The Flight to Safety and International Risk Sharing

Rohan Kekre and Moritz Lenel

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Rohan Kekre†      Moritz Lenel‡

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Abstract

We study a business cycle model of the international monetary system featuring a time-varying demand for safe dollar bonds, greater risk-bearing capacity in the U.S. than the rest of the world, and nominal rigidities. A flight to safety generates a dollar appreciation and decline in global output. Dollar bonds thus command a negative risk premium and the U.S. holds a levered portfolio of capital financed in dollars. We quantify the effects of safety shocks and heterogeneity in risk-bearing capacity for global macroeconomic volatility; U.S. external adjustment; and policy transmission, as of dollar swap lines.

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†Chicago Booth and NBER. Email: rohan.kekre@chicagobooth.edu.
‡Princeton University and NBER. Email: lenel@princeton.edu.
1 Introduction

The U.S. sits at the center of the international monetary system. At business cycle frequencies, there are two defining features of this role. The first concerns its currency. Relative to bonds denominated in the currencies of equally high-income countries, dollar bonds pay well when equities pay poorly, and have low expected returns when output has been declining. These imply that dollar bonds are a hedge whose value rises in bad times. The second concerns the U.S. international investment position. The U.S. is positively exposed to equities and negatively exposed to the dollar exchange rate. As such, it serves as the “world’s insurer” and transfers wealth to the rest of the world in bad times.

Despite substantial advances, the literature lacks a model of the international monetary system which can jointly capture these cyclical patterns and study their implications. One strand of the literature has emphasized the safety and liquidity value of U.S. Treasuries. While these features can rationalize patterns in currency markets, this literature has not yet traced out the implications for global business cycles, risk sharing, or risk premia. Another strand of the literature has argued that the U.S. has a greater capacity to bear risk than the rest of the world. This can explain patterns in U.S. net foreign assets, but has counterfactual asset pricing implications: given consumption home bias, the dollar should depreciate in bad times.

In this paper, we propose a business cycle model of the international monetary system which bridges these two perspectives. Our model features a time-varying demand for safe dollar bonds, greater risk-bearing capacity in the U.S. than the rest of the world, and nominal rigidities. A flight to safe dollar bonds — which we formalize as an increase in their non-pecuniary value — generates a stronger dollar and a decline in global output. Dollar bonds are thus an endogenous hedge and the U.S. finances a levered portfolio of capital in dollars. We discipline the time-varying demand for safe dollar bonds to match spreads in financial markets, and differences in risk tolerance across countries to match the sensitivity of U.S. net foreign assets to excess equity returns. The model generates untargeted comovements between relative bond returns, equity returns, output, and U.S. net foreign assets quantitatively in line with the data. We then trace out its macroeconomic and policy implications. Absent the time-varying demand for safe dollar bonds, global output would be roughly 15% less volatile, particularly so in the U.S. Absent the U.S.’ greater capacity to bear risk,
its net foreign assets would be only as volatile as net exports, but net exports would in
turn bear a greater burden in external adjustment and the U.S. would no longer earn
positive average returns on its external position. Both the flight to safety and greater
U.S. risk-bearing capacity played important roles in the Great Recession. Finally, the
creation of safe dollar liquidity, such as via the dollar swap lines employed by central
banks in recent crises, is globally stimulative but revalues wealth in the U.S.’ favor.

We study a workhorse open-economy New Keynesian environment extended to fea-
ture a non-pecuniary value of dollar bonds and heterogeneity in risk aversion. Agents
consume subject to home bias and supply labor domestically subject to adjustment
costs in nominal wages. They trade safe dollar bonds, other dollar bonds, foreign
bonds, and capital which can be deployed in either country. We associate safe dollar
bonds with Treasury bills and other money-like assets which are valued for their liq-
uidity or safety beyond their pecuniary return. The equilibrium non-pecuniary value
— described in the literature as a “convenience yield” — reflects both the latent de-
mand for these securities as well as their supply. We treat demand as a driving force
and term the associated shocks safety shocks. The model features three other sets of
shocks: to global productivity (including a rare disaster), to the disaster probability,
and to relative productivity across countries. We study unexpected shocks to the
supply of safe dollar bonds at the end of the paper.

Safety shocks and heterogeneity in risk aversion together generate a distinctive
pattern of comovements between excess foreign bond returns, equity returns, output,
and wealth in the global economy. A positive safety shock implies that the expected
return on all assets must rise relative to safe dollar bonds to keep agents indifferent
across assets. Absent nominal rigidity, this is achieved by deflation in the U.S. and
a decline in its real interest rate. With nominal rigidity and U.S. monetary policy
which does not lower nominal interest rates sufficiently in response, this instead is
achieved by a decline in global consumption and investment as well as immediate
dollar appreciation. The goods market and foreign exchange market responses are
linked by a larger fall in U.S. output than output abroad, appreciating the U.S. terms
of trade. As dollar bonds thus pay well in endogenously “bad” times, they earn a
negative risk premium versus foreign bonds, and relatively risk tolerant agents insure
the risk averse against such a shock. If agents in the U.S. are more risk tolerant than
those abroad, this implies that U.S. net foreign assets fall on impact of the shock. In
the periods which follow, the dollar depreciates, excess foreign bond and equity returns
are high, global output recovers, and U.S. net foreign assets improve. Consistent with the “reserve currency paradox” elucidated by Maggioredi (2017), productivity and disaster risk shocks are unable to deliver these comovements.

We calibrate the model to match observed portfolios and second moments in asset prices and real quantities. We use the yield spread between U.S. Treasuries and G10 government bonds swapped into dollars constructed by Du, Im, and Schreger (2018a) as a direct measure of safety shocks, up to its volatility; if swapped foreign government bonds are also partially valued for their liquidity or safety, the volatility of their yield difference versus Treasury bills will underestimate the volatility of safety shocks. We thus calibrate the volatility of safety shocks to match the observed (negative) risk premium on dollar bonds. We calibrate the volatility of global and relative productivity shocks to target volatilities in aggregate consumption and output. We calibrate the stochastic properties of disaster risk shocks to match the disaster risk series estimated by Barro and Liao (2021). The risk tolerance of Foreign is set to match the global equity premium. The risk tolerance of Home is set to match the positive exposure of U.S. net foreign assets to excess equity returns.

The model generates untargeted comovements quantitatively in line with the data. We focus on comovements involving excess foreign bond returns and the U.S. net foreign asset position which speak directly to the role of the dollar and U.S. economy in the international monetary system. As in the data, our model implies that (i) the year-over-year decline in U.S. output forecasts high future excess foreign bond returns; (ii) high global equity returns are accompanied by high excess foreign bond returns; and (iii) an increase in U.S. net foreign assets is accompanied by high excess foreign bond returns. Safety shocks are crucial for all of these, while greater risk-bearing capacity in the U.S. is crucial for the third.

We then use the model to quantify the roles of safety shocks and heterogeneity in risk-bearing capacity for global macroeconomic volatility and U.S. external adjustment. Safety shocks account for more than 25% of output volatility in the U.S. and 5% of output volatility in the rest of the world. Heterogeneity in risk-bearing capacity accounts for essentially all of the positive average return on the U.S. external position and the excess volatility of U.S. net foreign assets relative to net exports. While the U.S. external position would thus be less volatile if it did not serve as the world’s insurer, the share of innovations to net foreign assets rebalanced by future net exports would rise as valuation effects would no longer stabilize the U.S. external
position. These insights are obtained using simulations of the model’s driving forces over long time periods. We also feed in the observed sequence of safety and disaster risk innovations estimated by Du et al. (2018a) and Barro and Liao (2021) during the Great Recession. Together with the calibrated differences in risk tolerance across countries, these shocks alone generate a cumulative decline in U.S. output by 1.3%, Foreign output by 1.5%, and U.S. net foreign assets relative to output by 8.6% from the end of Q3 2007 through Q3 2009, versus 4.8%, 5.1%, and 10.0% in the data.

We finally use the model to trace out the transmission of shocks to the supply of safe dollar assets, as via dollar swap lines. An increased supply reduces the convenience yield like a negative safety shock. We simulate the Federal Reserve’s announcements to expand the availability and frequency of its swap line operations on March 19 and 20, 2020. The model generates a 100bp dollar depreciation and increase in the global equity return of 135bp on impact, comparable to the estimates obtained by Kekre and Lenel (2023) around the swap line announcements in the data. We then use the model to quantify the implications for real activity and wealth. The model implies an increase in U.S. output of 80bp, foreign output of 20bp, and U.S. net foreign assets relative to output of 440bp. We conclude that in recent crises, dollar swap lines have played a meaningful stabilization role and relaxed the U.S. external budget constraint by mitigating the flight to safety.

**Related literature**  Our model sits between and integrates two literatures. Our focus on the time-varying demand for safe dollar bonds builds on the rapidly growing literature studying convenience yields and safe assets (Engel (2016), Engel and Wu (2022), Jiang, Krishnamurthy, and Lustig (2021, 2022a, b), and Valchev (2020)).\(^1\) Relative to this literature, our contribution is to embed the convenience yield in a workhorse open economy New Keynesian model to trace out the implications for output, risk sharing, and risk premia. By accounting for greater risk-bearing capacity in the U.S. than rest of the world, we also build on a large literature studying international risk sharing in such an environment (Chien and Naknoi (2015), Dou and Verdelhan (2015), Gourinchas, Rey, and Govillot (2017), Maggiori (2017), and Sauzet (2023)). Relative to this literature, our model accommodates a time-varying demand for safe dollar bonds, production, and nominal rigidity, which together provide a

\(^1\)See DiTella (2020), Drechsler, Savov, and Schnabl (2017), Greenwood, Hanson, and Stein (2015), He, Krishnamurthy, and Milbradt (2019), Krishnamurthy and Vissing-Jorgensen (2012), and Nagel (2016) for related analyses of convenience yields and the dollar as safe asset.
resolution to the “reserve currency paradox” which has challenged this literature.

Our account of the cyclical properties of the dollar and the U.S. external balance sheet is distinct from others in the literature. Gourinchas et al. (2017) propose an increase in foreign risk aversion in times with elevated uncertainty, driving a higher demand for insurance from the U.S. Maggiori (2017) proposes an increase in trade costs which shifts demand to U.S. goods in crises. We instead emphasize an increase in the demand for safe dollar assets, disciplined by convenience yields and with distinct implications for policy, such as dollar swap lines. Jiang et al. (2022b) emphasize that the seignorage revenues earned by the U.S. upon a flight to safety drive an increase in U.S. wealth, such that there may be no paradox in accounting for a dollar appreciation after all. Our model also features this channel, but we relax the assumptions on risk neutrality and binding financial constraints made in their analysis and we provide a quantitative evaluation. We find that the seignorage gains of the U.S. upon a flight to safety are more than offset by losses on capital and foreign bonds. The dollar appreciation remains consistent with a fall in relative U.S. wealth because the flight to safety induces a larger reduction in the supply of U.S. goods. In Lucas tree environments, Dahlquist, Heyerdahl-Larsen, Pavlova, and Penasse (2023) and Sauzet (2023) also associate crises with a fall in the relative supply of U.S. goods, generating a dollar appreciation. Our model identifies a shock which endogenously has this feature in a production economy with nominal rigidities, the flight to safe dollar assets.\textsuperscript{2}

The effects of safety shocks in our model build most directly on Caballero and Farhi (2018) and Caballero, Farhi, and Gourinchas (2021). These authors demonstrate that an increase in the demand for safe assets reduces output and appreciates the exchange rate of safe asset issuers in the presence of nominal rigidities and a binding zero lower bound. We build on their work by demonstrating that these insights apply under conventional Taylor rules even if the zero lower bound is not binding; characterizing the ex-ante currency risk premia and international portfolios which arise in response; and quantifying the implications of the time-varying demand for safe dollar bonds for international business cycles, asset prices, and portfolios.

The effects of safety shocks contrast with the effects of other asset demand shocks in the literature which are “disconnected” from aggregates. Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021, 2023) study shocks to the demand for specific

\textsuperscript{2}In our analytical and quantitative results, we also speak to the empirical analyses in Dahlquist et al. (2023) and Sauzet (2023) concerning the dynamics of the U.S. wealth share over 2008-2009.
currencies that have to be intermediated by a subset of risk-averse agents, resulting in fluctuations in the exchange rate which may have little relationship to macroeconomic quantities. Our analysis differs from these papers in three ways. First, safety shocks pertain to the demand for a particular type of asset within a given a currency, as reflected in a time-varying convenience yield, whereas the “UIP shocks” studied in these papers reflect the demand for all assets of a given currency, as reflected in a time-varying currency risk premium. Second, safety shocks affect not only agents’ portfolio choice between bonds of different currencies, but also their portfolio choice between bonds and capital and their intertemporal decisions between consumption and saving. Third, we study an environment in which all agents can trade the same assets, whereas the aforementioned papers emphasize segmented financial markets in which only a small measure of agents trade the menu of assets. The latter two features combine to imply that safety shocks affect not only exchange rates and other asset prices, but also macroeconomic quantities and country-level portfolios.

Outline In section 2 we outline the environment. In section 3 we characterize the main mechanisms analytically in a limiting case. In section 4 we calibrate the full model and in section 5 we study its impulse responses and untargeted comovements versus the data. Having validated the model, in section 6 we study its macroeconomic and policy implications. Finally, in section 7 we conclude.

2 Model

There are two countries, Home and Foreign, comprised of measure one and \( \zeta^* \) households, respectively. We use asterisks to denote variables chosen by or endowed to Foreign households. For brevity, we focus on the optimization problems and policy at Home and only summarize the analogs in Foreign; a complete description is in appendix A. Since we will calibrate the model so that Home captures the U.S., we

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4In this sense, our analysis relates to the emerging literature on preferred habitats and exchange rates (Gourinchas, Ray, and Vayanos (2022) and Greenwood, Hanson, Stein, and Sunderam (2022)), which studies how shocks to the demand for specific assets (maturities) within a given currency transmits to exchange rates. The focus of these papers, however, remains on the quantity of risk borne by a subset of arbitrageurs in segmented markets. In our case, it is on the transmission of such asset demand shocks in general equilibrium in the presence of nominal rigidity.
refer to Home’s nominal unit of account as the dollar.

We add two essential ingredients to a workhorse open economy New Keynesian model with sticky nominal wages and capital: cross-country heterogeneity in risk-bearing capacity and a time-varying convenience yield on dollar-denominated government bonds. We model these via differences in risk tolerance and bonds in utility so that we can focus on their implications and interactions in the simplest possible environment. We expect our insights would extend to richer models of differences in risk-bearing capacity and convenience yields.

In addition to these essential ingredients, we add several features to isolate mechanisms and improve the model’s quantitative fit. In particular, Epstein-Zin preferences disentangle risk aversion from intertemporal substitution. A rare disaster with time-varying probability generates meaningful variation in risk premia.

2.1 Households

The representative household at Home has recursive preferences

\[ v_t = \left( (1 - \beta) (c_t \Phi(\ell_t) \Omega_t (B_{Ht,s}/P_t))^{1-1/\psi} + \beta \mathbb{E}_t [(v_{t+1})^{1-\gamma}]^{1+1/\psi} \right)^{1-1/\psi} \]  

over consumption \( c_t \), labor \( \ell_t \), and the real value of “safe” dollar bonds \( B_{Ht,s}/P_t \). Consumption \( c_t \) is a CES aggregator of Home- and Foreign-produced goods

\[ c_t = \left( \left( \frac{1}{1+\zeta^*} + \varsigma \right)^{\frac{1}{\sigma}} (c_{Ht})^{\frac{\sigma-1}{\sigma}} + \left( \frac{\zeta^*}{1+\zeta^*} - \varsigma \right)^{\frac{1}{\sigma}} (c_{Ft})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \]  

The disutility of labor follows Shimer (2010) and Trabandt and Uhlig (2011)

\[ \Phi(\ell_t) = \left( 1 + (1/\psi - 1) \tilde{\nu} \left( \frac{\ell_t}{1+1/\nu} \right)^{1+1/\psi} \right)^{1/1-1/\psi}. \]  

The utility provided by safe dollar bonds is analogous to the voluminous literature with money in the utility function since Sidrauski (1967). It captures the non-pecuniary value agents receive from the liquidity or perceived safety of these assets, and follows Krishnamurthy and Vissing-Jorgensen (2012) among many other papers in the recent literature on convenience yields. The household’s risk aversion is denoted
by $\gamma$, intertemporal elasticity of substitution as well as consumption-labor complementarity are jointly controlled by $\psi$, and discount rate is $\beta$. Home bias is controlled by $\varsigma$ and the trade elasticity by $\sigma$. Finally, $\bar{\nu}$ denotes the disutility of labor and $\nu$ controls the Frisch elasticity of labor supply. Each household supplies a continuum of labor varieties $j \in [0, 1]$, so $\ell_t = \int_0^1 \ell_t(j) dj$.

The household chooses one-period safe dollar bonds $B_{Ht,s}$ paying $i_t$ dollars at $t+1$; one-period other dollar bonds $B_{Ht,o}$ paying $\iota_t$ dollars at $t+1$; one-period Foreign nominal bonds $B_{Ft}$ paying $i^*_t$ in Foreign’s unit of account at $t+1$; and capital $k_t$ which trades at price $Q^k_t$ at $t$, pays dividends $\Pi_{t+1}$ per unit in $t+1$, and depreciates after its use at rate $\delta$. Without loss of generality, the price and return on the capital claim are written here in dollars. The rare disaster scales the capital stock by the stochastic term $\exp(\varphi_{t+1})$. We describe the effects of a disaster in more detail below.

Each period, the household supplies labor and chooses consumption and its portfolio subject to the resource constraint

$$P_{Ht}c_{Ht} + E_t^{-1} P^*_{Ft}c_{Ft} + B_{Ht,s} + B_{Ht,o} + E_t^{-1} B_{Ft} + Q^k_t k_t \leq (1+i_t-1)B_{Ht-1,s} + (1+\iota_t-1)B_{Ht-1,o} + E_t^{-1}(1+i^*_t-1)B_{Ft-1} + (\Pi_t + (1-\delta)Q^k_{t-1} k_{t-1} \exp(\varphi_t) + \int_0^1 W_t(j) \ell_t(j) dj - \int_0^1 AC^W_t(j) dj + T_t, \quad (4)$$

where $P_{Ht}$ and $P^*_{Ft}$ denote the prices of Home- and Foreign-produced goods in their domestic unit of accounts; $E_t$ is the nominal exchange rate in terms of Foreign’s unit of account per dollar; and we assume producer-currency pricing, implying that the law of one price holds. Each labor variety $j$ in the household earns a wage rate $W_t(j)$. Following Rotemberg (1982), the household pays a cost of setting such a wage

$$AC^W_t(j) = \frac{\chi^W}{2} W_t \ell_t \left( \frac{W_t(j)}{W_{t-1}(j) \exp(\varphi_t)} - 1 \right)^2, \quad (5)$$

where $\chi^W$ scales the adjustment costs and the aggregate wage bill $W_t \ell_t$ is defined below.$^5$ $^6$ Finally, the household receives a government transfer $T_t$.

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$^5$ We assume the adjustment cost is paid to the government and then rebated lump-sum so it does not mechanically affect our quantitative results (as on output volatility) later in the paper.

$^6$ The disaster enters into the denominator of the wage adjustment cost for computational simplicity, as it reduces the size of the grid of prior period real wages we need to consider in our numerical algorithm. We further view this as realistic, as richer models of nominal rigidity would imply that in response to large shocks, prices and wages indeed may be more flexible.
Households in Foreign face an analogous problem. Importantly, Foreign households also receive utility \( \Omega_t^* (B_{Ht,s}^*/(E_t^{-1}P_t^*)) \) from safe dollar bonds and their risk aversion \( \gamma^* \) can differ from that of Home households. We also allow their discount factor \( \beta^* \) to differ from that in Home so that we can match the level of net foreign assets in our calibration. Otherwise, they share the same intertemporal elasticity cum consumption-labor complementarity \( \psi \), home bias \( \varsigma \), trade elasticity \( \sigma \), and Frisch elasticity \( \nu \) as Home households. We further assume an identical degree of nominal wage rigidity \( \chi^W \) as in Home. We allow the disutility of labor \( \bar{v}^* \) to differ from that in Home only to normalize labor supply to one when calibrating the model.

2.2 Supply-side

**Labor unions** Home union \( j \) represents each variety \( j \) in Home households. Each period, it chooses the wage \( W_t(j) \) and labor supply \( \ell_t(j) \) to maximize the utilitarian social welfare of members. An analogous problem faces each Foreign union \( j^* \).

**Labor packer** A representative Home labor packer purchases varieties supplied by each union and combines them to produce a CES aggregate with elasticity of substitution \( \epsilon \) and sold at \( W_t \) to domestic firms. The labor packer thus earns

\[
W_t \left[ \int_0^1 \ell_t(j)^{(\epsilon-1)/\epsilon} \, dj \right]^{\epsilon/(\epsilon-1)} - \int_0^1 W_t(j)\ell_t(j) \, dj. \tag{6}
\]

An analogous problem faces the representative Foreign labor packer, and we assume that the elasticity of substitution across labor varieties is also \( \epsilon \).

**Production** A representative Home producer hires \( \ell_t \) units of labor from the domestic labor packer, rents \( \kappa_t \) units of capital on the international market, and produces the consumption good with productivity \( z_t \) and a constant-returns-to-scale technology with labor share \( 1 - \alpha \). The producer thus earns

\[
P_{Ht} (z_t\ell_t)^{1-\alpha} (\kappa_t)^\alpha - W_t\ell_t - \Pi_t\kappa_t. \tag{7}
\]

A symmetric problem faces the representative Foreign producer. Relative productivity in Foreign is stochastic and given by \( z_{Ft} \). We note that the return per unit
capital used in Foreign will still be $\Pi_t$ once expressed in dollars, reflecting the ability of households to freely deploy capital in either country, equating its rate of return.\footnote{This simplifies the model computation, as there is only a single aggregate capital state variable to keep track of $(\bar{k}_t-1)$. Recently, Atkeson, Heathcote, and Perri (2023) and Dahlquist et al. (2023) have emphasized the importance of heterogeneous returns on U.S. versus foreign equities. In section 3, we describe a model extension featuring distinct capital stocks in each country.}

Finally, a representative global capital producer uses $(\bar{k}_t/(\bar{k}_{t-1} \exp(\varphi_t))) x^x \ x_{Ht}$ units of the Home consumption good and $(\bar{k}_t/(\bar{k}_{t-1} \exp(\varphi_t))) x^x \ x_{Ft}$ units of the Foreign consumption good to produce

$$x_t = \left( \left( \frac{1}{1 + \zeta^*} \right)^{\frac{1}{\sigma}} \ x_{Ht} \right)^{\frac{\sigma-1}{\sigma}} + \left( \left( \frac{\zeta^*}{1 + \zeta^*} \right)^{\frac{1}{\sigma}} \ x_{Ft} \right)^{\frac{\sigma-1}{\sigma}}$$

(8)

new units of capital, where $\chi^x$ controls adjustment costs, global capital $\bar{k}_t$ is taken as given, and we assume investment is not subject to home bias. The producer earns

$$Q^x_t x_t - (\bar{k}_t/(\bar{k}_{t-1} \exp(\varphi_t))) x^x \ (P_{Ht} x_{Ht} + E_{t-1}^{-1} P_{Ft} x_{Ft})$$

(9)

which will be zero in equilibrium.

### 2.3 Policy

Monetary policy is characterized by a Taylor (1993) rule

$$1 + i_t = (1 + \tilde{i}) \left( \frac{P_t}{P_{t-1}} \right)^{\phi},$$

(10)

where $P_t$ is the ideal price index

$$P_t = \left[ \left( \frac{1}{1 + \zeta^*} + \zeta \right) P_{Ht}^{1-\sigma} + \left( \frac{\zeta^*}{1 + \zeta^*} - \zeta \right) (E_{t-1}^{-1} P_{Ft})^{1-\sigma} \right]^{\frac{1}{\sigma}}.$$  

(11)

An analogous Taylor rule in Foreign determines $i^*_t$ with the same coefficient $\phi$ on inflation in the Foreign ideal price index $P^*_t$. We focus on CPI-targeting Taylor rules anticipating our calibration to the U.S. and G10 currency countries.

Fiscal policy at Home is characterized by participation in the safe dollar bond market $B^d_{Ht,s}$ and lump-sum transfers. We assume that the government maintains a
constant ratio of safe dollar debt to global consumption

$$-B_{Ht,s}^g = \bar{b}^g(P_t c_t + \zeta^* E_{t}^{-1} P_t^* c_t^*),$$  \hspace{1cm} (12)$$

a specification we motivate in the next subsection. The empirically relevant case features $\bar{b}^g > 0$: the Home government borrows in safe dollar bonds, namely Treasury bills. The Home government then makes transfers to each household

$$T_t = \int_0^1 AC^W_t(j) dj + (1 + i_{t-1})B_{Ht-1,s}^g - B_{Ht,s}^g.$$  \hspace{1cm} (13)$$

We abstract from the Home government’s participation in asset markets other than safe dollar bonds because these do not provide non-pecuniary benefits and the government finances itself with lump-sum taxes, so Ricardian equivalence will apply. The Foreign government similarly provides wage subsidies and makes lump-sum transfers, but we abstract from its participation in asset markets because it is assumed to be unable to create safe dollar liquidity and thus Ricardian equivalence holds.

2.4 Non-pecuniary value of safe dollar bonds

The non-pecuniary value of safe dollar bonds is reflected in a wedge between the returns on safe dollar bonds and all other assets — a “convenience yield”. Among dollar-denominated bonds, this is particularly clear because both bonds pay in the same unit of account and are risk-free. Thus, investor indifference in Home requires

$$\frac{1 + i_t}{1 - c_t \Omega_t'(B_{Ht,s}/P_t)/\Omega_t(B_{Ht,s}/P_t)} = 1 + \iota_t.$$  \hspace{1cm} (14)$$

The left-hand side is the effective return on safe dollar bonds. The right-hand side is the return on other dollar bonds. Since an analogous condition must hold for Foreign agents, the non-pecuniary value of safe dollar bonds must be equated across agents on the margin, which we denote $\omega_t$:

$$\omega_t \equiv c_t \Omega_t'(B_{Ht,s}/P_t) = c_t^* \Omega_t^*(B_{Ht,s}^*/(E_{t-1}^{-1} P_t^*)) / \Omega_t^*(B_{Ht,s}^*/(E_{t-1}^{-1} P_t^*)).$$  \hspace{1cm} (15)$$

Equation (14) makes clear how the convenience yield $\omega_t$ can be estimated using spreads in financial markets, which we make use of in our quantitative analysis.
We now assume a particularly convenient functional form for $\Omega_t$:

$$
\Omega_t \left( \frac{B_{Ht,s}}{P_t} \right) = \exp \left( \omega_d^t \frac{B_{Ht,s}}{P_t \bar{c}_t} - \frac{1}{2} \epsilon_d \left( \frac{B_{Ht,s}}{P_t \bar{c}_t} \right)^2 - \left[ \omega_d \frac{\bar{B}_{Ht,s}}{P_t \bar{c}_t} - \frac{1}{2} \epsilon_d \left( \frac{\bar{B}_{Ht,s}}{P_t \bar{c}_t} \right)^2 \right] \right),
$$

where $\omega_d^t$ is an exogenous driving force, $\epsilon_d$ is a parameter, and all variables with bars are aggregates which the representative household takes as given.\(^8\) Given an analogous functional form in Foreign, appendix A proves that the second equality in (15) together with market clearing in safe dollar bonds implies

$$
\frac{B_{Ht,s}}{P_t \bar{c}_t} = \frac{B_{Ht,s}^*}{E_t^{-1} P_t^* c_t^*} = \frac{\left( -B_{Ht,s}^q \right)}{P_t \bar{c}_t + \zeta^* E_t^{-1} P_t^* c_t^*}.
$$

The first equality in (15) thus implies

$$
\omega_t = \omega_d^t - \frac{1}{\epsilon_d} \frac{\left( -B_{Ht,s}^q \right)}{P_t \bar{c}_t + \zeta^* E_t^{-1} P_t^* c_t^*}.
$$

Intuitively, the convenience yield is rising in private demand for safe dollar bonds $\omega_d^t$ and decreasing in public supply $-B_{Ht,s}^q$. The relative strength of the latter depends on $\epsilon_d$, the elasticity of demand to the non-pecuniary value. We treat $\omega_d^t$ as a driving force and refer to its innovations as “safety shocks”. Given our assumed supply of safe dollar debt (12), the convenience yield is effectively exogenous and inherits the stochastic properties of $\omega_d^t$. At the end of section 6, we instead study shocks to $B_{Ht,s}^q$.\(^9\)

### 2.5 Driving forces

Global productivity follows a unit root process subject to rare disasters

$$
\log(z_t) = \log(z_{t-1}) + \sigma^z \epsilon_t^z + \varphi_t,
$$

\(^8\)The rationale for this functional form is straightforward. Inside the parenthesis, the first two terms imply a time-varying non-pecuniary value of safe dollar bonds diminishing in the household’s position. The second two terms ensure that in equilibrium $\Omega_t(B_{Ht,s}/P_t) = 1$, so that the effects of a time-varying convenience yield do not arise from mechanical effects on stochastic discount factors.

\(^9\)We maintain the “cashless limit” of Woodford (2003). If money offers liquidity services which are neither substitutes nor complements with safe dollar bonds, this is innocuous. If dollar money and safe dollar bonds are perfect substitutes in liquidity provision, (17) would effectively be replaced by a condition relating $\omega_t$ to Home’s nominal rate. However, changes in the relative liquidity of safe dollar bonds versus money would still propagate like safety shocks in our baseline model.
where $\varphi_t$ is equal to zero with probability $1 - p_t$ and $\varphi < 0$ with probability $p_t$. The log disaster probability $p_t$ follows an AR(1) process

$$\log p_t - \log p = \rho^p (\log p_{t-1} - \log p) + \sigma^p \epsilon^p_t,$$

which we specify in terms of the log series to capture its skewness in the data. Similarly, the demand for safe dollar bonds is given by $\omega^d_t = \Delta^\omega + \tilde{\omega}^d_t$, where

$$\log \tilde{\omega}^d_t - \log \omega^d = \rho^\omega (\log \tilde{\omega}^d_{t-1} - \log \omega^d) + \sigma^\omega \epsilon^\omega_t.$$

This similarly captures the skewness of the convenience yield in the data, but we include the shift parameter $\Delta^\omega$ so that the mean of $\omega_t$ is zero, allowing us to make clear that all of the paper’s insights only rely on time-variation in the convenience yield. Finally, log relative productivity at Foreign $z_{Ft}$ follows

$$\log z_{Ft} = \rho^F \log z_{Ft-1} + \sigma^F \epsilon^F_t.$$

We assume that the innovations $\{\epsilon^z_t, \epsilon^p_t, \epsilon^\omega_t, \epsilon^F_t\}$ are each draws from a normal distribution with mean zero and variance one. We allow the shocks to disaster risk and the convenience yield to have correlation $\rho^{pz}$; a positive value (as we later estimate in the data) allows us to capture that the flight to safe dollar assets typically accompanies times of elevated global risk. We assume all other shock correlations are zero.

### 2.6 Equilibrium and solution

We provide the market clearing conditions in appendix A for brevity. The definition of equilibrium is standard and also provided in appendix A together with a characterization of agents’ first-order conditions. Since labor varieties are symmetric, $\ell_t(j) = \ell_t$, $\ell_t(j^*) = \ell_t^*$ and we drop the indices $j$ and $j^*$ going forward.

We globally solve a stationary transformation of the economy obtained by dividing all real variables (except labor) by $z_t$ and nominal variables by $P_t z_t$. As shown in appendix A, we obtain a recursive representation of equilibrium in which the aggregate state in period $t$ is given by the disaster probability $p_t$, convenience yield $\omega_t$, relative Foreign productivity $z_{Ft}$, scaled aggregate capital $\bar{k}_t - 1 / z_t$, scaled real wages $W_t / (P_t z_t)$ and $W^*_t / (E_t P_t z_t)$, and Home financial wealth share $\theta_t$. After scaling in this way, global
productivity shocks inclusive of disasters only govern the transition across states.

Appendix A also defines additional variables used in the remainder of the paper, including the real exchange rate \( q_t \) (so that an increase corresponds to a Home appreciation); real interest rates \( r_t \) and \( r_t^* \); real return on capital \( r_t^k \) (expressed in terms of the Home consumption bundle); Home’s real value of aggregate saving \( a_t \); and Home’s real net foreign assets \( nfa_t \). All of these definitions are standard. The appendix further defines the total positions of the Home and Foreign representative agents in dollar-denominated bonds

\[
B_{Ht} = (1 - \omega_t) (B_{Ht,s} + B_{Ht,s}^g) + B_{Ht,o},
\]

\[
B_{Ht}^* = (1 - \omega_t) B_{Ht,s}^* + B_{Ht,o}^*,
\]

each of which earn return \( 1 + \iota_t = \frac{1+i_t}{1-\omega_t} \). The composition of households’ dollar bond position is only relevant in determining the equilibrium seignorage earned by Home on the safe dollar debt held by Foreign which follows from (16).

3 Analytical insights

We first characterize the interactions between safety shocks, greater risk tolerance at Home, and nominal rigidities in a version of the model admitting analytical results. A positive safety shock generates a dollar appreciation and global recession. Dollar bonds earn a negative risk premium and the U.S. finances a levered capital portfolio in dollars. Several features of safety shocks and of the U.S. economy render these shocks particularly special for the global economy. We discuss the role of key model features in shaping our main results.

3.1 Parametric assumptions

We first describe the simplifying assumptions made in this section alone.

Definition 1. The simplified environment features:

- flexible wages or wages set one period in advance;
- a fixed global capital stock \( (\chi^x \to \infty, \delta \to 0) \);
• a unitary IES ($\psi = 1$), complete home bias ($\zeta \to \frac{\zeta^*}{1 + \zeta^*}$), and an infinite Frisch elasticity ($\nu \to 0$);

• no disaster risk ($p = 0$, $\sigma^p = 0$), constant relative productivity ($\sigma^F = 0$), and transitory safety ($\rho^\omega = 0$);

• identical per capita wealth across countries in the deterministic steady-state;

• identical discount factors ($\beta = \beta^*$).

The first assumption departs from the Rotemberg (1982) adjustment costs in the full model; together with the second assumption, this simplifies the dynamics. The next three assumptions simplify the algebra in the proofs. The final assumption ensures that the deterministic steady-state is well-defined. We study this environment using a perturbation approach around this steady-state. We emphasize that in the quantitative analysis in the subsequent sections, none of the above assumptions are made, and a global solution of the model is employed.

### 3.2 Effects of a safety shock

We now describe the effects of a safety shock. We use first-order approximations and $\cdot$ to denote log/level deviations from the deterministic steady-state, and variables without time subscripts to denote the deterministic steady-state.

We begin with the effects on prices and production, in which case the role of nominal rigidity is crucial. To most cleanly see this, we assume identical portfolios and zero safe debt issued by the Home government ($b^g_{H,s} \equiv B^g_{H,s}/P = 0$) in steady-state, eliminating any revaluation of wealth on impact of a safety shock:

**Proposition 1.** Consider the simplified environment and assume identical portfolios and $b^g_{H,s} = 0$ in the deterministic steady-state. If wages are flexible, then on impact of a positive safety shock:

• the Home real interest rate declines ($\mathbb{E}_t \hat{r}_{t+1} = -\hat{\omega}_t$);

• the Home CPI declines ($\Delta \hat{P}_t = -\frac{1}{\psi} \hat{\omega}_t$); and

• the Home real exchange rate and employment in each country are unchanged ($\hat{q}_t = \hat{\ell}_t = \hat{\ell}_t = 0$).
If wages are set one period in advance, then on impact of a positive safety shock:

- the Home real interest rate declines by less than above \( 0 > \hat{E}_{t}\hat{r}_{t+1} > -\hat{\omega}_t \);
- the Home CPI declines by less than above \( 0 > \Delta \hat{P}_t > -\frac{1}{\phi} \hat{\omega}_t \);
- the Home real exchange rate appreciates \( (\hat{q}_t \propto \hat{\omega}_t) \); and
- global employment falls, disproportionately so in Home \( (\frac{1}{1+\zeta} \hat{\ell}_t + \frac{\zeta^*}{1+\zeta} \hat{\ell}_t^* \propto -\hat{\omega}_t \) and \( \hat{\ell}_t - \hat{\ell}_t^* \propto -\hat{\omega}_t \).

The proof of this proposition, like all others, is provided in appendix B.

Intuitively, consider a positive safety shock \( \hat{\omega}_t > 0 \) in the Euler equation

\[
\hat{E}_{t}m_{t,t+1} \left( \frac{1 + r_{t+1}}{1 - \omega_t} \right) = 1,
\]

where \( m_{t,t+1} \) denotes the real pricing kernel of a Home household between \( t \) and \( t+1 \). Analogous conditions hold for Foreign households. Absent nominal rigidity, the flight to safe dollar bonds is met with a one-for-one decline in the Home expected real interest rate. With nominal bonds, this is achieved by an immediate dollar deflation which, under the assumed Taylor rule, results in a fall in the nominal interest rate. With nominal prices and interest rates at Home fully absorbing the increase in safe asset demand, there is no required adjustment in Foreign prices or interest rates to ensure that uncovered interest parity

\[
\hat{E}_{t}m_{t,t+1} \left[ \frac{q_t}{q_{t+1}} (1 + r_{t+1}^*) - \left( \frac{1 + r_{t+1}}{1 - \omega_t} \right) \right] = 0
\]

remains satisfied. There is thus no required adjustment in relative prices nor in production across countries.

In the presence of nominal rigidity and a monetary policy rule which does not sufficiently lower the nominal interest rate (as in the case of the conventional Taylor rule), real interest rates exceed those in the natural allocation and consumption demand is depressed, driving a global recession. In the foreign exchange market, the limited adjustment in real interest rates implies that the dollar must appreciate on impact so that it can be expected to depreciate going forward, ensuring uncovered interest parity holds. These goods market and foreign exchange market responses are linked by the relative supply response: the deflationary pressure particularly at Home
implies that product wages rise and output thus falls especially at Home, driving the appreciation in Home’s terms of trade and thus real exchange rate.\textsuperscript{10,11} The disproportionate recession borne by Home echoes the result in Caballero et al. (2021) that reserve asset issuers bear the disproportionate cost of “safety traps”. We demonstrate that this insight does not rely on the zero lower bound and is a consequence of any monetary policy rule which does not react one-for-one to safe asset demand.

We now turn to the predictions for realized and expected excess returns:

**Proposition 2.** Consider the simplified environment and assume identical portfolios and $b_{H,s}^0 = 0$ in the deterministic steady-state. Then on impact of a positive safety shock:

- the real return on dollar bonds rises ($\hat{r}_t \propto \omega_t$);
- the real return on capital is unaffected if wages are flexible ($\hat{r}_t^k = 0$) but falls if wages are set in advance ($\hat{r}_t^k \propto -\hat{\omega}_t$);
- the real return on Foreign bonds is unaffected if wages are flexible ($\hat{r}_t^* - \Delta \hat{q}_t = 0$) but falls if wages are set in advance ($\hat{r}_t^* - \Delta \hat{q}_t \propto -\hat{\omega}_t$);
- expected excess returns on capital and Foreign bonds are positive (up to first order, $E_t [\hat{r}_{t+1}^k - \hat{r}_{t+1}] = E_t [\hat{r}_{t+1}^* - \Delta \hat{q}_{t+1} - \hat{r}_{t+1}] = \hat{\omega}_t$).

Consider the excess returns on capital and Foreign bonds relative to safe dollar bonds. The realized excess returns on capital are negative due to a positive safety shock, both because of the deflation which raises the real return on dollar bonds (even absent nominal rigidity) and the decline in global production which reduces the return to capital (only with nominal rigidity). The realized excess returns on Foreign bonds are negative, again because of the higher real return on dollar bonds (even absent nominal rigidity) and the real dollar appreciation (only with nominal rigidity). Going forward, expected excess returns on capital and Foreign bonds are high so agents remain indifferent between safe dollar bonds and these other assets.

Finally, we turn to the predictions for wealth and net foreign assets, in which case the interaction between these dynamics of excess returns and heterogeneity in portfolios is crucial:

\textsuperscript{10}We note that these same results obtain if nominal prices rather than wages are sticky instead.
\textsuperscript{11}In the limit of complete home bias, Foreign output is in fact unaffected by a safety shock. Away from this limit, Foreign output will also fall on impact of a positive safety shock provided the trade elasticity $\sigma$ is not too high.
Proposition 3. Consider the simplified environment and assume portfolios are initially not too different from the symmetric benchmark and $b_{H,s}^0$ is not too different from zero. Then on impact of a positive safety shock:

- Home’s wealth share falls in its leverage in capital and Foreign bonds but rises in the safe debt issued by the Home government
  \[ \hat{\theta}_t = \left( \frac{q_k^H}{a} - 1 \right) \left( \hat{r}_t^k - \hat{r}_t \right) + \frac{b_F}{a} \left( \hat{r}_t^* - \Delta \hat{q}_t - \hat{r}_t \right) - \beta \left( \frac{c^*}{1+\zeta} - b_{H,s}^0 \right) \omega_t, \]
- revaluing Home’s net foreign assets in the same way.

The fact that Home’s wealth falls in its capital and Foreign bond positions is a straightforward consequence of Proposition 2. The fact that its wealth rises in the safe debt issued by the Home government reflects the seignorage revenue Home earns on the share of this debt owned by the rest of the world.

3.3 Portfolios and risk premia

We now characterize the equilibrium portfolios actually chosen by agents and the risk premium on Foreign bonds versus dollar bonds. Following Devereux and Sutherland (2011), these can be characterized using a second-order approximation around the deterministic steady-state. Because the simplified environment is only subject to global productivity and safety shocks, the three available assets implement efficient risk sharing around the steady-state (it is “locally complete” as defined by Coeurdacier and Gourinchas (2016)). We focus on the case with wages set one period in advance.

The equilibrium portfolios reflect the issuance of safe dollar debt by the Home government, differences in risk tolerance between Home and Foreign, and agents’ hedging demands given non-traded labor income, real exchange rate risk, and the disutility of labor. In appendix B, we characterize each of these forces in closed form. We focus here on comparative statics with respect to Home’s safe debt supply and heterogeneity in risk tolerance alone:

Proposition 4. Consider the simplified environment with wages set one period in advance and the same, positive steady-state labor wedge in each country. At least around the case with symmetric country portfolios:

- Home’s portfolio share in capital (dollar bonds) is unaffected (falls) with $-b_{H,s}^0$; and
• Home’s portfolio share in capital (dollar bonds) rises (falls) with \( \frac{\gamma}{\gamma^*} \), holding 
\( \gamma + \frac{1}{\varepsilon} \gamma^* \) fixed.

Intuitively, agents face two sources of risk: global productivity and safe asset demand. The former affects consumption holding fixed labor, and both affect labor in the presence of nominal rigidity. As Home’s government borrows more in safe dollar bonds, Home receives more seignorage on impact of a positive safety shock, rendering it a natural insurer of this shock. It can do so without loading up on productivity risk by borrowing more in dollar bonds to hold Foreign bonds. In contrast, as Home gets more risk tolerant than Foreign, it will provide insurance against both negative productivity shocks and positive safety shocks.\(^{12}\) It does so by holding more capital and borrowing more in dollar bonds. It is in this sense that greater risk tolerance is necessary to explain why the U.S. takes a disproportionate exposure to equity returns.

The risk premium on Foreign bonds versus dollar bonds reflects these risk factors and country-level portfolios. As our final analytical result makes clear, the presence of safety shocks has a crucial effect on the sign of the risk premium:

**Proposition 5.** Consider the same environment as in Proposition 4 and suppose safety and productivity shocks are independent. Then at least around the case with symmetric country portfolios:

\[
\text{Cov}_t \left( -\hat{m}_{t+1}, \hat{r}_{t+1}^* - \Delta \hat{q}_{t+1} - \hat{r}_{t+1} \right) \propto \gamma - \gamma^* \text{ if } \sigma_\omega = 0; \text{ and}
\]

\[
\text{Cov}_t \left( -\hat{m}_{t+1} + \hat{r}_{t+1}^* - \Delta \hat{q}_{t+1} - \hat{r}_{t+1} \right) \text{ is rising in } \sigma_\omega.
\]

This result holds as well for the pricing kernel of a Foreign household.

The first part of this result indicates that, absent safety shocks, Foreign bonds would earn a negative risk premium versus dollar bonds if Home is more risk tolerant than Foreign. This is because the dollar would appreciate in “good” times, when productivity is high, U.S. wealth rises (as it is levered in capital), and thus U.S. consumption rises. This indicates that the “reserve currency paradox” characterized in Maggiori (2017) is robust to endogenous production and nominal rigidities. The second part of this result indicates that safety shocks can provide a resolution to this paradox. Because safety shocks instead imply that the dollar appreciates in “bad”

\(^{12}\) The latter result relies on a positive labor wedge in steady-state: only in this case will risk tolerant agents insure risk averse agents against states of the world in which labor falls.
times, when safe asset demand is high and global employment declines, sufficiently volatile safety shocks imply that Foreign bonds instead earn a positive risk premium.

We finally emphasize where safety shocks and cross-country heterogeneity in risk tolerance interact in our results. The propagation of safety shocks to output and asset prices in Propositions 1 and 2 holds even in the absence of differences in risk tolerance. As a result, the variance of safety shocks raises the ex-ante risk premium on Foreign bonds versus dollar bonds in Proposition 5 even in the absence of differences in risk tolerance. Where these model ingredients interact is in the determination of international portfolios and the associated valuation effects. In particular, because safety shocks induce a negative beta of dollar bonds, the U.S. borrows more in dollar bonds as it grows more risk tolerant as a means to insure the rest of the world (Proposition 4). This in turn implies a larger loss in wealth upon a flight to safety (Proposition 3). In the absence of safety shocks, the U.S. would still be levered in capital as the more risk tolerant investor, but there would be no reason to finance this portfolio in dollar bonds as opposed to Foreign bonds. In the absence of differences in risk aversion, the U.S. would remain the natural insurer of safety shocks owing to the seignorage it earns, but its wealth share and net foreign assets would not fall upon these shocks because the seignorage gains would offset the losses on its Foreign bond portfolio, and it would not take a levered position in global capital.

3.4 The specialness of safe dollar bonds

Extensions of the model clarify several dimensions in which the demand for safe dollar bonds may be particularly special relative to the demand for other assets.

Zero net supply The demand for safe dollar bonds triggers a Keynesian recession because these assets are in zero net supply. If agents’ demand for capital instead increases — formally, capital also enters into utility and its non-pecuniary value increases on the margin — this would induce an increase in the price of capital, a rise in household wealth, and thus an increase in consumption demand and aggregate output. Relaxing the assumption of a fixed global capital stock, the demand for capital would also stimulate output via increased investment. This underscores the importance of distinguishing between dollar convenience yields in bond versus equity markets, as in the work of Koijen and Yogo (2020), to understand their macroeconomic effects.
Country size  The large size of the U.S. economy renders the demand for safe dollar bonds particularly special vis-à-vis the bonds of other reserve issuers. In particular, consider augmenting the model with a third country which is an infinitesimal part of the global economy (for concreteness, Switzerland). An increase in the portfolio demand for safe dollar bonds and Swiss franc bonds would generate a global recession and appreciation in both currencies relative to others in the global economy, ascribing both a negative beta. However, only the portfolio demand for safe dollar bonds would be responsible for generating the recession; the Swiss franc deflation would drive a decline in Swiss production, but this would have a negligible effect on global demand and production. This echoes the message of Hassan (2013) that the U.S.’ relative size in the global economy may be an important contributor to the dollar’s reserve status.

Currency of invoicing  The widespread use of the dollar in pricing further renders the demand for safe dollar bonds particularly special vis-à-vis the currencies of other large countries. For instance, suppose nominal wages even in Foreign are denominated and sticky in dollars. Then the global decline in employment following a positive safety shock is exacerbated relative to the baseline model. Intuitively, the dollar deflation now implies that product wages in both countries are too high, generating a more severe and uniform global recession. Similar results are obtained when we assume dollar pricing of exports as in Gopinath (2015), Gopinath, Boz, Casas, Diez, Gourinchas, and Plagborg-Moller (2020), and Mukhin (2022). Indeed, dollar pricing in trade flows among countries not involving the U.S. is analogous to an internal price in Foreign (such as the nominal wage) being denominated in dollars in our two-country set-up. Our model thus suggests potentially rich interactions between the dollar’s role in financial and goods markets.

3.5 Discussion of key model features

We finally provide additional discussion of the role of several model features in shaping the effects of safety shocks, greater risk tolerance at Home, and their interaction.

Efficient risk sharing and the law of one price  Our results imply that the dollar appreciation in bad times is induced by a shock — a safety shock — which
reduces output more in Home than Foreign.\textsuperscript{13} Here we demonstrate that this result is a natural consequence of efficient risk sharing and the law of one price.

Consider in particular the $\gamma = \gamma^* = 1$ case, which is convenient to make preferences time separable recalling our maintained assumption in this section of unitary elasticities of intertemporal substitution. Then efficient risk sharing requires

$$\hat{c}_t^* - \hat{c}_t = \hat{q}_t,$$

assuming no shocks prior to period $t$ for simplicity. Intratemporal optimality between Home and Foreign-produced goods in each country, together with goods market clearing, requires

$$\varsigma \frac{1 + \zeta^*}{\zeta^*} (\hat{c}_t^* - \hat{c}_t) + \left( 1 + \varsigma \frac{1 + \zeta^*}{\zeta^*} \right) \left( 1 - \varsigma \frac{1 + \zeta^*}{\zeta^*} \right) \frac{1}{\varsigma \frac{1 + \zeta^*}{\zeta^*}} \sigma \hat{y}_t = \hat{y}_t^* - \hat{y}_t,$$

where we allow an arbitrary degree of home bias $\varsigma$ to emphasize the generality of the result. Combining these implies

$$\hat{q}_t = \frac{1}{\varsigma \frac{1 + \zeta^*}{\zeta^*} + \left( 1 + \varsigma \frac{1 + \zeta^*}{\zeta^*} \right) \left( 1 - \varsigma \frac{1 + \zeta^*}{\zeta^*} \right) \frac{1}{\varsigma \frac{1 + \zeta^*}{\zeta^*}} \sigma} (\hat{y}_t^* - \hat{y}_t).$$

Thus, given consumption home bias $\varsigma > 0$, any shock which appreciates the Home real exchange rate must be reflected in a relative fall in Home output, both because efficient risk sharing calls for relative Home consumption to fall and because the associated appreciation of Home’s terms of trade will lead to a global expenditure switch away from Home-produced goods. Relaxing the law of one price, as in the case of sticky local currency prices, can decouple the Home real exchange rate from the relative price of goods faced by consumers, eliminating the second effect. But the first will remain provided that risk sharing is efficient.

This result helps to relate our paper to others in the literature. Sauzet (2023) demonstrates that a fall in relative U.S. output in crises can resolve the reserve currency paradox precisely following the logic above. Our paper identifies a shock in a production economy which will have this feature, namely the flight to safe dollar assets. Maggiori (2017) proposes instead to resolve the paradox by considering an

\textsuperscript{13}We emphasize that this does not require that Home output is more volatile than Foreign output in an unconditional sense; indeed, in our quantitative analysis, Home output is less volatile.
expenditure shift toward U.S. goods in crises. This introduces a shock in the second equation above, implying that the dollar can appreciate in crises even if relative output is unchanged. Dahlquist et al. (2023) generate a dollar appreciation from a fall in U.S. output as in Sauzet (2023), but augment the model with deep habits. This implies that the decline in U.S. output also generates a relative demand shift towards U.S. goods, further appreciating the dollar as in Maggiori (2017).

The effect of safety shocks on relative output also has implications for the U.S. wealth share. Proposition 4 focuses on the comparative statics of efficient portfolios with respect to heterogeneity in risk tolerance and U.S. safe dollar debt issuance. But it is useful to also note that, in the benchmark with identical risk tolerance across countries, efficient risk sharing calls for the Home wealth share to increase upon a flight to safety provided risk aversion exceeds unity. This is a standard implication of the motive to hedge real exchange rate risk in response to relative supply shocks: when risk aversion exceeds unity, the domestic wealth share should rise in response to shocks that cause the local currency to appreciate. Away from the limit of complete home bias and with a trade elasticity above one, the Home wealth share should also rise upon this shock to hedge the relative decline in Home labor income. Hence, if the U.S. is only slightly more risk tolerant than the rest of the world, it is ambiguous whether its wealth share will rise or fall upon a flight to safety. This relates to the mixed empirical findings in Dahlquist et al. (2023) and Sauzet (2023) regarding the U.S. wealth share dynamics in 2008-2009. Our analysis clarifies that the sign of the wealth share response in the data cannot, on its own, validate or reject the hypothesis that the U.S. provides insurance to the rest of the world, to the extent that this period was characterized by relative supply shocks as induced by the flight to safety.

**Monetary policy response** Essential to the non-neutrality of safety shocks for macroeconomic quantities is that U.S. monetary policy does not sufficiently respond to these shocks. This will be true not only for the conventional Taylor rule assumed in our paper, but any monetary policy rule that does not respond with a unitary coefficient on the safety shock itself. Of course, ignoring the seignorage effects of safety shocks and assuming that the flexible wage allocation is efficient (as in the absence of steady-state markups), the globally optimal response of U.S. monetary policy in our model would be to fully neutralize the effects of safety shocks.

Our focus on conventional Taylor rules rather than optimal monetary policy fol-
lows in the tradition of business cycle models seeking to describe the conduct of actual monetary policy, particularly among the U.S. and G10 currency countries which are our focus. We note that our analysis is particularly consistent with the large New Keynesian literature studying risk premium shocks, in which the important role of risk premium fluctuations in business cycle dynamics owes to an inadequate response of monetary policy to track the natural rate.

**Capital mobility and relative capital returns**  Our model assumes for computational simplicity that capital can be deployed in either country, implying that the returns to capital used in Home and Foreign are equated at all dates and states.

With sticky wages, having distinct capital stocks at Home and Foreign turns out not to matter for our results. Consider for concreteness the case with symmetric portfolios at Home and Foreign, as assumed in Propositions 1-3. Then the returns to capital at Home and Foreign are still equated in response to safety shocks; in other words, capital would not be reallocated across borders even if it could. Intuitively, any increase in Foreign output relative to Home output induces an appreciation of the Home real exchange rate which fully offsets the increase in the Foreign return to capital. This sharp result is no longer obtained away from the limit of complete home bias (provided the trade elasticity $\sigma \neq 1$), but it illustrates that our assumptions on capital mobility are not crucial for our results. With sticky prices, we can obtain richer results with distinct capital stocks at Home and Foreign. Most results, such as the propagation of safety shocks to macroeconomic quantities, remain unchanged. But now, a positive safety shock raises the return to Home capital relative to Foreign capital. This is because a positive safety shock induces a rise in Home mark-ups under sticky prices. This can help to account for the rise in relative U.S. equity returns in crises documented in Dahlquist et al. (2023).

**Global demand for safe dollar bonds**  We finally comment on the assumption that the demand for safe dollar bonds enters symmetrically for Home and Foreign households. Up to seignorage effects, this assumption does not affect the propagation.

\[\text{This is both true of models introducing such shocks as ad-hoc wedges, as in Smets and Wouters (2007), as well as those tracing them to underlying changes in the price or quantity of risk, as in Iliant and Schneider (2014), Fernandez-Villaverde, Guerrero-Quintana, Kuester, and Rubio-Ramirez (2015), Basu and Bundick (2017), Caballero and Simsek (2020), and Kekre and Lenel (2022).}\]

\[\text{See appendix B for a full description of this environment and formal statements of these results.}\]
of safety shocks as well as the ex-ante implications for risk premia or portfolios.

Concretely, suppose only Foreign households receive utility from safe dollar bonds, and we augment the model with the (realistic and now relevant) constraint that Home households cannot short safe dollar bonds. In equilibrium, Home households will be at the corner of holding zero safe dollar bonds, and the equilibrium convenience yield in (17) will now reflect only the marginal valuation of Foreign households (and still the supply of these bonds by the Home government). Considering for concreteness the case with \( b_{H,s}^0 = 0 \) so that we can abstract from seignorage, an increase in Foreign demand for safe dollar bonds propagates exactly like a safety shock in our baseline model. Intuitively, at unchanged interest rates and prices, foreigners would seek to borrow in non-safe dollar bonds to hold safe dollar bonds, allowing them to capture the convenience benefits of safe dollar bonds without changing their currency risk exposure. This would induce an increase in the interest rate \( \iota \) in the absence of a decline in the policy rate \( \iota_t \), making saving more attractive for Home households and inducing the same effects on quantities and relative prices as in the baseline model.17

4 Parameterization

In the rest of the paper we return to the full model and quantify the effects of safety shocks and heterogeneity in risk-bearing capacity in the global economy. In this section, we parameterize the model. We associate Home with the U.S. and Foreign with the G10 currency countries.18 A period is one quarter.

4.1 Data sources

Unless otherwise noted, we use data over 1995-2019, and we estimate moments for Foreign using a simple average of moments for each of the G10 currency countries.

In terms of business cycle moments, interest rates, and equity prices, we use Bu-

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16See appendix B for formal statements of the results which follow.
17It is useful to relate this discussion to the finding in Eichenbaum, Johannsen, and Rebelo (2021) that shocks to the foreign demand for dollar-denominated bonds can generate fluctuations in exchange rates disconnected from aggregates. This reflects the absence of a distinction between safe and other dollar-denominated bonds in their analysis. If they modeled shocks to the foreign demand for safe dollar bonds in particular, these shocks would affect output as described here.
18The G10 refers to Australia, Canada, Denmark, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, and the United Kingdom, following common convention.
reau of Economic Analysis, Bureau of Labor Statistics, and OECD data on consumption, investment, real GDP, and the working age population to estimate quarterly per capita growth rates in those series. We use three-month government bond yields from Bloomberg and the Center for Research in Security Prices as measures of nominal interest rates. We use the MSCI ACWI as our measure of the equity claim.

In terms of exchange rates, wealth, and portfolios, we use end-of-month nominal exchange rates vis-à-vis the dollar from the Federal Reserve Board, and we construct end-of-month real exchange rates using these data and the consumer price indices from the OECD. We measure net foreign assets using the Bureau of Economic Analysis International Investment Position (BEA IIP). Foreign-owned Treasury bills and central bank liquidity swap line usage are reported by the Treasury International Capital (TIC) System and Federal Reserve Board, respectively.

4.2 Externally set parameters

A subset of model parameters summarized in Table 1 are first set externally.

Among the model’s preference parameters, we set $\psi$ to 0.75, consistent with evidence on the consumption responses to changes in interest rates as well as consumption-labor complementarity. We set $\sigma = 1.5$, consistent with the trade elasticity estimated by Backus, Kehoe, and Kydland (1994) and widely used in the literature. We set a home bias parameter of $\varsigma = 0.4$, so that (given our calibration of $\zeta^*$ described in the next subsection) the expenditure share on domestically produced goods is 80% at Home, consistent with the U.S. evidence in Eaton, Kortum, and Neiman (2016). The Frisch elasticity of labor supply is set to $\nu = 0.75$, consistent with the micro evidence for aggregate hours surveyed in Chetty, Guren, Manoli, and Weber (2011).

Among the model’s technology and policy parameters, we choose $\alpha = 0.33$ for the capital share of production and a quarterly depreciation rate of 2.5%, standard values in the literature. We choose an elasticity of substitution across worker varieties $\epsilon = 20$ and, absent compelling evidence on heterogeneity in wage stickiness across countries, Rotemberg wage adjustment costs in each country of $\chi_W = \chi_W^* = 400$. Together these imply a Calvo (1983)-equivalent frequency of wage adjustment around 5 quarters, consistent with the U.S. evidence in Grigsby, Hurst, and Yildirimaz (2021). We assume a standard Taylor coefficient on inflation in each country of 1.5.

Finally, in terms of driving forces, we set $p$ so that the average quarterly global
disaster probability is 0.5% and the depth of the disaster to $\varphi = -10\%$, consistent with Barro (2006) and Nakamura, Steinsson, Barro, and Ursua (2013). The quarterly autocorrelation of the log probability is 0.75 and the standard deviation of shocks is $\sigma^p = 0.55$, consistent with the autocorrelation and standard deviation of the probability in levels in Barro and Liao (2021).

Following (14), the convenience yield is given by the spread between safe and other dollar bonds. One natural measure is the spread between three-month Treasury bills and three-month AA commercial paper. Another is the spread between three-month Treasury bills and three-month government bonds in the G10 currencies swapped into dollars, as constructed by Du et al. (2018a).\(^{19,20}\) As is evident in Figure 1, both series comove and spike in times of market turmoil. We calibrate the stochastic properties of $\omega_t^d$ to match the swapped G10/T-bill spread given our global focus.\(^{21}\) We set $\omega^d = 0.002$ to match the skewness of 6.1, $\rho^d = 0.4$ to match the autocorrelation (in levels) of 0.3, and $\rho^{p\omega} = 0.5$ to match the correlation with the Barro and Liao (2021) series.

\(^{19}\)We refrain from calling this a deviation from covered interest parity (CIP) because private agents cannot borrow at the U.S. Treasury bill rate and, relatedly, this spread is distinct from Libor-based CIP deviations which exhibited a structural break in 2008 (see Du, Tepper, and Verdelhan (2018b)).

\(^{20}\)We use an updated series through 2020 and thank Wenxin Du for sharing it with us.

\(^{21}\)We assume that variation in the convenience yield arises solely from changes in safe asset demand (except in section 6.4), while in the data it may also arise from shocks to supply. This source of misspecification would have minimal effects on our quantitative results: all that matters is that the equilibrium $\omega_t$ is consistent with the observed properties of the convenience yield.
<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$ IES</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>$\sigma$ trade elasticity</td>
<td>1.5</td>
<td>Backus et al. (1994)</td>
</tr>
<tr>
<td>$\varsigma$ home bias</td>
<td>0.4</td>
<td>Eaton et al. (2016)</td>
</tr>
<tr>
<td>$\nu$ Frisch elasticity</td>
<td>0.75</td>
<td>Chetty et al. (2011)</td>
</tr>
<tr>
<td>$\alpha$ 1 - labor share</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>$\delta$ depreciation rate</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>$\epsilon$ elast. of subs. across workers</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>$\chi^W$ Rotemberg wage adj. costs</td>
<td>400</td>
<td>$\approx P(\text{adjust}) = 5 \text{ qtrs}$</td>
</tr>
<tr>
<td>$\phi$ Taylor coeff. on inflation</td>
<td>1.5</td>
<td>Taylor (1993)</td>
</tr>
<tr>
<td>$\varphi$ disaster shock</td>
<td>-0.10</td>
<td>Nakamura et al. (2013)</td>
</tr>
<tr>
<td>$p$ disaster risk</td>
<td>0.4%</td>
<td>$E[p] = 0.5%$ (Barro (2006))</td>
</tr>
<tr>
<td>$\rho^p$ dis. risk persistence</td>
<td>0.75</td>
<td>$\rho(p) = 0.7$</td>
</tr>
<tr>
<td>$\sigma^p$ dis. risk std. dev.</td>
<td>0.55</td>
<td>$\sigma(p)/E[p] = 1$</td>
</tr>
<tr>
<td>$\omega^d$ safety skewness</td>
<td>0.002</td>
<td>$skew(\omega) = 6.1$</td>
</tr>
<tr>
<td>$\rho^\omega$ safety persistence</td>
<td>0.4</td>
<td>$\rho(\omega) = 0.3$</td>
</tr>
<tr>
<td>$\rho^{p\omega}$ corr. safety, disaster</td>
<td>0.5</td>
<td>$\rho(p, \omega) = 0.4$</td>
</tr>
</tbody>
</table>

Table 1: externally set parameters

We calibrate $\sigma^\omega$ in the next subsection to match the conditional correlation between equity returns and excess foreign bond returns; following Jiang et al. (2021), the standard deviation of the swapped G10/T-bill spread may understate the volatility of $\omega$ if swapped G10 bonds are also partially valued for their liquidity or safety.

### 4.3 Calibrated parameters

We calibrate the remaining model parameters to match evidence on the business cycle, asset prices, and cross-border wealth and portfolios. Table 2 reports the moment in model and data that each parameter is most closely linked to.

In terms of output and the business cycle, the population in Foreign is set to 1.6 to match the fact that the G10 plus other euro area countries’ GDP was on average 1.6 times that of the U.S. over the sample period. The standard deviation of

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22The other euro area countries included besides Germany are Austria, Belgium, Spain, Finland, France, Greece, Ireland, Italy, Luxembourg, Netherlands, and Portugal. We use the annual GDP.
<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Moment</th>
<th>Target</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta^*$ rel. pop.</td>
<td>1.6</td>
<td>$y^*/(sy)$</td>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td>$\sigma^z$ std. dev. prod.</td>
<td>0.002</td>
<td>$\sigma(\Delta \log c)$</td>
<td>0.5%</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\sigma^F$ std. dev. rel. prod.</td>
<td>0.007</td>
<td>$\sigma(\Delta \log y^*)$</td>
<td>0.8%</td>
<td>0.8%</td>
</tr>
<tr>
<td>$\rho^F$ persist. rel. prod.</td>
<td>0.9</td>
<td>$\rho(y^<em>/y, y^</em>/y-1/y-1)$</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>$\chi^z$ capital adj cost</td>
<td>3</td>
<td>$\sigma(\Delta \log x)$</td>
<td>1.6%</td>
<td>1.6%</td>
</tr>
<tr>
<td>$\beta^*$ disc. fac. Foreign</td>
<td>0.9892</td>
<td>$4r$</td>
<td>2.0%</td>
<td>2.0%</td>
</tr>
<tr>
<td>$\beta$ disc. fac. Home</td>
<td>0.9887</td>
<td>$nfa/(4y)$</td>
<td>-23%</td>
<td>-23%</td>
</tr>
<tr>
<td>$\sigma^{\omega}$ std. dev. safety</td>
<td>0.91</td>
<td>$\rho_{-1}(r^e, r^* + \Delta \log q - r)$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$\gamma^*$ RRA Foreign</td>
<td>24</td>
<td>$4[r^e - r]$</td>
<td>5.1%</td>
<td>5.2%</td>
</tr>
<tr>
<td>$\gamma$ RRA Home</td>
<td>21</td>
<td>$\beta((\Delta nfa)/y, r^e - r)$</td>
<td>0.5</td>
<td>0.6</td>
</tr>
<tr>
<td>$b^g$ safe debt/agg. cons.</td>
<td>0.13</td>
<td>$b_{H,s}^*(4y)$</td>
<td>3.8%</td>
<td>3.8%</td>
</tr>
<tr>
<td>$\bar{v}$ $\ell$ disutility</td>
<td>0.73</td>
<td>$\ell$</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$\bar{v}^<em>$ $\ell^</em>$ disutility</td>
<td>0.71</td>
<td>$\ell^*$</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$i$ Taylor intercept Home</td>
<td>0.49%</td>
<td>log $P/P_{-1}$</td>
<td>0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>$i^*$ Taylor intercept Foreign</td>
<td>0.47%</td>
<td>log $P^<em>/P^</em>_{-1}$</td>
<td>0%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Table 2: targeted moments and calibrated parameters

Notes: second moments are reported over quarterly frequency. Data moments are estimated over Q1 1995 – Q4 2019. Model moments are computed by (i) simulating model for 20,000 quarters and discarding first 10,000 quarters; (ii) drawing 100 starting points from remaining 10,000 quarters; (iii) simulating 100 samples beginning from these starting points, with no disaster realizations in sample; (iv) computing moments for each sample and averaging across samples.

global productivity shocks is set to 0.2% to target U.S. quarterly consumption growth volatility of 0.5%. The capital adjustment cost is set to 3 to match U.S. investment growth volatility of 1.6%. The standard deviation of relative productivity shocks is set to 0.7% to match the average output growth volatility of the G10 countries of 0.8%. The autocorrelation is set to 0.9 to match the average year-over-year autocorrelation of G10 countries’ GDP relative to U.S. GDP, linearly detrended, of 0.6.

In terms of asset prices, wealth, and portfolios, Foreign households’ discount factor is set to 0.9892 to target a 2% annualized real interest rate, and Home households’ discount factor is very slightly lower at 0.9887 to target the U.S.’ average net foreign measures in dollars reported by Lane and Milesi-Ferretti (2018).

23Throughout the rest of the paper, we work with log returns in both data and model, though in a slight abuse of notation we continue to write these as $r_t$, $r_t^*$, and so on.
asset position relative to annual GDP of -23% over 1995-2019. The volatility of safety shocks is set to match the conditional correlation of the MSCI ACWI equity return and excess foreign bond return of 0.5. The resulting parameter implies that \( \omega_t \) has a quarterly standard deviation of 0.4% in levels. Foreign households have risk aversion of 24 to target the excess annualized real returns on the MSCI ACWI index of 5.1% over this period. Home households have risk aversion of 21 to target the 0.5pp by which U.S. net foreign assets to GDP rises when excess equity returns rise by 1pp. That is, Home agents must be more risk tolerant than Foreign to match the U.S.’ levered position in capital, consistent with Proposition 4. Finally, \( \overline{b} \) is set so that the level of safe dollar debt owned by Foreign (given by (16)) is 3.8% of annual Home output. This is the average ratio of foreign-owned Treasury bills plus central bank liquidity swaps relative to annual U.S. GDP over 2003-2019.

Lastly, we set agents’ disutility of labor \( \overline{\nu} \) and \( \overline{\nu}^* \) to target average labor supply of one in each country, and the Taylor rule intercepts \( \overline{i} \) and \( \overline{i}^* \) to target average inflation rates of zero in each country, convenient normalizations.

5 Impulse responses and validation

We now summarize the model’s key impulse responses and demonstrate that it matches a number of untargeted comovements between excess foreign bond returns, excess equity returns, output, and U.S. net foreign assets in the data.

24To ensure the ergodic wealth distribution is well centered and does not require an excessively large grid, we further assume that the discount factors are a shallow function of Home’s wealth share (\( \beta_t = \beta - 0.001(\theta_t - \theta) \) and \( \beta_t^* = \beta^* + 0.001(\theta_t - \theta) \)) not internalized by agents in their optimization.

25This conditional correlation is closely related to risk premium on foreign bonds versus dollar bonds. We prefer to match it rather than the average realized excess return because the latter is highly sensitive to the time period used, given the large volatility of realized excess returns. As further described in appendix D, we estimate the conditional correlation following Maggiori (2013).

26The model counterpart to the real MSCI ACWI return, \( r^e \), is the real return on a levered claim on capital with a debt/equity ratio of 0.5, and where the debt is comprised of a fraction \( \frac{1}{1+\zeta} \) 5-year dollar bonds and \( \frac{\zeta}{1+\zeta} \) 5-year Foreign bonds. The 5-year duration of debt is consistent with maturity of outstanding U.S. and European corporate debt in S&P Global (2021). We price a 5-year bond in each currency, even though such assets are not traded, by defining the price at each point in the state space to be what the highest valuation agent would be willing to pay.

27In this regression, we also condition on the contemporaneous excess foreign bond return so that we can isolate the marginal exposure to equity returns.
5.1 Impulse responses to disaster risk shock

We begin by evaluating the responses to a disaster probability shock to provide a benchmark against which to compare the effects of safety shocks.

Figure 2 summarizes a subset of the impulse responses; a full set of responses is provided in appendix C. As demonstrated in the second panel of the top row, realized excess equity returns are negative on impact and then high in the quarters which follow, reflecting a decline in the price of capital on impact and an increase in the risk premium. Because Home agents are more risk tolerant than in Foreign, on aggregate they hold a levered portfolio in capital. The dynamics of excess equity returns thus lower Home’s wealth share initially but then lead to an increase over time, as shown in the second panel of the bottom row. With home bias in consumption, these same dynamics are reflected in relative consumption demand for Home goods and thus the Home real exchange rate in the first panel of the bottom row. In the third panel of the top row, the realized excess return on Foreign bonds is thus positive on impact, while it is negative in the subsequent months: since a disaster would similarly induce a positive excess return on Foreign bonds, the risk premium on Foreign bonds falls when disaster risk is elevated. On the production side, as demonstrated in the third panel of the bottom row, Home output falls (as it does in Foreign, not shown) because the increase in precautionary savings is not met with a sufficient decline in real interest rates. Taken together, the disaster probability shock implies that excess foreign bond returns comove negatively with output, equity returns, and U.S. wealth.

These dynamics extend Maggiori (2017)’s “reserve currency paradox” to a setting with endogenous production and nominal frictions: as in his endowment economy, in the presence of home bias, the relatively risk tolerant country’s currency must depreciate in bad times because equilibrium risk sharing implies that its consumption must disproportionately fall. In appendix C we demonstrate that this holds not just for disaster risk but also global and Foreign productivity shocks.

5.2 Impulse responses to safety shock

We now turn to the impulse responses to a safety shock.

Figure 3 summarizes a subset of impulse responses in the calibrated model as well as two alternative parameterizations which help to isolate the role of nominal rigidity and heterogeneity in risk-bearing capacity; a full set of impulse responses is again
Figure 2: effects of increase in disaster probability

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.

provided in appendix C. When there are no nominal frictions and no heterogeneity in risk tolerance across countries (the light blue responses), the expected real interest rate at Home simply falls to accommodate the increase in safe asset demand, consistent with Proposition 1.\textsuperscript{28} This is achieved by an immediate Home deflation and resulting negative excess equity and Foreign bond return on impact. Since Home is actually long dollar bonds and short Foreign bonds in this case — which hedges the relative labor income and real exchange rate effects of Foreign productivity shocks — it experiences an increase in wealth. This implies a persistent but mild real appreciation of the dollar due to home bias in consumption. Next we introduce nominal rigidity (the medium blue responses), in which case the deflationary pressure underlies a global recession, more severe at Home than Foreign (not shown for brevity), as in Proposition 1. Indeed, the relative decline in Home output is what rationalizes a more dramatic immediate appreciation in Home’s terms of trade and real exchange rate, which absorbs the safety shock when the response of real interest rates is muted

\textsuperscript{28}In this figure and all subsequent tables, we also set $\beta = \beta^*$ whenever we set $\gamma = \gamma^*$. Since the latter is crucial for the economics whereas the former is not, we simply use the label $\gamma = \gamma^*$.\textsuperscript{32}
due to nominal rigidity. With identical risk tolerance, however, the implied patterns in excess returns have only small effects on Home wealth (and thus net foreign assets). With greater risk tolerance at Home (the dark blue responses), the response of Home’s wealth share (and net foreign assets) flips. Home now takes a levered position in capital and Foreign bonds financed by dollar bonds, so it suffers a valuation loss followed by wealth accumulation over time, as in Proposition 3.

Safety shocks thus provide a resolution to the reserve currency paradox: following a safety shock, excess foreign bond returns are high as excess equity returns are high, output rises, and U.S. wealth and thus net foreign assets rise. In appendix D we estimate the effects of a shock to the swapped G10/T-bill spread in the data, finding effects which are consistent with these responses.

\footnote{Consistent with the discussion in section 3.5, these responses to safety shocks are robust to two generalizations of the monetary policy rule (10) in appendix C: we allow the nominal rate in each country to respond to domestic output and we consider inertia in the nominal rate. The first modification slightly dampens the exchange rate and output effects of safety shocks, while the second amplifies the effects on impact. The overall responses remain comparable to the baseline policy rule.}
\[ \omega = \gamma^* \]

\[ \beta(r^*_{t+1} - \Delta \log q_{t+1} - r_{t+1}, \log y_t - \log y_{t-4}) = -0.17 \pm 0.11 \]

\[ \beta(r^*_{t+1} - \Delta \log q_{t+1} - r_{t+1}, r^*_{t+1}) = 0.23 \pm 0.04 \]

\[ \beta((\Delta nfa_{t+1})/y_t, r^*_{t+1} - \Delta \log q_{t+1} - r_{t+1}) = 1.38 \pm 0.30 \]

\begin{tabular}{l|cccc}
& Data & Model & No & $\gamma = \gamma^*$ \\
\hline
\( \beta(r^*_{t+1} - \Delta \log q_{t+1} - r_{t+1}, \log y_t - \log y_{t-4}) \) & -0.17 & -0.11 & 0.00 & -0.11 \\
\( (0.11) \) & & & & \\
\( \beta(r^*_{t+1} - \Delta \log q_{t+1} - r_{t+1}, r^*_{t+1}) \) & 0.23 & 0.06 & -0.00 & 0.06 \\
\( (0.04) \) & & & & \\
\( \beta((\Delta nfa_{t+1})/y_t, r^*_{t+1} - \Delta \log q_{t+1} - r_{t+1}) \) & 1.38 & 1.45 & -3.38 & 0.25 \\
\( (0.30) \) & & & & \\
\hline
\textit{Memo:} \((k - \kappa)/(4y)\) & 60\% & 50\% & 0\% \\
\( b_H/(4y) \) & -103\% & 151\% & 14\% \\
\( b_F/(4y) \) & 20\% & -225\% & -16\% \\
\end{tabular}

Table 3: comovements in the international monetary system

Notes: data moments estimated over 1995 - 2019. Standard errors are given in parenthesis. First two rows use monthly data and thus Hansen and Hodrick (1980) standard errors with 4 lags to correct for overlapping observations. Model moments are computed as described in note to Table 2.

### 5.3 Comovements in the international monetary system

The previous subsections demonstrated that safety shocks as well as greater risk tolerance in the U.S. generate a distinctive set of international comovements. We now demonstrate that, quantitatively, the model matches these in the data.

Table 3 summarizes the key moments. In the data, a 1\( pp \) year-over-year decline in U.S. industrial production forecasts a 0.2\( pp \) higher quarterly excess return on foreign bonds, consistent with the evidence in Lustig, Roussanov, and Verdelhan (2014); a 1\( pp \) higher equity return is associated with a contemporaneous 0.2\( pp \) higher excess return on foreign bonds, consistent with the evidence in Lilley, Maggiori, Neiman, and Schreger (2020); and a 1\( pp \) higher excess return on foreign bonds is associated with a 1.4\( pp \) rise in U.S. net foreign assets to GDP, consistent with the exposure of the U.S. to the dollar exchange rate estimated by Tille (2003) and Gourinchas and Rey (2007a).\(^{30}\) These patterns imply that dollar bonds are a hedge and the U.S. insures the rest of the world by being short the dollar. In model-generated data, these same coefficients are 0.1\( pp \), 0.1\( pp \), and 1.5\( pp \), respectively.

Safety shocks are essential for the model’s success in each dimension. The third column of Table 3 eliminates safety shocks from the model. In this case, excess

\(^{30}\) Consistent with footnote 27, in this regression we also condition on the contemporaneous excess equity return so that we can isolate the marginal exposure to relative bond returns.
Foreign bond returns are essentially unpredictable by output, and the reserve currency paradox implies that excess Foreign bond returns are high when equity returns are low. Moreover, Home net foreign assets comove negatively with excess Foreign bond returns. This is because, as is evident from the bottom panel, the desire to hedge the relative labor income and real exchange rate risk induced by relative productivity shocks induces Home to go long dollar bonds financed by Foreign bonds.

Heterogeneity in risk tolerance is also quantitatively important for the last moment. The fourth column of Table 3 assumes identical risk tolerance across countries. Relative to the baseline model, there would be substantially less trade in assets across countries, evident again from the bottom panel. Home’s net foreign assets in this case comove positively with excess Foreign bond returns, but not enough to match the degree of comovement observed in the data.\(^{31}\)

The fact that safety shocks alone are important for the first two moments, while their interaction with heterogeneity in risk tolerance is important for the last moment, is consistent with our analytical results in section 3. In particular, safety shocks alone imply that the dollar has a negative beta, but it is the interaction with heterogeneity in risk tolerance which shapes international portfolios and in particular means that the U.S. net foreign assets deteriorate when excess Foreign bond returns fall.

Appendix C provides a decomposition of the role of individual model parameters beyond \(\sigma_\omega\) and \(\gamma\) in determining international portfolios and the currency risk premium. Consistent with Proposition 4, the safe dollar debt issued by the Home government \(\bar{b}_g\) pushes up the average Foreign bond holdings of Home since seignorage revenues can offset the carry trade losses upon a safety shock. However, because the supply of Treasury bills relative to aggregate wealth is relatively small in the data, we find this channel to be modest. By contrast, the correlation between safety shocks and disaster risk \(\rho_{p_\omega}\) is quantitatively quite important in accounting for the currency composition of the U.S. external balance sheet and positive risk premium on Foreign bonds. It raises the risk premium because it makes a decline in excess Foreign bond returns more likely when disaster risk is elevated and thus global marginal utility is high. It also induces the U.S. as the relatively risk tolerant investor to take a leveraged

\(^{31}\)The positive comovement reflects the decoupling of net foreign assets and wealth shares in response to relative productivity shocks given identical risk tolerance. A decline in Foreign productivity raises excess Foreign bond returns and raises Home’s net foreign assets as it lends to Foreign so that the latter can smooth consumption intertemporally. This obtains even though Home’s wealth share falls because it is long dollar bonds financed by Foreign bonds.
position in Foreign bonds to insure Foreign against such states of the world.

5.4 Additional untargeted second moments

The previous subsection focused on moments which speak directly to the role of the U.S. in the international monetary system. Here we summarize additional moments of interest regarding returns and exchange rates.

The first panel of Table 4 demonstrates that the model successfully generates excess equity return volatility which is several times that of real interest rate volatility, which in turn is close to the data. In contrast, the model substantially undershoots the volatility of excess foreign bond returns, which the second panel demonstrates is because it undershoots the volatility of exchange rates. The third panel also demonstrates that the model does not resolve the “Backus and Smith (1993) puzzle”, the observation that the data features a low correlation between real exchange rate movements and relative consumption growth, whereas efficient risk sharing in most models implies that this correlation is close to 1. Our model features recursive utility and markets which are not fully complete, but innovations in the real exchange rate remain tightly related to innovations in relative consumption.

Adding to the present framework a distinct set of asset demand shocks, pure shocks

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32 The remaining gap in excess equity return volatility between model and data could be closed if we assume a higher elasticity of intertemporal substitution (above 1). This is the case in most other papers studying production economies with time-varying disaster risk, such as Gourio (2012). We prefer to keep the EIS below 1, both given microeconomic evidence and because an EIS above 1 breaks the comovement of consumption, investment, and output on impact of an increase in risk.
Data Model No
\[ \omega \gamma = \gamma \sigma (\Delta \log y_t) \]

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>No ( \omega )</th>
<th>( \gamma = \gamma^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma(\Delta \log y_t) )</td>
<td>0.59%</td>
<td>0.61%</td>
<td>0.44%</td>
<td>0.60%</td>
</tr>
<tr>
<td>( \sigma(\Delta \log y^*_t) )</td>
<td>0.81%</td>
<td>0.80%</td>
<td>0.75%</td>
<td>0.81%</td>
</tr>
</tbody>
</table>

Table 5: output volatility

Notes: data moments estimated over Q1 1995 - Q4 2019. Model moments are computed as described in note to Table 2.

to the demand for bonds in different currencies as in Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2021), may be useful to resolve both shortcomings. Such shocks could induce additional exchange rate volatility disconnected from macroeconomic aggregates, pushing toward zero the correlation between real exchange rates and relative consumption. We leave it to future work to enrich our framework with these shocks, as the segmentation assumptions typically required to study them would substantially complicate the present analysis. But we note that, to the extent our model is missing such shocks generating disconnected fluctuations in exchange rates, this should not affect the macro-financial comovements which are our focus.\(^33\)

6 Macroeconomic and policy implications

Having used asset price data to validate the model’s core comovements, we now quantify its macroeconomic and policy implications. Safety shocks are an important contributor to global macroeconomic volatility. Greater risk tolerance in the U.S. has both destabilizing and stabilizing effects on U.S. external adjustment. Both of these features played important roles in the Great Recession. Dollar swap lines stimulate output globally and revalue wealth in the U.S.’ favor by mitigating the flight to safety.

6.1 Output volatility

The model first implies that safety shocks are an important contributor to global macroeconomic volatility.

Table 5 summarizes the volatilities of Home and Foreign output. In both data and model, output volatility is higher in Foreign than Home. Comparing the third column

\(^{33}\)See Fukui, Namakura, and Steinsson (2023) for recent work accounting for properties of exchange rates using multiple types of financial shocks.
with the second, the model implies that safety shocks account for more than 25% of the output volatility at Home and 5% of the volatility in Foreign. In other words, safety shocks are meaningful contributors to global volatility, especially so in the U.S. The disproportionate effects of safety shocks on Home output are consistent with the mechanisms described around Proposition 1 and Figure 3: given nominal rigidities, the dollar deflation and appreciation on impact of a flight to safety induces a more severe Keynesian recession at Home. The fourth column indicates that, by contrast, heterogeneity in risk-bearing capacity has a minimal effect on output volatilities. As we show in the next subsection, however, this feature of the global economy is quite important for U.S. external adjustment.

6.2 U.S. external adjustment

Absent safety shocks and especially heterogeneity in risk-bearing capacity, the model implies that U.S. net foreign assets would be substantially less volatile. At the same time, the U.S. would not earn such high returns on its external position, and net exports would bear a greater burden in external adjustment.

We can study these issues by focusing on the country-level budget constraint

$$\Delta nfa_t = nx_t + r^k_t nfa_{t-1} + val_t,$$

(22)

where $nfa_t$ denotes the real value of Home net foreign assets at the end of period $t$, $nx_t$ denotes net exports during period $t$, $r^k_t nfa_{t-1}$ denotes net foreign income if all assets paid the return on capital, and $val_t$ denotes the excess returns arising from relative returns and the composition of Home’s net foreign assets. Appendix C defines each of these terms in our model environment.

The first panel of Table 6 summarizes the volatilities of the components in (22) after scaling by output. In both data and model, the volatility of the change in net foreign assets is substantially larger than the volatility of net exports, though the model understates the difference (because it understates the volatility of excess returns). The third column indicates that safety shocks are an important contributor to volatility in net foreign assets, operating largely through asset returns rather than net exports. The fourth column indicates that greater risk tolerance in the U.S. is especially important to volatility in net foreign assets. Absent its greater risk tolerance, the U.S. would not take large balance sheet exposure to relative returns,
\[ \sigma(\Delta nfa_t/y_t) = \gamma \]

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>No ( \omega )</th>
<th>( \gamma = \gamma^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma(nx_t/y_t) )</td>
<td>11.0%</td>
<td>3.3%</td>
<td>1.6%</td>
<td>0.8%</td>
</tr>
<tr>
<td>( \sigma((\Delta nfa_t - nx_t)/y_t) )</td>
<td>10.9%</td>
<td>3.1%</td>
<td>1.8%</td>
<td>0.2%</td>
</tr>
<tr>
<td>( \Delta nfa/y )</td>
<td>-2.8%</td>
<td>0.2%</td>
<td>0.1%</td>
<td>0.0%</td>
</tr>
<tr>
<td>( nx/y )</td>
<td>-3.2%</td>
<td>-0.6%</td>
<td>-0.2%</td>
<td>0.1%</td>
</tr>
<tr>
<td>( (\Delta nfa - nx)/y )</td>
<td>0.4%</td>
<td>0.7%</td>
<td>0.3%</td>
<td>-0.1%</td>
</tr>
</tbody>
</table>

Table 6: U.S. net foreign asset volatility and means

Notes: volatilities in data estimated over Q1 2006 - Q4 2019 since BEA IIP data is available quarterly only after that date; means are estimated using annual data over Q1 1995 - Q4 2019. Model moments are computed as described in note to Table 2.

and thus net foreign asset volatility would essentially equal that of net exports.

The counterpart to the higher volatility in U.S. net foreign assets is the high returns it earns on its external balance sheet as a levered investor in equities and foreign bonds, related to the empirical literature on the “exorbitant privilege”\(^{34}\). The second panel of Table 6 demonstrates that in both data and model, the U.S. earns positive net foreign income despite running trade deficits on average\(^{35}\). In the absence of safety shocks, the third column indicates the average returns earned by the U.S. would fall by half as there would be less risk in the global economy. In the absence of greater risk tolerance in the U.S., the fourth column indicates that the positive average returns earned by the U.S. would be eliminated altogether.

We can further use the model to understand the dynamics of U.S. external adjustment arising from its levered international portfolio. Iterating on (22) and evaluating news at any date \( t \) (defining for brevity the operator \( \mathbb{E}^{t-1}_t \equiv \mathbb{E}_t - \mathbb{E}_{t-1} \)), we have that

\[
\mathbb{E}^{t-1}_t nfa_t = -\mathbb{E}^{t-1}_t \sum_{h=1}^H \left( \prod_{i=1}^h \frac{1}{1 + r_{t+i}^k} \right) n_{x_{t+h}}
- \mathbb{E}^{t-1}_t \sum_{h=1}^H \left( \prod_{i=1}^h \frac{1}{1 + r_{t+i}^k} \right) val_{t+h} + \mathbb{E}^{t-1}_t \left( \prod_{i=1}^H \frac{1}{1 + r_{t+i}^k} \right) n_{fa_{t+H}}.
\]

\(^{34}\)See for instance Gourinchas and Rey (2007a) and Curcuru, Dvorak, and Warnock (2008).

\(^{35}\)The average U.S. trade deficit to output is of course larger in the data, reflecting the secular decline in net foreign assets which is absent in the model. In the model, the average change in net foreign assets is in fact slightly positive because there are no disaster realizations in sample.
This identity says that a negative innovation in net foreign assets at $t$ must be rebalanced by news about future trade surpluses through period $t + H$ (the trade channel), news about excess returns through period $t + H$ (the valuation channel), or news about a higher net foreign asset position at $t + H$. Taking a large value of $H$ and the covariance of both sides with innovations to net foreign assets, we can decompose U.S. external adjustment into the trade and valuation channels. This is closely related to the decomposition in Gourinchas and Rey (2007b) but does not use linearizations.

Table 7 decomposes the process of external adjustment in the model. The first column indicates that a substantial fraction of U.S. external adjustment in the model occurs via the valuation channel. This primarily reflects the role of excess capital returns: the U.S. is levered in capital financed by dollar bonds and time-varying disaster risk induces time-varying expected excess returns on capital. On impact of an increase in disaster risk, U.S. net foreign assets decline but subsequently rise rapidly as the U.S. earns higher excess returns on its capital position. Absent heterogeneity in risk-bearing capacity in the third column, the U.S. would not have disproportionate balance sheet exposure to disaster risk and net exports would bear essentially all of the burden in external adjustment. Safety shocks play a secondary role in generating a valuation channel, through two mechanisms. First, a flight to safety generates a decline in U.S. wealth which is partially rebalanced in future periods through higher seignorage revenues. Second, the existence of safety shocks implies that the U.S. is slightly more levered in capital (as demonstrated in the bottom panel of Table 3),

Table 7: understanding U.S. external adjustment

<table>
<thead>
<tr>
<th>Model</th>
<th>No $\omega$</th>
<th>$\gamma = \gamma^n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Cov}(\mathbb{E}_t^{t-1}, nfa_t)$</td>
<td>$32%$</td>
<td>$76%$</td>
</tr>
<tr>
<td>$\text{Cov}(\mathbb{E}<em>t^{t-1}, \sum</em>{h=1}^{500} \left( \prod_{i=1}^{h} \frac{1}{1+r_{t+h}^{i}} \right) nx_{t+h}, \mathbb{E}_t^{t-1} nfa_t)$</td>
<td>$68%$</td>
<td>$24%$</td>
</tr>
<tr>
<td>$\text{Cov}(\mathbb{E}<em>t^{t-1}, \sum</em>{h=1}^{500} \left( \prod_{i=1}^{h} \frac{1}{1+r_{t+h}^{i}} \right) val_{t+h}, \mathbb{E}_t^{t-1} nfa_t)$</td>
<td>$0%$</td>
<td>$0%$</td>
</tr>
</tbody>
</table>

Notes: moments are computed as described in note to Table 2, but including disaster realizations.

36 Quantitatively, the role of the valuation channel in our model exceeds the roughly 30% estimated by Gourinchas and Rey (2007b) and Gourinchas, Rey, and Sauzet (2019). Adding other standard business cycle shocks to our model, essentially all of which induce fluctuations without movements in expected excess returns, would bring the valuation channel in our model closer to these estimates.
implying stronger valuation effects resulting from disaster risk shocks.

6.3 Great Recession

The previous two subsections use long, simulated time-series to demonstrate the quantitative importance of safety shocks and greater risk-bearing capacity for macroeconomic outcomes in the global economy. We now use the model to quantify the importance of these features during the Great Recession.

For $p$ we feed in the series estimated by Barro and Liao (2021) and for $\omega$ (via $\omega^d$) we feed in the series estimated by Du et al. (2018a). We assume zero innovations to global productivity $z$ and relative productivity $z_F$. We begin the simulation in 1995 but focus in Figure 4 on the 2006-2011 period of interest.

Just with the observed disaster risk and safety series, the model generates sizable movements in output and net foreign assets relative to the data. In particular, these shocks and model features generate a cumulative decline in Home output of 1.3%, Foreign output of 1.5%, and net foreign assets relative to output of 8.6% from the end of Q3 2007 through Q3 2009. These compare with 4.8%, 5.1%, and 10.0% (not detrended) in the data. Moreover, as the last two panels of the figure make clear, as in the data the change in net foreign assets is not primarily due to net exports but rather returns on the U.S. external position.

Appendix C decomposes the role of each driving force in generating these dynamics. Both play an important role. The flight to safety is important in generating output declines in late 2008, particularly for the U.S. However, the increase in disaster risk is more important in accounting for the persistence of the output decline, particularly in Foreign. Relatedly, the flight to safety plays a key role in generating the U.S. valuation losses in late 2008, but it is the elevated disaster risk thereafter which delivers high excess returns on the U.S. external position.

Appendix C also compares additional variables of interest between model and data. First, nominal interest rates globally (and especially in the U.S.) fall well below zero in the model, while they were constrained by the zero lower bound in the data. While this is consistent with the decline in “shadow rates” in practice, owing to policies such as quantitative easing which are outside the model, this suggests that the model may understate the effects of disaster risk and safety shocks during this period, if anything. Second, the U.S. wealth share in fact slightly rises over 2008-2009
Figure 4: simulation using observed $p$ and $\omega$ series

Notes: $p$ is from Barro and Liao (2021) and $\omega$ is from Du et al. (2018a) (demeaned). Both are scaled to match volatilities of $p$ and $\omega$ in model. Given these shocks, figure depicts average paths starting from 100 points drawn from ergodic distribution as described in note to Table 2.

in the model. While both the increase in disaster risk and flight to safety lower the U.S. wealth share on impact, the elevated disaster risk induces a rise in the wealth share thereafter as the U.S. earns high excess equity returns, while the flight to safety dissipates. This relates to the mixed empirical findings in Dahlquist et al. (2023) and Sauzet (2023) regarding the U.S. wealth share dynamics during this period. Our model generates effects well within the range estimated by these papers, and clarifies that it is fully consistent for the U.S. wealth share to rise over the 2008-2009 period even if, on impact, both shocks reduce it.

6.4 Dollar swap lines

We finally turn from macroeconomic outcomes to policy. We focus on changes in the supply of safe dollar bonds, as through the swap of Foreign bonds for safe dollar bonds by central banks in recent crises.\textsuperscript{37} By (17), such a policy would reduce the

\textsuperscript{37}Strictly speaking, dollar swap lines by the Federal Reserve have involved the issuance of dollars, not safe dollar bonds. Extending the model to feature dollar money which provides liquidity services
value of safety/liquidity $\omega_t$. In this subsection, we quantify its effects.

In particular, we compare the model predictions to the estimated effects of the March 19 and 20, 2020 announcements of expanded dollar swap lines studied in Kekre and Lenel (2023). On March 19, the Federal Reserve announced it would temporarily expand from 5 to 14 the number of central banks which could access its swap lines, and on March 20, it announced it would increase the frequency of its standing swap line operations from weekly to daily. Kekre and Lenel (2023) use high frequency event studies around the announcements to identify their asset pricing effects. In particular, they estimate that the (annualized) three-month Libor rate fell by 12bp, the current and three-month ahead Fed funds rates were essentially unchanged, the dollar depreciated by roughly 70bp versus the G10 currencies, and the S&P 500 rose by roughly 150bp, due to these announcements.

To simulate these announcements in the model, we first need to map the response of the three-month Libor rate into a decline in $\omega_t$. Assuming that the three-month Treasury bill yield was constant around the announcements, the announcement effect on the three-month Libor amounts to a 12bp decline in the Libor/Tbill spread (the “Ted spread”). The volatility of $\omega_t$ in the model is roughly 1.2 times the volatility of the (annualized) Ted spread in the data. Our interpretation of this moment is that intraday funds lent between financial institutions provide some liquidity services, so they understate the true volatility of the liquidity premium $\omega_t$. We thus multiply the 12bp decline in the annualized Ted spread by 1.2 to obtain a 14bp decline in $\omega_t$.

We thus simulate a 14bp decline in $\omega_t$ in the model resulting from the expanded dollar swap lines. Appendix D describes how the decline in $\omega_t$ can be used to estimate a range for the elasticity of safe asset demand $\epsilon^d$ in (17) given a plausible range for the news regarding the expanded supply of safe dollar assets in these announcements. We assume that the swap line usage implies the same persistence in $\omega_t$ as safety shocks in the model (0.4), consistent with the fact that swap line usage fell back to roughly alongside safe dollar bonds (as in Kekre and Lenel (2023)), an increase in the supply of dollar money will reduce the convenience yield $\omega_t$ like an increase in the supply of safe dollar bonds. For simplicity we thus conceptualize swap lines as an increase in the supply of safe dollar bonds here. Its intraday response is not available in Kekre and Lenel (2023).

This is analogous to our interpretation that swapped G10 government bonds also provide some liquidity/safety, so the swapped G10/Tbill spread understates $\omega_t$. However, we acknowledge that the Ted spread may also reflect default risk, not just a liquidity premium. We would have preferred to use the response of the swapped G10/Tbill spread for this reason, but its intraday response is not available in Kekre and Lenel (2023).
Notes: data column are cumulative estimates from Kekre and Lenel (2023) for March 19-20, 2020 announcements (Table 1 in that paper). Model columns simulate a decrease in $\omega_t$ of 14bp starting from the average of the model’s ergodic distribution.

We consider two assumptions on the monetary policy response. Our baseline assumption is that the U.S. nominal interest rate is fixed in the first quarter and follows the Taylor rule thereafter, consistent with the findings in Kekre and Lenel (2023) that near-term Fed funds futures do not respond to the announcements. We consider an alternative assumption that the U.S. nominal interest rate follows the Taylor rule in all periods. In both cases, Foreign follows its standard Taylor rule.

The first two rows of Table 8 present the asset pricing effects of the swap line announcements in data and under these model scenarios. Kekre and Lenel (2023) estimate that the March 19-20 announcements generated a roughly 70bp nominal depreciation of the dollar versus G10 currencies and 150bp increase in the (nominal value of the) S&P 500. With a constant nominal rate in the first quarter, the present model implies a 100bp nominal dollar depreciation and 135bp increase in the dollar value of equities on impact, remarkably consistent with the estimated responses. The last column indicates that, with an active Taylor rule even in the first quarter, these asset pricing responses would be substantially diminished as the central bank tightens the policy rate upon the reduction in $\omega_t$.

The subsequent rows of Table 8 use our structural model to quantify the implied
effects for the real economy, which the empirical analysis in Kekre and Lenel (2023) is unable to do. Focusing on the case with a constant nominal interest rate in the first quarter, the swap line announcements in the model generate a peak increase in U.S. output of roughly 80bp and G10 output of roughly 20bp, indicating that the dollar swap lines played a meaningful macroeconomic stabilization role during the Covid-19 pandemic. The announcements further generate an increase in U.S. net foreign assets to output of roughly 440bp, most of which is accounted for by valuation effects, indicating that the swap lines relaxed the U.S. external budget constraint owing to their effects on asset prices and the exposures of the U.S. balance sheet.

7 Conclusion

In this paper we have proposed a business cycle model of the international monetary system emphasizing a time-varying demand for safe dollar bonds, greater risk-bearing capacity in the U.S. than the rest of the world, and nominal rigidities.

A flight to safety triggers a dollar appreciation and decline in global output. Dollar bonds thus command a negative risk premium and the U.S. insures the rest of the world against such shocks. Quantitatively, the model matches untargeted co-movements between relative bond returns, equity returns, output, and wealth in the global economy. It in turn clarifies that safety shocks are an important driver of global macroeconomic volatility. Heterogeneity in risk-bearing capacity amplifies U.S. net foreign asset volatility but raises the average return earned by the U.S. on its external position and reduces the required role of net exports in external adjustment. Both safety shocks and heterogeneity in risk-bearing capacity were important during the Great Recession. Dollar swap lines are globally stimulative and relax the U.S. external budget constraint by mitigating the flight to safety.

We have deliberately made minimal departures from a workhorse two-country open economy model to focus on the consequences of a time-varying demand for safe dollar bonds and greater U.S. risk-bearing capacity in this context. We view the introduction of additional heterogeneity within countries, particularly in the form of financial intermediaries, as among the most fruitful ways to enrich this framework going forward. Shocks in the interbank market or to financial constraints facing banks may provide a deeper microfoundation of the flight to safe dollar bonds,\footnote{See Bianchi, Bigio, and Engel (2022) and Devereux, Engel, and Wu (2023) for recent work in} and...
currency intermediation by these banks could bring the volatilities of exchange rates closer to the data. The negative beta of the dollar could also help explain why dollar funding is an important feature of bank balance sheets.\textsuperscript{41} The effects of the dollar exchange rate on intermediary wealth may in turn help to account for the effects of U.S. monetary policy on global financial conditions and especially on emerging markets.\textsuperscript{42} We leave these exciting questions for future work.

References


\textsuperscript{41}See Adrian, Etula, and Shin (2010) and Bruno and Shin (2015a,b) for such evidence.


Appendix for Online Publication

A Equilibrium

In this appendix we provide additional details on the equilibrium excluded from the main text for brevity. We first specify the optimization problems and policy in Foreign. We then outline the market clearing conditions. Finally, we define the equilibrium and characterize the model’s equilibrium conditions and solution.

A.1 Optimization problems and policy in Foreign

Households  The representative Foreign household seeks to maximize

$$v_t^* = \left(1 - \beta^*\right) \left(c_t^* \Phi^* (\ell_t^*) \Omega_t^* \left(B_{Ht,s}^*/\left(E_t P_t^*\right)\right)\right)^{1-1/\psi} + \beta^* E_t \left[\left(v_{t+1}^*\right)^{1-1/\psi}\right],$$

subject to the consumption aggregator

$$c_t^* = \left(\frac{1}{1 + \zeta^*} - \frac{\zeta}{\xi^*}\right)^{\frac{1}{\sigma}} \left(c_{Ht}^*\right)^{\frac{1}{\sigma} - 1} + \left(\frac{\zeta^*}{1 + \zeta^*} + \frac{\zeta^*}{\xi^*}\right)^{\frac{1}{\sigma}} \left(c_{Ft}^*\right)^{\frac{1}{\sigma} - 1},$$

the disutility of labor

$$\Phi^* (\ell_t^*) = \left(1 + (1/\psi - 1) \nu^* (\ell_t^*)^{1+1/\nu} \right)^{1/\psi} \left(1 + 1/\nu\right),$$

the utility from safe dollar bonds

$$\Omega_t^* \left(\frac{B_{Ht,s}^*}{E_t^{-1} P_t^*}\right) = \exp \left(\omega_t d B_{Ht,s}^* \left(\frac{B_{Ht,s}^*}{E_t^{-1} P_t^* c_t^*}\right)^{2} - \frac{1}{2} e^d \left(\frac{B_{Ht,s}^*}{E_t^{-1} P_t^* c_t^*}\right)^{2}\right),$$

and the resource constraint

$$E_t P_{Ht} c_{Ht}^* + P_{Ft} c_{Ft}^* + E_t B_{Ht,s}^* + E_t B_{Ht,o}^* + B_{Ft}^* + E_t Q_t^* k_t^* \leq$$
\[ E_t(1+i_{t-1})B^*_{Ht-1,s} + E_t(1+i_{t-1})B^*_{Ht-1,o} + (1+i_{t-1})B^*_{Ft-1} + E_t(\Pi_t + (1-\delta)Q^h_t)k^*_t \exp(\varphi_t) + \int_0^1 W_t^*(j^*)e_t(j^*)dj^* - \int_0^1 AC_t^W(W^*(j^*))dj^* + T^*_t, \]

where the cost of setting wages is given by

\[ AC_t^W(W^*(j^*)) = \frac{\chi}{2} W_t^* \ell_t^* \left( \frac{W_t^*(j^*)}{W_{t-1}^*(j^*) \exp(\varphi_t)} - 1 \right)^2. \]

**Labor unions** Foreign union \( j^* \) chooses the wage \( W_t^*(j^*) \) and labor supply \( \ell_t^*(j^*) \) to maximize the utilitarian social welfare of union members.

**Labor packer** A representative Foreign labor packer purchases varieties supplied by each union and combines them to produce a CES aggregate with elasticity of substitution \( \epsilon \) and sold at \( W_t^* \) to domestic firms. The labor packer thus earns

\[ W_t^* \xi^* \left[ \int_0^1 \ell_t^*(j^*)^{(\epsilon-1)/\epsilon} \right]^{\epsilon/(\epsilon-1)} - \int_0^1 W_t^*(j^*)\xi^* \ell_t^*(j^*)dj^*. \]

**Production** The representative Foreign producer hires \( \ell_t^* \) units of labor and rents \( \kappa_t^* \) units of capital to maximize

\[ P_t^*(\zeta_t\zeta_t^*\ell_t^*)^{-\alpha} (\kappa_t^*)^\alpha - W_t^* \xi^* \ell_t^* - E_t \Pi_t \kappa_t^*. \]

**Policy** Monetary policy is characterized by a Taylor rule

\[ 1 + i_t^* = (1 + \bar{i}^*) \left( \frac{P_t^*}{P_{t-1}^*} \right)^\phi, \]

where \( P_t^* \) is the ideal price index

\[ P_t^* = \left[ \left( \frac{1}{1+\zeta^*} - \frac{\zeta_t}{\zeta^*} \right)(E_tP_{Ht})^{1-\sigma} + \left( \frac{\zeta^*}{1+\zeta^*} + \frac{\zeta_t}{\zeta^*} \right)P_{Ft}^{1-\sigma} \right]^{1/\sigma}. \]

Fiscal policy is characterized by lump-sum transfers

\[ T_t^* = \int_0^1 AC_t^W(j^*)dj^*. \]
A.2 Market clearing

Market clearing in goods each period is

\[ c_{Ht} + \zeta c^*_{Ht} + \left( \frac{\bar{k}_t}{\bar{k}_{t-1} \exp(\phi_t)} \right)^\chi x_{Ht} = (z_t\ell_t)^{1-\alpha} (\kappa_t)^\alpha, \]  

(23)

\[ c_{Ft} + \zeta c^*_{Ft} + \left( \frac{\bar{k}_t}{\bar{k}_{t-1} \exp(\phi_t)} \right)^\chi x_{Ft} = (z_t\zeta F_t\ell^*_t)^{1-\alpha} (\kappa^*_t)^\alpha, \]  

(24)

in labor is

\[
\left[ \int_0^1 \ell_t(j)^{(\epsilon-1)/\epsilon} dj \right]^{\epsilon/(\epsilon-1)} = \ell_t, 
\]

(25)

\[
\left[ \int_0^1 \ell^*_t(j^*)^{(\epsilon-1)/\epsilon} dj^* \right]^{\epsilon/(\epsilon-1)} = \ell^*_t, 
\]

(26)

in the capital rental market is

\[ \kappa_t + \kappa^*_t = \bar{k}_{t-1} \exp(\phi_t), \]  

(27)

in the capital market is

\[ k_{t-1} + \zeta^* k^*_{t-1} = \bar{k}_{t-1}, \]  

(28)

\[ (1-\delta)\bar{k}_{t-1} \exp(\phi_t) + x_t = \bar{k}_t, \]  

(29)

and in bonds is

\[ B^*_{Ht,s} + \zeta^* B^*_{Ht,s} + B^*_{Ht,o} = 0, \]  

(30)

\[ B^*_{Ht,o} + \zeta^* B^*_{Ht,o} = 0, \]  

(31)

\[ B^*_{Ft} + \zeta^* B^*_{Ft} = 0. \]  

(32)

A.3 Definition of equilibrium

Definition 2. An equilibrium is a sequence of prices and policies such that:

- each Home representative household chooses \( \{ c_{Ht}, c_{Ft}, B^*_{Ht,s}, B^*_{Ht,o}, B^*_{Ft}, k_t \} \) to maximize (1) subject to (2)-(5) and analogously in Foreign;
• each Home union $j$ chooses $\{W_t(j), \ell_t(j)\}$ to maximize the utilitarian social welfare of its members subject to (5), and analogously in Foreign;

• the representative Home labor packer chooses $\{\ell_t(j)\}$ to maximize profits (6) and analogously in Foreign;

• the representative Home producer chooses $\{\ell_t, \kappa_t\}$ to maximize profits (7) and analogously in Foreign;

• the representative global capital producer chooses $\{x_{Ht}, x_{Ft}, x_t\}$ to maximize profits (9) subject to (8);

• the Home government sets $B^g_{Ht,s}$ according to (12) and $\{i_t, \{T_t\}\}$ according to (10) and (13), and the Foreign government analogously does the latter;

• the goods, factor, and asset markets clear according to (23)-(32).

### A.4 Additional variables

Before turning to the model analysis, defining several additional variables will be helpful. Except for the nominal interest rates $i_t$ and $i^*_t$, we use lower-case variables to denote real variables.

We first define several important relative prices: the real exchange rate

$$q_t \equiv \frac{E_t P_t}{P^*_t},$$

the real interest rates

$$1 + r_{t+1} \equiv (1 + i_t) \frac{P_t}{P_{t+1}},$$

$$1 + r^*_{t+1} \equiv (1 + i^*_t) \frac{P_t}{P^*_{t+1}},$$

and the real return to capital (expressed in Home consumption goods)

$$1 + r^k_{t+1} \equiv \frac{(\Pi_{t+1} + (1 - \delta)Q^k_{t+1})}{Q^k_t} \frac{P_t}{P_{t+1}} \exp(\varphi_{t+1}).$$
We then define several important quantities: at Home (with analogous definitions in Foreign), output
\[ y_t \equiv (z_t f_t)^{1-\alpha} (\kappa_t)^{\alpha}, \]
the real value of aggregate saving
\[ a_t \equiv \frac{1}{P_t} \left( B_{Ht} + E_t^{-1} B_{Ft} + Q_t^k k_t \right), \]
and the real value of net foreign assets
\[ nfa_t \equiv a_t - Q_t^k \kappa_t+1 \exp(-\varphi_{t+1}), \]
where we define all of these variables at the end of the period, consistent with the way they are measured in the data.\(^{43,44}\)

### A.5 First-order conditions

#### A.5.1 Households

The representative Home household’s intratemporal optimality is characterized by
\[ \frac{c_{Ht}}{c_{Ft}} = \left( \frac{1}{1+\varsigma} + \varsigma \right) s_t^{-\sigma}, \]
where we denote the terms of trade
\[ s_t \equiv \frac{E_t P_{Ht}}{P_{Ft}^*}. \]

Given the household’s pricing kernel
\[ m_{t,t+1} = \beta \frac{c_t}{c_{t+1}} \left( \frac{c_{t+1} \Phi(\ell_{t+1})}{c_t \Phi(\ell_t)} \right)^{1-1/\psi} \left( \frac{v_{t+1}}{ce_t} \right)^{1/\psi-\gamma}, \]

\(^{43}\)While \( \kappa_{t+1} \) and \( \varphi_{t+1} \) are only known after shocks have realized at \( t+1 \), we still date net foreign assets as of \( t \). This is sensible if shocks are realized “just after” a period starts.

\(^{44}\)By multiplying \( \kappa_{t+1} \) by \( \exp(-\varphi_{t+1}) \) in the definition of net foreign assets, we are undoing the effect of capital destruction at \( t+1 \) and thus appropriately comparing how much capital is used in production at Home with the capital owned by Home residents.
(where we have used that \( \Omega_t(B_{Ht,s}/P_t) = 1 \) at all dates and states) as well as the certainty equivalent

\[
cc_t = \mathbb{E}_t \left[(v_{t+1})^{1-\gamma} \right]^{\frac{1}{1-\gamma}},
\]

its intertemporal optimality is characterized by

\[
1 = \mathbb{E}_t m_{t,t+1} \left( \frac{1 + r_{t+1}}{1 - \omega_t} \right),
\]

\[
1 = \mathbb{E}_t m_{t,t+1} \frac{q_t}{q_{t+1}} (1 + r^*_t),
\]

\[
1 = \mathbb{E}_t m_{t,t+1} (1 + r^k_t),
\]

where the first equation is implied by optimality in either safe dollar bonds or other dollar bonds given the definition of \( \omega_t \) in (15) in the main text. Substituting in government transfers (13) into the resource constraint (4), dividing by \( P_t \), and denoting \( b_{Ht,s} \equiv B_{Ht,s} P_t \), \( b_{gHt,s} \equiv B_{gHt,s} P_t \), \( b_{Ht,o} \equiv B_{Ht,o} P_t \), \( b_{Ft} \equiv B_{Ft} P^* \), \( q_t \equiv Q_t P_t \), \( \pi_t \equiv \Pi_t P_t \), and \( w_t \equiv W_t P_t \), the household’s resource constraint becomes

\[
c_t + b_{Ht,s} + b_{gHt,s} + b_{Ht,o} + q_t b_{Ft} + q_t^k k_t = w_t l_t +
\]

\[
(1 + r_t) (b_{Ht-1,s} + b_{gHt-1,s}) + \left( \frac{1 + r_t}{1 - \omega_{t-1}} \right) b_{Ht-1,o} + q_t^{-1} (1 + r^*_t) b_{Ft-1} + 
\]

\[
(\pi_t + (1 - \delta) q_t^k) k_{t-1} \exp(\varphi_t).
\]

Households’ first-order conditions in Foreign are analogous. Their resource constraint becomes

\[
c_t^* + q_t b_{Ht,s}^* + q_t b_{Ht,o}^* + b_{Ft}^* + q_t q_t^k k_t = q_t w_t^* l_t^* +
\]

\[
q_t (1 + r_t) b_{Ht-1,s}^* + q_t \left( \frac{1 + r_t}{1 - \omega_{t-1}} \right) b_{Ht-1,o}^* + (1 + r^*_t) b_{Ft-1}^* + 
\]

\[
q_t (\pi_t + (1 - \delta) q_t^k) k_{t-1} \exp(\varphi_t),
\]

where \( b_{Ht,s}^* \equiv B_{Ht,s}^* P_t^* \), \( b_{Ht,o}^* \equiv B_{Ht,o}^* P_t^* \), \( b_{Ft}^* \equiv B_{Ft}^* P_t^* \), and \( w_t^* \equiv W_t^* E_t P_t^* \).

Now consider households’ optimal choice of safe dollar bonds. Given the assumed
functional forms of \( \Omega_t \) and \( \Omega_t^* \), we have by (15) that

\[
\omega_t = \omega_t^d - \frac{1}{c_t} \frac{B_{Ht,s}}{P_t c_t} = \omega_t^d - \frac{1}{c_t} \frac{B_{Ht,s}^*}{E_t^{-1} P_t^* c_t^*}
\]

and thus

\[
\frac{B_{Ht,s}}{P_t c_t} = \frac{B_{Ht,s}^*}{E_t^{-1} P_t^* c_t^*}.
\]

Combining this with global market clearing in safe dollar bonds, straightforward algebra yields

\[
B_{Ht,s} = \frac{P_t c_t}{P_t c_t + \zeta^* E_t^{-1} P_t^* c_t^*} (-B_{Ht,s}^g),
\]

\[
B_{Ht,s}^* = \frac{E_t^{-1} P_t^* c_t^*}{P_t c_t + \zeta^* E_t^{-1} P_t^* c_t^*} (-B_{Ht,s}^g).
\]

It follows that, now in real terms, we can re-write the Home household’s resource constraint as

\[
c_t + \omega_t \frac{\zeta^* q_t^{-1} c_t^*}{c_t + \zeta^* q_t^{-1} c_t^*} b_{Ht,s}^g + (1 - \omega_t) \left( b_{Ht,s} + b_{Ht,o}^g \right) + b_{Ht,o} + q_t^{-1} b_{Ft} + q_t^k k_t = w_t \ell_t + (1 + r_t) \left( b_{Ht-1,s} + b_{Ht-1,o}^g \right) + \left( \frac{1 + r_t}{1 - \omega_{t-1}} \right) b_{Ht-1,o} + q_t^{-1} (1 + r_t^*) b_{Ft-1} + (\pi_t + (1 - \delta) q_t^k)^{k_{t-1}} \exp(\varphi_t)
\]

and the Foreign household’s resource constraint as

\[
c_t^* - q_t \omega_t \frac{q_t^{-1} c_t^*}{c_t + \zeta^* q_t^{-1} c_t^*} b_{Ht,s}^g + q_t (1 - \omega_t) b_{Ht,s}^* + q_t b_{Ht,o} + b_{Ft}^* + q_t q_t^k k_t = q_t w_t^* \ell_t^* + q_t (1 + r_t) b_{Ht-1,s} + q_t \left( \frac{1 + r_t}{1 - \omega_{t-1}} \right) b_{Ht-1,o}^* + (1 + r_t^*) b_{Ft-1}^* + q_t (\pi_t + (1 - \delta) q_t^k)^{k_{t-1}} \exp(\varphi_t).
\]

Defining households’ net positions in dollar-denominated bonds

\[
b_{Ht} \equiv (1 - \omega_t) \left( b_{Ht,s} + b_{Ht,o}^g \right) + b_{Ht,o},
\]

\[
b_{Ht}^* \equiv (1 - \omega_t) b_{Ht,s}^* + b_{Ht,o}^*.
\]
their positions in safe dollar bonds are only relevant insofar as they determine the seignorage earned by Home from the safe dollar bonds purchased by Foreign, given by the second term in each resource constraint.

### A.5.2 Unions

The representative union’s first-order condition is

\[
 w_t - \frac{c_t \Phi'(\ell_t)}{\Phi(\ell_t)} + w_t \frac{\chi^W}{\epsilon} \left[ \frac{w_t}{w_{t-1} \exp(\varphi_t)} \frac{P_t}{P_{t-1}} \left( \frac{w_t}{w_{t-1} \exp(\varphi_t)} \frac{P_t}{P_{t-1}} - 1 \right) \right.

- \left. \mathbb{E}_t m_{t+1} \left( \frac{w_{t+1}}{w_t \exp(\varphi_{t+1})} \right)^2 \frac{P_{t+1}}{P_t} \frac{\ell_{t+1}}{\ell_t} \left( \frac{w_{t+1}}{w_t \exp(\varphi_{t+1})} \frac{P_{t+1}}{P_t} - 1 \right) \right] = 0,
\]

The representative union’s first-order condition in Foreign is analogous.

### A.5.3 Producers

The representative Home producer’s first-order conditions are

\[
 w_t = \frac{P_{Ht}}{P_t} (1 - \alpha) z_t^{1-\alpha} \ell_t^{-\alpha} \kappa_t^\alpha, \\
 \pi_t = \frac{P_{Ht}}{P_t} \alpha z_t^{1-\alpha} \ell_t^{-\alpha} \kappa_t^{\alpha-1}.
\]

The representative Foreign producer’s first-order conditions are

\[
 w_t^* = q_t^{\alpha} \frac{P_{Ft}}{P_t} (1 - \alpha) (z_t z_{Ft})^{1-\alpha} (\zeta_t^* \ell_t^*)^{-\alpha} \kappa_t^\alpha, \\
 \pi_t = q_t^{1-\alpha} \frac{P_{Ft}}{P_t} \alpha (z_t z_{Ft})^{1-\alpha} (\zeta_t^* \ell_t^*)^{1-\alpha} \kappa_t^{\alpha-1}.
\]

Finally, the representative producer of capital’s first-order conditions are

\[
 \frac{x_{Ht}}{x_{Ft}} = \frac{1}{\zeta^* s_t^\sigma}, \\
 q_t^k = \left( \frac{\bar{k}_t}{k_{t-1} \exp(\varphi_t)} \right)^{\chi^*} \left( \frac{1}{1 + \zeta^*} \left( \frac{P_{Ht}}{P_t} \right)^{1-\sigma} + \left( \frac{\zeta^*}{1 + \zeta^*} \right) \left( \frac{P_{Ft}}{P_t} \right)^{1-\sigma} \right)^{-\frac{1}{1-\sigma}}.
\]
Define the re-scaled variables

\begin{align*}
\tilde{c}_t &\equiv \frac{c_t}{z_t}, \quad \tilde{c}_{Ht} \equiv \frac{c_{Ht}}{z_t}, \quad \tilde{c}_{Ft} \equiv \frac{c_{Ft}}{z_t}, \quad \tilde{\tilde{c}}_t \equiv \frac{c_{\tilde{t}}}{z_t}, \quad \tilde{m}_{t,t+1} \equiv m_{t,t+1} \left( \frac{z_{t+1}}{z_t} \right)^\gamma,
\end{align*}

\begin{align*}
\tilde{b}_{Ht} &\equiv \frac{b_{Ht}}{z_t}, \quad \tilde{b}_{Ht,s} \equiv \frac{b_{Ht,s}}{z_t}, \quad \tilde{b}_{Ft} \equiv \frac{b_{Ft}}{z_t}, \quad \tilde{\tilde{b}}_{Ht-1} \equiv \frac{\tilde{b}_{Ht-1}}{\exp(\sigma^2 \epsilon_t^2 + \varphi_t)}, \quad \tilde{\tilde{b}}_{Ft-1} \equiv \frac{\tilde{b}_{Ft-1}}{\exp(\sigma^2 \epsilon_t^2 + \varphi_t)},
\end{align*}

\begin{align*}
\tilde{k}_t &\equiv \frac{k_t}{z_t}, \quad \tilde{k}_{t-1} \equiv \frac{k_{t-1}}{z_t}, \quad \tilde{\tilde{k}}_t \equiv \frac{\tilde{k}_t}{\exp(\sigma^2 \epsilon_t^2)}, \quad \tilde{\tilde{\tilde{k}}}_{t-1} \equiv \frac{\tilde{k}_{t-1}}{\exp(\sigma^2 \epsilon_t^2)},
\end{align*}

\begin{align*}
\tilde{w}_t &\equiv \frac{w_t}{z_t}, \quad \tilde{\tilde{w}}_{t-1} \equiv \frac{\tilde{w}_{t-1}}{\exp(\sigma^2 \epsilon_t^2)}.
\end{align*}

The re-scaled Home household first-order conditions and constraints are:

\begin{align*}
\bar{v}_t &= \left( (1 - \beta) (\tilde{c}_t \Phi(\ell_t))^{1-1/\psi} + \beta (\tilde{\tilde{c}}_t) \right)^{1/\psi}, \quad \text{(33)}
\end{align*}

\begin{align*}
\bar{\tilde{c}}_t &= \mathbb{E}_t \left[ \exp \left( \left( (1 - \gamma) \left[ \sigma^2 \epsilon_{t+1}^2 + \varphi_{t+1} \right] \right) \tilde{v}_{t+1} \right) \right]^{1/\gamma}, \quad \text{(34)}
\end{align*}

\begin{align*}
\bar{\tilde{c}}_t &= \left( \left( \frac{1}{1 + \zeta^*} + \varsigma \right)^{1/\sigma} + \left( \frac{\zeta^*}{1 + \zeta^*} - \varsigma \right)^{1/\sigma} \left( \tilde{c}_{Ft} \right)^{\sigma-1} \right)^{\sigma^{\sigma-1}}, \quad \text{(35)}
\end{align*}

\begin{align*}
\bar{\tilde{c}}_{Ht} &= \frac{1}{1 + \zeta^* + \varsigma} s_t^{-\sigma}, \quad \text{(36)}
\end{align*}

\begin{align*}
\bar{\tilde{m}}_{t,t+1} &= \beta \left( \bar{\tilde{c}}_{t+1} \Phi(\ell_{t+1}) \right)^{1-1/\psi} \left( \bar{\tilde{v}}_{t+1} \right)^{1/\psi - \gamma}, \quad \text{(37)}
\end{align*}

\begin{align*}
1 &= \mathbb{E}_t \tilde{m}_{t,t+1} \exp \left( -\gamma \left[ \sigma^2 \epsilon_{t+1}^2 + \varphi_{t+1} \right] \right) \left( 1 + r_{t+1} \right) \left( 1 - \omega_t \right), \quad \text{(38)}
\end{align*}

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\[ 1 = \mathbb{E}_{t} \tilde{m}_{t+1} \exp \left( -\gamma \left[ \sigma^{*} \epsilon_{t+1} + \varphi_{t+1} \right] \right) \frac{q_{t+1}}{q_{t}} (1 + r_{t+1}), \]
\[ 1 = \mathbb{E}_{t} \tilde{m}_{t+1} \exp \left( -\gamma \left[ \sigma^{*} \epsilon_{t+1} + \varphi_{t+1} \right] \right) (1 + r_{t+1}), \]
\[ \ddot{c}_{t} + \ddot{b}_{Ht} + q_{t}^{-1} \ddot{b}_{Ft} + q_{t}^{k} \ddot{k}_{t} = \ddot{w}_{t} \ell_{t} + \theta_{t} (\pi_{t} + (1 - \delta) q_{t}^{k}) \ddot{k}_{t-1}. \]

The re-scaled Foreign household first-order conditions and constraints are:

\[ \ddot{c}_{t}^{*} = \left( \left( 1 - \beta \right) (\ddot{c}_{t}^{*} \Phi^{*}(\ell_{t}^{*}))^{(1-1/\psi)} + \beta (\ddot{c}_{t}^{*})^{(1-1/\psi)} \right)^{1/(1-\psi)}, \]
\[ \ddot{c}_{t}^{*} = \mathbb{E}_{t} \left[ \exp \left( (1 - \gamma^{*}) \left[ \sigma^{*} \epsilon_{t+1}^{*} + \varphi_{t+1}^{*} \right] \right) (\ddot{c}_{t+1}^{*})^{(1-\gamma^{*})} \right]^{1/(1-\gamma^{*})}, \]
\[ \ddot{c}_{t}^{*} = \left( \left( \frac{1}{1 + \zeta^{*}} - \frac{\zeta}{\zeta^{*}} \right)^{\frac{1}{2}} (\ddot{c}_{Ht}^{*})^{\frac{1}{2}} \right) \left( \left( \frac{1}{1 + \zeta^{*}} + \frac{\zeta}{\zeta^{*}} \right)^{\frac{1}{2}} (\ddot{c}_{Ft}^{*})^{\frac{1}{2}} \right), \]
\[ \frac{\ddot{c}_{Ht}^{*}}{\ddot{c}_{Ft}^{*}} = \frac{1 + \zeta - \frac{\zeta}{\zeta^{*}}} {1 + \zeta^{*} + \frac{\zeta}{\zeta^{*}}} \left[ \frac{\ddot{c}_{Ht+1}^{*} \Phi^{*}(\ell_{t+1})}{\ddot{c}_{Ft+1}^{*} \Phi^{*}(\ell_{t+1})} \right]^{1/2} \left( \left( \frac{\ddot{c}_{Ht+1}^{*}}{\ddot{c}_{Ft+1}^{*}} \right)^{(1-\gamma^{*})} \right), \]
\[ 1 = \mathbb{E}_{t} \tilde{m}_{t+1}^{*} \exp \left( -\gamma^{*} \left[ \sigma^{*} \epsilon_{t+1}^{*} + \varphi_{t+1}^{*} \right] \right) \frac{q_{t+1}}{q_{t}} (1 + r_{t+1}), \]
\[ 1 = \mathbb{E}_{t} \tilde{m}_{t+1}^{*} \exp \left( -\gamma^{*} \left[ \sigma^{*} \epsilon_{t+1}^{*} + \varphi_{t+1}^{*} \right] \right) (1 + r_{t+1}), \]
\[ 1 = \mathbb{E}_{t} \tilde{m}_{t+1}^{*} \exp \left( -\gamma^{*} \left[ \sigma^{*} \epsilon_{t+1}^{*} + \varphi_{t+1}^{*} \right] \right) \frac{q_{t+1}}{q_{t}} (1 + r_{t+1}), \]
\[ q_{t}^{-1} \ddot{c}_{t}^{*} + \ddot{b}_{Ht}^{*} + q_{t}^{-1} \ddot{b}_{Ft}^{*} + q_{t}^{k} \ddot{k}_{t}^{*} = \ddot{w}_{t} \ell_{t}^{*} + \frac{1}{\zeta^{*}} (1 - \theta_{t}) (\pi_{t} + (1 - \delta) q_{t}^{k}) \ddot{k}_{t-1}. \]

The global wealth share of Home households, inclusive of seignorage, is

\[ \theta_{t+1} = \frac{1}{(\pi_{t+1} + (1 - \delta) q_{t+1}^{k}) \ddot{k}_{t}} \left[ \frac{1 + r_{t+1}}{1 - \omega_{t}} \ddot{b}_{Ht}^{*} + \frac{1}{q_{t+1}} (1 + r_{t+1}) \ddot{b}_{Ft}^{*} \right. \]
\[ + \left. (\pi_{t+1} + (1 - \delta) q_{t+1}^{k}) \ddot{k}_{t} - \omega_{t+1} \frac{\zeta^{*} q_{t+1}^{k} \ddot{k}_{t+1}}{\ddot{c}_{t+1}^{*} + \zeta^{*} q_{t+1}^{k} \ddot{c}_{t+1}^{*}} \ddot{b}_{Ht+1}^{*} \right]. \]

Supply-side optimality requires:

\[ \ddot{w}_{t} - \ddot{c}_{t} \Phi^{*}(\ell_{t}) + \ddot{w}_{t} \frac{\chi^{W}}{\Phi(\ell_{t})} \left[ \frac{\ddot{w}_{t}}{\ddot{w}_{t-1}} \frac{P_{t}}{P_{t-1}} \left( \frac{\ddot{w}_{t}}{\ddot{w}_{t-1}} \frac{P_{t}}{P_{t-1}} - 1 \right) \right] \]
\[- \mathbb{E}_t m_{t,t+1} \exp \left( -\gamma \left[ \sigma \epsilon_{t+1}^* + \varphi_{t+1} \right] \right) \times \]
\[
\frac{(\tilde{w}_{t+1})}{\tilde{w}_t} = \frac{P_{t+1} \ell_{t+1}}{P_t \ell_t} \left( \frac{\tilde{w}_{t+1} P_{t+1}}{P_t} - 1 \right) \]
\[
(52) \]
\[
\bar{\bar{w}}_t^* = q_t^{-1} \tilde{c}_t^* \Phi^\prime(\ell_t^*) + \tilde{w}_t^* \frac{\chi^W}{\epsilon} \left[ \frac{\tilde{w}_t^*}{\tilde{w}_{t-1}^*} \frac{E_t P_t}{E_{t-1} P_{t-1}} \left( \frac{\tilde{w}_t^*}{\tilde{w}_{t-1}^*} \frac{E_t P_t}{E_{t-1} P_{t-1}} - 1 \right) \right] = 0 \]
\[
(53) \]
\[
\bar{\bar{w}}_t = \frac{P_{Ht}}{P_t} (1 - \alpha) \ell_t^{-\alpha} \tilde{\kappa}_t^\alpha, \]
\[
(54) \]
\[
\bar{\bar{w}}_t = q_t^{-1} \frac{P_{Ht}^*}{P_t^*} (1 - \alpha) z_{Ft}^1 (\zeta^* \ell_t^*)^{-\alpha} \tilde{\kappa}_t^* \alpha, \]
\[
(55) \]
\[
\bar{\bar{w}}_t = \frac{P_{Ht}}{P_t} \alpha \ell_t^{1-\alpha} \tilde{\kappa}_t^{1-\alpha}, \]
\[
(56) \]
\[
\bar{\bar{w}}_t = q_t^{-1} \frac{P_{Ht}^*}{P_t^*} \alpha (z_{Ft}^* \zeta^* \ell_t^*)^{1-\alpha} \tilde{\kappa}_t^{1-\alpha}, \]
\[
(57) \]
\[
\bar{\bar{w}}_t \left( \frac{1}{1 + \zeta} \right) \left( \frac{\tilde{\kappa}_t}{\tilde{k}_{t-1}} \right) \left( \frac{\tilde{w}_t^*}{\tilde{w}_{t-1}^*} \right) \left( \frac{E_t P_t}{E_{t-1} P_{t-1}} \left( \frac{E_t P_t}{E_{t-1} P_{t-1}} - 1 \right) \right) = 0 \]
\[
(58) \]
\[
\bar{\bar{w}}_t = \frac{1}{\zeta^*} \tilde{\kappa}_t^{1-\sigma}, \]
\[
(59) \]
\[
q_t^k = \left( \frac{\tilde{\kappa}_t}{\tilde{k}_{t-1}} \right) \left( \frac{1}{1 + \zeta^*} \left( \frac{P_{Ht}}{P_t} \right)^{1-\sigma} + \left( \frac{\zeta^*}{1 + \zeta^*} \right) \left( \frac{P_{Ft}}{P_t} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \]
\[
(60) \]

Market clearing requires

\[
\bar{\bar{c}}_H t + \zeta^* \bar{\bar{c}}_H t + \left( \frac{\tilde{\kappa}_t}{\tilde{k}_{t-1}} \right) \left( \frac{1}{1 + \zeta^*} \left( \frac{P_{Ht}}{P_t} \right)^{1-\sigma} + \left( \frac{\zeta^*}{1 + \zeta^*} \right) \left( \frac{P_{Ft}}{P_t} \right)^{1-\sigma} \right) \]
\[
(61) \]
\[
\bar{\bar{c}}_F t + \zeta^* \bar{\bar{c}}_F t + \left( \frac{\tilde{\kappa}_t}{\tilde{k}_{t-1}} \right) \left( \frac{1}{1 + \zeta^*} \left( \frac{P_{Ht}}{P_t} \right)^{1-\sigma} + \left( \frac{\zeta^*}{1 + \zeta^*} \right) \left( \frac{P_{Ft}}{P_t} \right)^{1-\sigma} \right) \]
\[
(62) \]
\[
\tilde{\kappa}_t + \tilde{\kappa}_t^* = \tilde{k}_{t-1}, \]
\[
(63) \]
\[
\tilde{\kappa}_t + \zeta^* \tilde{\kappa}_t^* = \tilde{k}_t, \]
\[
(64) \]
\[
(1 - \delta) \tilde{k}_{t-1} + \tilde{x}_t = \tilde{k}_t, \]
\[
(65) \]
\[ \dot{b}_{Ht} + \zeta^* \ddot{b}_{Ht} = 0. \]  \hfill (66)

The definitions of returns are

\[
1 + r_{t+1} = (1 + \bar{i}_t) \frac{P_t}{P_{t+1}},
\]

\[
1 + r^*_t = (1 + \bar{i}^*_t) \frac{P^*_t}{P^*_{t+1}},
\]

\[
1 + r^k_t = \left( \pi_{t+1} + (1 - \delta)q^k_{t+1} \right) \frac{1}{q^k_t} \exp(\varphi_{t+1}).
\]  \hfill (69)

Finally, the definitions of prices imply

\[
\frac{P_t}{P^*_{Ht}} = \left[ \left( \frac{1}{1 + \zeta} + \varsigma \right) + \left( \frac{\zeta^*}{1 + \zeta^*} - \varsigma \right) s_t^{\sigma - 1} \right]^{\frac{1}{1 - \sigma}},
\]

\[
\frac{P^*_t}{P^*_{Ft}} = \left[ \left( \frac{1}{1 + \zeta^*} - \frac{\varsigma}{\zeta^*} \right) s_t^{1 - \sigma} + \left( \frac{\zeta^*}{1 + \zeta^*} + \frac{\varsigma}{\zeta^*} \right) \right]^{\frac{1}{1 - \sigma}},
\]

\[
q_t = \frac{E_t P^*_{Ht} P_t / P_{Ht}}{P^*_{Ft} P_t / P^*_{Ft}} = s_t \left[ \frac{\left( \frac{1}{1 + \zeta} + \varsigma \right) + \left( \frac{\zeta^*}{1 + \zeta^*} - \varsigma \right) s_t^{\sigma - 1}}{\frac{1}{1 + \zeta^*} - \frac{\varsigma}{\zeta^*} s_t^{1 - \sigma} + \left( \frac{\zeta^*}{1 + \zeta^*} + \frac{\varsigma}{\zeta^*} \right) s_t^{\sigma - 1}} \right]^{\frac{1}{1 - \sigma}}.
\]  \hfill (72)

Together with the Taylor and fiscal rules and specification of driving forces, (33)-(72) define the equilibrium. Note that by Walras’ Law, the Foreign bond market clears as well. As is evident, this environment features 7 state variables:

\[ \{p, \omega, z_F, \theta, \tilde{k}_{-1}, \tilde{w}_{-1}, \tilde{w}^*_{-1}\}. \]

### A.7 Solution algorithm

We solve the model globally. We use anistropic, sparse grids as described in Judd, Maliar, Maliar, and Valero (2014). When forming expectations, we use Gauss-Hermite quadrature and interpolate with Chebyshev polynomials for states off the grid. The stochastic equilibrium is determined through backward iteration, while dampening the updating of asset prices and individuals’ expectations over the dynamics of the aggregate states. Further details are provided in the document SolutionAlgorithm.pdf in our online replication package.
B Analytical insights

In this appendix we provide supplemental analytical results for the simplified environment described in the main text. We work with the equilibrium conditions (33)-(72) under the parametric conditions in definition 1.

The first subsection characterizes the impulse responses to safety and productivity shocks. The second subsection characterizes agents’ pricing kernels and provides a general characterization of equilibrium portfolios and risk premia. The third subsection proves each of Propositions 1-5. The fourth subsection demonstrates these results are robust to asymmetric demand shocks for safe dollar bonds in Foreign and Home. The final subsection outlines an alternative environment without capital mobility and with sticky prices, and presents analogs of our baseline results in this environment.

B.1 Impulse responses

Without loss of generality, we characterize the impulse responses to shocks in period 1, assuming that the economy was in steady-state in period 0 and there are no other shocks from period 2 onwards. We employ the parametric assumptions in definition 1 except that we allow for a general $\varsigma$ so that the role of home bias is clear.

B.1.1 Dynamics from period 2 onwards

Since there are no shocks from period 2 onwards, under the parametric conditions assumed in definition 1 it is straightforward to use the equilibrium conditions from period 2 onwards to show

\begin{align}
E_1\hat{c}_2 &= \hat{\alpha}E_1\hat{\theta}_2 + \hat{\alpha}k_1, \\
E_1\hat{c}^*_2 &= -\frac{1}{\zeta^*}\hat{\alpha}E_1\hat{\theta}_2 + \hat{\alpha}k_1, \\
E_1\hat{s}_2 &= \frac{\varsigma \left( \frac{1+\zeta}{\zeta^*} \right)^2}{\frac{\alpha}{1-\alpha} + \sigma \left( 1 - \left( \frac{1+\varsigma}{\varsigma^*} \right)^2 \right)}\hat{\alpha}E_1\hat{\theta}_2, \\
E_1\hat{q}_2 &= \varsigma \frac{1+\varsigma}{\varsigma^*}E_1\hat{s}_2, \\
E_1\hat{v}_2 &= E_1\hat{c}_2 = \hat{\alpha}E_1\hat{\theta}_2 + \hat{\alpha}k_1,
\end{align}
\[ E_1 \hat{c}_2^* = E_1 \hat{c}_2 = -\frac{1}{\zeta^*} \hat{\alpha} E_1 \hat{\theta}_2 + \alpha \hat{k}_1, \]  

(78)

where

\[ \hat{\alpha} \equiv \frac{\alpha}{1 - \left( \frac{1}{1 + \zeta^*} - \xi \right) \frac{\zeta (1 + \frac{1}{\zeta^*})^2}{1 - \alpha + \sigma (1 - \xi) \zeta^*}}. \]

These are the only conditions we need to solve for the equilibrium in period 1, to which we now turn.

**B.1.2 Log-linearized conditions in period 1**

Log-linearizing the definition of the real exchange rate implies

\[ \hat{q}_1 = \frac{1 + \zeta^*}{\zeta^*} \hat{s}_1, \]  

(79)

a relationship we use repeatedly in what follows.

Log-linearizing the intratemporal allocation of consumption, the equilibrium factor prices, the resource constraints, and goods market clearing yields

\[ \frac{1}{1 + \zeta^*} \hat{c}_1 + \frac{\zeta^*}{1 + \zeta^*} \hat{c}_2 = (1 - \alpha) \left[ \frac{1}{1 + \zeta^*} \hat{\ell}_1 + \frac{\zeta^*}{1 + \zeta^*} \hat{\ell}_2 \right] + \alpha \hat{k}_1 \]  

(80)

and

\[ \xi \frac{1 + \zeta^*}{\zeta^*} \left( \hat{c}_1 - \hat{c}_2 \right) = \left[ \frac{\alpha}{1 - \alpha} + \sigma \left( 1 - \left( \frac{1 + \zeta^*}{\zeta^*} \right)^2 \right) \right] \hat{s}_1 + \hat{\ell}_1 - \hat{\ell}_2. \]  

(81)

Log-linearizing the Euler equations yields

\[ \Delta E_1 \hat{c}_2 = \Delta E_1 \hat{c}_2^* - \frac{1 + \zeta^*}{\zeta^*} \Delta E_1 \hat{s}_2, \]  

(82)

\[ \Delta E_1 \hat{c}_2 = E_1 \hat{r}_2^k, \]  

(83)

\[ E_1 \hat{r}_2^k = E_1 \hat{r}_2 + \hat{\omega}_1, \]  

(84)

\[ E_1 \hat{r}_2^* = E_1 \hat{r}_2 + \hat{\omega}_1 + \xi \frac{1 + \zeta^*}{\zeta^*} \Delta E_1 \hat{s}_2, \]  

(85)

where we have used (76).

Log-linearizing the expected evolution of Home’s wealth share, using the equilib-
rium factor prices and Home resource constraint, implies

$$\mathbb{E}_1 \hat{\theta}_2 = \frac{1}{\beta \alpha} \left[ (1 - \beta) \left( \frac{\zeta^*}{1 + \zeta^*} - \varsigma \right) \hat{s}_1 + (1 - \beta)(1 - \alpha) \hat{l}_1 + (1 - \beta) \alpha \hat{k}_1 + \alpha \hat{\theta}_1 - (1 - \beta) \hat{c}_1 \right].$$ \hfill (86)

Linearizing the definition of Home net foreign assets, using the equilibrium factor prices, Home resource constraint, and capital allocation across countries, implies

$$\widetilde{nf}_{a_1} = a \left[ \frac{1}{\beta \alpha} \left( (1 - \beta) \left( \frac{\zeta^*}{1 + \zeta^*} - \varsigma \right) \hat{s}_1 + (1 - \beta)(1 - \alpha) \hat{l}_1 + \alpha \hat{k}_1 + \alpha \hat{\theta}_1 - (1 - \beta) \hat{c}_1 \right] - \frac{\zeta^*}{1 + \zeta^*} \frac{1}{1 - \alpha} \hat{s}_2 - \hat{k}_1. \hfill (87)$$

Log-linearizing the Fisher equations and Taylor rules yields

$$\mathbb{E}_1 \hat{r}_2 = \hat{i}_1, \hfill (88)$$
$$\mathbb{E}_1 \hat{r}^*_2 = \hat{i}^*_1, \hfill (89)$$
$$\hat{i}_1 = \phi \Delta \hat{P}_1, \hfill (90)$$
$$\hat{i}^*_1 = \phi \Delta \hat{P}^*_1, \hfill (91)$$

where we use that the Taylor rules implement $\Delta \hat{P}_2 = \Delta \hat{P}^*_2 = 0$.

Log-linearizing the realized evolution of Home’s wealth share implies

$$\hat{\theta}_1 = \left( \frac{q^k k}{a} - 1 \right) \left( \hat{r}_1 - \hat{r}_1 \right) + \frac{b_F}{a} \left( \hat{r}_1 - \hat{q}_1 - \hat{r}_1 \right) - \beta \frac{\zeta^*}{1 + \zeta^*} \frac{b^p_{H,a}}{a} \hat{\omega}_1. \hfill (92)$$

Log-linearizing the realized returns on capital, using the equilibrium profits, yields

$$\hat{r}_1^k = (1 - \beta) \left[ -\varsigma \hat{s}_1 + (1 - \alpha) \left[ \frac{1}{1 + \zeta^*} \hat{\ell}_1 + \frac{\zeta^*}{1 + \zeta^*} \hat{\ell}^*_1 \right] - (1 - \alpha) \hat{k}_1 \right] + \beta \hat{q}_1^k. \hfill (93)$$

Log-linearizing the expected returns on capital implies

$$\hat{q}_1^k = -\varsigma \mathbb{E}_1 \hat{s}_2 - (1 - \alpha) \hat{k}_1 - \mathbb{E}_1 \hat{r}_2^k. \hfill (94)$$
Log-linearizing the realized returns on dollar and Foreign bonds yields

\[ \hat{r}_1 = -\Delta \hat{P}_1, \quad (95) \]
\[ \hat{r}^*_1 = -\Delta \hat{P}^*_1. \quad (96) \]

Finally, with flexible wages and an infinite Frisch elasticity \( \nu \to 0 \), it is clear from the union’s wage-setting condition that

\[ \hat{\ell}_1 = \hat{\ell}^*_1 = 0. \]

Alternatively, if wages are set one period in advance, it is straightforward to show that up to first-order

\[ \hat{\omega}_1 = -\Delta \hat{P}_1 - \sigma^z \hat{\epsilon}^*_1, \]
\[ \hat{\omega}^*_1 + \hat{q}_1 = -\Delta \hat{P}^*_1 - \sigma^z \hat{\epsilon}^*_1. \]

Combining these with log-linearized labor demand of firms yields

\[ \left[ \left( \frac{\zeta^*}{1 + \zeta^*} - \zeta \right) + \frac{\zeta^*}{1 + \zeta^*} \frac{\alpha}{1 - \alpha} \right] \hat{s}_1 - \alpha \left( \frac{1}{1 + \zeta^*} \hat{\ell}_1 + \frac{\zeta^*}{1 + \zeta^*} \hat{\ell}^*_1 \right) - (1 - \alpha) \hat{\bar{k}}_1 = -\Delta \hat{P}_1 \quad (97) \]
\[ - \left[ \left( \frac{1}{1 + \zeta^*} - \frac{\zeta}{\zeta^*} \right) + \frac{1}{1 + \zeta^*} \frac{\alpha}{1 - \alpha} \right] \hat{s}_1 - \alpha \left( \frac{1}{1 + \zeta^*} \hat{\ell}_1 + \frac{\zeta^*}{1 + \zeta^*} \hat{\ell}^*_1 \right) - (1 - \alpha) \hat{\bar{k}}_1 = -\Delta \hat{P}^*_1. \quad (98) \]

We now combine these log-linearized conditions to facilitate the proof of the results provided in the main text.

(80) implies

\[ \hat{c}^*_1 = -\frac{1}{\zeta^*} \hat{c}_1 + \frac{1 + \zeta^*}{\zeta^*} \left[ (1 - \alpha) \left( \frac{1}{1 + \zeta^*} \hat{\ell}_1 + \frac{\zeta^*}{1 + \zeta^*} \hat{\ell}^*_1 \right) + \alpha \hat{\bar{k}}_1 \right]. \]

Substituting in (81) implies
\[ \hat{s}_1 = \frac{1}{\frac{\alpha}{1-\alpha} + \sigma \left(1 - \left(\frac{1+\zeta^*}{\zeta^*}\right)^2\right)} \left[ \left(\hat{\ell}_1^* - \hat{\ell}_1\right) + \zeta \left(\frac{1+\zeta^*}{\zeta^*}\right)^2 \left(\hat{c}_1 - \left(1 - \alpha \right) \left(\frac{1}{1+\zeta^*} \hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*} \hat{\ell}_1^*\right) + \hat{\alpha} k_1\right)\right] . \]

Combining these with (73), (74), (75), and (82) implies

\[ \hat{c}_1 = \hat{\alpha} E_1 \hat{\theta}_2 + \left(1 - \alpha\right) \left(\frac{1}{1+\zeta^*} \hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*} \hat{\ell}_1^*\right) + \alpha \hat{\zeta} - \frac{\zeta}{\frac{\alpha}{1-\alpha} + \sigma (1 - \sigma) \left(\frac{1+\zeta^*}{\zeta^*}\right)^2 \left(\hat{\ell}_1^* - \hat{\ell}_1\right)} , \]

which we can substitute into the previous result to give

\[ \hat{s}_1 = \frac{1}{\frac{\alpha}{1-\alpha} + \sigma + (1 - \sigma) \left(\frac{1+\zeta^*}{\zeta^*}\right)^2 \left(\hat{\ell}_1^* - \hat{\ell}_1\right)} \left(\frac{1+\zeta^*}{\zeta^*}\right)^2 \left(\hat{\theta}_1 + \left(1 - \beta\right) \frac{\zeta^*}{1+\zeta^*} (\sigma - 1) \frac{1-\alpha}{\alpha} \frac{1-\left(\frac{1+\zeta^*}{\zeta^*}\right)^2}{\frac{\alpha}{1-\alpha} + \sigma + (1 - \sigma) \left(\frac{1+\zeta^*}{\zeta^*}\right)^2} \hat{\ell}_1^* - \hat{\ell}_1\right) . \]

Substituting these into (86) implies

\[ E_1 \hat{\theta}_2 = \hat{\theta}_1 + \left(1 - \beta\right) \frac{\zeta^*}{1+\zeta^*} (\sigma - 1) \frac{1-\alpha}{\alpha} \frac{1-\left(\frac{1+\zeta^*}{\zeta^*}\right)^2}{\frac{\alpha}{1-\alpha} + \sigma + (1 - \sigma) \left(\frac{1+\zeta^*}{\zeta^*}\right)^2} \hat{\ell}_1^* - \hat{\ell}_1 , \]

while substituting these into (87) implies

\[ \hat{n} f a_1 = a \left[ \hat{\theta}_1 + \left(1 - \beta\right) \frac{\zeta^*}{1+\zeta^*} (\sigma - 1) \frac{1-\alpha}{\alpha} \frac{1-\left(\frac{1+\zeta^*}{\zeta^*}\right)^2}{\frac{\alpha}{1-\alpha} + \sigma + (1 - \sigma) \left(\frac{1+\zeta^*}{\zeta^*}\right)^2} \hat{\ell}_1^* - \hat{\ell}_1 \right] - \frac{\zeta^*}{1+\zeta^*} \frac{1}{1-\alpha} \hat{s}_2 . \]
and substituting these into (83) and making use of (73) implies

\[
\mathbb{E}_t r^k_2 = -(1 - \alpha) \left( \frac{1}{1 + \zeta^*} \hat{\ell}_1 + \frac{\zeta^*}{1 + \zeta^*} \hat{\ell}_1^* \right) + \frac{\zeta}{\frac{\alpha}{1-\alpha} + \sigma + (1 - \sigma) \left( \frac{1 + \zeta^*}{\zeta^*} \right)^2 (\hat{\ell}_1^* - \hat{\ell}_1) .
\]

Then (88)-(91) imply

\[
\Delta \hat{P}_1 = \frac{1}{\phi} \left( \mathbb{E}_t r^k_2 - \hat{\omega}_1 \right),
\]

\[
\Delta \hat{P}_1^* = \frac{1}{\phi} \left( \mathbb{E}_t r^k_2 - \frac{1 + \zeta^*}{\zeta^*} \frac{\zeta}{\frac{\alpha}{1-\alpha} + \sigma + (1 - \sigma) \left( \frac{1 + \zeta^*}{\zeta^*} \right)^2 (\hat{\ell}_1^* - \hat{\ell}_1) \right),
\]

so (95) together with the first implies

\[
\hat{r}_1 = -\frac{1}{\phi} \left( - (1 - \alpha) \left( \frac{1}{1 + \zeta^*} \hat{\ell}_1 + \frac{\zeta^*}{1 + \zeta^*} \hat{\ell}_1^* \right) + \frac{\zeta}{\frac{\alpha}{1-\alpha} + \sigma + (1 - \sigma) \left( \frac{1 + \zeta^*}{\zeta^*} \right)^2 (\hat{\ell}_1^* - \hat{\ell}_1) - \hat{\omega}_1 \right),
\]

while (96) together with the second implies

\[
\hat{r}_1^* = -\frac{1}{\phi} \left( - (1 - \alpha) \left( \frac{1}{1 + \zeta^*} \hat{\ell}_1 + \frac{\zeta^*}{1 + \zeta^*} \hat{\ell}_1^* \right) - \frac{1}{\zeta^*} \frac{\zeta}{\frac{\alpha}{1-\alpha} + \sigma + (1 - \sigma) \left( \frac{1 + \zeta^*}{\zeta^*} \right)^2 (\hat{\ell}_1^* - \hat{\ell}_1) \right),
\]

Moreover, (93) and (94) imply
\\[
\hat{\gamma}_k = -\frac{\xi}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)} \left( \hat{\ell}_1 - \hat{\ell}_1 \right) - \frac{\left( \frac{1}{1+\zeta} \right)^2}{\frac{\alpha}{1-\alpha} + \sigma \left( 1 - \frac{1}{1+\zeta} \right)^2} \tilde{\alpha} E_1 \hat{\theta}_2 + (1-\alpha) \left( 1 + \frac{1}{1+\zeta} \hat{\ell}_1 + \frac{\zeta}{1+\zeta} \hat{\ell}_1 \right) - (1-\alpha) \hat{k}_1.
\]

B.2 Pricing kernels, portfolios, and risk premia

Now a second order approximation of the optimal portfolio choice conditions in period 0 implies

\[
E_0 [\hat{r}_1 - \hat{r}_1] + \text{Jensen terms} = \hat{\omega}_0 - E_0 \left[ m_{0,1} - \gamma \sigma^2 \hat{\varepsilon}_1^z \right] [\hat{r}_1 - \hat{r}_1],
\]

\[
E_0 [\hat{r}_1^* - \Delta \hat{q}_1 - \hat{r}_1] + \text{Jensen terms} = \hat{\omega}_0 - E_0 \left[ m_{0,1} - \gamma \sigma^2 \hat{\varepsilon}_1^z \right] [\hat{r}_1 - \Delta \hat{q}_1 - \hat{r}_1],
\]

and analogously in Foreign, where *Jensen terms* reflect the component of excess returns which do not reflect safety shocks nor the covariance with agents’ pricing kernels (and instead reflect the variance of returns).

In period 1, the log deviation in the representative Home household’s pricing kernel is given by

\[
\hat{m}_{0,1} = -\hat{c}_1 + (1-\gamma)\hat{v}_1.
\]

Now,

\[
\hat{v}_1 = (1-\beta)\hat{c}_1 - (1-\beta)(1-\tau)(1-\alpha)\hat{l}_1 + \beta \hat{c}_1 e_1,
\]

where \(\tau\) denotes the labor wedge in the deterministic steady-state. By the results of the previous subsection,

\[
\hat{c}_1 = \tilde{\alpha} E_1 \hat{\theta}_2 + (1-\alpha) \left( \frac{1}{1+\zeta} \hat{\ell}_1 + \frac{\zeta}{1+\zeta} \hat{\ell}_1 \right) + \alpha \hat{k}_1 - \frac{\xi}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)} \left( \frac{1}{1+\zeta} \hat{\ell}_1 + \frac{\zeta}{1+\zeta} \hat{\ell}_1 \right),
\]

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while the log-linearized certainty equivalent is given by

$$\hat{c}_1 = E_1 \hat{v}_2,$$

$$= \tilde{\alpha} E_1 \hat{\theta}_2 + \alpha \hat{k}_1,$$

where the second equality uses (77). Combining these implies

$$\hat{v}_1 = \tilde{\alpha} E_1 \hat{\theta}_2 - \alpha \sigma \tilde{\epsilon}_1 +$$

$$\tau(1-\beta)(1-\alpha)\hat{\ell}_1 + (1-\beta) \left( (1-\alpha) \frac{\zeta*}{1+\zeta*} - \frac{\varsigma}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)\left(\frac{\zeta*^{1+\varsigma*}}{\zeta*}\right)^2} \right) \left( \hat{\ell}_1^* - \hat{\ell}_1 \right).$$

Combining the previous results, we obtain

$$\hat{m}_{0,1} - \gamma \sigma \tilde{\epsilon}_1 = -\gamma \left[ \tilde{\alpha} E_1 \hat{\theta}_2 + (1-\alpha)\sigma \tilde{\epsilon}_1 \right]$$

$$- (1-\alpha) \left( \frac{1}{1+\zeta*} \hat{\ell}_1 + \frac{\zeta*}{1+\zeta*} \hat{\ell}_1^* \right) + \frac{\varsigma}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)\left(\frac{\zeta*^{1+\varsigma*}}{\zeta*}\right)^2} \left( \hat{\ell}_1^* - \hat{\ell}_1 \right)$$

$$+ (1-\gamma)(1-\beta) \left[ \tau(1-\alpha)\hat{\ell}_1 + \left( (1-\alpha) \frac{\zeta*}{1+\zeta*} - \frac{\varsigma}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)\left(\frac{\zeta*^{1+\varsigma*}}{\zeta*}\right)^2} \right) \left( \hat{\ell}_1^* - \hat{\ell}_1 \right) \right].$$

Analogous steps in Foreign yield

$$\hat{m}_{0,1} - \gamma^* \sigma \tilde{\epsilon}_1 = -\gamma^* \left[ \frac{1}{\zeta*} \tilde{\alpha} E_1 \hat{\theta}_2 + (1-\alpha)\sigma \tilde{\epsilon}_1 \right]$$

$$- (1-\alpha) \left( \frac{1}{1+\zeta*} \hat{\ell}_1 + \frac{\zeta*}{1+\zeta*} \hat{\ell}_1^* \right) - \frac{\varsigma}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)\left(\frac{\zeta*^{1+\varsigma*}}{\zeta*}\right)^2} \left( \hat{\ell}_1^* - \hat{\ell}_1 \right)$$

$$+ (1-\gamma^*)(1-\beta) \left[ \tau(1-\alpha)\hat{\ell}_1^* - \left( (1-\alpha) \frac{1}{1+\zeta*} - \frac{1}{\zeta*} \frac{\varsigma}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)\left(\frac{\zeta*^{1+\varsigma*}}{\zeta*}\right)^2} \right) \left( \hat{\ell}_1^* - \hat{\ell}_1 \right) \right].$$

Now, the present environment is **locally complete** as defined by Coeurdacier and Gourinchas (2016). It follows that the equilibrium portfolios ensure that

$$\hat{m}_{0,1} - \gamma \sigma \tilde{\epsilon}_1 = \hat{m}_{0,1}^* - \gamma^* \sigma \tilde{\epsilon}_1^* + \Delta \hat{q}_1.$$

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Substituting in using the above results and those of the previous section and collecting terms, we obtain

\[
\left( \frac{q^k}{a} - 1 \right) \left( \hat{r}^k - \hat{r}_1 \right) + \frac{b^F}{a} \left( \hat{r}^* - \hat{q}_1 - \hat{r}_1 \right) - \beta \frac{\zeta^*}{1 + \zeta^*} \frac{b_{H,s}^2}{a} \hat{\omega}_1 = \frac{1}{\Gamma}(\gamma^* - \gamma)(1 - \alpha)\sigma^* \hat{\epsilon}_1 +
\]

\[
\frac{1}{\Gamma}(\gamma^* - \gamma) \tau (1 - \beta)(1 - \alpha) \left( \frac{1}{1 + \zeta^*} \hat{\ell}_1 + \frac{\zeta^*}{1 + \zeta^*} \hat{\ell}_1 \right) +
\]

\[
(1 - \beta) \frac{\zeta^*}{1 + \zeta^*} (\sigma - 1) \frac{1 - \alpha}{\alpha} \frac{1}{\Gamma} \left( \frac{1}{1 - \alpha} + \sigma + (1 - \sigma) \left( \frac{1}{1 + \zeta^*} \right)^2 \right)^2 \left( \hat{\ell}_1 - \hat{\ell}_1 \right) -
\]

\[
\frac{1}{\Gamma} \left( \frac{1}{\zeta^*} (\gamma^* - 1) + (\gamma - 1) \right) (1 - \beta)(1 - \tau)(1 - \alpha) \frac{\zeta^*}{1 + \zeta^*} \left( \hat{\ell}_1 - \hat{\ell}_1 \right) +
\]

\[
\frac{1}{\Gamma} \left( \frac{1}{\zeta^*} (\gamma^* - 1) + (\gamma - 1) \right) (1 - \beta) \frac{\zeta}{\frac{1}{1 - \alpha} + \sigma + (1 - \sigma) \left( \frac{1}{1 + \zeta^*} \right)^2} \left( \hat{\ell}_1 - \hat{\ell}_1 \right), \tag{99}
\]

where

\[
\Gamma \equiv \left[ \gamma + \frac{1}{\zeta^*} \right] + \frac{\zeta^2 \left( \frac{1}{1 + \zeta^*} \right)^3}{\frac{1}{1 - \alpha} + \sigma + \left( 1 - \left( \frac{1}{1 + \zeta^*} \right)^2 \right) \zeta^*}
\]

Thus, international risk sharing calls for Home wealth (on the left-hand side) to rise with:

- **productivity**, provided \( \gamma^* > \gamma \): since a positive TFP shock raises aggregate production and thus consumption;

- **aggregate employment**, provided \( \tau (\gamma^* - \gamma) > 0 \): since an increase in labor raises welfare;

- **Foreign employment less Home employment**, if:

\[(\sigma - 1) \left( 1 - \left( \frac{1}{1 + \zeta^*} \right)^2 \right) > 0 \text{: since this implies that Foreign labor income rises relative to Home labor income; or}
\]

\[\left( \frac{1}{\zeta^*} (\gamma^* - 1) + (\gamma - 1) \right) \left( \frac{\zeta}{\frac{1}{1 - \alpha} + \sigma + (1 - \sigma) \left( \frac{1}{1 + \zeta^*} \right)^2} - (1 - \tau)(1 - \alpha) \frac{\zeta^*}{1 + \zeta^*} \right) > 0 \text{: since this implies that Home requires more wealth when its real exchange rate}
\]
appreciates, despite the larger disutility of labor in Foreign.

Finally, the equilibrium risk premium on Foreign bonds relative to dollar bonds is given by the covariance of (the negative of) the log deviation in any agent’s pricing kernel with the excess log return.

**B.3 Proofs**

**B.3.1 Propositions 1-3**

Now consider the case with identical portfolios (so \( q^k k = a, \) \( b_F = 0, \) and \( b_H = 0 \)) and zero safe debt issued by the Home government (\( b_{H,s}^g = 0 \)) assumed in Propositions 1 and 2. Thus \( \dot{\theta}_1 = 0. \) Further, since \( \varsigma \to \frac{\zeta^*}{1 + \zeta^*} \), we have that \( E_{\hat{1}} \dot{\theta}_2 = 0. \)

Then in the further case absent nominal rigidity and with \( \nu \to 0, \) the claims follow immediately from the above results given \( \hat{\ell}_1 = 0. \) Alternatively in the case with wages set one period ahead, we can substitute the above results into (97) and (98) and solve for \( \hat{\ell}_1 \) and \( \hat{\ell}_1^* \), yielding

\[
\hat{\ell}_1 = -\frac{1}{\alpha + \frac{1}{\phi} (1 - \alpha)} \frac{1}{\phi} \hat{\omega}_1 - \frac{1}{\alpha + \frac{1}{\phi} (1 - \alpha)} (1 - \alpha) \hat{k}_1,
\]

\[
\hat{\ell}_1^* = -\frac{1}{\alpha + \frac{1}{\phi} (1 - \alpha)} (1 - \alpha) \hat{k}_1.
\]

Thus, in response to a safety shock, \( \frac{1}{1 + \zeta^*} \hat{\ell}_1 + \frac{\zeta^*}{1 + \zeta^*} \hat{\ell}_1^* \propto -\hat{\omega}_1 \) and \( \hat{\ell}_1 - \hat{\ell}_1^* \propto \hat{\omega}_1 \) as claimed. We note that the limit of complete home bias \( \varsigma \to \frac{\zeta^*}{1 + \zeta^*} \) implies that \( \hat{\ell}_1^* \) is unaffected by a safety shock (up to first order), but for \( \varsigma < \frac{\zeta^*}{1 + \zeta^*} \) it is straightforward to show that \( \hat{\ell}_1^* \propto -\hat{\omega}_1 \) for \( \sigma \) sufficiently low and \( \hat{\ell}_1^* \propto \hat{\omega}_1 \) for \( \sigma \) sufficiently high.

**B.3.2 Propositions 4-5**

When \( \varsigma \to \frac{\zeta^*}{1 + \zeta^*} \), the international portfolios solve

\[
\left( \frac{q^k k}{a} - 1 \right) \left( \hat{r}_1 - \hat{r}_1 \right) + \frac{b_F}{a} \left( \hat{r}_1^* - \hat{q}_1 - \hat{r}_1 \right) - \beta \frac{\zeta^*}{1 + \zeta^*} \frac{b_{H,s}^g}{a} \hat{\omega}_1 = \frac{1}{1} (\gamma - \gamma)(1 - \alpha) \sigma^* \hat{\epsilon}_1^* + \\
\frac{1}{1} (\gamma^* - \gamma) \tau (1 - \beta)(1 - \alpha) \left( \frac{1}{1 + \zeta^*} \hat{\ell}_1 + \frac{\zeta^*}{1 + \zeta^*} \hat{\ell}_1^* \right) +
\]

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For arbitrary portfolios, it is straightforward to show that

$$
\frac{1}{1 + \zeta*} \hat{\ell}_1 + \frac{\zeta*}{1 + \zeta*} \hat{\ell}_1 = -\frac{1}{\alpha + \frac{1}{\phi}(1 - \alpha)} \frac{1}{\phi} + \frac{\zeta*}{1 + \zeta*} \hat{\omega}_1 + \frac{1}{\alpha + \frac{1}{\phi}(1 - \alpha)}(1 - \alpha)\sigma^z \hat{e}_1^z,
$$

$$
\hat{\ell}_1 - \hat{\ell}_1 = \frac{1}{\alpha + \frac{1}{\phi}(1 - \alpha)} \frac{1}{\phi} \hat{\omega}_1 - \frac{1}{\alpha + \frac{1}{\phi}(1 - \alpha)} \frac{1 + \zeta*}{\zeta*} \alpha \bar{E}_1 \hat{\theta}_2,
$$

generalizing the results given in the proof of Proposition 1 to arbitrary portfolios.

Substituting these into the expression for international portfolios, and using that

$$
\bar{E}_1 \hat{\theta}_2 = \left( \frac{q^k}{a} - 1 \right) \left( \hat{r}_1^k - \hat{r}_1 \right) + \frac{b_{F-1}}{a} \left( \hat{r}_1^* - \hat{q}_1 - \hat{r}_1 \right) - \beta \frac{\zeta*}{1 + \zeta*} \frac{b_{H,s}^* \hat{\omega}_1}{a},
$$

we obtain

$$
\left( \frac{q^k}{a} - 1 \right) \left( \hat{r}_1^k - \hat{r}_1 \right) + \frac{b_{F-1}}{a} \left( \hat{r}_1^* - \hat{q}_1 - \hat{r}_1 \right) - \beta \frac{\zeta*}{1 + \zeta*} \frac{b_{H,s}^* \hat{\omega}_1}{a} =
$$

$$
\frac{1}{\Gamma'}(\gamma^* - \gamma)(1 - \alpha) \left[ 1 + \tau(1 - \beta) \frac{1}{\alpha + \frac{1}{\phi}(1 - \alpha)} (1 - \alpha) \right] \sigma^z \hat{e}_1^z - \frac{1}{\Gamma'}(\gamma^* - \gamma) \tau(1 - \beta)(1 - \alpha) \frac{1}{\alpha + \frac{1}{\phi}(1 - \alpha)} \frac{1}{\phi} \frac{1}{1 + \zeta*} \hat{\omega}_1 + \frac{1}{\Gamma'} \frac{1}{\zeta*}(\gamma^* - 1) + (\gamma - 1) \left(1 - \beta\right)(1 - \alpha) \frac{\zeta*}{1 + \zeta*} \frac{1}{\alpha + \frac{1}{\phi}(1 - \alpha)} \frac{1}{\phi} \hat{\omega}_1,
$$

where

$$
\Gamma' \equiv \Gamma + \left( \frac{1}{\zeta*}(\gamma^* - 1) + (\gamma - 1) \right) (1 - \beta)(1 - \alpha) \frac{1}{\alpha + \frac{1}{\phi}(1 - \alpha)} \alpha > 0.
$$

Thus, in comparative statics with respect to \( \gamma + \frac{1}{\zeta*} \gamma^* \), on the right-hand side it is clear that only the first two terms vary; the third, capturing the effect of the relative labor response on international portfolios, is constant. When \( \zeta \to \frac{\zeta*}{1 + \zeta*} \), it is straightforward to show that the earlier results imply

$$
\hat{r}_1^* - \hat{q}_1 - \hat{r}_1 = -\left( \frac{1}{\phi} + \left( 1 - \frac{1}{\phi} \right) \frac{1 - \alpha}{\alpha + \frac{1}{\phi}(1 - \alpha)} \frac{1}{\phi} \right) \hat{\omega}_1 - \frac{1}{\alpha + \frac{1}{\phi}(1 - \alpha)} \frac{1}{\phi} \hat{\omega}_1 \frac{1}{\phi} (1 - \alpha) \frac{\zeta*}{\zeta*} \alpha \bar{E}_1 \hat{\theta}_2,
$$

$$
\hat{r}_1^k - \hat{r}_1 = -\left( \frac{1}{\phi} + \left( 1 - \frac{1}{\phi} \right) \frac{1 - \alpha}{\alpha + \frac{1}{\phi}(1 - \alpha)} \frac{1}{\phi} \right) \hat{\omega}_1 + \frac{1}{\phi} \frac{1}{\phi} (1 - \alpha) \frac{\zeta*}{\zeta*} \alpha \bar{E}_1 \hat{\theta}_2,
$$

75
\[
\left[ (1 - \alpha) + \left(1 - \frac{1}{\phi}\right)(1 - \alpha)^2 \frac{1}{\alpha + \frac{1}{\phi}(1 - \alpha)} \right] \sigma^z \hat{z} + \\
\frac{\frac{1}{\phi}(1 - \alpha)}{\alpha + \frac{1}{\phi}(1 - \alpha)} \mathbb{E}_1 \hat{\theta}_2,
\]

where we define \( \Delta \equiv 1 + \left[ \left( \frac{q^k}{a} - 1 \right) + \frac{b_F}{a} \right] \left[ \frac{1}{\phi} \frac{(1 - \alpha)}{\alpha + \frac{1}{\phi}(1 - \alpha)} \right] \), which evaluates to one in the case with symmetric portfolios. By the method of undetermined coefficients, it follows that at the point of symmetric portfolios,

\[
\frac{dk}{db} H, s = 0, \quad \frac{db_H}{db_{H, s}} > 0,
\]

where the second line holds \( \gamma + \frac{1}{\zeta} \gamma^* \) fixed and assumes \( \tau > 0 \).

Finally, consider

\[-\mathbb{E}_0 \left[ \hat{m}_{0, 1} - \gamma \sigma^z \hat{z} \right] \left[ \hat{r}_1^* - \hat{q}_1 - \hat{r}_1 \right].\]

Assuming that productivity and safety are independent, this can be expressed as a linear combination of \( (\sigma^z)^2 \) and \( (\sigma^w)^2 \). The above results imply that the coefficient on the former takes the sign of \( \gamma - \gamma^* \) and the coefficient on the latter is positive, completing the claim.

### B.4 Foreign-only demand for safe dollar bonds

We now demonstrate that our analysis is robust to Foreign-only demand for safe dollar bonds, as discussed in section 3.5.

We augment the model with a non-negativity constraint on households’ positions in the safe dollar bond, reflecting the assumption that they cannot create these safe assets (only the Home government can). This is irrelevant when the demand shock
for the safe dollar bond is global, since in that case all agents hold the outstanding supply of safe dollar debt issued by the Home government.

The Home representative agent’s FOC for safe dollar bonds, other dollar bonds, Foreign bonds, and capital are now

\[
1 - \mu_t = \mathbb{E}_t m_{t,t+1} (1 + i_t) \frac{P_t}{P_{t+1}},
\]

\[
1 = \mathbb{E}_t m_{t,t+1} (1 + i_t) \frac{P_t}{P_{t+1}},
\]

\[
1 = \mathbb{E}_t m_{t,t+1} \frac{q_t}{q_{t+1}} (1 + r_{t+1}^*),
\]

\[
1 = \mathbb{E}_t m_{t,t+1} (1 + r_{t+1}^k),
\]

where \( \mu_t \geq 0 \) is the (scaled) multiplier on the non-negativity constraint on safe dollar bonds and the last three FOCs are as in the baseline model.

The Foreign representative agent’s FOC for safe dollar bonds, other dollar bonds, Foreign bonds, and capital are

\[
1 - c_t^* \Omega_t' (B_{Ht,s}^*/(E_t^{-1} P_t^*)) / \Omega_t^* (B_{Ht,s}^*/(E_t^{-1} P_t^*)) = \mathbb{E}_t m_{t,t+1}^* \frac{q_{t+1}}{q_t} (1 + i_t) \frac{P_t}{P_{t+1}},
\]

\[
1 = \mathbb{E}_t m_{t,t+1}^* \frac{q_{t+1}}{q_t} (1 + i_t) \frac{P_t}{P_{t+1}},
\]

\[
1 = \mathbb{E}_t m_{t,t+1}^* (1 + r_{t+1}^*),
\]

\[
1 = \mathbb{E}_t m_{t,t+1}^* \frac{q_{t+1}}{q_t} (1 + r_{t+1}^k),
\]

as in the baseline model. Given the same functional form for \( \Omega_t \) as in the baseline model,

\[
c_t^* \Omega_t' (B_{Ht,s}^*/(E_t^{-1} P_t^*)) / \Omega_t^* (B_{Ht,s}^*/(E_t^{-1} P_t^*)) = \omega_t^{ds} - \frac{B_{Ht,s}^*}{E_t^{-1} P_t^* c_t^*},
\]

where \( \omega_t^{ds} \) is the Foreign latent demand shock for safe dollar bonds. We assume for expositional simplicity that the non-negativity constraint on safe dollar bonds for Foreign agents does not bind, which will be the case when \( \omega_t^{ds} \) is sufficiently high.

Now note that each agent’s FOCs for safe and other dollar bonds imply

\[
\frac{1 + i_t}{1 - \mu_t} = 1 + \nu_t = \frac{1 + i_t}{1 - \left( \omega_t^{ds} - \frac{B_{Ht,s}^*}{E_t^{-1} P_t^* c_t^*} \right)}.
\]
It follows that
\[ \mu_t = \omega_t^{ds} - \frac{B_{Ht,s}^*}{E_t^{-1} P_t^* c_t^*} \equiv \omega_t. \]

That is, the Lagrange multiplier on the non-negativity constraint for safe dollar bonds for Home agents must be equated to the convenience yield perceived by Foreign agents for these bonds. The equilibrium system is thus exactly as in the baseline model, except with \( B_{Ht,s} = 0 \) (zero holdings of safe dollar bonds by Home agents). It follows that the propagation of Foreign demand shocks for safe dollar bonds is exactly like global demand shocks for these assets, up to any differences in seignorage because in this case only Foreign agents hold them.\(^{45}\)

Note in particular that despite the fact that only foreigners have a special demand for safe dollar bonds, the international risk sharing conditions are unchanged. That is, we still have that
\[
\begin{align*}
\mathbb{E}_t m_{t,t+1} \frac{1 + r_{t+1}}{1 - \omega_t} &= \mathbb{E}_t m_{t,t+1}^* \frac{q_{t+1}}{q_t} \frac{1 + r_{t+1}}{1 - \omega_t}, \\
\mathbb{E}_t m_{t,t+1} \frac{q_t}{q_{t+1}} (1 + r_{t+1}^*) &= \mathbb{E}_t m_{t,t+1}^* (1 + r_{t+1}^*), \\
\mathbb{E}_t m_{t,t+1} (1 + r_{t+1}^k) &= \mathbb{E}_t m_{t,t+1}^* \frac{q_{t+1}}{q_t} (1 + r_{t+1}^k),
\end{align*}
\]

With one more asset than shocks, this simplified environment is locally complete as in the prior subsections.

### B.5 Distinct capital stocks and sticky prices

We finally provide more detail on the sensitivity of our results to the alternative environment with distinct capital stocks and sticky prices discussed in section 3.5.

#### B.5.1 Environment

We first outline the changes relative to the baseline environment to accommodate distinct capital stocks and sticky prices.

The representative Home household’s budget constraint is now
\[
P_{Ht} c_{Ht} + E_t^{-1} P_{Ft}^* c_{Ft} + B_{Ht,s} + B_{Ht,o} + E_t^{-1} B_{Ft} + Q_t^k k_{Ht} + E_t^{-1} Q_t^k k_{Ft} \leq
\]

\(^{45}\)When the Home government issues zero debt \( B_{Ht,s}^g = 0 \), there is no seignorage and hence the propagation of Foreign demand shocks is identical to global demand shocks.
\[(1 + i_{t-1})B_{Ht-1,s} + (1 + i_{t-1})B_{Ht-1,o} + E^{-1}_t(1 + i^*_{t-1})B_{Ft-1} +
\]
\[(\Pi_t + Q^*_t)k_{Ht-1} + E^{-1}_t(\Pi^*_t + Q^{k*}_t)k_{Ft-1} + W_t\ell_t + T_t,
\]

and the representative Foreign household’s budget constraint is analogous.\footnote{We ignore disaster risk and depreciation because we are studying this environment analytically in the absence of such features, following the maintained assumptions of section 3. It would be straightforward to add them in a quantitative analysis of this environment.} In particular, Home agents now choose distinct positions in capital used at Home, \(k_{Ht}\), and capital used at Foreign, \(k_{Ft}\), which offer distinct payoffs and trade at distinct prices. Consumption of goods produced in each country is now a CES aggregator across varieties with elasticity of substitution \(\epsilon\). We no longer need to assume distinct labor varieties.

In each country, there are now intermediate good producers and final goods retailers.\footnote{Again, we abstract from capital good producers because we are studying this environment analytically in the absence of capital accumulation, following the assumptions of section 3.} The representative Home intermediate good producer earns profits

\[P^i_t (z_t\ell_t)^{1-\alpha} (\tilde{k})^{1-\alpha} - W_t\ell_t - \Pi^i_t \tilde{k}, \quad (100)\]

where \(P^i_t\) is the price of the intermediate good, \(\Pi^i_t\) is the rental rate on capital, and \(\tilde{k}\) now denotes the fixed capital stock at Home. A unit measure of monopolistically competitive retailers at Home (indexed by \(j\)) purchase the domestic intermediate good and earn a discounted stream of profits

\[J_t(j) = \Pi_t(j) + \mathbb{E}_t M^{f}_{t,t+1} J_{t+1}(j),\]

where the stochastic discount factor \(M^{f}_{t,t+1}\) is described below. Flow profits are

\[\Pi_t(j) = (P_{Ht}(j) - P^i_t) y_{Ht}(j)\]

given the retailer’s global demand (which it internalizes)

\[y_{Ht}(j) = \left(\frac{P_{Ht}(j)}{P_t}\right)^{-\epsilon} (c_{Ht} + \zeta^* c^*_H),\]

and we already make use of the assumption that the retailer engages in producer currency pricing. Retailers either set prices flexibly or one period in advance. The
problems of intermediate good producers and final good retailers in Foreign are analo-
gous given Foreign capital stock $\bar{k}$, except that Foreign intermediate good producers
are also subject to an additional productivity shock $z_{Ft}$. Relative to the baseline
environment studied in section 3, we need one more shock to pin down portfolios
given that there is trade in one more asset. We assume this productivity shock is
fully transitory ($\rho^F = 0$) analogous to our assumption for safety shocks in section 3.

We assume that households own a share in all retailers from a given country in
proportion to their ownership of capital in that country. That is, the aggregate profits
earned by owners of Home capital are

$$\Pi_t \bar{k} = \Pi_t^H \bar{k} + \int_0^1 \Pi_t(j) dj,$$

and analogously for owners of Foreign capital. With incomplete markets, there is the
usual problem that there is no standard way to describe firms’ stochastic discount
factor in making dynamic decisions. But because asset markets are locally complete
in this environment studied analytically, the results which follow will be the same
using any owner’s stochastic discount factor (or weighted average).

Intermediate good market clearing is now

$$\int_0^1 y_{Ht}(j) dj = (z_t \ell_t)^{1-\alpha} (\bar{k})^{1-\alpha},$$

$$\int_0^1 y_{Ft}^*(j^*) dj^* = (z_t z_{Ft} \zeta^* \ell_t^*)^{1-\alpha} (\bar{k}^*)^{1-\alpha},$$

final good market clearing is now

$$c_{Ht}(j) + \zeta^* c_{Ht}^*(j) = y_{Ht}(j), \ \forall j,$$

$$c_{Ft}(j^*) + \zeta^* c_{Ft}^*(j^*) = y_{Ft}^*(j^*), \ \forall j^*,$$

and capital market clearing is now

$$k_{Ht} + \zeta^* k_{Ht}^* = \bar{k},$$

$$k_{Ft} + \zeta^* k_{Ft}^* = \bar{k}^*.$$

All other features of the environment are unchanged from that studied in section 3,
except we do not impose the limit of an infinite Frisch elasticity $\nu$. We refer to this
as the simplified environment with distinct capital shocks in what follows.

**B.5.2 Results**

We now provide analogs of Propositions 1-5 in this environment. We exclude the proofs for brevity but they are available on request.

Propositions 6 and 7 characterize the dynamics of prices, quantities, and wealth provided that $\mathbb{E}_t \hat{\theta}_{t+1} = 0$ on impact of all shocks at $t$. Proposition 8 demonstrates that $\mathbb{E}_t \hat{\theta}_{t+1} = 0$ is implied by efficient risk sharing in the natural benchmark with $\gamma = \gamma^* = 1$, so that the earlier results apply at least in neighborhood of this benchmark. Finally, Propositions 9 and 10 characterize the comparative statics of portfolios and risk premia around this benchmark.

We first consider the analog of Proposition 1 in this environment:

**Proposition 6.** Consider the simplified environment with distinct capital stocks and assume $\mathbb{E}_t \hat{\theta}_{t+1} = 0$ on impact of all shocks at $t$. If prices are flexible, then on impact of a positive safety shock:

- the Home CPI declines ($\Delta \hat{P}_t = -\frac{1}{\phi} \hat{\omega}_t$); and
- the Home real interest rate declines ($\mathbb{E}_t \hat{r}_{t+1} = -\hat{\omega}_t$);
- the Home real exchange rate and employment in each country are unchanged ($\hat{q}_t = \hat{\ell}_t = \hat{\ell}_t^* = 0$).

If prices are set one period in advance, then on impact of a positive safety shock:

- the Home CPI is unchanged ($\Delta \hat{P}_t = 0$);
- the Home real interest rate is unchanged ($\mathbb{E}_t \hat{r}_{t+1} = 0$);
- the Home real exchange rate appreciates ($\hat{q}_t = \hat{\omega}_t$); and
- global employment falls, disproportionately so in Home ($\frac{1}{1+\zeta} \hat{\ell}_t + \frac{\zeta^*}{1+\zeta^*} \hat{\ell}_t^* = -\frac{1}{1-\alpha} \hat{\omega}_t$ and $\hat{\ell}_t - \hat{\ell}_t^* = -\frac{1}{1-\alpha} \hat{\omega}_t$).

Thus, the transmission of safety shocks to prices and quantities is largely unchanged from Proposition 1.
We next consider the analog of Proposition 2 in this environment, defining the real returns to each capital stock as

\[ 1 + r^k_t = \frac{\Pi_t + Q^k_t P_{t-1}}{Q^k_{t-1}}, \]
\[ 1 + r^{k^*}_t = \frac{\Pi^{k^*}_t + Q^{k^*}_t P^{*}_{t-1}}{Q^{k^*}_{t-1}}, \]

We obtain:

**Proposition 7.** Consider the simplified environment with distinct capital stocks and assume \( E_t \hat{\theta}_{t+1} = 0 \) on impact of all shocks at \( t \). Then on impact of a positive safety shock:

- the real return on dollar bonds rises if prices are flexible (\( \hat{r}_t = \frac{1}{\phi} \omega_t \)) but is unchanged if prices are set in advance (\( \hat{r}_t = 0 \));
- the real return on Foreign bonds is unaffected if prices are flexible (\( \hat{r}^{k^*}_t - \hat{q}_t = 0 \)) but falls if prices are set in advance (\( \hat{r}^{k^*}_t - \Delta \hat{q}_t = -\hat{\omega}_t \));
- the real return on Home capital is unaffected if prices are flexible (\( \hat{r}^k_t = 0 \)) but may rise or fall if prices are set in advance (\( \hat{r}^k_t = \left[ \frac{(1-\beta)(1-\tau)(\frac{1}{\gamma} + 1)}{1-(1-\alpha)(1-\tau)} - 1 \right] \hat{\omega}_t \));
- the real return on Foreign capital is unaffected if prices are flexible (\( \hat{r}^{k^*}_{t} - \hat{q}_t = 0 \)) but falls if prices are set in advance (\( \hat{r}^{k^*}_t - \hat{q}_t = -\hat{\omega}_t \)).

The ambiguous response of the real return on Home capital owes to competing effects of a safety shock given sticky prices: it reduces Home output, but raises the Home mark-up \( P_{Hi}/P^i_t \). What is definitive, however, is that the real return on Home capital exceeds that on Foreign capital (\( \hat{r}^k_t - \hat{r}^{k^*}_t + \hat{q}_t \propto \hat{\omega}_t \)).

This plays an important role in the following new result:

**Proposition 8.** Consider the simplified environment with distinct capital stocks and prices set one period in advance. When \( \gamma = \gamma^* = 1 \), the equilibrium portfolios are:

- \( \frac{\hat{q}^{k^*}_H}{\alpha} = 1, \ \frac{\hat{q}^{k^*}_F}{\alpha} = 0, \ \frac{b_F}{\alpha} = -\beta \frac{\zeta^*}{1+\zeta^*} \frac{b_H}{\alpha}, \) and \( \frac{b_H}{\alpha} = \beta \frac{\zeta^*}{1+\zeta^*} \frac{b_H}{\alpha} \);
- implying that Home’s financial wealth share \( \theta_t \) rises on impact of a positive safety shock, falls on impact of a positive Foreign productivity shock, and is unchanged on impact of a global productivity shock at \( t \);
while its net foreign assets $nfa_t$ and expected future wealth share $E_t\theta_{t+1}$ are unchanged on impact of all shocks at $t$.

In this benchmark case efficient risk sharing calls for Home’s overall wealth share (inclusive of labor income) to be constant in response to all shocks.\footnote{We need to assume not only that risk aversions are the same across countries, but that they are 1, for this result. This is because, consistent with (99) in the baseline simplified environment, unitary risk aversions imply that equilibrium financial portfolios hedge only labor income risk.} When agents fully own their domestic capital stock, this ensures that returns on financial assets offset the changes in labor income in response to safety and Foreign productivity shocks, and wealth does not redistribute across countries in response to global productivity shocks. This builds on Engel and Matsumoto (2009) and Coeurdacier and Gourinchas (2016). And if the Home government earns seignorage revenues from safety shocks, an offsetting position of Home agents long Foreign bonds by shorting dollar bonds can neutralize these revenues.

The next result characterizes portfolios around this benchmark, an analog of Proposition 4 in this environment:

**Proposition 9.** Consider the simplified environment with distinct capital stocks and prices set one period in advance. At least around the case with $\gamma = \gamma^* = 1$, $b_{H,s}^0 < 0$, and the same, positive steady-state labor wedge in each country:

- Home’s portfolio share in Home capital (Foreign capital, dollar bonds) is unaffected (unaffected, falls) with $-b_{H,s}^0$; and

- Home’s portfolio share in Home capital (Foreign capital, dollar bonds) rises (rises, falls) with $\frac{\gamma}{\tilde{\gamma}}$, holding $\gamma + \frac{1}{\tilde{\gamma}}\gamma^*$ fixed.

That is, at least around the benchmark with $\gamma = \gamma^* = 1$ and $b_{H,s}^0 < 0$, efficient risk sharing calls for Home’s overall wealth share to fall upon a positive safety shock as it gets more risk tolerant versus Foreign. This is implemented by Home owning a leveraged portfolio of Home capital and Foreign capital financed by dollar bonds.

An interesting implication of the last two results is that even if Home’s overall wealth share and thus net foreign assets fall upon a safety shock because it is insuring Foreign, it can still be the case that its financial wealth share $\theta_t$ rises on impact because the return on Home capital outperforms Foreign capital.

The final result characterizes the currency risk premium around the $\gamma = \gamma^* = 1$ and $b_{H,s}^0 < 0$ benchmark, essentially identical to Proposition 5:

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Proposition 10. Consider the same environment as in Proposition 9 and suppose safety and productivity shocks are independent. Then at least around the case with \( \gamma = \gamma^r = 1 \):

- \( \text{Cov}_t (\hat{m}_{t+1}, \hat{r}^*_{t+1} - \hat{r}_{t+1}) \propto \gamma - \gamma^r \) if \( \sigma^\omega = 0 \); and
- \( \text{Cov}_t (\hat{m}_{t+1}, \hat{r}^*_{t+1} - \hat{r}_{t+1}) \) is rising in \( \sigma^\omega \).

This result holds as well for the pricing kernel of a Foreign household.

C Additional quantitative results

In this appendix we provide supplementary material accompanying the quantitative results in the paper. We first provide the complete set of model impulse responses. We next study how the transmission of safety shocks depends on the parameters of monetary policy rules in each country. We then isolate the effects of individual model parameters on equilibrium portfolios and the risk premium on Foreign bonds. We provide additional detail on our analysis of U.S. external adjustment. Finally, we decompose the role of each driving force in our simulation of the Great Recession.

C.1 Impulse responses

The responses to an increase in disaster risk are provided at the end of this appendix in Figures 11 and 12. The responses to a disaster realization are provided in Figures 13 and 14. The responses to a negative global productivity shock are provided in Figures 15 and 16. The responses to a negative Foreign productivity shock are provided in Figures 17 and 18. Finally, the responses to a positive safety shock are provided in Figures 19 and 20.

C.2 Sensitivity to monetary policy rules

Consider the following generalization of the monetary policy rule at Home (10)

\[
1 + i_t = (1 + \tilde{i})(1 + i_{t-1})^{\rho_i} \left[ \frac{P_t}{P_{t-1}} \right]^{\psi_i} \left[ \frac{y_t}{z_t} \right]^{\phi_i} 1^{-\rho_i},
\]

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and analogously in Foreign. Now the nominal interest rate can respond to output relative to trend with elasticity \( \phi^y \), and there may be inertia in the nominal interest rate as captured by the parameter \( \rho^i \).

Figure 5 depicts the response to a safety shock in the baseline model with \( \phi^\pi = 1.5, \phi^y = 0, \) and \( \rho^i = 0 \) (same as the dark blue line of Figure 3); an alternative specification with \( \phi^\pi = 1.5, \phi^y = 0.5/4, \) and \( \rho^i = 0 \) (as in Gali (2008)); and an alternative specification with \( \phi^\pi = 1.5, \phi^y = 0.5/4, \) and \( \rho^i = 0.5 \). In each case the other model parameters are recalibrated to match the same targets in Table 2 (leaving the other parameters unchanged does not change the results which follow). As is evident, when the central bank also responds to output, it slightly dampens the exchange rate and output effects of safety shocks, but quantitatively not by much. Adding interest rate inertia amplifies the exchange rate and output effects of safety shocks. With this moderate amount of inertia, the impact responses are in fact larger than in the baseline model. Taken together, we conclude that the effects of safety shocks are broadly robust to monetary policy rules characterizing the U.S. and G10 economies that have been studied in the literature.

Given this result, the main comovements of interest in Table 9 (the same as Table 3 in the main text) are also robust to these alternative policy rules.

\[ \beta(r^*_t - \Delta \log q_{t+1} - r_{t+1}, \log y_t - \log y_{t-4}) \]
\[ \beta(r^*_t - \Delta \log q_{t+1} - r_{t+1}, r^e_{t+1}) \]
\[ \beta((\Delta nfa_{t+1})/y_t, r^*_t - \Delta \log q_{t+1} - r_{t+1}) \]

<table>
<thead>
<tr>
<th>Model</th>
<th>( \phi^y = 0.5/4 )</th>
<th>( \phi^y = 0.5/4, \rho^i = 0.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( -0.11 )</td>
<td>( -0.10 )</td>
<td>( -0.13 )</td>
</tr>
<tr>
<td>( 0.06 )</td>
<td>( 0.06 )</td>
<td>( 0.08 )</td>
</tr>
<tr>
<td>( 1.45 )</td>
<td>( 1.30 )</td>
<td>( 0.85 )</td>
</tr>
</tbody>
</table>

Memo: \( (k - \kappa)/(4y) \)
\[ b_H/(4y) \]
\[ b_F/(4y) \]

60% 52% 38%
-103% -127% -73%
20% 52% 13%

Table 9: comovements under alternative monetary policy rules

Notes: moments are computed as described in note to Table 2.

\[ 49 \text{Recall that trend output is proportional to productivity } z_t \text{ since the latter follows a unit root.} \]
C.3 Determinants of portfolios and currency risk premium

Table 10 examines the sensitivity of Home’s portfolio shares and the conditional correlation of excess Foreign bond returns with each country’s pricing kernel to model parameters. Holding all other parameters as in the calibrated model, we vary \(\gamma, \chi, \sigma^\omega, \bar{b}_g, \rho^\omega\) one at a time. This contains similar insights as, but in a more granular manner than, Table 3 in the main text. The results also illustrate the usefulness of Propositions 4 and 5 in the simplified environment.

Beginning with identical risk tolerance and no safety shocks (column 1), agents hold identical per-capita positions in capital but Home is long dollar bonds financed by Foreign bonds to hedge the effects of relative productivity shocks. While our analytical results did not include this shock, it is consistent with the demands to hedge labor income risk and real exchange rate risk in (99): a negative Foreign productivity shock generates a relative decrease in Foreign labor income and real depreciation of the dollar, so efficient risk sharing calls for Home financial wealth to fall on impact. This is achieved by Home being long dollar bonds, financed by Foreign bonds.
Table 10: portfolios and risk premium

Notes: model moments are computed as described in note to Table 2.

Furthermore, because the relative productivity shock implies that the dollar depreciates in bad times (when Foreign productivity is low), excess Foreign bond returns are high when marginal utility is high. We note that the small magnitude of the correlation coefficients is because most of the volatility in pricing kernels is due to time varying disaster risk, not productivity shocks.

Making Home more risk tolerant than Foreign (column 2) implies that it increases its exposure to capital, financed by less positive / more negative positions in both bonds. Consistent with Proposition 4 in the paper, differences in risk tolerance are essential to rationalize Home’s disproportionate exposure to capital. Consistent with Proposition 5 in the paper, differences in risk tolerance alone exacerbate the reserve currency paradox, as they imply that the dollar depreciates in bad times (because Home consumption disproportionately falls). Hence, the correlation between excess Foreign bond returns and marginal utility rises in both countries.

Introducing safety shocks (column 3) implies that Home substantially reduces its position in dollar bonds and raises its position in Foreign bonds, again consistent with Proposition 4 in the paper. However, because of relative productivity shocks the sign of these positions does not yet fully switch in this column. Consistent with Proposition 5 in the paper, the presence of safety shocks mitigates the reserve currency paradox, as it pushes downward the correlation between excess Foreign bond returns and marginal utility. But again, the sign of this correlation does not yet switch.
Introducing Home government issuance of safe dollar bonds (column 4) implies that Home further reduces its position in dollar bonds and raises its position in Foreign bonds, since it can naturally insure against safety shocks due to the seignorage revenue it receives. This is again consistent with Proposition 4 in the paper, but is quantitatively small since the supply of Treasury bills relative to aggregate wealth is small in the data. Relatedly, the effects on asset price comovements are essentially unaffected in the last two rows.

Finally, matching the correlation between safety shocks and disaster risk in the data (column 5) further reduces Home’s position in dollar bonds and raises its position in Foreign bonds, since it means that the dollar is more likely to appreciate in bad times when disaster risk is high. Relatedly, this model feature pushes the correlation between excess Foreign bond returns and marginal utility in both countries to be negative. That is, we have resolved the reserve currency paradox, as dollar bonds pay relatively better when marginal utility globally is high. Moreover, this correlation is now large in magnitude, since disaster risk most drives variation in pricing kernels.

**C.4 U.S. external adjustment**

It is useful to first review the timing of events within a model period:

1. Exogenous driving forces are realized, including a rare disaster which destroys capital.

2. Production:
   
   (a) Firms hire domestic labor and import capital in excess of that supplied by domestic households.
   
   (b) Firms produce, pay workers, pay dividends to capital owners, and export undepreciated capital in excess of that supplied by domestic households.

3. Consumption, savings, and capital production:
   
   (a) Households close nominal positions from the previous period, consume domestically produced and imported goods, and trade new nominal claims and capital.
   
   (b) Global capital producers import goods from Home and Foreign and export capital to capital owners.
Net foreign assets dated in period $t$ are measured accounting for capital used in domestic production in step #2(a) of period $t + 1$, appropriately undoing the effect of capital destruction that occurs at $t + 1$. Hence, Home’s net foreign assets dated in period $t$ are

$$na_t = b_{Ht} + q_t^{-1}b_{Ft} + q_t^k (k_t - \kappa_{t+1} \exp(-\varphi_{t+1})),$$

where we use the lower-case notation for real variables introduced in appendix A. We similarly assume exports and imports dated in period $t$ measure all transactions from the beginning of step #2(b) in period $t$ through the end of step #2(a) in period $t + 1$, thus obtaining:

$$nx_t \equiv \frac{P_{Ht}}{P_t} \zeta^* c_{Ht} + \frac{P_{Ht}}{P_t} \left( \frac{\bar{k}_t}{k_{t-1} \exp(\varphi_t)} \right) x_{Ht} + q_t^k ((1 - \delta)\kappa_t - \kappa_{t+1} \exp(-\varphi_{t+1})) - q_t^{-1} \frac{P_{Ft}}{P_t} c_{Ft}.$$

It is then straightforward to use the model’s resource constraints to obtain the accounting identity

$$\Delta na_t = nx_t + r_t^k na_{t-1} + val_t,$$

where

$$val_t \equiv - \left( r_t^k - \left( \frac{1 + r_t}{1 - \omega_{t-1}} - 1 \right) \right) (b_{Ht-1} + b_{Ft-1}) + \left( q_t^{-1} (1 + r_t^*) - q_t^{-1} - \left( \frac{1 + r_t}{1 - \omega_{t-1}} - 1 \right) \right) b_{Ft-1} - \omega_t \frac{\zeta^* q_t^{-1} c_t^*}{c_t} b_{Ht,s}.$$

That is, the change in net foreign assets equals net exports plus interest income at $r_t^k$ and excess returns. The latter are collected in the term $val_t$.

### C.5 Great Recession

Figure 6 decomposes the role of each driving force in our simulation of the Great Recession. It shuts down each driving force (holding it at its mean) and simulates the effects of the other alone. It demonstrates that both safety and disaster risk shocks play important roles in our simulation of the Great Recession. The flight to safety is important in generating a U.S. output decline and valuation loss in late 2008. However, the increase in disaster risk is important in accounting for the persistence
of the output decline, particularly in Foreign, as well as high excess returns through 2009 on the U.S. external position.

Figure 7 depicts additional variables of interest. The first two panels report nominal interest rates compared to their empirical counterparts (recalling that the latter are three-month government bond yields). Nominal interest rates globally (and especially in the U.S.) fall well below zero in the model, while they were constrained by the zero lower bound in the data. While this is consistent with the decline in “shadow rates” in practice (Wu and Xia (2016)), owing to policies such as quantitative easing which are outside the model, this suggests that the model may understate the effects of disaster risk and safety shocks during this period, if anything.

The third panel of Figure 7 reports the U.S. financial wealth share in the model. While both the increase in disaster risk and flight to safety lower the U.S. wealth share on impact, the elevated disaster risk induces a rise in the wealth share thereafter as the U.S. earns high excess equity returns, while the flight to safety dissipates. Hence, the U.S. wealth share in fact slightly rises from Q1 2008 through Q1 2010 in the model. Dahlquist et al. (2023) estimate a rise in the U.S. wealth share over this period, while Sauzet (2023) estimates a decline, reflecting the difficulty in measuring
market values of wealth in a comprehensive way across countries. The model-implied change in the wealth share over this period is well within the range estimated by these papers. The model further clarifies that it is fully consistent for the U.S. wealth share to rise over the 2008-2009 period even if, on impact, both an increase in disaster risk and flight to safety reduce it.

D Empirical estimates

In this appendix we provide additional detail on empirical estimates which inform or validate the model. We first estimate the conditional correlation between global equity returns and excess G10 currency bond returns, used to calibrate the magnitude of safety shocks in the model. We then provide evidence on the effects of safety shocks in the data. We finally describe how the evidence on swap line announcement effects can be used to discipline $\epsilon_d$ in the model.

D.1 Equity returns and excess foreign bond returns

We first estimate the conditional correlation between global equity returns and excess returns on G10 currency bonds versus Treasuries. Our approach builds on that in Maggiori (2013). As we use monthly data, in this subsection we write $t$ to mean a month in time but everywhere use three-month interest rates, as in the model.

We first estimate unexpected return innovations over the next three months by
running the regressions

\[ r_t^e = \alpha_0^e + \alpha_1^e dp_{t-3} + \varepsilon_t^e, \]
\[ r_t^F = \alpha_0^F + \alpha_1^F (i_{t-3}^* - i_{t-3}) + \alpha_2^F (\log y_{t-3} - \log y_{t-15}) + \varepsilon_t^F. \]

Here, \( r_t^e \) is the real return on global equities from month \( t - 3 \) to \( t \) and \( r_t^F \equiv i_{t-3}^* - (\log E_t - \log E_{t-3}) - i_{t-3} \) is the return on a position short 3-month U.S. Treasury bills and long 3-month G10 currency bonds from month \( t - 3 \) to \( t \). The variables known at \( t - 3 \) used to predict returns are the dividend-price ratio on the global equity index \( dp_{t-3} \), the interest rate differential \( i_{t-3}^* - i_{t-3} \), and the year-over-year change in U.S. industrial production \( \log y_{t-3} - \log y_{t-15} \). The first regression is a standard predictability regression for equity returns. The second regression is consistent with Lustig et al. (2014). The estimated coefficients are provided in Table 11.

The resulting estimated return innovations are given by the estimated residuals \( \hat{\varepsilon}_t^e \) and \( \hat{\varepsilon}_t^F \). A time-series of their product is given in Figure 8. As argued in Maggiori (2013), the disproportionately positive values imply in a wide class of environments a positive risk premium on foreign bonds relative to U.S. bonds. Consistent with the “exchange rate reconnect” emphasized by Lilley et al. (2020), the values are more consistently positive after 2008. We use as our calibration target in the model the correlation of \( \hat{\varepsilon}_t^e \) and \( \hat{\varepsilon}_t^F \) over the entire period, 0.5. We obtain quantitatively similar results if we include additional conditioning variables in the predictability regressions (101) and (102) such as lagged returns or the VIX.

Table 11: predicting global equity and excess foreign bond returns

<table>
<thead>
<tr>
<th></th>
<th>( r_t^e )</th>
<th>( r_t^F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( dp_{t-3} )</td>
<td>2.3</td>
<td>(1.7)</td>
</tr>
<tr>
<td>( i_{t-3}^* - i_{t-3} )</td>
<td>1.8</td>
<td>(1.5)</td>
</tr>
<tr>
<td>( \log y_{t-3} - \log y_{t+15} )</td>
<td>-0.1</td>
<td>(0.1)</td>
</tr>
</tbody>
</table>

Notes: sample period is 1/1995 - 12/2019. Standard errors are given in parenthesis and follow Hansen and Hodrick (1980) with 4 lags to correct for overlapping observations.
Notes: $\hat{\varepsilon}_t$ and $\hat{\varepsilon}_t^F$ are residuals from the specifications estimated in Table 11. Each is expressed in percentage points.

D.2 Estimated effects of safety shocks

We now provide direct evidence on the effects of safety shocks in the data. We compute the simple average of the log real exchange rate, the three-month interest rate differential, and the difference in log industrial production between the U.S. and each of the G10 countries. Over January 1995 through December 2019, we then run a six-variable, four-lag recursive VAR with the swapped G10/T-bill spread (from Du et al. (2018a)), log real exchange rate, interest rate differential, global equity returns, log U.S. industrial production, and difference in log industrial production. We identify the effects of a safety shock by ordering the swapped G10/T-bill spread first in the VAR, so other variables can respond contemporaneously to it. This is consistent with our assumption that safety shocks are an exogenous driving force.

Figure 9 summarizes the results. As in Jiang et al. (2021) as well as our model, a positive innovation to the yield on swapped G10 bonds relative to T-bills leads to a dollar appreciation and increase in the foreign interest rate relative to U.S. interest rate. More novel, we find that a positive innovation leads to an initial decline followed by sustained increase in excess returns on the MSCI ACWI; a decline in U.S. industrial
production; and an increase in foreign production relative to U.S. production. All of these are consistent with our model.

Figure 10 quantitatively compares the empirical and model impulse responses. We first re-estimate the effects of safety shocks using quarterly data, since our model is solved at a quarterly frequency. We run a one-lag recursive VAR over Q1 1995 through Q4 2019 with the same variables as above.\(^{50}\) We then simulate a safety shock in the model, setting the initial innovation to equal the estimated innovation in the swapped G10/T-bill spread, multiplied by the ratio of the unconditional volatilities of \(\omega_t\) to the swapped G10/T-bill spread.\(^{51}\) The top right panel reflects that U.S. monetary policy is too responsive to safety shocks in the model. This is consistent with the model undershooting the effects on other asset prices and quantities in all other panels. We thus conclude that the model may be conservative, if anything, in quantifying the

\(^{50}\)The only exception is that we replace the one-month excess equity return with the three-month excess equity return.

\(^{51}\)Recall that the swapped G10/T-bill spread understates the volatility of \(\omega_t\) if swapped G10 bonds are also partially valued for their liquidity or safety, so we calibrate \(\sigma_{\omega}\) to match the conditional correlation between equity returns and excess foreign bond returns in the data.
importance of safety shocks for asset prices and real fluctuations.

D.3 Estimating $\epsilon^d$ from swap line announcements

We finally describe how the estimated announcement effects of swap lines can be used to discipline $\epsilon^d$ in our model.

Section 6.4 describes how we translate the announcement effects estimated in Kekre and Lenel (2023) into a 14bp decline in $\omega_t$. Given this decline in $\omega_t$, we can estimate the elasticity of safe asset demand $\epsilon^d$ in (17) given an assumption on the news regarding the expanded supply of safe dollar assets contained in these announcements. A plausible range is $50bn - 450bn$. The lower end corresponds to the assumption that only the March 19 announcement contained news about incremental swap line usage (of $50bn$ in the subsequent weeks, by the central banks granted temporary swap lines$^{52}$), since the March 20 announcement only pertained to the frequency of

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52See Chart 3 of Choi, Goldberg, Lerman, and Ravazzolo (2022).
swap line operations. The upper end corresponds to the assumption that all $450bn in swap line usage in the subsequent weeks was communicated to the public in the March 19-20 announcements. Since a range of $50bn − $450bn corresponds to roughly 0.25% − 2.25% of annual U.S. GDP, and annual Home GDP in the model is roughly 2 times quarterly global consumption, this implies $b^g_t \equiv -B^g_{Ht,s}/(P_t c_t + \zeta E_t^{-1} P^c c_t)$ rose by 50 − 450bp. With a 14bp resulting decline in $\omega_t$, equation (17) then implies that $\epsilon^d$ is between 4 and 32.

By construction, the scenarios of a $50bn increase in $-B^g_{Ht,s}$ and $\epsilon^d = 4$, and a $450bn increase in $-B^g_{Ht,s}$ and $\epsilon^d = 32$, induce the same increase in $\omega_t$. They only differ in the implications for seignorage earned by the U.S. However, the latter is small relative to the general equilibrium effects of the change in $\omega_t$. For instance, even in the case of a $450bn increase in $-B^g_{Ht,s}$ fully absorbed by foreigners (2.25% of annual U.S. GDP), given an initial value of $\omega_t$ of say 1%, the seignorage earned by the U.S. in the first period would be 2.25bp of annual U.S. GDP.53 This compares to the roughly 440bp response in U.S. NFA from the decline in $\omega_t$ reported in section 6.4. This is why in the main text we simply simulate a shock to $\omega_t$ of $-14bp$ in the first period, with the understanding that this corresponds to a shock to $B^g_{Ht,s}$ in the background and appropriate value of $\epsilon^d$.

53An initial value of $\omega_t = 1\%$ is consistent with the annualized swapped G10/Tbill spread of 0.74% as of March 19, 2020, multiplied by the ratio of the volatility of $\omega_t$ relative to the swapped G10/Tbill spread in the data.
Figure 11: effects of disaster risk (1/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.
Figure 12: effects of disaster risk (2/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.
Figure 13: effects of disaster realization (1/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.
Figure 14: effects of disaster realization (2/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.
Figure 15: effects of global productivity shock (1/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.
Figure 16: effects of global productivity shock (2/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.
Figure 17: effects of relative productivity shock (1/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.
Figure 18: effects of relative productivity shock (2/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.
Figure 19: effects of safety shock (1/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.
Figure 20: effects of safety shock (2/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.