Random Search and Over-the-Counter Markets

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Over-the-Counter Markets

- many financial markets rely on bilateral trading
  - e.g., bonds, derivatives, currencies

- often modeled as a core-periphery network
  - each peripheral trader is linked to one core trader
  - core traders are linked to each other
  - typically a static environment

- random search approach
  - explicitly dynamic
  - allow for periphery-periphery trades
  - allow for peripheral traders to link with multiple core traders
  - natural framework for exploring frictionless limit
  - explore more complicated network structures and why they arise

- follow Duffie-Gârleanu-Pedersen (*Econometrica* 2005) to start
Investors

- measure one, risk-neutral, discount rate $r$

- two preference states ("intrinsic type"), $\{h, l\}$
  - switches from $l$ to $h$ at rate $\gamma_u$, from $h$ to $l$ at rate $\gamma_d$
  - independent switches across all pairs of investors

- two asset holdings, $\{o, n\}$ (risk-neutrality, quantity limited to $[0, 1]$)
  - high-type investor gets 1 from holding the asset
  - low-type investor gets $1 - \delta$

- total asset supply $s \in (0, 1)$
  - $\mu_{ho}(t) + \mu_{hn}(t) + \mu_{lo}(t) + \mu_{ln}(t) = 1$
  - $\mu_{ho}(t) + \mu_{lo}(t) = s$
Trading

- trades between investors
  - an investor meets another investor at rate $\lambda$
  - uniformly at random from investor population
  - independent of preference switches
  - swap asset for outside consumption good

- trades with marketmakers
  - an investor meets a marketmaker at rate $\rho$
  - marketmakers have instantaneous access to an interdealer market
  - marketmakers do not hold any inventories
  - swap asset for outside consumption good

- terms of trade set by Nash bargaining
  - two investors: seller has bargaining power $q$
  - investor-marketmaker: maker-maker has bargaining power $z$
Evolution of State Variables

- bilateral trades between $hn$ and $lo$
- $hn$ and $lo$ trade with middlemen whenever possible

\[
\dot{\mu}_{lo}(t) = -\left( \lambda \mu_{hn}(t) \mu_{lo}(t) + \rho \min\{\mu_{lo}(t), \mu_{hn}(t)\} \right) - \gamma_u \mu_{lo}(t) + \gamma_d \mu_{ho}(t)
\]
\[
\dot{\mu}_{hn}(t) = -\left( \lambda \mu_{hn}(t) \mu_{lo}(t) + \rho \min\{\mu_{lo}(t), \mu_{hn}(t)\} \right) - \gamma_d \mu_{hn}(t) + \gamma_u \mu_{ln}(t)
\]
\[
\dot{\mu}_{ln}(t) = \left( \lambda \mu_{hn}(t) \mu_{lo}(t) + \rho \min\{\mu_{lo}(t), \mu_{hn}(t)\} \right) - \gamma_u \mu_{ln}(t) + \gamma_d \mu_{hn}(t)
\]
\[
\dot{\mu}_{ho}(t) = \left( \lambda \mu_{hn}(t) \mu_{lo}(t) + \rho \min\{\mu_{lo}(t), \mu_{hn}(t)\} \right) - \gamma_d \mu_{ho}(t) + \gamma_u \mu_{lo}(t)
\]

- DGP prove convergence to steady state
Value Functions

- bid price $B$, ask price $A$, bilateral price $P$

\[ rV_{lo} - \dot{V}_{lo} = (1 - \delta) + \gamma_u (V_{ho} - V_{lo}) + \lambda \mu_{hn} (V_{ln} - V_{lo} + P) \]
\[ + \rho \min\{1, \mu_{hn}/\mu_{lo}\} (V_{ln} - V_{lo} + B) \]

\[ rV_{hn} - \dot{V}_{hn} = \gamma_d (V_{ln} - V_{hn}) + \lambda \mu_{lo} (V_{ho} - V_{hn} - P) \]
\[ + \rho \min\{\mu_{lo}/\mu_{hn}, 1\} (V_{ho} - V_{hn} - A) \]

\[ rV_{ln} - \dot{V}_{ln} = \gamma_u (V_{hn} - V_{ln}) \]

\[ rV_{ho} - \dot{V}_{ho} = 1 + \gamma_d (V_{lo} - V_{ho}) \]
Prices

- \[ P = (1 - q)(V_{lo} - V_{ln}) + q(V_{ho} - V_{hn}) \]
- \[ A = z(V_{ho} - V_{hn}) + (1 - z)M \]
- \[ B = z(V_{lo} - V_{ln}) + (1 - z)M \]

- \( M \) is the price in the interdealer market
  - if \( \mu_{lo} > \mu_{hn} \), \( M = V_{lo} - V_{ln} = B \)
  - if \( \mu_{hn} > \mu_{lo} \), \( M = V_{ho} - V_{hn} = A \)
  - if \( \mu_{hn} = \mu_{lo} \), \( M \) lies between these bounds
Equilibrium Definition

- similar notion to mean-field game
  1. state variables satisfy Kolmogorov forward equations
  2. values satisfy HJB equations given prices and state variables
  3. prices satisfy Nash bargaining and interdealer market clears

- see paper for explicit characterization of steady state
Walrasian Limit

- fast trades, either through marketmakers or bilaterally

- equilibrium depends on whether \( s \geq \gamma_u/(\gamma_u + \gamma_d) \) (share of \( h \))
  - small \( s \): marginal buyer is \( h \), price is \( P^* = 1/r \)
  - large \( s \): marginal buyer is \( l \), price is \( P^* = (1 - \delta)/r \)
  - \( s = \gamma_u/(\gamma_u + \gamma_d) \): any price in this interval

- let \( \lambda \to \infty \) with \( 0 < q < 1 \). \( P, A, B \to P^* \)
- let \( \rho \to \infty \) with \( z < 1 \). \( P, A, B \to P^* \)
- if \( z = 1 \), \( A - B \) is increasing in \( \rho \)
  - monopolist marketmaker eliminates bilateral trading opportunities
Taking Stock

 recall key features of random search environment
  ▶ explicitly dynamic
  ▶ allow for periphery-periphery trades
  ▶ allow for peripheral traders to link with multiple core traders
  ▶ natural framework for exploring frictionless limit
  ▶ explore more complicated network structures and why they arise

 Farboodi-Jarosch-Shimer (2022)
Modifications

- **Simplifications**
  - half the traders are $h$, half are $l$: $\gamma_d = \gamma_u = \gamma$
  - stock of assets is $s = 1/2$
  - symmetric bargaining power: $q = 1/2$
  - there are no marketmakers: $\rho = 0$

- **Modification**: traders die at rate $r$, replaced by newborn

- **Complication**: newborn traders choose $\lambda \in \mathcal{X}$ at cost $C(\lambda)$
  - bounded $\mathcal{X}$ is technically useful when $C$ is (asymptotically) linear
● meet investors at a rate proportional to their contact rate
  ▶ but conditionally independent of their asset holdings and preferences

● counterparty measure $\mu_F(S)$: probability that, conditional on a meeting, the counterparty's contact rate is some $\lambda' \in S$
  ▶ associated CDF $F$

● contact rate measure $\mu_G(S)$: measure of traders whose contact rate is some $\lambda' \in S$
  ▶ associated CDF $G$

\[
\mu_F(S) \equiv \frac{\int_S \lambda d\mu_G(\lambda)}{\int_{\mathcal{X}} \lambda d\mu_G(\lambda)}
\]
\[
\Lambda \equiv \int_{\mathcal{X}} \lambda d\mu_G(\lambda)
\]
Trading and Prices

- trading is possible iff individuals have different asset holdings

- *misaligned* asset position: \( hn \) and \( lo \)
  - \( m_\lambda \): fraction of misaligned type \( \lambda \) traders

- types of trades:
  1. fundamental trade: \( hn \) buys from \( lo \).
  2. *intermediation*: \( ln \) buys from \( lo \)
     \( hn \) buys from \( ho \)

- symmetric Nash bargaining splits the gains from trade
underlying environment is symmetric

look for equilibrium where trade is symmetric:
  - $\left(\lambda, ho\right)$ sells to $\left(\lambda', hn\right)$ $\iff$ $\left(\lambda, ln\right)$ buys from $\left(\lambda', ho\right)$

this implies the misalignment rate is symmetric:
  - half of all misaligned traders of type $\lambda$ are in state $hn$
Joint Surplus

- value functions:
  - \( v_{\lambda,0} \equiv \frac{1}{2}(v_{\lambda,hn} + v_{\lambda,lo}) \): average value if misaligned
  - \( v_{\lambda,1} \equiv \frac{1}{2}(v_{\lambda,ho} + v_{\lambda,ln}) \): average value if well-aligned
  - \( s(\lambda) \equiv v_{\lambda,1} - v_{\lambda,0} \): joint surplus

\[
rv_{\lambda,0} = \frac{1}{2}(1 - \delta) + \gamma s(\lambda) \\
+ \frac{\lambda}{4} \int_{\lambda'} \left( (s(\lambda) + s(\lambda'))^+ m_{\lambda'} + (s(\lambda) - s(\lambda'))^+(1 - m_{\lambda'}) \right) dF(\lambda'),
\]

\[
rv_{\lambda,1} = \frac{1}{2} - \gamma s(\lambda) \\
+ \frac{\lambda}{4} \int_{\lambda'} \left( (s(\lambda') - s(\lambda))^+ m_{\lambda'} + (-s(\lambda) - s(\lambda'))^+(1 - m_{\lambda'}) \right) dF(\lambda'),
\]

- \( s(\lambda) \) governs the direction of trade
- when \( s(\lambda) > s(\lambda') \), \( \lambda' \) takes on misalignment from \( \lambda \)
Definition of Equilibrium: $\mu_F$, $m$, $s$ such that

1. the surplus function is given by

$$
(r + 2\gamma)s(\lambda) = \frac{\delta}{2} + \frac{\lambda}{4} \int_{\mathcal{X}} \left( ((s(\lambda) + s(\lambda'))^+ - (s(\lambda') - s(\lambda))^+) m_{\lambda'} + ((s(\lambda) - s(\lambda'))^+ - (-s(\lambda) - s(\lambda'))^+) (1 - m_{\lambda'}) \right) d\mu_F(\lambda')
$$

2. balanced inflow-outflow in equilibrium

$$
\left( r + \gamma + \frac{\lambda}{2} \int_{\mathcal{X}} \left( \begin{array}{c} \mathbb{I}_{s(\lambda) + s(\lambda') > 0} m_{\lambda'} + \mathbb{I}_{s(\lambda) > s(\lambda')} (1 - m_{\lambda'}) \end{array} \right) d\mu_F(\lambda') \right) m_{\lambda}

= \left( \gamma + \frac{\lambda}{2} \int_{\mathcal{X}} \left( \begin{array}{c} \mathbb{I}_{s(\lambda) < s(\lambda')} m_{\lambda'} + \mathbb{I}_{s(\lambda) + s(\lambda') < 0} (1 - m_{\lambda'}) \end{array} \right) d\mu_F(\lambda') \right) (1 - m_{\lambda})
$$

3. optimal investment: $\mu_F(\mathcal{Y}) = 1$, where $\mathcal{Y} = \arg\max_{\lambda \in \mathcal{X}} \pi_{\lambda}$,

$$
r_{\pi_{\lambda}} = \frac{1}{2} - \gamma s(\lambda) + \frac{\lambda}{4} \int_{\mathcal{X}} \left( (s(\lambda') - s(\lambda))^+ m_{\lambda'} + (-s(\lambda) - s(\lambda'))^+ (1 - m_{\lambda'}) \right) d\mu_F(\lambda') - rC(\lambda)
$$
Lemma

The surplus function \( s(\lambda) \) is positive-valued and strictly decreasing. When two traders with opposite asset positions meet they

1. always trade the asset if both are misaligned;
2. never trade the asset if both are well-aligned;
3. trade the asset if one is misaligned and the other is well-aligned and the well-aligned trader has the higher contact rate.

- faster traders act as intermediaries for slower traders
- special case of trading patterns in Neklyudov (2019) and Üslü (2019)
Dispersion in Contact Rates

Proposition

Assume $C$ is differentiable and $C'$ is Lipschitz continuous. Then the equilibrium counterparty distribution $F$ and contact rate distribution $G$ are absolutely continuous on $[0, \bar{\lambda})$. If additionally $C$ is weakly convex, $C'(0) < \frac{\delta \gamma^2}{4r(r+2\gamma)^3}$, and $C'(\bar{\lambda}) \geq \frac{2\gamma \delta}{r \lambda^2}$, then a positive measure of traders choose a contact rate in the interval $(0, \bar{\lambda})$ in any equilibrium.

- proof shows that mass points create kinks in the incentive to invest
  - slightly slower than mass point: they can all intermediate for you
  - slightly faster than mass point: you can intermediate for all of them
Special case: Degenerate Equilibrium

- single mass point at $\Lambda$
- $\lambda > \Lambda$: convex profit function
- $\lambda < \Lambda$: concave profit function
- *convex kink* at $\lambda = \Lambda$

![Graph showing well-aligned value $v_{\lambda,1}$ vs contact rate $\lambda$ with a convex kink at $\lambda = \Lambda$.]
Comparison to other core-periphery predictions

1. continuous heterogeneity

- Fricke and Lux (2015): estimate a network model that allows for a continuous notion of “coreness” (Italian interbank market)

- vast amount of heterogeneity, inconsistent with a binary classification

- in’t Veld and van Lelyveld (2014) (Dutch interbank market), Di Maggio, Kermani and Song (2017) (US corporate bond market)
Comparison to other core-periphery predictions

2. widespread intermediation

- Craig and Von Peter (2014): classify 2.7% of the banks in German interbank market as the core, while 92.7% of banks occasionally intermediate.


• assume $C(\lambda) = c\lambda$
  ▶ if $\mathcal{X} = [0, \bar{\lambda}]$ and $c$ is not too large, $\mu_F(\bar{\lambda}) > 0$
  ▶ sequence of economies with $\bar{\lambda} \to \infty$: true in the limit as well
  ▶ zero measure of investors account for a positive fraction of trades
    ★ DGP marketmakers? We call them middlemen

• solution method: reduce to a pair of first order differential equations
\[ C(\lambda) = c\lambda, \quad \lambda \to \infty \]

- assume \( c < \frac{\gamma \delta}{16r(r+\gamma)(r+2\gamma)} \)

**Proposition**

There are middlemen \( \lim_{\lambda \to \infty} F(\lambda) < 1 \); and the contact rate distribution has a Pareto tail with index 2 \( \lim_{\lambda \to \infty} \lambda^2(1 - G(\lambda)) \) is positive and finite.

**Corollary**

The fraction of trades with middlemen is strictly positive; and the trading rate distribution has a Pareto tail with index 2.
Empirical Content

1 small, highly connected core representing a strictly positive measure of trades

- Craig and Von Peter (2014): 2.7% of banks as the core (German interbank market)
- in’t Veld and van Lelyveld (2014): 13 percent of banks in the core (Dutch interbank market)
- Hollifield, Neklyudov and Spatt (2017): core of 6 – 10% of dealers, do 60 – 70% of trades (inter-dealer derivatives market)
- Di Maggio, Kermani and Song (2017): core of top 50 dealers, 80% of transactions (corporate bond market)

2 Pareto tail

Frictionless Limit, \( C(\lambda) = c\lambda \)

**Proposition**

Assume \( C(\lambda) = c\lambda \). Consider a sequence of limiting equilibria as \( c \) converges to zero. The aggregate trading volume \( V \) converges to approximately \( 2.46 \gamma \) and can be decomposed as follows: middlemen’s purchases from other middlemen account for a volume \( V_{mm} = \frac{1}{2} \gamma \); middlemen’s purchases from non-middlemen account for a volume of \( V_{mn} = \frac{1}{2} \gamma \); non-middlemen’s purchases from middlemen account for a volume \( V_{nm} = \frac{1}{2} \gamma \); and non-middlemen’s purchases from non-middlemen account for a volume \( V_{nn} \approx 0.96 \gamma \).

- (almost) no misalignment, (almost) all misalignment is in middlemen
- (almost) all fundamental trades are between middlemen
- intermediation chains and heterogeneity remain prominent in limit
Pareto Optimal Allocation

- planner chooses the distribution of meeting rates along with the set of admissible trades to maximize steady state utility net of meeting costs

\[ 1 - \frac{\delta}{2} \int \chi m_\lambda d\mu_G(\lambda) - r \int \chi C(\lambda)d\mu_G(\lambda) \]

- planner faces same constraint on evolution of misaligned rate

- we impose the same symmetry on planner as in equilibrium

- Lagrangian multipliers play the role of decentralized Bellman values
The characterization of equilibrium above applies to the social planner’s problem, except non-middlemen trade less frequently with each other.

- Two externalities in equilibrium:
  - Only capture half the private surplus from own meetings
  - Take away meetings between others

- Pigouvian tax: improve terms of trade for slow traders
\( c/\delta = 0.0005, \ r = 0.05, \ \gamma = 2.75 \)

- **Contact Rate Distribution**
  - Survivor function \( 1 - G(\lambda) \)
  - Contact rate \( \lambda \)

- **Counterparty Distribution**
  - Distribution \( F(\lambda) \)
  - Contact rate \( \lambda \)

- **Trading Rate Distribution**
  - Survivor function \( 1 - \hat{G}(\alpha) \)
  - Trading rate \( \alpha = \lambda \rho_\lambda \)

- **Misalignment Rate**
  - Misalignment rate \( m_\lambda \)
  - Contact rate \( \lambda \)
Excess Equilibrium Trade

- Lower Bound $\lambda$
- Average Contact Rate $\Lambda/\bar{\lambda}$
- Share of Contacts with Middlemen
- Volume

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p.29
random search provides a theory of networks in OTC markets

straightforward framework for modeling the decision to form links

not (yet) amenable to choices about which links to form
  ▶ thus doesn’t capture repeated interactions very well
  ▶ directed or partially directed search?
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