Digital Currencies: What does the Future Hold?
Macro Finance Research Program Summer Session

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August 1st, 2022
The Future
This talk: general remarks plus taste of my research

1. The battlefield.

2. Private cryptocurrencies:
   - Bitcoin and Blockchains.
     - Schilling-Uhlig, “Currency Substitution Under Transaction Costs”.
   - Big Players: e.g. Facebook.
     - Benigno-Schilling-Uhlig, “... Impossible Trinity”.
     - Uhlig-Xie, “Parallel Digital Currencies and Sticky Prices”.
   - DeFi, Smart Contracts and Stablecoins.
     - Uhlig, “A Luna-tic Stablecoin Crash”

3. Central bank digital currencies:
   - Pros and Cons.
     - Schilling - Fernández-Villaverde - Uhlig, “CBDC: when Price and Bank Stability Collide”.

1. The battlefield

- Privately issued cryptocurrencies:
  - New technology: the blockchain.
  - Today: several thousand cryptocurrencies.
  - Entry by “big players:” FaceBook for now.

- Central bank digital currencies:
  - Response to the competition of private cryptocurrencies.

- Traditional means of payments:
  - Cash.
  - Deposit accounts.
  - Credit cards.
  - PayPal.
  - Fast retail payment systems.

- Privacy vs criminal activity. KYC, “know your customer”.
2a. Bitcoin and Blockchains.

**New technology!** Smart contracts, NFT (“non-fungible token”).
Bitcoin Quantity, 2009-01-09 to 2022-07-29

Total Circulating Bitcoin

The total number of mined bitcoin that are currently circulating on the network.

Source: https://www.blockchain.com/charts/total-bitcoins

Source: https://coinmarketcap.com/currencies/bitcoin/
Log Bitcoin Price, US $, 2014-11-03 to 2022-03-09

Source: https://coinmarketcap.com/currencies/bitcoin/
Schilling - Uhlig, “Some Simple Bitcoin Economics”

Key Questions:
1. What determines the Bitcoin price? $P > NPV(\text{Dividends}) = 0$.
2. Can Bitcoin serve as medium of exchange, despite price volatility?
3. What are monetary policy implications?

Key Insights:
1. A novel model of an endowment economy with two intrinsically worthless currencies (Dollar, Bitcoin) as medium of exchange.
2. “Fundamental pricing equation”. Special case: Bitcoin price is martingale.
   - Kareken and Wallace (1981)
   - Manuelli and Peck (1990)
3. “No speculation” theorem.
4. Volatility does not invalidate medium-of-exchange function.
5. Monetary policy implications:
   - Bitcoin block rewards are not a tax on Bitcoin holders: they are financed with a Dollar tax.
   - Interaction of monetary policy vs Bitcoin price: Two Perspectives.
The Schilling - Uhlig (2019) model

- Time: discrete, infinite $t = 0, 1, 2, \ldots$
- Randomness: $\theta_t$, at beginning of period. History: $\theta^t$.
- One perishable consumption good per period.
- Two monies: Bitcoins $B_t$ and Dollars $D_t$ (aggregates).
- A central bank steers quantity of Dollars per lump sum transfers:
  \[ D_{t+1} = D_t + \tau_{t+1}, \quad \tau_{t+1} \in \mathbb{R}. \] Goal: exogenous price path $P_t$.
- Bitcoin quantity: deterministic
  \[ B_{t+1} = B_t + A_t, \quad A_t \geq 0 \] (endowment or “mining”)
- $P_t$ price of consumption good in $\$: exogenous.
- $Q_t = Q(\theta^t)$ price of Bitcoins in $\$: endogenous.
- Two types of agents: “red” and “green”. “Red” agents consume in odd periods and have endowments in even periods. “Green” agents other way around.
- Goods are traded for monies. Agents do not need to spend all money (“hodlers”) or accept all money. But: “No speculation” result: they will!
The Fundamental Pricing Equation

Compare to Kareken-Wallace (1981), Manuelli-Peck (1990)

**Proposition 1**

Assume agents use both Dollars and Bitcoins to buy goods at $t$ and $t + 1$. Then

$$
E_t \left[ u'(c_{t+1}) \frac{P_t}{P_{t+1}} \right] = E_t \left[ u'(c_{t+1}) \frac{Q_{t+1}/P_{t+1}}{Q_t/P_t} \right]
$$

(1)

If production (consumption) is constant at $t + 1$ or if agents are risk-neutral, and if further $Q_{t+1}$ and $1/\pi_{t+1}$ are conditionally uncorrelated, then the Bitcoin price $Q_t$ in Dollar is a martingale,

$$Q_t = E_t[Q_{t+1}]$$
Bitcoin block rewards are financed by Dollar taxes

Consider two economies, which differ in the growth paths for the Bitcoin quantity.

- The central bank seeks to achieve the same path for prices.
- Quantity theory:
  \[ P_t y_t = D_t + Q_t B_t \]
- More Bitcoins \( B_t \) means less \( D_t \), keeping everything else the same.
- Same equilibrium can obtain, otherwise.
Monetary Policy vs Bitcoin Price

Reminder: traditional quantity of money without Bitcoins or \( Q = 0 \),

\[
P_t y_t = D_t, \quad \text{velocity} = 1
\]  \hspace{1cm} (2)

Here: Via market clearing:

\[
P_t y_t = D_t + Q_t B_t
\]

Recall: \( y_t, P_t, B_t \) exogenous. \( D_t, Q_t \) endogenous

Suppose CB loss function: \( \mathcal{L}(P_t, Q_t) = -(P_t - P^*_t)^2 - (Q_t - Q^*_t)^2 \),

Two perspectives (or: equilibrium selection principles):

1. **Conventional**: Given \( Q_t \), solve for \( D_t \).
2. **Unconventional**: Given \( D_t \), solve for \( Q_t \).
Monetary Policy with endog $P_t$: equil. sel.

for $D<D'$

for $D<D'$
Monetary Policy with exog $P_t$. 

Graph showing the relationship between $D$, $Py$, $D'$, $D$, $Q'$, $Q$, and $A$.
2a. Private cryptocurrencies.

Number of cryptocurrencies worldwide from 2013 to February 2022

Source: https://www.statista.com/statistics/863917/number-crypto-coins-tokens/

Source
CoinMarketCap
© Statista 2022

Additional Information:
Worldwide; 2013 to 2022

Source: https://www.statista.com/statistics/863917/number-crypto-coins-tokens/
Date: July 29th, 2022. Current total market cap: $ 1106 B
Compare to currency in circulation in U.S.: $ 2279 B, July 2022
Source: coinmarketcap.com/charts/, fred.stlouisfed.org/series/CURRCIR
Private cryptocurrencies by Market Cap.

<table>
<thead>
<tr>
<th>Coin</th>
<th>Market Cap</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>BTC</td>
<td>$23,899.03</td>
<td>+0.37%</td>
</tr>
<tr>
<td>ETH</td>
<td>$1,721.39</td>
<td>+0.01%</td>
</tr>
<tr>
<td>BNB</td>
<td>$296.935</td>
<td>+7.73%</td>
</tr>
<tr>
<td>USDT</td>
<td>$1.001</td>
<td>+0.03%</td>
</tr>
</tbody>
</table>

Source: coin360.com, July 29th, 2022
Private cryptocurrencies by Volume.

Source: coin360.com, July 29, 2022
2b. Big Players

Libra:
- Originally to be issued by a FaceBook-led consortium in 2020.
- permissioned blockchain digital currency.
- backed by a basket of financial assets: e.g. regular currencies (50% US $, 18% Euro, 14% Yen, 11% Pound Sterling, 7% Singapore $.), U.S. treasuries.
- Fierce resistance by regulators.
- Multiple companies left in October 2019.

Diem (or Libra 2.0):
- Rebranding of Libra in December 2020. 27 member consortium.
- Moved from Switzerland to U.S. to get regulators on board
- Dollar-backed stable-coin.
- Close competitors already exists: Tether, Paypal.
- More sophisticated blockchain, phased approach.

Digital currency: private competition to central banks.

  - Focus on “medium of exchange” role of money.
  - Bare-bones model of two countries and three currencies.
    - Two national currencies (n.c.), issued by the two central bank.
    - One global currency (g.c.). Perfect substitute in either country to n.c..
  - If nat currency drops in value rel to global; it will not be used.
  - Main result 1: mon. pol. synchronization or n.c. is no longer used.
  - Main result 2: if g.c. is “asset backed,” narrow range for mon pol.

- **Uhlig-Xie, “Parallel Digital Currencies and Sticky Prices,”** draft.
  - Focus on “unit of account” role of money.
  - New Keynesian model, two currencies, one issued by central bank.
  - Firms set sticky prices in one of the two currencies.
  - Main result: martingale exchange rate fluctuations create new source of macro uncertainty. Challenge to central bank!

- **Upshot:** large privately issued cryptocurrencies will be competition and headaches for central banks.
Question: What are the monetary policy implications of introducing global currencies?

Focus on: medium of exchange (not: store of value, unit of account).

Answer:

- **Old:** “Impossible Trinity” (Mundell-Fleming). With free capital flows, one cannot both have independent monetary policy and a pegged exchange rate.

- **New, here:** With free capital flows and a global currency circulating alongside national currencies, the monetary policy interest rates are equalized and the exchange rates are risk-adjusted martingales.

- **Crypto-Enforced Monetary Policy Synchronization** or **CEMPS**.

- Escape options unpleasant: ZLB or abandon national currency.

- Additional restrictions arise, if the global currency is asset backed.

- The “Impossible Trinity” becomes even less reconcilable.
The Model: A Minimalistic, General Structure

- discrete time, \( t = 0, 1, 2 \ldots \)
- 2 countries
- 3 currencies: home H, foreign F, global G.

Four assumptions:

1. Nominal stochastic discount factors in each country.
2. Central banks set nominal interest rates for national currencies.
3. Free (or: complete) capital markets.
4. Money offers liquidity services.
1. Nominal stochastic discount factors

2. CBs set nom. interest rates.

Assume: nominal stochastic discount factors:

\[ M_{t+1} \quad M_{t+1}^* \]

Asset Pricing: Let \( R_{t+1} \) be the stochastic return between \( t \) and \( t + 1 \) on some asset, denominated in H. Likewise \( R_{t+1}^* \) in F. Then

\[
1 = \mathbb{E}_t[M_{t+1} R_{t+1}] \\
1 = \mathbb{E}_t[M_{t+1}^* R_{t+1}^*]
\]

Nominal Interest Rates, set by CBs.

- \( i_t \) on one-period safe bond in H(ome),
- \( i_t^* \) on one-period safe bond in F(oreign)

Asset Pricing implies:

\[
\frac{1}{1 + i_t} = \mathbb{E}_t[M_{t+1}] \\
\frac{1}{1 + i_t^*} = \mathbb{E}_t[M_{t+1}^*]
\]
3. Complete Capital Markets

Define: exchange rates
- $S_t$: price of one F in terms of H ("Dollar per Yen"),
- $Q_t$: price of one G in terms of H ("Dollar per Bitcoin"),
- $Q_t^*$: price of one G in terms of F ("Yen per Bitcoin"),

Assume: Complete Markets,

$$\mathcal{M}_{t+1} = \mathcal{M}_{t+1}^* \frac{S_t}{S_{t+1}}$$ (5)

Implication: Stochastic UIP,

$$\tilde{E}_t [S_{t+1}] := \frac{E_t [\mathcal{M}_{t+1} S_{t+1}]}{E_t [\mathcal{M}_{t+1}]} = \frac{1 + i_t}{1 + i^*_t} S_t$$ (6)

$$\tilde{E}^*_t [S^*_{t+1}] := \frac{E_t [\mathcal{M}^*_{t+1} S^*_{t+1}]}{E_t [\mathcal{M}^*_{t+1}]} = \frac{1 + i^*_t}{1 + i_t} S^*_t$$ (7)
4. Liquidity Services: Money as Medium-of-Exchange

Assume: If currencies are used, they provide liquidity services or l.s.
- If H is used at home: one H provides \( L_t \geq 0 \) units of l.s.
- If G is used at home: one G provides \( L_t Q_t \) units of l.s.
- If F is used abroad: one F provides \( L^*_t \geq 0 \) units of l.s.
- If G used abroad: one G provides \( L^*_t Q^*_t \) units of l.s.

Currency pricing:

**Home:**
\[
1 \geq L_t + \mathbb{E}_t[M_{t+1}]
\]
\[
1 \geq L_t + \mathbb{E}_t \left[ M_{t+1} \frac{Q_{t+1}}{Q_t} \right]
\]

**Foreign:**
\[
1 \geq L^*_t + \mathbb{E}_t[M^*_{t+1}]
\]
\[
1 \geq L^*_t + \mathbb{E}_t \left[ M^*_{t+1} \frac{Q^*_{t+1}}{Q^*_t} \right]
\]

Interpretation:
- “=”: if currency is **used** at home resp. abroad.
- “>”: then, currency is **not used** or is **abandoned**.
- Convention: ignore knife-edge case, i.e. “=”, but not used.
Main Result

Suppose:

- The national currencies are used in their countries.
- Global currency is valued $Q_t, Q_t^* > 0$.
- Global currency used in both countries.

Proposition 0.1 (Crypto-Enforced Monetary Policy Synchronization)

- The liquidity services in Home and Foreign are equal $L_t = L_t^*$
- The nominal interest rates on bonds are equal $i_t = i_t^*$
- The nominal exchange rate between home and foreign currency is a martingale wrt to the risk-neutral probabilities

\[ \tilde{E}_t[S_{t+1}] := \frac{E_t[M_{t+1}S_{t+1}]}{E_t[M_{t+1}]} = S_t \]  \hspace{1cm} (12)

\[ \tilde{E}_t^*[S_{t+1}^*] := \frac{E_t[M_{t+1}^*S_{t+1}^*]}{E_t[M_{t+1}^*]} = S_t^* \]  \hspace{1cm} (13)

Furthermore,

\[ \tilde{E}_t[Q_{t+1}] = Q_t \quad \text{and} \quad \tilde{E}_t^*[Q_{t+1}^*] = Q_t^* \]  \hspace{1cm} (14)
The key argument

1 Global currency is used in both:

\[ 1 = E_t \left[ M_{t+1} \frac{Q_{t+1}}{Q_t} \right] + L_t \quad \text{and} \quad 1 = E_t \left[ M_{t+1}^* \frac{Q_{t+1}^*}{Q_t^*} \right] + L_t^* \]

2 Complete Markets:

\[ M_{t+1} \frac{Q_{t+1}}{Q_t} = M_{t+1}^* \frac{Q_{t+1}^*}{Q_t^*} \]

Thus, \( L_t = L_t^* \).

3 H and F is used in both:

\[ 1 = E_t \left[ M_{t+1} \right] + L_t \quad \text{and} \quad 1 = E_t \left[ M_{t+1}^* \right] + L_t^* \]

Thus, \( E_t \left[ M_{t+1} \right] = E_t \left[ M_{t+1}^* \right] \).

4 Nominal interest rates must be equal:

\[ \frac{1}{1 + i_t} = E_t \left[ M_{t+1} \right] = E_t \left[ M_{t+1}^* \right] = \frac{1}{1 + i_t^*} \]
Economic Mechanism

A INTRODUCTION OF GLOBAL CURRENCY CREATES GLOBAL COMPETITION BETWEEN NATIONAL CURRENCIES

- Currency competition at home: Home ⇔ Global
- Currency competition abroad: Foreign ⇔ Global
- Transnational currency competition: Home ⇔ Foreign (through Global)

B DIRECT COMPETITION BETWEEN BONDS

- Local competition: Home currency ⇔ home bond
- Local competition: Foreign currency ⇔ foreign bond
- Global competition: Home bond ⇔ Foreign bond \((i = i^*)\)
2c. DeFi, Smart Contracts and Stablecoins

- **DeFi**: “Decentralized Finance”.
- “Smart contracts”: automatic execution of contractual arrangements encoded on a blockchain.
- Ethereum. Solidity is “Turing complete”. ERC-20 tokens.
- Key issue: making payments in Dollars or equivalent.
- Stablecoins arrangements:
  - as narrow banks
  - as money market funds
  - Algorithmic stablecoins
- Accounting? Safeguards?
- Stablecoin regulatory discussions in EU, US as of summer 2022.
Stablecoins, total market cap, 2022-07-29

Source: https://btctools.io/stats/market-cap
# Largest Stablecoins: 2022-07-29

This page lists the most valuable stablecoins. They are listed by market capitalization with the largest first and then descending in order.

<table>
<thead>
<tr>
<th>#</th>
<th>Name</th>
<th>Price</th>
<th>1h %</th>
<th>24h %</th>
<th>7d %</th>
<th>Market Cap</th>
<th>Trading Volume</th>
<th>Circulating Supply</th>
<th>Last 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Tether USDT</td>
<td>$1.00</td>
<td>-0.00%</td>
<td>0.00%</td>
<td>0.01%</td>
<td>$65,949,584,366</td>
<td>$73,065,414,423</td>
<td>65,933,977,708 USDT</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>USD Coin USDC</td>
<td>$1.00</td>
<td>0.01%</td>
<td>0.07%</td>
<td>0.04%</td>
<td>$54,643,617,511</td>
<td>$8,793,338,372</td>
<td>54,622,858,666 USDC</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Binance USD BUSD</td>
<td>$1.00</td>
<td>0.19%</td>
<td>0.19%</td>
<td>0.15%</td>
<td>$17,807,720,134</td>
<td>$9,034,333,998</td>
<td>17,825,514,573 BUSD</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Dai DAI</td>
<td>$0.9996</td>
<td>-0.01%</td>
<td>0.03%</td>
<td>0.04%</td>
<td>$7,482,311,595</td>
<td>$755,852,996</td>
<td>7,483,990,981 DAI</td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>TrueUSD TUSD</td>
<td>$0.9998</td>
<td>-0.01%</td>
<td>0.04%</td>
<td>-0.03%</td>
<td>$1,188,922,916</td>
<td>$1,144,437,860</td>
<td>1,188,758,816 TUSD</td>
<td></td>
</tr>
<tr>
<td>53</td>
<td>Pax Dollar USDP</td>
<td>$1.00</td>
<td>0.04%</td>
<td>0.20%</td>
<td>0.16%</td>
<td>$945,654,679</td>
<td>$5,530,045</td>
<td>945,642,940 USDP</td>
<td></td>
</tr>
</tbody>
</table>

Source: https://coinmarketcap.com/view/stablecoin/
Uhlig, “A Luna-tic Stablecoin crash”

- After remaining close to 1 US Dollar, the algorithmic stablecoin Terra UST lost more than 75 percent of its value in the two weeks of May 9th to May 15th, 2022, leading to a price collapse of the underlying LUNA token of more than 99.9 percent, an increase in LUNA supply by a factor of 19,000 and the erasure of more than 50 Billion U.S. Dollar.

- The system worked by allowing traders to convert a Terra UST coin into 1 U.S. Dollar worth of LUNA tokens and vice versa.

- Eventually, a sustained outflow or “burning” of UST coins into LUNA tokens resulted in a massive price decline of LUNA and suspension of convertibility.
LUNA and TERRA UST market cap

Market Capitalization

Market Cap

Jul 2021 Jan 2022 time

Jul 2021 Jan 2022

time

×10^{10}

The gradual unfolding of the crash

Number of UST tokens burnt VS LUNA price

# UST tokens
# UST burnt
LUNA price

May 08 May 09 May 10 May 11 May 12

2022

-1
0
1
2
3
4
5
6

$10\times 10^8$

$\times 10^8$

May 08 May 09 May 10 May 11 May 12

time

May 12
2022

0
10
20
30
40
50
60
70

38 / 63
Theory

- At $t = 0$: “MIT shock”, burning begins.
- At $t$, and within the next time interval $\Delta$,
  - burning of $b_t \Delta$ UST coins into $b_t \Delta / Q_t$ LUNA tokens.
  - with probability $\lambda_t \Delta$: the burning of UST stops and LUNA has market cap $n_{t+\Delta}$.
  - with probability $1 - \lambda_t \Delta$: the burning of UST continues.
- Rational traders price LUNA tokens, taking dilution into account.
- When LUNA price reaches $\epsilon > 0$: suspension of convertibility, market cap $m_T$ for LUNA.
- UST holder $i$ burns UST, if $P(\text{“suspension”}) \geq P_i$.
- For $\Delta \to 0$: system of ODEs.
- Closed-form solution, when $b_t \equiv b$, $\lambda_t \equiv \lambda$, $n_t \equiv n$.
- **Method of quantitative interpretation**: use data to back out $n_t$, $m_T$, $\lambda_t$.
- Scenario A: $n_t \equiv n$. Scenario B: $\lambda_t \equiv \lambda$. 
Quantitative Interpretation

**Scenario A:** $\lambda_t$

**Scenario B:** $n_{t+\Delta}$

---

**Implied exit probability**

- **Scenario A:** $\lambda_t$
  - May 08 to May 12, 2022
  - Probability values range from 0 to 0.9

- **Scenario B:** $n_{t+\Delta}$
  - May 08 to May 12, 2022
  - Market cap values range from $10^9$ to $8 	imes 10^{10}$

---

**Implied exit market cap**

- **Scenario A:** $\lambda_t$
  - Time series plot

- **Scenario B:** $n_{t+\Delta}$
  - Time series plot
  - Market cap values represented with a solid line
  - Reference market cap represented with a dashed line
Distribution of threshold $P$ for burning UST

Scenario A

Scenario B
3. Central bank digital currencies

CBDC research and pilots around the world

BS = The Bahamas; ECCB = Eastern Caribbean Central Bank; HK = Hong Kong SAR; JM = Jamaica; SG = Singapore. The use of this map does not constitute, and should not be construed as constituting, an expression of a position by the BIS regarding the legal status of, or sovereignty of any territory or its authorities, to the delimitation of international frontiers and boundaries and/or to the name and designation of any territory, city or area.


Source: https://www.bis.org/publ/work880.htm
Chinese CBDC may replace SWIFT

China's 6 state owned banks part of M-CBDC Bridge SWIFT replacement

Source: https://www.ledgerinsights.com, November 4th, 2021
Why do agents accept fiat money?
- Agents do not care for fiat money per se, but for real goods.
- They accept money as a temporary store of value.
- **Monetary trust:** money can purchase goods in the future.
- Trust may evaporate and agents may abandon the currency.

Objectives
1. Commit to **price stability** under many circumstances.
2. **Efficiency**, i.e. implement the socially optimal allocation.
3. Assure **monetary trust**.
   - This paper: these are separate objectives.
   - Caveat: could fold all three into “efficiency”. We do not.

General or specific to CBDC?
- General, in principle.
- Particularly salient for CBDC:
  - Speed of transactions.
  - Transformation of the financial system.
Central Bank Digital Currency or CBDC

- A CBDC is an (interest bearing) account held by households at the central bank. (Barrdear and Kumhof, 2016)
- Likely to be introduced widely. Bahamas, China, Sweden.
- "Financial inclusion": good!
- But Disintermediation Threat: if HH hold CBDC rather than deposits, banks cannot fund firms ...
  1. ... unless HH re-invest CBDC at banks (Duffie) or ...
  2. ... Central Bank re-funds banks or projects (Brunnermeier-Niepelt).
The CBDC Trilemma

In our model: Only HH, CB, projects. CB is financial intermediary.

Key Mechanism
- Nominal Diamond-Dybvig (1983) model for a CB and its CBDC.
- Central bank can always deliver on its nominal obligations.
- But: CB runs can happen: “spending run” on available goods.

Three competing objectives:
1. Traditional CB objective: commitment to **Price Stability**
2. Social optimum, optimal risk sharing: **Efficiency**
3. Absence of runs, financial stability: **Monetary Trust**

Key Result: **CBDC Trilemma**

Of the three objectives, the central bank can only achieve two.
A run on the central bank

Ruble plummets amid global sanctions, sending Russians to banks and ATMs

The model: the real portion is Diamond-Dybvig, 1983

- time \( t = 0, 1, 2 \).
- Continuum \([0, 1]\) of agents:
  - \( t = 0 \): symmetric, endowed with one unit of a real good
  - \( t = 1 \): types reveal: “impatient” \( \lambda \), “patient” \( 1 - \lambda \).
  - Impatient agents: have to consume in \( t = 1 \).
  - \( u(\cdot) \) strictly increasing, concave, RRA greater than one,
    \( -x \cdot u''(x)/u'(x) > 1 \).
- Real Technology:
  - long term: \( 1 \rightarrow 1 \rightarrow R \)
  - storage \( t = 1 \rightarrow t = 2 \), available to all: \( 1 \rightarrow 1 \)
- Optimal solution:
  \[
  \max \lambda u(x_1) + (1 - \lambda) u(x_2) \quad \text{s.t.} \quad \lambda x_1 + (1 - \lambda) \frac{x_2}{R} = 1
  \]
  Unique solution, where \( u'(x_1^*) = Ru'(x_2^*) \)
- With that: \( x_1^* > 1 \). (Diamond and Dybvig, 1983)
The model: the nominal portion introduces CBDC.

**Definition 1**

A central bank policy is a triple \((M, y(\cdot), i(\cdot))\), where \(y : [0, 1] \rightarrow [0, 1]\) is the central bank’s liquidation policy for every observed fraction \(n\) of spending agents, and \(i : [0, 1] \rightarrow [-1, \infty)\) is the nominal interest rate policy.

\[ t = 0: \]
- Agents sell goods to CB for \(M\) CBDC units in \(t = 1\).
- CB: invests all received real goods in projects.

\[ t = 1: \]
- Agents learn type. Impatient agents spend \(M\).
- Patient agents may. Total fraction: \(\lambda \leq n \leq 1\).
- CB observes agg. spending fraction \(n\).
- CB liquidates fraction \(y = y(n) \in [0, 1]\) of projects.
- CB sells goods \(y\). Market clearing price \(P_1\).

\[ t = 2: \]
- Remaining agents spend \((1 + i(n))M\).
- CB sells remaining project payoffs \(R(1 - y)\).
- Market clearing price \(P_2\).
A central bank policy is a triple $(M, y(\cdot), i(\cdot))$: 

A liquidation policy

An interest rate policy

Set $M$ so that $P_1 = 1$ clears the market, if $n = \lambda$ agents spend in $t = 1$. 
**Definition 2**

Given a central bank policy \((M, y(\cdot), i(\cdot))\), an equilibrium \((n, P_1, P_2)\) is aggregate spending behavior \(n \in [0, 1]\), and price levels \(P_1\) and \(P_2\) such that:

1. The indiv. consumer’s spending decisions are optimal, given aggregate spending \(n\), the central bank’s policy \((M, y(\cdot), i(\cdot))\), the price level sequence \((P_1, P_2)\).
2. Given the aggregate spending realization \(n\), the central bank liquidates \(y(n)\) and sets the nominal interest rate \(i(n)\).
3. Given the realization \((n, y(n), i(n))\) and \(M\), the price levels \((P_1, P_2)\) clear the goods market in each period;
Market Clearing

\[ nM = P_1 y(n) \]
\[ (1 - n)(1 + i(n))M = P_2 R(1 - y(n)), \]

⇒ \( n, y(n), i(n) \) pin down the price levels \( P_1, P_2 \).

\[
P_1(n) = \frac{nM}{y(n)} \quad \text{and} \quad P_2(n) = \frac{(1 - n)(1 + i(n))M}{R(1 - y(n))}
\]

Note: \( P_2(n) \) can be “anything” per \( i(n) \), but \( i(n) \) does not affect \( P_1(n) \).

Real allocation: only depends on \( n \) via \( y(n) \):

\[
x_1(n) = \frac{M}{P_1} = \frac{y(n)}{n} \quad \text{and} \quad x_2(n) = \frac{(1 + i(n))M}{P_2} = \frac{1 - y(n)}{1 - n} R
\]

Given \( n \), patient agents run iff \( x_1(n) \geq x_2(n) \).
Objective 2: Optimal Risk Sharing

The social optimum \((x_1^*, x_2^*)\) is an equilibrium, if \(y(\lambda) = y^* = \lambda x_1^*\).
Objective 3: Absence of Runs

A Run on the Central Bank is a Spending Run:

**Definition 3**
A run occurs if $n > \lambda$: patient agents also spend.

**CBDC looses its ‘store of value’ function.**
- Patient agents purchase goods instantaneously even though they do not need to consume
- Enable future consumption by storing toilet paper and other goods at home rather than storing value in form of CBDC
- Trust in monetary system and CBDC evaporates.
- Monetary instability.

Compare to:
- temporary pandemic stockouts.
- hyperinflations.
- currency crises.
No run

$t=0$

$t=1$

$t=2$
The policy is “run-proof”, if $n \neq \lambda$ is “off equilibrium”, i.e. if $x_1(n) < x_2(n)$ for all $n$, i.e. $y(n) < \bar{y}(n) = \frac{nR}{1 + n(R - 1)}$.

Example: the policy $y(n) \equiv y^*$ is run-proof.
Run-Proof Policies and Price Implications

E.g. for \( y(n) \equiv y^* \), we have \( P_1(n) = n \frac{M}{y^*} \)

- These two policies violate the price stability objective for \( P_1(n) \).
- The problem only arises “off equilibrium.”
- Commitment-issue / credibility / sub-game perfection: should \( n \neq \lambda \) arise, a price-stability oriented Central Bank may not stick to the “threat” of letting the price \( P_1 \) move far from the target.
- Remark: objective for \( P_2(n) \) can always be achieved via \( i(n) \).
Objective 1: Price Stability

<table>
<thead>
<tr>
<th>Definition 4</th>
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<tbody>
<tr>
<td>1. A central bank policy is <strong>fully price stable</strong>, if it achieves $P_1(n) \equiv \bar{P}$ for all $n$.</td>
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<tr>
<td>2. A central bank policy is <strong>partially price stable</strong>, if it achieves either $P_1(n) = \bar{P}$ or there is full liquidation, $y(n) = 1$, for all $n$.</td>
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(In the paper: extend to period 2, pick the right interest rate policy.)

Recall Market Clearing:

$$P_1(n) = \frac{nM}{y(n)}$$

Therefore,

**fully price stable:** $y(n) = \frac{nM}{\bar{P}}$

**partially price stable:** $y(n) = \min\left\{ \frac{nM}{\bar{P}} \right\}$
Prices are fully stable and runs are avoided on green line.

But: no longer efficient at $n = \lambda$!

At best: green line = 45 degree line.
Prices partially stable and efficiency on blue line.

But: no longer run-proof. Runs may happen!
Why not change the money supply in $t = 1$?

- **Also allow** for state-contingent $M(n)$ in $t = 1$.
- Suppose $y(n) \equiv y^*$. To maintain price stability at some $\bar{P}$:
  \[
  nM(n) = \bar{P}y^* = \lambda M(\lambda)
  \] (15)

- Total money spent in $t = 1$ is constant. Money spent per agent decreases with $n$. CB commits itself to **reduce** quantity of money in response to demand shock and spending spree.

Implementations:
- State-contingent money supply.
- Taxation of individual money holdings.
- Suspension of spending: only some of the money can be used.
- Change $i(n)$? Or OMOs, i.e. sell bonds? Won’t do the trick: $i(n)$ does not impact real allocation.

“Suspension Of Convertibility” becomes “**Suspension Of Spendability**”

- No runs, stable prices! Problem solved?
- Doubtful. **SOS will undermine trust in monetary system**.
- With general $y(n)$: run issues as before. Agents only care about real allocation. Money is neutral.
4. An Assessment

- The currency landscape is changing dramatically.
- Bitcoin has shown that privately issued currencies are possible.
- Crypto market cap already similar to currency in circulation for US.
- Big players, foreign countries are interested, will introduce.
- Central banks face competition, will have to act: CBDC.
- Privacy concerns: not just criminals value privacy.
- Private crypto-currencies will continue to exist and flourish.
- Ecosystem and technological possibilities:
  - DAO: decentralized autonomous organization.
  - DeFi: decentralized finance.
- Challenges to monetary policy, financial stability and regulation.
- But: do not be afraid! This will improve our lives.