasing the disparity in reoffending rates, suggesting that such violations are less informative predictions of risk for Black individuals on probation. Feigenberg and Miller (2020) show that Black motorists in Texas are stopped at higher rates than white motorists without any commensurate increase in contraband hit rates, suggesting that the racial disparity in search rates is inefficient.

from the pretrial context include Stevenson and Doleac (2019) and Albright (2019).

Methodologically, this paper adds to a recent literature on estimating average treatment effects (ATEs) with multiple discrete instruments (Kowalski, 2016; Brinch, Mogstad and Wiswall, 2017; Mogstad, Santos and Torgovitsky, 2018; Hull, 2020). The average misconduct risk parameters can be seen as race-specific ATEs, of pretrial release on pretrial misconduct, with a similar interpretation for the covariance parameters. Importantly, our approach to estimating these ATEs does not impose the usual assumption of first-stage monotonicity (Imbens and Angrist, 1994; Heckman and Vytlacil, 2005), which has received recent scrutiny both in general (Mogstad, Torgovitsky and Walters, 2019) and in the specific context of judge decision-making (Mueller-Smith, 2015; Frandsen, Lefgren and Leslie, 2019; Norris, 2019). Our approach is closely related to Hull (2020), who considers non-parametric extrapolations of quasi-experimental moments in the spirit of “identification at infinity” in conventional sample selection models (Chamberlain, 1986; Heckman, 1990; Andrews and Schafgans, 1998).

The remainder of the paper is organized as follows. Section 2 presents a general empirical framework for defining and measuring algorithmic discrimination. Section 3 summarizes our data on pretrial bail decisions and our algorithmic predictions of pretrial misconduct risk. Section 4 applies our framework and presents our findings. Section 5 concludes.

## 2 Empirical Framework

### 2.1 Setting

We consider a binary classification problem, in which a population of individuals  $i$  is differentiated by their race  $R_i \in \{w, b\}$  (either white or Black) and a latent variable  $Y_i^* \in \{0, 1\}$  which indicates their qualification for a binary treatment. In the pretrial context,  $Y_i^* = 1$  may indicate that defendant  $i$  would engage in pretrial misconduct (i.e., fail to appear in court or be rearrested for a new crime) if she were released before trial. In a medical testing context,  $Y_i^* = 1$  may indicate the latent disease state of patient  $i$ . The objective is to align decisions with qualification status: e.g., releasing defendants with a low risk of pretrial misconduct or subjecting patients with a high risk of disease to costly testing.<sup>2</sup>

We suppose that an algorithm attempts to predict individual qualification from some observables  $X_i$  and returns a treatment recommendation  $T_i \in \{0, 1\}$ . We leave the details of this algorithmic recommendation process unspecified, requiring only the observability of  $T_i$ . In the pretrial context, we may have  $T_i = \mathbf{1}[p(X_i) \leq \tau]$  where  $X_i$  is a set of observed defendant and case characteristics,  $p(X_i)$

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<sup>2</sup>While we consider binary  $Y_i^*$  here, our approach can be extended to multivalued or continuous qualification states; see Arnold, Dobbie and Hull (2020) for details.

is a statistical (not necessarily unbiased) prediction of pretrial misconduct potential, and  $\tau$  is a risk tolerance. Here  $T_i = 1$  indicates an algorithmic recommendation of pretrial release, with  $T_i = 0$  for defendants who are recommended to be detained before trial. We emphasize that the algorithm may or may not train directly on race; i.e.,  $X_i$  need not include  $R_i$ .

Building on Arnold, Dobbie and Hull (2020), we measure discrimination in the algorithmic recommendations  $T_i$  by the implied treatment disparity among equally-qualified white and Black individuals:

$$\Delta = E[E[T_i | R_i = w, Y_i^*] - E[T_i | R_i = b, Y_i^*]] \quad (1)$$

The inner difference in  $\Delta$  compares the average treatment recommendation for white and Black individuals, holding fixed their true qualification  $Y_i^*$ . The outer expectation in  $\Delta$  averages this comparison over the marginal qualification distribution. We say there is algorithmic discrimination against Black individuals when  $\Delta > 0$ , that there is algorithmic discrimination against white individuals when  $\Delta < 0$ , and that there is no white/Black algorithmic discrimination when  $\Delta = 0$ . In the pretrial context,  $\Delta > 0$  may mean the algorithm recommends white defendants be released at a higher rate than Black defendants with equal misconduct potential, on average.

Our definition of algorithmic discrimination relates to idea of “conditional procedure accuracy equality” or “equalized odds” in the computer science literature (Zafar et al., 2017; Berk et al., 2018). In the language of binary classification problems, this fairness condition imposes the equality of true- and false-negative rates across race.<sup>3</sup> Here  $\Delta$  is an weighted average of racial disparities in true-negative rates  $\delta_r^T = Pr(T_i = 1 | Y_i^* = 1, R_i = r)$  and false negative rates  $\delta_r^F = Pr(T_i = 1 | Y_i^* = 0, R_i = r)$ , where we interpret (as in the pretrial setting)  $Y_i^* = 1$  as an adverse state:

$$\Delta = (\delta_w^T - \delta_b^T)\bar{\mu} + (\delta_w^F - \delta_b^F)(1 - \bar{\mu}) \quad (2)$$

where the weight  $\bar{\mu} = E[Y_i^*]$  is given by the average qualification rate in the population.<sup>4</sup>

Our  $\Delta$  measure also aligns with the proposed definition of labor market discrimination in Aigner and Cain (1977), which compares the treatment of white and Black workers with the same objective level of productivity. We analogously compare the recommended release rates of white and Black defendants with the same objective potential for pretrial misconduct,  $Y_i^*$ . We show in Arnold, Dobbie

<sup>3</sup>An alternative measure of fairness, when  $T_i = \mathbf{1}[p(X_i) \leq \tau]$ , is the conditional independence of  $p(X_i)$  and  $R_i$  given  $Y_i^*$ : see, e.g., Agarwal, Dudik and Wu (2019). The approach we develop here can be extended to measure deviations from this condition, via similar extrapolations of reduced-form variation.

<sup>4</sup>Other notions of algorithmic fairness include the racial equality of false-negative rates only (Hardt, Price and Srebro, 2016) and the racial equality of positive and negative predictive values (Zafar et al., 2017). We show below how our framework can also be used to quantify these alternative measures; see Kleinberg, Mullainathan and Raghavan (2017) for a discussion of various inherent tradeoffs between them.

and Hull (2020) that parameters like  $\Delta$  capture a broad notion of discrimination arising from both accurate statistical discrimination (e.g., Aigner and Cain, 1977) and racially biased preferences or beliefs (e.g., Becker, 1957; Bordalo et al., 2016). We further show that  $\Delta \neq 0$  can arise either when release recommendations are directly based on race (i.e.,  $R_i$  is included in the algorithmic input  $X_i$ ) or because release decisions are based on observable characteristics that are correlated with race (i.e., variables correlated with  $R_i$  are included in the algorithm’s feature set  $X_i$ ).<sup>5</sup>

Estimating algorithmic discrimination  $\Delta$  is often challenging because individual qualification  $Y_i^*$  is often only selectively observed. Formally, we observe a censored outcome  $Y_i = D_i Y_i^*$ , where  $D_i \in \{0, 1\}$  indicates the treatment decision of an existing human or algorithmic decision-maker. In the context of bail decisions, for example, pretrial misconduct potential  $Y_i^*$  is only observed among defendants selected by a judge for release ( $D_i = 1$ ). Individuals who are detained before trial ( $D_i = 0$ ) cannot engage in pretrial misconduct and so  $Y_i = 0$ . In the medical testing setting, patients who are tested ( $D_i = 1$ ) have their disease state revealed but untested patients do not. This nonrandom selection can bias algorithmic predictions of  $Y_i^*$ , by causing  $p(X_i) = E[Y_i^* | X_i, D_i = 1]$  to diverge from accurate predictions  $E[Y_i^* | X_i]$ .

Kleinberg et al. (2018) refer to the endogenous observability of  $Y_i^*$  in such settings as the “selective labels problem.” They consider how algorithmic predictions of qualification can be compared to, and in some cases improve, human decision-making in light of this problem. We instead consider how selection complicates measurement of algorithmic discrimination and derive a new approach to overcome this challenge. Formally, nonrandom selection may cause a feasible measure of discrimination,

$$\Delta^S = E[E[T_i | R_i = w, Y_i^*, D_i = 1] - E[T_i | R_i = b, Y_i^*, D_i = 1] | D_i = 1] \quad (3)$$

to diverge from  $\Delta$ .<sup>6</sup> We next present our approach to this selection challenge.

## 2.2 Identification and Estimation

Our approach to estimating algorithmic discrimination proceeds in two steps. We first show that the challenge of selectively observed qualification reduces to a challenge of identifying four race-specific moments. These moments capture the average qualification rate for each race and how qualification

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<sup>5</sup>A finding of  $\Delta \neq 0$  may indicate unlawful discrimination in many settings. For example, Title VII of the 1964 Civil Rights Acts prohibits employment decisions that have a disparate impact by race. In many other contexts, including bail decisions, the Equal Protection Clause of the 14th Amendment prohibits the intentional unequal treatment of equally-qualified white and Black individuals (Yang and Dobbie, 2020).

<sup>6</sup>Coston et al. (2020) discuss conditions for fairness metrics like  $\Delta$  to be identified by selected metrics like  $\Delta^S$ . These conditions are typically strong and unlikely to hold in practice.

covaries with the algorithmic recommendations within race. We use the fact that the true- and false-negative rates which enter  $\Delta$  can be written:

$$\delta_r^T = \frac{E[T_i Y_i^* | R_i = r]}{E[Y_i^* | R_i = r]} = \frac{\rho_r}{\mu_r} \quad (4)$$

$$\text{and } \delta_r^F = \frac{E[T_i(1 - Y_i^*) | R_i = r]}{E[(1 - Y_i^*) | R_i = r]} = \frac{E[T_i | R_i = r] - \rho_r}{1 - \mu_r} \quad (5)$$

where  $\mu_r = E[Y_i^* | R_i = r]$  denotes the average qualification rate among race- $r$  individuals and  $\rho_r = E[T_i Y_i^* | R_i = r]$  captures the race-specific second moment of algorithmic recommendations and individual qualification. The weights in  $\Delta$  can further be written  $\bar{\mu} = \mu_w p_w + \mu_b p_b$  where  $p_r = Pr(R_i = r)$ . Since these racial shares and the race-specific average recommendation  $E[T_i | R_i = r]$  are identified, these expressions show that the missing information in  $\Delta$  are the four race-specific parameters  $\{\mu_w, \mu_b, \rho_w, \rho_b\}$ . Algorithmic discrimination can thus be measured by estimating these four parameters, without needing to measure individual qualification directly.

We next show how the key four moments (and thus  $\Delta$ ) can be estimated by extrapolating reduced-form variation across as-good-as-randomly assigned decision-makers, such as bail judges in the pretrial setting. Under random assignment, each judge  $j$  makes treatment decisions  $D_{ij}$  among a comparable group of individuals  $i$  of each race. We can therefore estimate a series of judge-specific misconduct rates among the defendants, of each race, that a judge releases before trial,  $\tilde{\mu}_{jR_i} \equiv E[Y_i | D_{ij} = 1, R_i] = E[Y_i^* | D_{ij} = 1, R_i]$ , as well as a series of judges' race-specific release rates  $\pi_{jR_i} \equiv Pr(D_{ij} = 1 | R_i)$ . We show in Arnold, Dobbie and Hull (2020) how estimates of these differentially-selected samples of race-specific average misconduct risk can be extrapolated towards judges with high release rates in order to estimate the average misconduct risk parameter  $E[Y_i^* | R_i] = \mu_{R_i}$ . Our insight here is that the same logic can be applied to estimate the second moments  $\rho_{R_i}$ . Instead of the released misconduct rates, we estimate and extrapolate, for each race, the judge-specific released second moments  $\tilde{\rho}_{jR_i} \equiv E[T_i Y_i | D_{ij} = 1, R_i] = E[T_i Y_i^* | D_{ij} = 1, R_i]$  towards judges with high release rates.<sup>7</sup>

To build intuition for our estimation approach, it is helpful to first consider a hypothetical ‘‘supremely lenient’’ bail judge  $j^*$  who releases nearly all defendants assigned to her of each race. This judge’s race-specific release rates are close to one, i.e.,  $\pi_{j^*R_i} \approx 1$ , so by quasi-random assignment her race-specific released first and second moments are both close to the unselected moments:  $\tilde{\mu}_{j^*R_i} \approx \mu_{R_i}$  and  $\tilde{\rho}_{j^*R_i} \approx \rho_{R_i}$ . The decisions of a supremely lenient and quasi-randomly assigned judge can therefore be used to estimate the four parameters that enter our discrimination measure.

<sup>7</sup>This second set of extrapolations is not needed to estimate discrimination in a judge’s own decisions, as in Arnold, Dobbie and Hull (2020), since if  $T_i = D_{ij}$  then  $\rho_{R_i} = E[D_{ij} Y_i^* | R_i] = E[Y_i | D_{ij} = 1, R_i] Pr(D_{ij} | R_i)$  is directly estimable for each judge  $j$ .

In the absence of a supremely lenient judge, these parameters can instead be extrapolated from the variation in  $\tilde{\mu}_{jR_i}$  and  $\tilde{\rho}_{jR_i}$  across quasi-randomly assigned judges  $j$  within race. This approach is analogous to a standard regression discontinuity design, in which average potential outcomes are extrapolated to a treatment cutoff from nearby observations.<sup>8</sup> Here, selected moments are extrapolated from quasi-randomly assigned judges to the release rate cutoff of one to estimate unselected moments. Estimates may, for example, come from the vertical intercept of linear, quadratic, or local linear regressions of the selected moment estimates on estimated release rates. Crucially, as discussed in Arnold, Dobbie and Hull (2020), such extrapolation can be conducted flexibly without assuming a model of judge decision-making or imposing the strong assumption of first-stage monotonicity often used with quasi-random judge assignment.

### 3 Data

We analyze algorithmic discrimination in the NYC pretrial system, one of the largest in the country. Bail conditions in NYC are set by a judge at an arraignment hearing, held shortly after an arrest. Bail hearings usually last a few minutes. The judge receives detailed information on the defendant’s current offense and criminal record and decides on one of several possible bail conditions. First, she can release defendants who show minimal risk on a promise to return for all court appearances, known broadly as release on recognizance (ROR). Second, she can require defendants to post some sort of bail to be released. The judge can also send higher-risk defendants to a supervised release program as an alternative to cash bail. Finally, she can detain defendants pending trial by denying bail altogether. Bail judges are granted considerable discretion in determining who should be released before trial, but they cannot discriminate against minorities and other protected classes. Judges may consider the risk that defendants will not appear for a required court appearance (a so-called failure to appear, or FTA) or that they will engage in new criminal activity if released.

Our analysis sample is drawn from the universe of NYC arraignments made between November 1, 2008 and November 1, 2013. We describe the construction of this sample in Arnold, Dobbie and Hull (2020), where we also give more detail on the institutional background. The sample consists of 595,186 cases involving 367,434 white or Black defendants. Each case is assigned to one of 268 judges, each of whom sees at least 100 cases. We drop cases where the defendant is not charged with a felony or misdemeanor and cases that were disposed at arraignment or adjourned in contemplation

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<sup>8</sup>Formally, this approach draws on recent advances in average treatment effect extrapolation with multiple discrete instruments (Brinch, Mogstad and Wiswall, 2017; Hull, 2020) and a classic literature on identification “at infinity” in sample selection models (Heckman, 1990; Andrews and Schafgans, 1998).

of dismissal, which are likely to be dismissed by virtually every judge.<sup>9</sup>

Table 1 summarizes the analysis sample, both overall and by race. Panel A shows that 73.0 percent of defendants are released before trial ( $D_i = 1$ ). The vast majority of pretrial releases are without conditions (ROR), with only 14.4 percent of defendants being released by posting an assigned bail amount. White defendants are more likely to be released than Black defendants (76.7 percent versus 69.5 percent release rate) but among released defendants, the distribution of release conditions is virtually identical. Panel B of Table 1 shows that Black defendants are 4.9 percentage points more likely to have been arrested for a new crime before trial in the past year compared to white defendants, as well as 3.0 percentage points more likely to have a prior FTA in the past year. Panel C further shows that Black defendants are 1.3 percentage points more likely to have been charged with a felony compared to white defendants, as well as 3.6 percentage points more likely to have been charged with a violent crime. Finally, Panel D shows that Black defendants who are released are 6.6 percentage points more likely to be rearrested or have an FTA than white defendants who are released (though the composition of such misconduct is similar). Importantly, and in contrast to the other statistics in Table 1, these rates of pretrial misconduct ( $Y_i^*$ ) are only measured among released defendants.

Our approach exploits the quasi-random assignment of bail judges in NYC. As detailed in Arnold, Dobbie and Hull (2020), NYC uses a rotation calendar system to assign judges to arraignment shifts in each of the five county courthouses in the city, generating quasi-random variation in bail judge assignment for defendants arrested at the same time and in the same place. Appendix Table A3 verifies the conditional randomness in assignment by regressing leave-one-out estimates of judge leniency on various defendant and case characteristics, controlling for court-by-time fixed effects. Most coefficients in this balance table are small and not statistically significantly different from zero, both overall and by defendant race, and joint  $F$ -tests fail to reject the null of quasi-random assignment.

Our approach further exploits first-stage variation in judge leniency. Appendix Table A4 verifies that differential judge assignment meaningfully affects the probability an individual is released before trial, by regressing  $D_i$  on leave-one-out estimates of judge leniency and court-by-time fixed effects. A one percentage point increase in the predicted leniency of an individual's judge leads to a 0.96 percentage point increase in the probability of release, with a somewhat smaller first-stage effect for white defendants and a somewhat larger effect for Black defendants.

Our baseline analysis measures racial discrimination in algorithmic release recommendations that are based on machine learning predictions of pretrial misconduct potential. The predictions come from

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<sup>9</sup>Appendix Table A1 compares the full sample of NYC bail cases to our estimation sample. Appendix Table A2 confirms that the quasi-random judge assignment variation we exploit in estimation is not systematically related to case disposal or dismissal. Both tables are taken from Arnold, Dobbie and Hull (2020).

a gradient boosted decision tree estimated in the sample of released defendants, following Kleinberg et al. (2018). The features  $X_i$  include a number of characteristics of the current offense and prior criminal history, but exclude certain demographic variables such as race, ethnicity, and gender. Appendix B.1 details our estimation of this model. The model yields algorithmic risk predictions  $p(X_i)$  for each defendant  $i$ . We use these predictions to form release recommendations by  $T_i = \mathbf{1}[p(X_i) < \tau]$  for different risk thresholds  $\tau$ . Our benchmark analysis sets  $\tau$  to equalize the recommended average release rate and the actual NYC release rate of 73 percent.

Appendix Figure A1 shows that the model reliably predicts pretrial misconduct potential in the sample of released defendants. We plot true misconduct risk against the risk predictions of the machine learning model across 1,000 equal-sized bins of predicted risk, along with a local linear curve of best fit. We next discuss how we evaluate racial discrimination in recommendations based on these predictions.

## 4 Results

### 4.1 Parameter Estimates

Figures 1 and 2 show our extrapolation-based estimation of the race-specific mean risk and second moment parameters,  $\mu_r$  and  $\rho_r$ , for the baseline algorithmic recommendations. The horizontal axis of both figures shows estimates of judge- and race-specific release rates  $\pi_{jr}$ , obtained from ordinary least squares (OLS) estimates of

$$D_i = \sum_j \alpha_j W_i Z_{ij} + \sum_j \phi_j Z_{ij} + X_i' \beta + \epsilon_i \quad (6)$$

where  $D_i$  indicates pretrial release of defendant  $i$ ,  $W_i = \mathbf{1}[R_i = w]$  indicates that defendant  $i$  is white, and the  $Z_{ij}$  indicate assignment of defendant  $i$  to judge  $j$ . The control vector  $X_i$  includes court-by-time fixed effects that control for the level of quasi-experimental bail judge assignment; we demean this vector in order to include all judge indicators. The vertical axis of Figure 1 shows the corresponding estimates of judge- and race-specific misconduct rates among released defendants  $\mu_{jr}$ , obtained from OLS estimates of

$$Y_i = \sum_j \delta_j W_i Z_{ij} + \sum_j \psi_j Z_{ij} + X_i' \gamma + u_i \quad (7)$$

among released ( $D_i = 1$ ) individuals. Finally, the vertical axis of Figure 2 shows the corresponding estimates of judge- and race-specific second moments among released defendants  $\rho_{jr}$ , obtained from

OLS estimates of

$$T_i Y_i = \sum_j \omega_j W_i Z_{ij} + \sum_j \varphi_j Z_{ij} + X_i' \theta + v_i \quad (8)$$

again among released individuals. These specifications leverage an auxiliary assumption of linear conditional expectations of  $D_{ij}$  and  $Y_i^*$  in order to tractably accommodate the conditional random assignment of bail judges in this setting.<sup>10</sup>

Parameter estimates come from the vertical intercept, at one, of each race-specific extrapolation of the quasi-experimental variation in these figures. We consider linear, quadratic, and local linear extrapolations and report the corresponding parameter estimates in Panels A and B of Table 2. Our mean risk estimates match those of Arnold, Dobbie and Hull (2020); in the most flexible local linear extrapolation we estimate mean risk as  $\mu_w = 0.346$  and  $\mu_b = 0.436$ , both with standard errors of 0.016.<sup>11</sup> These estimates suggest that white defendants in the population are on average 0.9 percentage points less likely to engage in pretrial misconduct. The corresponding local linear estimates of the second moments are more similar, at  $\rho_w = 0.226$  and  $\rho_b = 0.213$ , with standard errors of 0.012 and 0.017. We obtain broadly similar estimates with the linear and quadratic extrapolations. Released misconduct rates across judges trend upwards with their release rates with a relatively constant slope, while the relationship between released second moments and release rates is flatter.<sup>12</sup>

## 4.2 Algorithmic Discrimination

Panel C of Table 2 reports our estimates of algorithmic discrimination,  $\Delta$ , for the different parameter estimates in Panels A and B. We obtain the discrimination estimates by applying Equations (2), (4), and (5) to the first-step parameter estimates. Our most conservative estimate comes from the local linear extrapolation, which yields an estimate of  $\Delta = 0.079$  with a standard error of 0.07. The linear and quadratic extrapolations yield a slightly higher estimate of  $\Delta = 0.086$  and  $\Delta = 0.080$ , with standard errors of 0.003 and 0.011 respectively.

Figure 3 shows how our estimate of algorithmic discrimination varies with the risk threshold  $\tau$ , which controls the algorithm’s average release rate, and compares these estimates to the unadjusted

<sup>10</sup>If  $Z_i$  is independent of  $(T_i, Y_i^*, D_{i1}, \dots, D_{iJ}, R_i)$  given  $X_i$  and  $E[T_i Y_i^* | D_{ij} = 1, R_i = r, X_i] = \omega_{jr} + X_i' \theta$ , then  $E[T_i Y_i | R_i, Z_i, X_i, D_i = 1]$  is linear in  $(W_i Z_{i1}, \dots, W_i Z_{iJ}, Z_{i1}, \dots, Z_{iJ}, X_i')'$ , as in Equation (8). The same logic holds for Equations (6) and (7) under analogous linearity assumptions.

<sup>11</sup>We obtain standard errors by a bootstrapping procedure, in which first-step estimates from Figures 1 and 2 are redrawn according to their estimated asymptotic distribution. Standard errors in second-step parameters like  $\mu_r$ ,  $\rho_r$ , and  $\Delta$  are then given by the standard deviation the bootstrapped estimates. First-step asymptotics are robust to two-way clustering by the defendant and judge.

<sup>12</sup>Appendix Figure A2 shows that the local linear parameter estimates imply a stronger (more negative) covariance of true pretrial misconduct potential and algorithmic release recommendations. See Appendix B.3 for details.

racial disparity in release rates. The baseline estimate of 7.9, at the average release rate in NYC, is a large share (76.0 percent) of the unadjusted disparity (10.4 percentage points). The magnitude of algorithmic discrimination rises as release rates fall, remaining a roughly equal share of the unadjusted disparity. Only at thresholds that essentially release all defendants do we fail to find a statistically significant level of algorithmic discrimination.

### 4.3 Robustness and Extensions

Appendix Figures A3 and A4 show that our finding of significant discrimination in algorithmic bail decisions is not driven by the specific machine learning algorithm that predicts pretrial misconduct risk. We obtain similar estimates of the second moments  $\rho_w$  and  $\rho_b$ , and correspondingly similar (local linear) estimates of algorithmic discrimination  $\Delta$ , using simpler regression-based predictions of pretrial misconduct risk. These predictions, detailed in Appendix B.2, are inspired by a widely used pretrial risk assessment tool originally developed by the Laura and John Arnold Foundation. At the baseline release rate of 73 percent, we find a 6.7 percentage point disparity in the recommended release rates of white and Black defendants with the same potential for pretrial misconduct. This discriminatory disparity is again a large share (73.6 percent) of the unadjusted release rate disparity in algorithmic recommendations (9.1 percentage points), again increases as release rates fall, and is again statistically distinguishable from zero at all but the highest release rates.

Appendix Figure A5 shows how our baseline estimates of algorithmic discrimination compare with naive estimates computed on the selected sample of release defendants (i.e., estimates of Equation (3)). This comparison reveals the extent of bias due to selective labels. The selected  $\Delta^S$  estimates are of similar magnitude as our selection-corrected  $\Delta$  estimates, and similarly increase with the algorithm’s average leniency. At the baseline NYC release rate the selected estimate is lower by 1.2 percentage points, a difference just at the margin of conventional statistical significance levels. Thus, while in theory the selective labels problem can induce bias in observable measures of algorithmic discrimination, we find by computing  $\Delta$  in this setting that the scope for such bias is small.

Finally, Appendix Figures A6 and A7 compute alternative measures of discrimination in our baseline algorithm. We first estimate the racial disparities in true- and false-negative rates that Equation (2) show are averaged together in  $\Delta$ . Racial equality in false-negative rates can be seen as satisfying what is known in the computer science literature as “equality of opportunity” (Hardt, Price and Srebro, 2016), meaning that qualified white and Black defendants without pretrial misconduct potential are released at the same rate. Appendix Figure A6 shows a disparity in false-negative rates that is

large and roughly constant across different release rates, while the racial disparity in true-negative rates (i.e., the release rate differential among defendants without misconduct potential) is only statistically significantly different from zero at low release rates. These results suggest that a measure of “inequality of opportunity” (that  $\delta_w^T - \delta_b^T \neq 0$ ) could fail to detect overall racial discrimination (that  $\Delta \neq 0$ ) in this setting. In Appendix Figure A7 we consider departures from what is known in the computer science literature as “sufficiency” (Zafar et al., 2017), meaning the racial equality of positive and negative predictive values (see Appendix B.3 for details). We estimate a positive and relatively constant degree of “insufficiency” with our baseline parameter estimates, suggesting that this alternative measure of algorithmic unfairness and our baseline  $\Delta$  measure qualitatively agree.

## 5 Conclusion

Algorithmic discrimination is an increasingly widespread concern in many settings, but its measurement is often hampered by a fundamental selection challenge. We show that this challenge can be overcome by estimating four race-specific parameters involving algorithmic recommendations and an individual’s selectively observed qualification. We further show that these parameters can be estimated by extrapolating reduced-form variation across as-good-as-randomly assigned decision-makers. We illustrate this approach in the NYC pretrial setting, where we find large and pervasive discrimination in algorithmic release recommendations that do not directly use information on race.

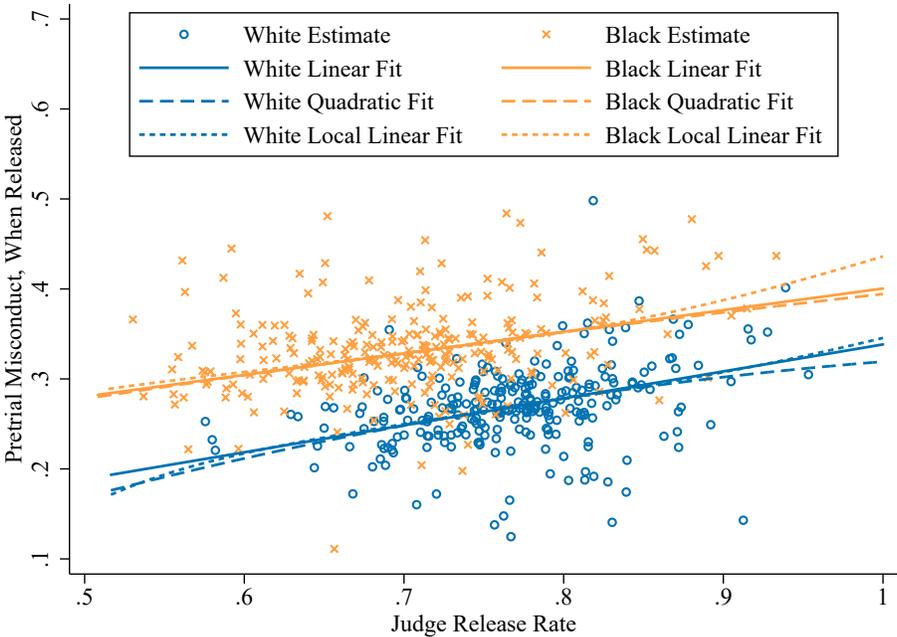
We conclude by noting that the methods we develop to study racial discrimination in algorithmic bail decisions may prove useful for measuring unfairness in several other high-stakes settings, both within and outside of the criminal justice system. One key requirement is the quasi-random assignment of decision-makers, such as judges, police officers, employers, government benefits examiners, or medical providers. A second requirement is that an individual’s qualification of treatment is measurable among a subset of individuals that the decision-maker endogenously selects. Mapping these settings to the quasi-experimental approach in this paper can overcome fundamental selection challenges and bring a large theoretical literature on algorithmic fairness to data.

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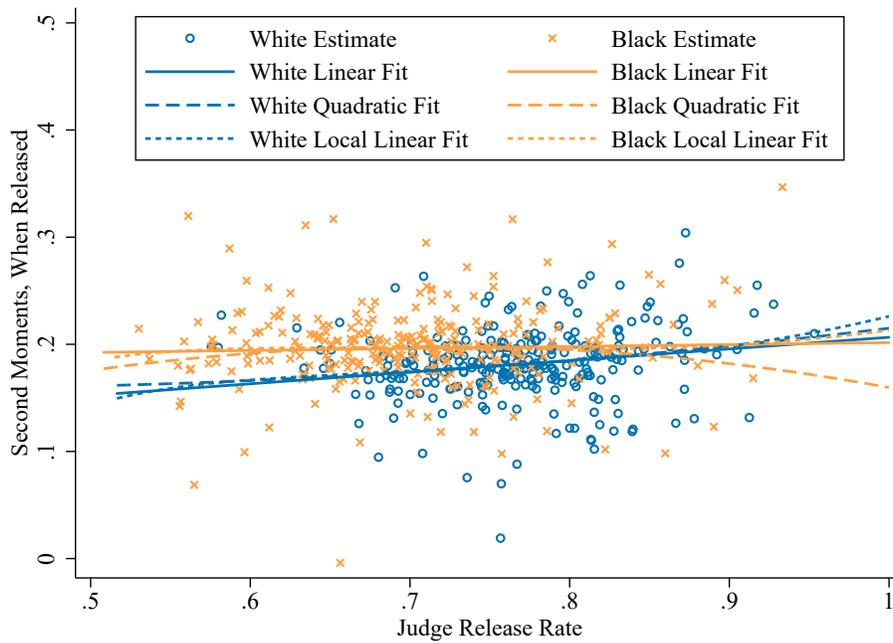
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Figure 1: Extrapolating Released Misconduct Rates Across Bail Judges



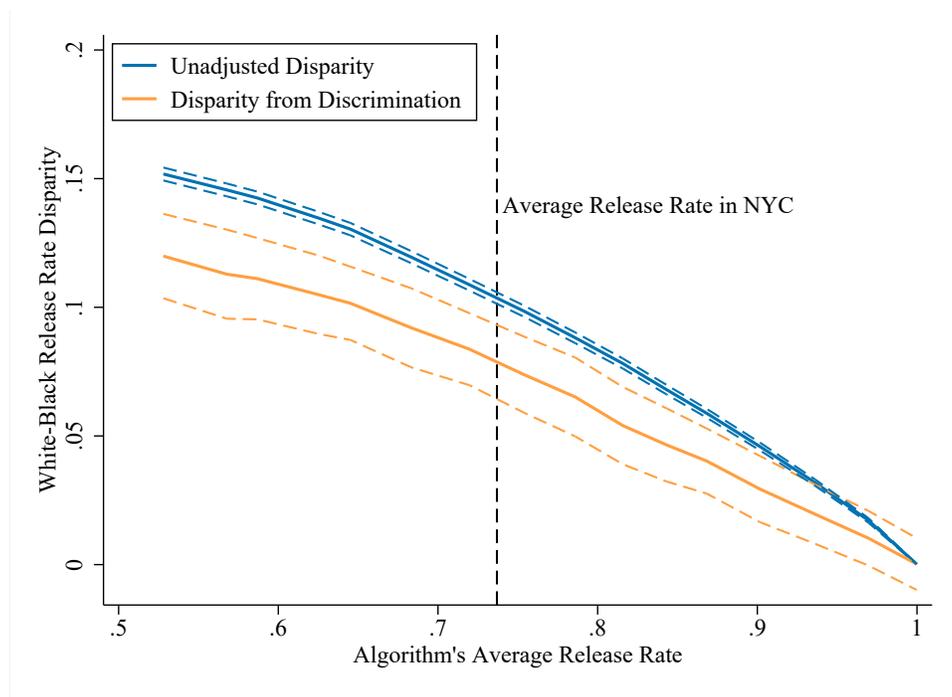
*Notes.* This figure plots race-specific release rates for the 268 judges in our sample against rates of pretrial misconduct among the set of released defendants. All estimates adjust for court-by-time fixed effects. The figure also plots race-specific linear, quadratic, and local linear curves of best fit, obtained from judge-level regressions that inverse-weight by the variance of the estimated misconduct rate among released defendants. The local linear regressions use a Gaussian kernel with a race-specific rule-of-thumb bandwidth.

Figure 2: Extrapolating Second Moments Across Bail Judges



*Notes.* This figure plots race-specific release rates for the 268 judges in our sample against the uncentered second moments of pretrial misconduct and algorithmic recommendations among the set of released defendants. Algorithmic recommendations are from our baseline gradient-boosted decision tree model with a risk threshold calibrated to equalize the average recommended release rate and the average release rate in NYC. All estimates adjust for court-by-time fixed effects. The figure also plots race-specific linear, quadratic, and local linear curves of best fit, obtained from judge-level regressions that inverse-weight by the variance of the estimated misconduct rate among released defendants. The local linear regressions use a Gaussian kernel with a race-specific rule-of-thumb bandwidth.

Figure 3: Discrimination in Algorithmic Bail Decisions



*Notes.* This figure plots the range of unadjusted racial disparities in algorithmic release rate recommendations, for different average release rates, along with the range of disparities due to racial discrimination. Algorithmic recommendations are from our baseline gradient-boosted decision tree model. Disparities from discrimination are computed as described in the text, using local linear estimates of the race-specific first and second moments. Dashed lines indicate pointwise 95 percent confidence intervals, computed by the bootstrapping procedure described in the text.

Table 1: Descriptive Statistics

	All Defendants	White Defendants	Black Defendants
	(1)	(2)	(3)
<i>Panel A: Pretrial Release</i>			
Released Before Trial	0.730	0.767	0.695
Share ROR	0.852	0.852	0.851
Share Money Bail	0.144	0.144	0.145
Share Other Bail Type	0.004	0.004	0.004
Share Remanded	0.000	0.000	0.000
<i>Panel B: Defendant Characteristics</i>			
White	0.478	1.000	0.000
Male	0.821	0.839	0.804
Age at Arrest	31.97	32.06	31.89
Prior Rearrest	0.229	0.204	0.253
Prior FTA	0.103	0.087	0.117
<i>Panel C: Charge Characteristics</i>			
Number of Charges	1.150	1.184	1.118
Felony Charge	0.362	0.355	0.368
Misdemeanor Charge	0.638	0.645	0.632
Any Drug Charge	0.256	0.257	0.256
Any DUI Charge	0.046	0.067	0.027
Any Violent Charge	0.143	0.124	0.160
Any Property Charge	0.136	0.127	0.144
<i>Panel D: Pretrial Misconduct, When Released</i>			
Pretrial Misconduct	0.299	0.266	0.332
Share Rearrest Only	0.499	0.498	0.499
Share FTA Only	0.281	0.296	0.269
Share Rearrest and FTA	0.220	0.205	0.232
Total Cases	595,186	284,598	310,588
Cases with Defendant Released	434,201	218,256	215,945

*Notes.* This table summarizes the NYC analysis sample. The sample consists of bail hearings that were quasi-randomly assigned to judges between November 1, 2008 and November 1, 2013, as described in the text. Information on demographics and criminal outcomes is derived from court records as described in the text. Pretrial release is defined as meeting the bail conditions set by the first assigned bail judge. ROR (released on recognizance) is defined as being released without any conditions. FTA (failure to appear) is defined as failing to appear at a mandated court date.

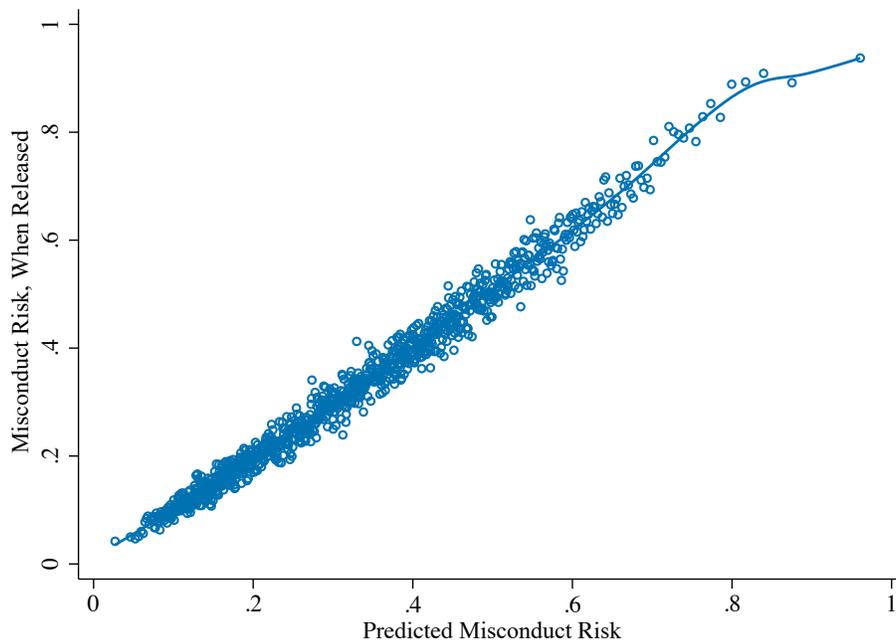
Table 2: Parameter and Discrimination Estimates

	Linear Extrapolation	Quadratic Extrapolation	Local Linear Extrapolation
	(1)	(2)	(3)
<i>Panel A: Mean Misconduct Risk</i>			
White Defendants	0.338 (0.007)	0.319 (0.022)	0.346 (0.016)
Black Defendants	0.400 (0.006)	0.394 (0.020)	0.436 (0.016)
<i>Panel B: Misconduct/Recommendation Second Moment</i>			
White Defendants	0.207 (0.006)	0.215 (0.019)	0.226 (0.012)
Black Defendants	0.202 (0.006)	0.160 (0.016)	0.213 (0.017)
<i>Panel C: Algorithmic Discrimination</i>			
Release Rate Disparity	0.086 (0.003)	0.080 (0.011)	0.079 (0.007)

*Notes.* Panels A and B of this table summarize estimates of race-specific mean risk and second moments of misconduct potential and the algorithmic release recommendation from different extrapolations of the variation in Figures 1 and 2. Panel C reports corresponding estimates of algorithmic discrimination, as defined in the text. Column 1 uses a linear extrapolation of the variation, while column 2 uses a quadratic extrapolation and column 3 uses a local linear extrapolation with a Gaussian kernel and a rule-of-thumb bandwidth. Robust standard errors, two-way clustered at the individual and judge level, are obtained by a bootstrapping procedure and appear in parentheses.

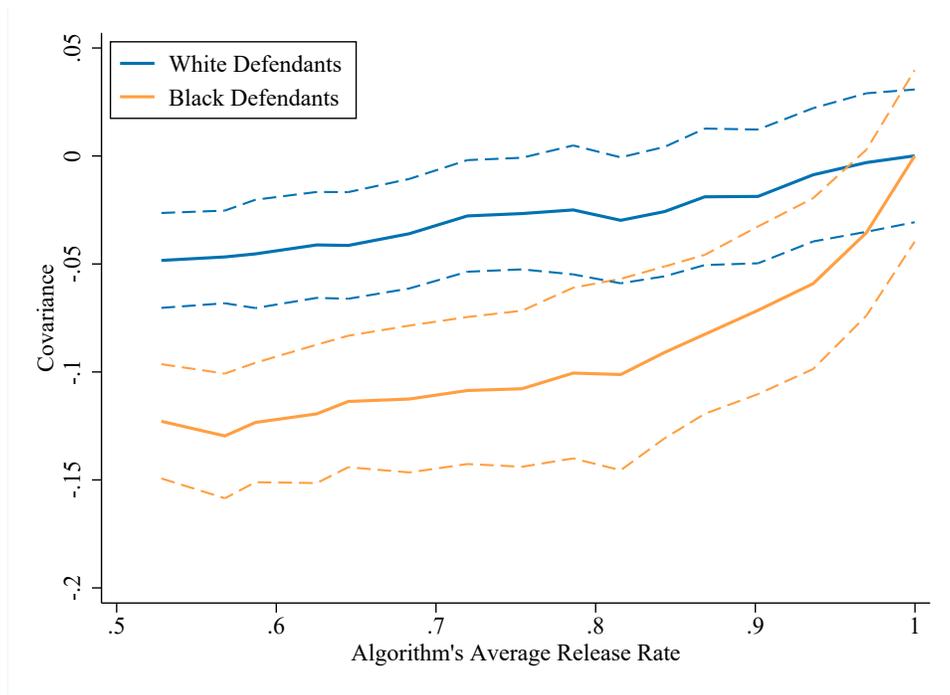
## A Appendix Figures and Tables

Appendix Figure A1: Algorithmic Predictions of Pretrial Misconduct Risk



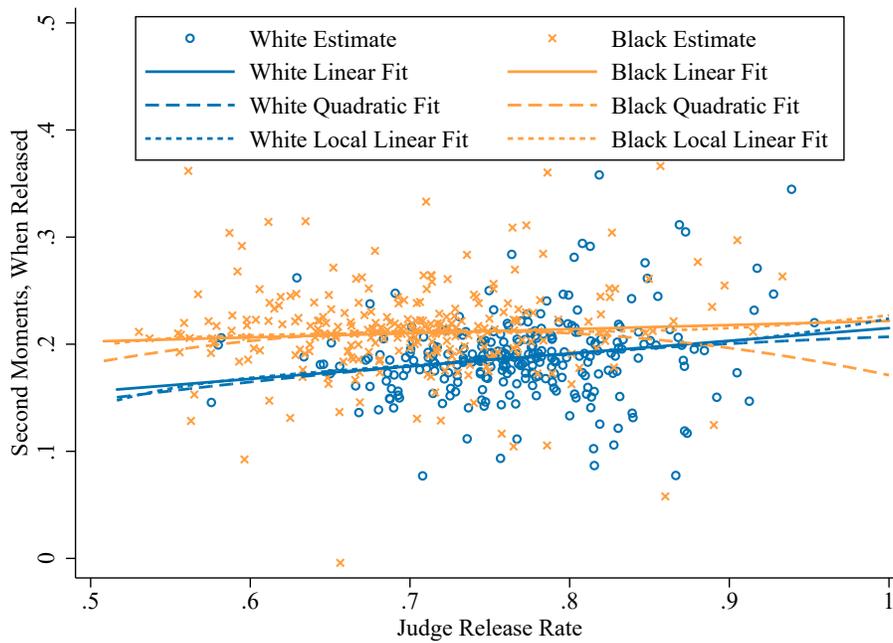
*Notes.* This figure plots true pretrial misconduct risk against the risk predictions of our baseline gradient-boosted decision tree algorithm among the set of released defendants in our sample. True risk is computed within 1,000 equal-sized bins of predicted risk. The curve of best fit comes from a local linear regression with a Gaussian kernel and rule-of-thumb bandwidth.

Appendix Figure A2: Covariance of Pretrial Misconduct and Algorithmic Release Recommendations



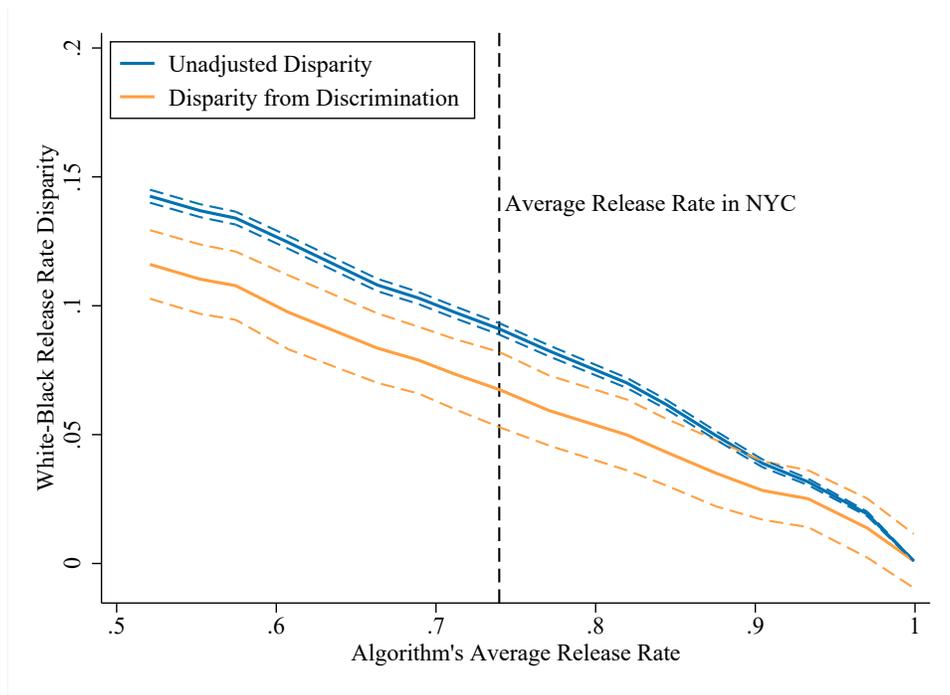
*Notes.* This figure plots the range of race-specific covariance between pretrial misconduct potential and algorithmic release recommendations for different average release rates. Algorithmic recommendations are from our baseline gradient-boosted decision tree model. Covariances are computed by using local linear estimates of the race-specific first and second moments. Dashed lines indicate pointwise 95 percent confidence intervals, computed by the bootstrapping procedure described in the text.

Appendix Figure A3: Extrapolating Regression-Based Second Moments Across Bail Judges



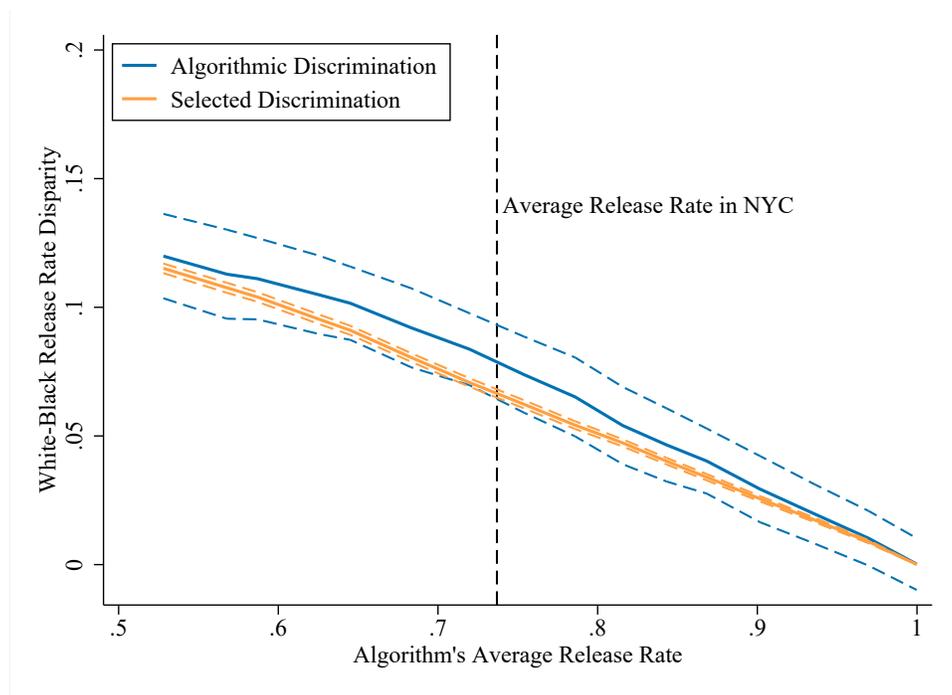
*Notes.* This figure plots race-specific release rates for the 268 judges in our sample against the uncentered second moments of pretrial misconduct and algorithmic recommendations among the set of released defendants. Algorithmic recommendations are from the regression model described in the text with a risk threshold calibrated to equalize the average recommended release rate with the average release rate in NYC. All estimates adjust for court-by-time fixed effects. The figure also plots race-specific linear, quadratic, and local linear curves of best fit, obtained from judge-level regressions that inverse-weight by the variance of the estimated misconduct rate among released defendants. The local linear regressions use a Gaussian kernel with a race-specific rule-of-thumb bandwidth.

Appendix Figure A4: Discrimination in Regression-Based Algorithmic Bail Decisions



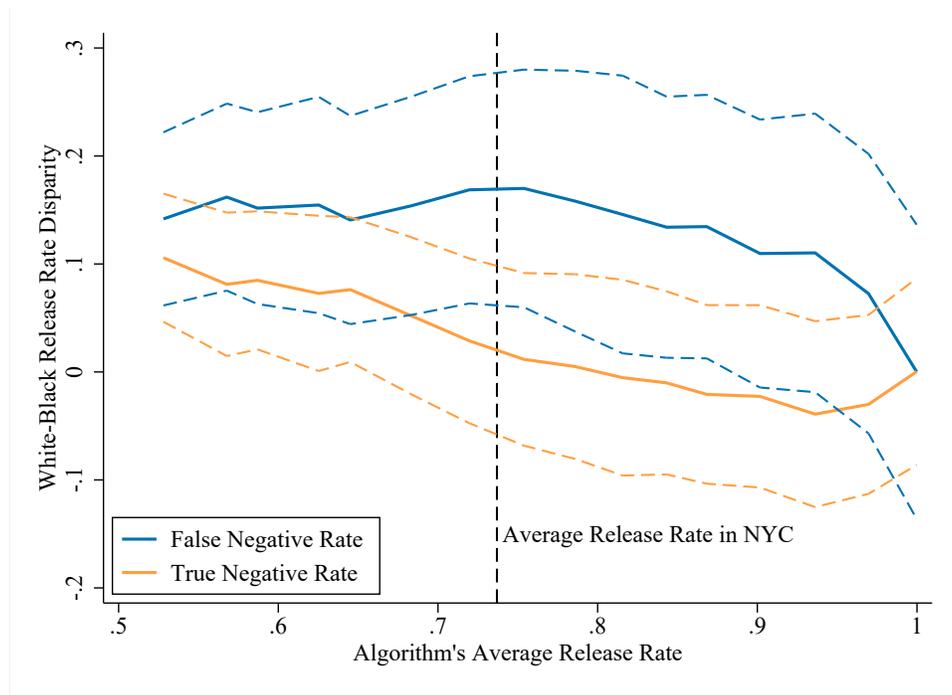
*Notes.* This figure plots the range of unadjusted racial disparities in algorithmic release rate recommendations, for different average release rates, along with the range of disparities due to racial discrimination. Algorithmic recommendations are from the regression model described in the text. Disparities from discrimination are computed as described in the text, using local linear estimates of the race-specific first and second moments. Dashed lines indicate pointwise 95 percent confidence intervals, computed by the bootstrapping procedure described in the text.

Appendix Figure A5: Quantifying the Bias from Selective Labels



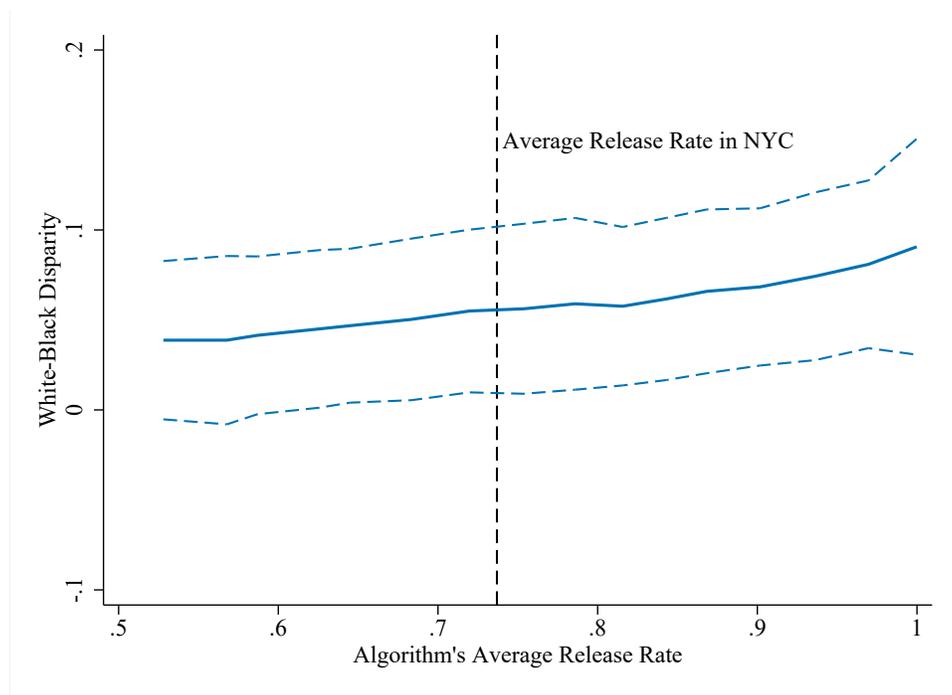
*Notes.* This figure plots our main discrimination estimates against a potentially biased measure of discrimination that is estimated on the subsample of defendants released before trial. Algorithmic recommendations are from our baseline gradient-boosted decision tree model. Disparities from discrimination are computed as described in the text, using local linear estimates of the race-specific first and second moments. Dashed lines indicate pointwise 95 percent confidence intervals, computed by the bootstrapping procedure described in the text.

Appendix Figure A6: Decomposition of Algorithmic Discrimination



*Notes.* This figure plots the range of racial disparities in true and false negative rates, for different average release rates, which make up the disparities due to racial discrimination. Algorithmic recommendations are from our baseline gradient-boosted decision tree model. Disparities are computed as described in the text, using local linear estimates of the race-specific first and second moments. Dashed lines indicate pointwise 95 percent confidence intervals, computed by the bootstrapping procedure described in the text.

Appendix Figure A7: Measuring Discrimination by Sufficiency of Algorithmic Recommendations



*Notes.* This figure plots the range of racial disparities in average positive and negative predictive values, or the sufficiency of algorithmic release rate recommendations, for different average release rates. Algorithmic recommendations are from our baseline gradient-boosted decision tree model. Disparities are computed as described in the text, using local linear estimates of the race-specific first and second moments. Dashed lines indicate pointwise 95 percent confidence intervals, computed by the bootstrapping procedure described in the text.

Appendix Table A1: Descriptive Statistics by Sample

	All Defendants		White Defendants		Black Defendants	
	Full	Estimation	Full	Estimation	Full	Estimation
	Sample	Sample	Sample	Sample	Sample	Sample
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Panel A: Pretrial Release</i>						
Released Before Trial	0.852	0.730	0.872	0.767	0.832	0.695
Share ROR	0.601	0.852	0.616	0.852	0.586	0.851
Share Disposed	0.301	0.000	0.274	0.000	0.327	0.000
Share Adjourned	0.191	0.000	0.199	0.000	0.183	0.000
Share Money Bail	0.068	0.144	0.070	0.144	0.066	0.145
Share Other Bail Type	0.332	0.004	0.314	0.004	0.348	0.004
Share Remanded	0.000	0.000	0.000	0.000	0.000	0.000
<i>Panel B: Defendant Characteristics</i>						
White	0.483	0.478	1.000	1.000	0.000	0.000
Male	0.822	0.821	0.831	0.839	0.813	0.804
Age at Arrest	31.819	31.969	31.540	32.055	32.080	31.890
Prior Rearrest	0.192	0.229	0.168	0.204	0.214	0.253
Prior FTA	0.085	0.103	0.071	0.087	0.099	0.117
<i>Panel C: Charge Characteristics</i>						
Number of Charges	1.094	1.150	1.111	1.184	1.078	1.118
Felony Charge	0.184	0.362	0.181	0.355	0.188	0.368
Misdemeanor Charge	0.816	0.638	0.819	0.645	0.812	0.632
Any Drug Charge	0.347	0.256	0.342	0.257	0.352	0.256
Any DUI Charge	0.031	0.046	0.046	0.067	0.017	0.027
Any Violent Charge	0.072	0.143	0.062	0.124	0.081	0.160
Any Property Charge	0.217	0.136	0.209	0.127	0.226	0.144
Cases	1,358,278	595,186	656,711	284,598	701,567	310,588

*Notes.* This table summarizes the difference between the NYC analysis sample and the full sample of NYC arraignments. The full sample consists of all bail hearings between November 1, 2008 and November 1, 2013. The analysis sample consists of bail hearings that were quasi-randomly assigned to judges between November 1, 2008 and November 1, 2013, as described in the text. Information on demographics and criminal outcomes is derived from court records as described in the text. Pretrial release is defined as meeting the bail conditions set by the first assigned bail judge. ROR (released on Recognizance) is defined as being released without any conditions. FTA (failure to appear) is defined as failing to appear at a mandated court date.

Appendix Table A2: Judge Leniency and Sample Attrition

	All Defendants	White Defendants	Black Defendants
	(1)	(2)	(3)
Dropped from Sample	0.00007 (0.00012)	0.00003 (0.00013)	0.00012 (0.00014)
Court x Time FE	Yes	Yes	Yes
Mean Sample Attrition	0.416	0.409	0.424
Cases	1,425,652	726,284	697,597

*Notes.* This table reports OLS estimates of regressions of judge leniency on an indicator for leaving the sample due to case adjournment or case disposal and court-by-time fixed effects. The regressions are estimated on the sample of all arraignments made in NYC between November 1, 2008 and November 1, 2013. Judge leniency is estimated using data from other cases assigned to a given bail judge. Robust standard errors, two-way clustered at the individual and the judge level, are reported in parentheses.

Appendix Table A3: Tests of Quasi-Random Judge Assignment

	All Defendants (1)	White Defendants (2)	Black Defendants (3)
White	0.00013 (0.00009)		
Male	0.00003 (0.00014)	0.00003 (0.00019)	0.00004 (0.00018)
Age at Arrest	-0.00011 (0.00004)	-0.00015 (0.00006)	-0.00008 (0.00005)
Prior Rearrest	-0.00021 (0.00011)	0.00006 (0.00018)	-0.00042 (0.00015)
Prior FTA	0.00016 (0.00016)	-0.00011 (0.00024)	0.00036 (0.00023)
Number of Charges	-0.00001 (0.00001)	-0.00001 (0.00001)	-0.00001 (0.00003)
Felony Charge	0.00025 (0.00020)	0.00011 (0.00023)	0.00039 (0.00025)
Any Drug Charge	-0.00022 (0.00016)	-0.00017 (0.00021)	-0.00027 (0.00018)
Any DUI Charge	0.00045 (0.00027)	0.00051 (0.00032)	0.00008 (0.00045)
Any Violent Charge	-0.00008 (0.00023)	-0.00023 (0.00033)	0.00001 (0.00025)
Any Property Charge	-0.00033 (0.00018)	-0.00028 (0.00019)	-0.00036 (0.00027)
Joint p-value	[0.10689]	[0.29792]	[0.10136]
Court x Time FE	Yes	Yes	Yes
Cases	595,186	284,598	310,588

*Notes.* This table reports OLS estimates of regressions of judge leniency on defendant characteristics. The regressions are estimated on the sample described in Table 1. Judge leniency is estimated using data from other cases assigned to a given bail judge. All regressions control for court-by-time fixed effects. The p-values reported at the bottom of each column are from F-tests of the joint significance of the variables listed in the rows. Robust standard errors, two-way clustered at the individual and the judge level, are reported in parentheses.

Appendix Table A4: First Stage Effects of Judge Leniency

	All Defendants	White Defendants	Black Defendants
	(1)	(2)	(3)
Judge Leniency	0.960 (0.025)	0.788 (0.029)	1.104 (0.033)
Court x Time FE	Yes	Yes	Yes
Mean Release Rate	0.730	0.767	0.695
Cases	595,186	284,598	310,588

*Notes.* This table reports OLS estimates of regressions of an indicator for pretrial release on judge leniency. The regressions are estimated on the sample described in Table 1. Judge leniency is estimated using data from other cases assigned to a bail judge. All regressions control for court-by-time fixed effects. Robust standard errors, two-way clustered at the individual and the judge level, are reported in parentheses.

## B Econometric Appendix

### B.1 Algorithm Estimation Details

This appendix details our baseline algorithmic predictions of pretrial misconduct risk. We use a gradient boosted decision tree model, based on the model Kleinberg et al. (2018) develop for the NYC pretrial system. We use the same feature set  $X_i$ , which includes a total of 38 variables summarizing prior criminal history, charge characteristics, and demographic variables such as the age of the defendant. The outcome variable  $Y_i$  is an indicator for pretrial misconduct, defined as either a failure to appear or being arrested for a new crime.

Gradient boosting is an ensemble method that aggregates a number of “weak learners” in an iterative fashion. Here, the weak learners are decision trees, which divide the data through a sequence of binary splits based on the feature set. The algorithm averages multiple decision trees built sequentially on the data, with subsequent iterations up-weighting the observations predicted most poorly by the preceding sequence of trees. The complexity of the gradient boosting algorithm depends on the “depth” of each tree and a “shrinkage” parameter which governs how trees are averaged together.

Following Kleinberg et al. (2018), we choose the model hyperparameters by k-fold cross-validation with five folds. We first select a random 80 percent sample of released defendants, which we take as the training dataset. Applying the cross-validation procedure to this dataset yields an optimal tree depth of 4 and shrinkage parameter of 0.05. We then use the full training set and the remaining 20 percent of released defendants (the test dataset) to fit the gradient boosted decision tree model with these hyperparameters. Finally, we apply the model to the full sample (including defendants detained before trial) to compute risk predictions  $p(X_i)$ . We use the complete sample to estimate algorithmic discrimination for consistency with Arnold, Dobbie and Hull (2020) and to maximize precision.

### B.2 Alternative Regression-Based Risk Predictions

This appendix measures algorithmic discrimination in a simpler regression-based prediction of pretrial misconduct risk, inspired by the Laura and John Arnold Foundation Public Safety Assessment tool (LJAF PSA). The LJAF PSA is used in a number of states and cities to assist bail judges in making pretrial release decisions. LJAF PSA scores are based on nine defendant and case observables: the defendant’s age; an indicator for a violent crime charge; an indicator for a pending charge at the time of offense; indicators for a prior misdemeanor, felony, or violent crime conviction; the number of previous failures to appear over the last two years; an indicator for a failure to appear more than two

years ago; and an indicator for prior incarceration.

We construct ordinary least squares risk predictions  $\hat{Y}_i$  by regressing, in the sample of released defendants, an indicator for pretrial misconduct (either a failure to appear or being rearrested for a new crime) on a set of observed characteristics based on the LJAF PSA inputs. For most characteristics, we are able to match the inputs exactly. We do not observe whether a defendant has a pending charge, however, so we exclude this input. We also do not observe prior incarceration, so we instead use an indicator for prior arrest. As with the main algorithmic predictions, we use these  $\hat{Y}_i$  and a range of risk thresholds  $\tau$  to form release recommendations  $T_i = \mathbf{1}[\hat{Y}_i < \tau]$  for all defendants in the sample.

Appendix Figure A3 shows our extrapolation-based estimation of the key race-specific second moments  $\rho_w$  and  $\rho_b$  when using the regression-based prediction of pretrial misconduct. Appendix Figure A4 plots the corresponding range of estimated measures of algorithmic discrimination for the regression-based prediction of pretrial misconduct. As with our baseline gradient boosted decision tree algorithm, the regression-based algorithmic recommendations yield similar second-moment estimates for white and Black defendants (of around 0.2) at the average release rate in NYC (73 percent). These estimates and the common mean risk estimates yield a 6.7 percentage point disparity in the recommended release rates of white and Black defendants with the same potential for pretrial misconduct. This discriminatory disparity is a large share (73.6 percent) of the unadjusted release rate disparity in algorithmic recommendations (9.1 percentage points), and a similar share as with our baseline gradient boosted decision tree algorithm. We again find algorithmic discrimination over a wide range of potential release rates, with the estimated  $\Delta$  statistically distinguishable from zero at all but the highest release rates.

### B.3 Alternative Discrimination Measures

This appendix shows how our estimates of race-specific parameters  $\{\mu_w, \mu_b, \rho_w, \rho_b\}$  can be used to construct alternative measures of algorithmic discrimination in the NYC pretrial setting. We first estimate race-specific covariances of misconduct potential  $Y_i^*$  and algorithmic release recommendations  $T_i$ . We then estimate racial disparities in true- and false-negative rates,  $\delta_r^T$  and  $\delta_r^F$ , which enter our average discrimination measure  $\Delta$ . Racial equality in false-negative rates can be seen as satisfying what is known in the computer science literature as “equality of opportunity” (Hardt, Price and Srebro, 2016), meaning that “qualified” white and Black defendants without pretrial misconduct potential are released at the same rate. We also show that our estimates can be used to detect departures from what is known in the computer science literature as “sufficiency” (Zafar et al., 2017), and what Kleinberg,

Mullainathan and Raghavan (2017) refer to as “calibration,” meaning the racial equality of positive and negative predictive values.

Appendix Figure A2 first plots our estimates of race-specific covariances of misconduct potential and algorithmic release recommendations across a range of release rates. These estimates are obtained by  $Cov(Y_i^*, T_i | R_i) = \rho_{R_i} - \mu_{R_i} \times E[T_i | R_i]$ . We tend to find a stronger (more negative) covariance for Black defendants than white defendants. This results from the fact that we estimate a the higher mean risk  $\mu_{R_i}$  for Black defendants than for white defendants, while we tend to obtain similar estimates of the second moment  $\rho_{R_i}$  and somewhat higher release rates  $E[T_i | R_i]$  for white defendants.

Appendix Figure A6 next plots our estimates of racial disparities in true- and false-negative rates. These estimates are obtained by the formulas for  $\delta_r^T$  and  $\delta_r^F$  in the main text. We find a disparity in false-negative rates that is large and roughly constant across different release rates, where white defendants with misconduct potential tend to be released at a higher rate than Black defendants with misconduct potential. In contrast, the racial disparity in true-negative rates (i.e., the release rate differential among defendants without misconduct potential) is only statistically significantly different from zero at low release rates. These results suggest that a measure of “inequality of opportunity” (that  $\delta_w^T - \delta_b^T \neq 0$ ) could fail to detect overall racial discrimination (that  $\Delta \neq 0$ ) in this setting.

Finally, Appendix Figure A7 plots estimates of algorithmic “insufficiency.” Paralleling our main discrimination measure  $\Delta$ , we define insufficiency as:

$$\Sigma = E[E[1 - Y_i^* | R_i = w, T_i] - E[1 - Y_i^* | R_i = b, T_i]]$$

Here, the inner difference compares the non-misconduct rate for white and Black defendants, holding fixed the algorithmic recommendation  $T_i$ . The outer expectation averages this comparison over the recommendation distribution. A finding of  $\Sigma > 0$  indicates that white individuals tend to be less risky than Black defendants with identical algorithmic recommendations. As with  $\Delta$ , this measure can be decomposed as:

$$\Sigma = (\sigma_w^R - \sigma_b^R)E[T_i] + (\sigma_w^D - \sigma_b^D)(1 - E[T_i])$$

where  $\sigma_r^R = E[1 - Y_i^* | R_i = r, T_i = 1]$  is the non-misconduct rate among released individuals of race  $r$  and  $\sigma_r^D = E[1 - Y_i^* | R_i = r, T_i = 0]$  is the non-misconduct rate among detained individuals of race  $r$ . Here,  $\sigma_r^R$  can be interpreted as the negative predictive value of race  $r$  and  $1 - \sigma_r^D$  can be interpreted as the positive predictive value of race  $r$ , such that  $\Sigma$  captures racial disparities in these values. To

estimate  $\Sigma$ , we use this decomposition and the fact that:

$$\begin{aligned}\sigma_r^+ &= 1 - \frac{E[Y_i^* T_i | R_i = r]}{E[T_i | R_i = r]} = 1 - \frac{\rho_r}{E[T_i | R_i = r]} \\ \sigma_r^- &= 1 - \frac{E[Y_i^* (1 - T_i) | R_i = r]}{E[(1 - T_i) | R_i = r]} = 1 - \frac{\mu_r - \rho_r}{1 - E[T_i | R_i = r]}\end{aligned}$$

We find a generally positive  $\Sigma$  across a wide range of algorithmic release rates when we use our estimates of first- and second-moments as inputs to these formulas. White defendants tend to have lower pretrial misconduct rates than black defendants conditional on the algorithm's release recommendation. This result suggests that the insufficiency measure and our algorithmic discrimination measure  $\Delta$  qualitatively agree in this setting.