Random Search and Over-the-Counter Markets

Robert Shimer

Macro Finance Research Program

August 2, 2022

Over-the-Counter Markets

- many financial markets rely on bilateral trading
  - e.g., bonds, derivatives, currencies
- often modeled as a core-periphery network
  - each peripheral trader is linked to one core trader
  - core traders are linked to each other
  - typically a static environment
- random search approach
  - explicitly dynamic
  - allow for periphery-periphery trades
  - allow for peripheral traders to link with multiple core traders
  - natural framework for exploring frictionless limit
  - explore more complicated network structures and why they arise
- follow Duffie-Gârleanu-Pedersen (*Econometrica* 2005) to start

Investors

- measure one, risk-neutral, discount rate r
- two preference states ("intrinsic type"), $\{h, l\}$
  - switches from l to h at rate γ_u, from h to l at rate γ_d
  - independent switches across all pairs of investors
- two asset holdings, $\{o, n\}$ (risk-neutrality, quantity limited to $[0, 1]$)
  - high-type investor gets 1 from holding the asset
  - low-type investor gets $1 - \delta$
- total asset supply $s \in (0, 1)$
  - $\mu_{ho}(t) + \mu_{hn}(t) + \mu_{lo}(t) + \mu_{ln}(t) = 1$
  - $\mu_{ho}(t) + \mu_{lo}(t) = s$

Trading

- trades between investors
  - an investor meets another investor at rate λ
  - uniformly at random from investor population
  - independent of preference switches
  - swap asset for outside consumption good
- trades with marketmakers
  - an investor meets a marketmaker at rate ρ
  - marketmakers have instantaneous access to an interdealer market
  - marketmakers do not hold any invetories
  - swap asset for outside consumption good
- terms of trade set by Nash bargaining
  - two investors: seller has bargaining power q
  - investor-marketmaker: maker-maker has bargaining power z

Evolution of State Variables

- bilateral trades between *hn* and *lo*
- *hn* and *lo* trade with middlemen whenever possible

$$\dot{\mu}_{lo}(t) = -\big(\lambda\mu_{hn}(t)\mu_{lo}(t) + \rho\min\{\mu_{lo}(t), \mu_{hn}(t)\}\big) - \gamma_u\mu_{lo}(t) + \gamma_d\mu_{ho}(t)$$

$$\dot{\mu}_{hn}(t) = -\big(\lambda\mu_{hn}(t)\mu_{lo}(t) + \rho\min\{\mu_{lo}(t), \mu_{hn}(t)\}\big) - \gamma_d\mu_{hn}(t) + \gamma_u\mu_{ln}(t)$$

$$\dot{\mu}_{ln}(t) = \big(\lambda\mu_{hn}(t)\mu_{lo}(t) + \rho\min\{\mu_{lo}(t), \mu_{hn}(t)\}\big) - \gamma_u\mu_{ln}(t) + \gamma_d\mu_{hn}(t)$$

$$\dot{\mu}_{ho}(t) = \big(\lambda\mu_{hn}(t)\mu_{lo}(t) + \rho\min\{\mu_{lo}(t), \mu_{hn}(t)\}\big) - \gamma_d\mu_{ho}(t) + \gamma_u\mu_{lo}(t)$$

- DGP prove convergence to steady state

Value Functions

- bid price B, ask price A, bilateral price P

$$
\begin{aligned}
rV_{lo} - \dot{V}_{lo} &= (1-\delta) + \gamma_u(V_{ho} - V_{lo}) + \lambda\mu_{hn}(V_{ln} - V_{lo} + P) \\
&\quad + \rho \min\{1, \mu_{hn}/\mu_{lo}\}\big)(V_{ln} - V_{lo} + B) \\
rV_{hn} - \dot{V}_{hn} &= \gamma_d(V_{ln} - V_{hn}) + \lambda\mu_{lo}(V_{ho} - V_{hn} - P) \\
&\quad + \rho \min\{\mu_{lo}/\mu_{hn}, 1\}(V_{ho} - V_{hn} - A) \\
rV_{ln} - \dot{V}_{ln} &= \gamma_u(V_{hn} - V_{ln}) \\
rV_{ho} - \dot{V}_{ho} &= 1 + \gamma_d(V_{lo} - V_{ho})
\end{aligned}
$$

Prices

- $P = (1-q)(V_{lo} - V_{ln}) + q(V_{ho} - V_{hn})$
- $A = z(V_{ho} - V_{hn}) + (1-z)M$
- $B = z(V_{lo} - V_{ln}) + (1-z)M$
- M is the price in the interdealer market
  - if $\mu_{lo} > \mu_{hn}$, $M = V_{lo} - V_{ln} = B$
  - if $\mu_{hn} > \mu_{lo}$, $M = V_{ho} - V_{hn} = A$
  - if $\mu_{hn} = \mu_{lo}$, M lies between these bounds

Equilibrium Definition

- similar notion to mean-field game
  1. state variables satisfy Kolmogorov forward equations
  2. values satisfy HJB equations given prices and state variables
  3. prices satisfy Nash bargaining and interdealer market clears

- see paper for explicit characterization of steady state

Walrasian Limit

- fast trades, either through marketmakers or bilaterally
- equilibrium depends on whether $s \gtrless \gamma_u/(\gamma_u + \gamma_d)$ (share of h)
  - small s: marginal buyer is h, price is $P^* = 1/r$
  - large s: marginal buyer is l, price is $P^* = (1-\delta)/r$
  - $s = \gamma_u/(\gamma_u + \gamma_d)$: any price in this interval
- let $\lambda \to \infty$ with $0 < q < 1$. $P, A, B \to P^*$
- let $\rho \to \infty$ with $z < 1$. $P, A, B \to P^*$
- if $z = 1$, $A - B$ is increasing in ρ
  - monopolist marketmaker eliminates bilateral trading opportunities

Taking Stock

- recall key features of random search environment
  - explicitly dynamic
  - allow for periphery-periphery trades
  - allow for peripheral traders to link with multiple core traders
  - natural framework for exploring frictionless limit
  - explore more complicated network structures and why they arise
- Farboodi-Jarosch-Shimer (2022)

Modifications

- Simplifications
  - half the traders are h, half are l: $\gamma_d = \gamma_u = \gamma$
  - stock of assets is $s = 1/2$
  - symmetric bargaining power: $q = 1/2$
  - there are no marketmakers: $\rho = 0$
- Modification: traders die at rate r, replaced by newborn
- Complication: newborn traders choose $\lambda \in \mathcal{X}$ at cost $C(\lambda)$
  - bounded $\mathcal{X}$ is technically useful when C is (asymptotically) linear

Meeting Technology

- meet investors at a rate proportional to their contact rate
  - but conditionally independent of their asset holdings and preferences

- counterparty measure $\mu_F(S)$: probability that, conditional on a meeting, the counterparty's contact rate is some $\lambda' \in S$
  - associated CDF F

- contact rate measure $\mu_G(S)$: measure of traders whose contact rate is some $\lambda' \in S$
  - associated CDF G

$$\mu_F(S) \equiv \frac{\int_S \lambda d\mu_G(\lambda)}{\int_{\mathcal{X}} \lambda d\mu_G(\lambda)}$$

$$\Lambda \equiv \int_{\mathcal{X}} \lambda d\mu_G(\lambda)$$

Trading and Prices

- trading is possible iff individuals have different asset holdings
- *misaligned* asset position: *hn* and *lo*
  - m_λ: fraction of misaligned type λ traders
- types of trades:
  1. fundamental trade: *hn* buys from *lo*.
  2. *intermediation*: *ln* buys from *lo*
     hn buys from *ho*
- symmetric Nash bargaining splits the gains from trade

Symmetry

- underlying environment is symmetric
- look for equilibrium where trade is symmetric:
  - (λ, ho) sells to $(\lambda', hn) \Leftrightarrow (\lambda, ln)$ buys from (λ', ho)
- this implies the misalignment rate is symmetric:
  - half of all misaligned traders of type λ are in state hn

Joint Surplus

- value functions:
  - $v_{\lambda,0} \equiv \frac{1}{2}(v_{\lambda,hn} + v_{\lambda,lo})$: average value if misaligned
  - $v_{\lambda,1} \equiv \frac{1}{2}(v_{\lambda,ho} + v_{\lambda,ln})$: average value if well-aligned
  - $s(\lambda) \equiv v_{\lambda,1} - v_{\lambda,0}$: joint surplus

$$
\begin{aligned}
rv_{\lambda,0} = & \frac{1}{2}(1-\delta) + \gamma s(\lambda) \\
& + \frac{\lambda}{4} \int_{\mathcal{X}} \left((s(\lambda) + s(\lambda'))^+ m_{\lambda'} + (s(\lambda) - s(\lambda'))^+ (1 - m_{\lambda'}) \right) dF(\lambda'),
\end{aligned}
$$

$$
\begin{aligned}
rv_{\lambda,1} = & \frac{1}{2} - \gamma s(\lambda) \\
& + \frac{\lambda}{4} \int_{\mathcal{X}} \left((s(\lambda') - s(\lambda))^+ m_{\lambda'} + (-s(\lambda) - s(\lambda'))^+ (1 - m_{\lambda'}) \right) dF(\lambda')
\end{aligned}
$$

- $s(\lambda)$ governs the direction of trade
- when $s(\lambda) > s(\lambda')$, λ' takes on misalignment from λ

Definition of Equilibrium: μ_F, m, s such that

1. the surplus function is given by

$$(r+2\gamma)s(\lambda) = \frac{\delta}{2} + \frac{\lambda}{4}\int_{\mathcal{X}} \Big(\big((s(\lambda)+s(\lambda'))^+ - (s(\lambda')-s(\lambda))^+\big) m_{\lambda'} + \big((s(\lambda)-s(\lambda'))^+ - (-s(\lambda)-s(\lambda'))^+\big)(1-m_{\lambda'}) \Big) d\mu_F(\lambda')$$

2. balanced inflow-outflow in equilibrium

$$\left(r+\gamma+\frac{\lambda}{2}\int_{\mathcal{X}} \left(\mathbb{I}_{s(\lambda)+s(\lambda')>0} m_{\lambda'} + \mathbb{I}_{s(\lambda)>s(\lambda')}(1-m_{\lambda'})\right) d\mu_F(\lambda')\right) m_\lambda = \left(\gamma+\frac{\lambda}{2}\int_{\mathcal{X}} \left(\mathbb{I}_{s(\lambda)<s(\lambda')} m_{\lambda'} + \mathbb{I}_{s(\lambda)+s(\lambda')<0}(1-m_{\lambda'})\right) d\mu_F(\lambda')\right)(1-m_\lambda)$$

3. optimal investment: $\mu_F(\mathcal{Y}) = 1$, where $\mathcal{Y} = \arg\max_{\lambda\in\mathcal{X}} \pi_\lambda$,

$$r\pi_\lambda = \frac{1}{2} - \gamma s(\lambda) + \frac{\lambda}{4}\int_{\mathcal{X}} \left((s(\lambda')-s(\lambda))^+ m_{\lambda'} + (-s(\lambda)-s(\lambda'))^+(1-m_{\lambda'})\right) d\mu_F(\lambda') - rC(\lambda)$$

Stepping Stone. Equilibrium Trading Patterns

Lemma

The surplus function $s(\lambda)$ is positive-valued and strictly decreasing. When two traders with opposite asset positions meet they

1. *always trade the asset if both are misaligned;*
2. *never trade the asset if both are well-aligned;*
3. *trade the asset if one is misaligned and the other is well-aligned and the well-aligned trader has the higher contact rate.*

- faster traders act as intermediaries for slower traders
- special case of trading patterns in Neklyudov (2019) and Üslü (2019)

Dispersion in Contact Rates

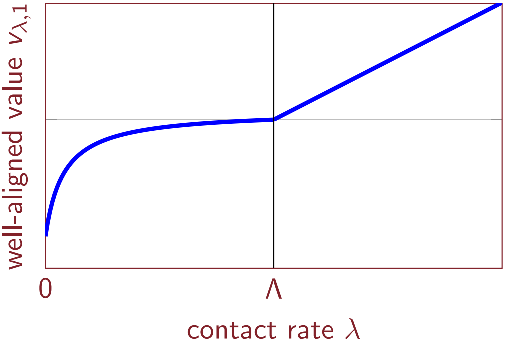
Proposition

Assume C is differentiable and C' is Lipschitz continuous. Then the equilibrium counterparty distribution F and contact rate distribution G are absolutely continuous on $[0, \bar{\lambda})$. If additionally C is weakly convex, $C'(0) < \frac{\delta\gamma^2}{4r(r+2\gamma)^3}$, and $C'(\bar{\lambda}) \geq \frac{2\gamma\delta}{r\bar{\lambda}^2}$, then a positive measure of traders choose a contact rate in the interval $(0, \bar{\lambda})$ in any equilibrium.

- proof shows that mass points create kinks in the incentive to invest
  - slightly slower than mass point: they can all intermediate for you
  - slightly faster than mass point: you can intermediate for all of them

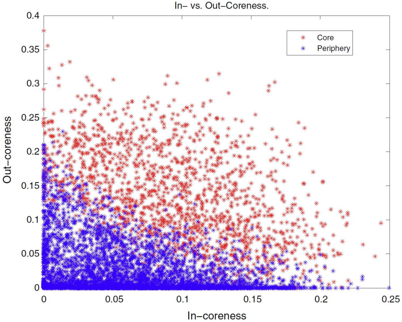
Special case: Degenerate Equilibrium

- single mass point at Λ
- $\lambda > \Lambda$: convex profit function
- $\lambda < \Lambda$: concave profit function
- *convex kink* at $\lambda = \Lambda$



Comparison to other core-periphery predictions

1. continuous heterogeneity
   - Fricke and Lux (2015): estimate a network model that allows for a continuous notion of "coreness" (Italian interbank market)



   - vast amount of heterogeneity, inconsistent with a binary classification
   - in't Veld and van Lelyveld (2014) (Dutch interbank market), Di Maggio, Kermani and Song (2017) (US corporate bond market)

Comparison to other core-periphery predictions

2. **widespread intermediation**

    - Craig and Von Peter (2014):

      classify 2.7% of the banks in German interbank market as the core, while 92.7% of banks occasionally intermediate

    - Bech and Atalay (2010):

      multi-tier fed fund market $\rightarrow$ many links within tiers (figure 4)

    - Hollifield, Neklyudov and Spatt (2017): multi-tier US market for asset-backed securities (figure 5)

      peripheral dealers frequently part of long intermediation chains with multiple dealers (Table 4)

Linear Cost Function

- assume $C(\lambda) = c\lambda$
  - if $\mathcal{X} = [0, \bar{\lambda}]$ and c is not too large, $\mu_F(\bar{\lambda}) > 0$
  - sequence of economies with $\bar{\lambda} \to \infty$: true in the limit as well
  - zero measure of investors account for a positive fraction of trades
    - ⋆ DGP marketmakers? We call them middlemen
- solution method: reduce to a pair of first order differential equations

$C(\lambda) = c\lambda$, $\bar{\lambda} \to \infty$

- assume $c < \frac{\gamma\delta}{16r(r+\gamma)(r+2\gamma)}$

Proposition

There are middlemen [$\lim_{\lambda\to\infty} F(\lambda) < 1$]; and the contact rate distribution has a Pareto tail with index 2 [$\lim_{\lambda\to\infty} \lambda^2(1 - G(\lambda))$ is positive and finite].

Corollary

The fraction of trades with middlemen is strictly positive; and the trading rate distribution has a Pareto tail with index 2

Empirical Content

1. **small, highly connected core representing a strictly positive measure of trades**
   - Craig and Von Peter (2014): 2.7% of banks as the core (German interbank market)

     in't Veld and van Lelyveld (2014): 13 percent of banks in the core (Dutch interbank market)
   - Hollifield, Neklyudov and Spatt (2017): core of $6-10\%$ of dealers, do $60-70\%$ of trades (inter-dealer derivatives market)

     Di Maggio, Kermani and Song (2017): core of top 50 dealers, 80% of transactions (corporate bond market)
2. **Pareto tail**
   - Li and Schurhoff (2019): municipal bonds market, Hollifield, Neklyudov and Spatt (2017): derivatives, Peltonen, Scheicher and Vuillemey (2014): credit default swap market, Bech and Atalay (2010): out-degree of banks in the federal funds market
   - Boss, Elsinger, Summer and Thurner (2004), De Masi, Iori, Precup, Gabbi and Caldarelli (2008), De Masi, Iori and Caldarelli (2006) for different European interbank markets

Frictionless Limit, $C(\lambda) = c\lambda$

Proposition

Assume $C(\lambda) = c\lambda$. Consider a sequence of limiting equilibria as c converges to zero. The aggregate trading volume $\mathcal{V}$ converges to approximately 2.46γ and can be decomposed as follows: middlemen's purchases from other middlemen account for a volume $\mathcal{V}_{mm} = \frac{1}{2}\gamma$; middlemen's purchases from non-middlemen account for a volume of $\mathcal{V}_{mn} = \frac{1}{2}\gamma$; non-middlemen's purchases from middlemen account for a volume $\mathcal{V}_{nm} = \frac{1}{2}\gamma$; and non-middlemen's purchases from non-middlemen account for a volume $\mathcal{V}_{nn} \approx 0.96\gamma$.

- (almost) no misalignment, (almost) all misalignment is in middlemen
- (almost) all fundamental trades are between middlemen
- intermediation chains and heterogeneity remain prominent in limit

Pareto Optimal Allocation

- planner chooses the distribution of meeting rates along with the set of admissible trades to maximize steady state utility net of meeting costs

$$1 - \frac{\delta}{2} \int_{\mathcal{X}} m_\lambda d\mu_G(\lambda) - r \int_{\mathcal{X}} C(\lambda) d\mu_G(\lambda)$$

- planner faces same constraint on evolution of misaligned rate
- we impose the same symmetry on planner as in equilibrium
- Lagrangian multipliers play the role of decentralized Bellman values

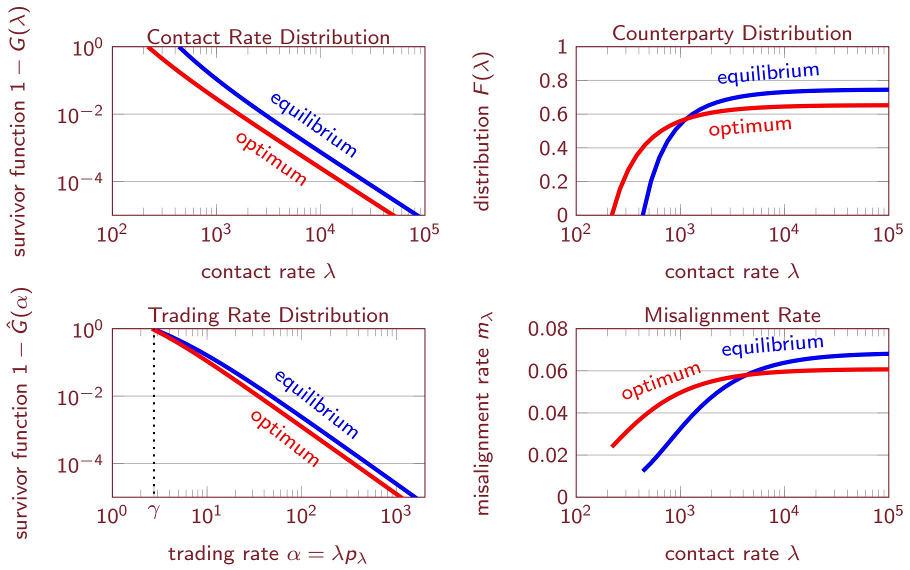
Theoretical Characterization

Proposition

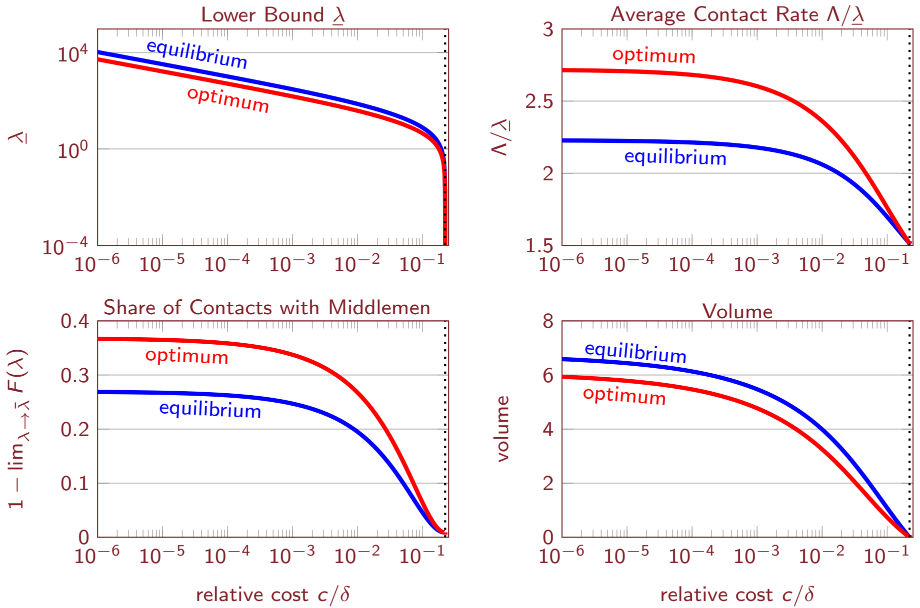
The characterization of equilibrium above applies to the social planner's problem, except non-middlemen trade less frequently with each other

- two externalities in equilibrium:
  - only capture half the private surplus from own meetings
  - take away meetings between others
- Pigouvian tax: improve terms of trade for slow traders

$c/\delta = 0.0005,\ r = 0.05,\ \gamma = 2.75$



Excess Equilibrium Trade



Summary

- random search provides a theory of networks in OTC markets
- straightforward framework for modeling the decision to form links
- not (yet) amenable to choices about which links to form
  - thus doesn't capture repeated interactions very well
  - directed or partially directed search?

Random Search and Over-the-Counter Markets

Robert Shimer

Macro Finance Research Program

August 2, 2022