The economy is changing. We need new tools.

- The largest firms are valued primarily for their data.
- Do the economics change? Or is data just new capital?

Challenges:

- Economic activity generates informative data. Production is a form of active experimentation.
- How do we value information used for multiple periods? Requires dynamic programming with information as a state variable.
- Data depreciation depends on economic fluctuations.
- Data is a non-rival good, whose value declines when it is sold.
Overview

- Model: recursive framework, as tractable as std DSGE.
  - A tool to value data and data-intensive firms.

- Long run growth
  - Diminishing returns

- Short run tools
  - Increasing returns

- Efficiency and externalities

- Imperfect Competition and Market Power
continuum of competitive firms $i$

each uses capital $k_{i,t}$ to produce $k_i^\alpha$ units of goods

these goods have quality $A_{i,t}$

Output / demand

\[ Y_t = \int_i A_{i,t} k_i^\alpha di \]

\[ P_t = \bar{P} Y_t^{-\gamma} \]
firm has one optimal technique: $\theta_t + \epsilon_{a,i,t}$

- $\theta_t$: AR(1), innovation $\eta_t \sim N(\mu, \sigma^2_{\theta})$

$$\theta_t = \bar{\theta} + \rho(\theta_{t-1} - \bar{\theta}) + \eta_t$$

- $\epsilon_{a,i,t} \sim N(0, \sigma^2_{\epsilon})$ is unlearnable and i.i.d.

- quality depends on chosen production technique $a_{i,t}$ and distance to optimum $(\theta_t + \epsilon_{a,i,t})$:

$$A_{i,t} = g \left( (a_{i,t} - \theta_t - \epsilon_{a,i,t})^2 \right)$$

- $g(.)$: monotonically decreasing (accuracy is good)
at time $t$, firm obtains $n_{i,t}$ data points about $\theta_{t+1}$

$\triangleright$ $n_{i,t} = z_i k_{i,t}^\alpha$

$\triangleright$ data is a byproduct of production with data-mining ability $z_i$

each data point $m \in [1 : n_{i,t}]$ reveals

$$s_{i,t,m} = \theta_{t+1} + \xi_{i,t,m} \quad \text{where} \quad \xi_{i,t,m} \sim N(0, \sigma^2_\epsilon)$$
Data Feedback Loop

More Data

More Transactions / Customers

Higher Quality / Efficiency
Model: Market for Data

- $\delta_{i,t}$: amount of data traded by firm $i$ at time $t$
  - $\delta_{i,t} > 0$: data purchases ($< 0$: data sales)
  - firm can buy or sell, not both

- data price $\pi_t$ clears the data market

- multi-use data: firm can sell it and still use it
  - $\iota$: fraction of sold data that is lost
  - We need $\iota > 0$ for a competitive equilibrium to exist.
  - Many data contracts include prohibitions on seller use. Or this captures imperfect competition.

- Data adjustment cost: $\Psi(\cdot)$: avoid 1-period convergence
Valuing and depreciating data as an asset.

What happens in the long run?
  ▶ Diminishing returns: no long-run growth without innovation.
  ▶ Endogenous growth: If data is used for R&D

What happens in the short run?
  ▶ Increasing returns, data poverty traps
  ▶ data barter and book-to-market dynamics

Welfare and business stealing
Solution: Depreciating Data with Bayes Law

- Goal is to forecast \( \theta_{t+1} = \rho \theta_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, \sigma^2_{\theta}). \)

- Priors: \( E[\theta_t|I_t] \) and \( V[\theta_t|I_t] := \Omega_t^{-1} \). \( \Omega_t \) is “stock of knowledge.”

- What are today’s beliefs about tomorrow’s state?

\[
E[\theta_{t+1}|I_t] = \rho E[\theta_t|I_t]
\]

\[
V[\theta_{t+1}|I_t] = \rho^2 \Omega_t^{-1} + \sigma^2_{\varepsilon}
\]

- Bayes law for normal variables says:
  posterior precision = prior precision + signal precision \( \sigma^{-2}_s \).

- A law of motion for stock of knowledge (Kalman filter, Ricatti eqn):

\[
\Omega_{t+1} = (\rho^2 \Omega_t^{-1} + \sigma^2_{\theta})^{-1} + \sigma^{-2}_s
\]

Discount more for low persistence \( \rho \) and volatile innovations \( \sigma^2_{\theta} \).
Semi-rival data as a negative bid-ask spread

- For data purchases ($\delta_{i,t} > 0$), data added to stock is $n_{i,t} + \delta_{i,t}$.

- For data sales ($\delta_{i,t} < 0$), net data additions are $n_{i,t} + \iota \delta_{i,t}$.

- An adjusted price of data: $\tilde{\pi}_{i,t} \equiv \pi / (1_{\omega_{i,t} > n_{i,t}} + \iota 1_{\omega_{i,t} < n_{i,t}})$.

- This price is higher for data sales than it is for data purchases. We can use tools from finance for modeling economies with bid-ask spreads.
Valuing Data: A Recursive Solution

- $a_{i,t}^* = \mathbb{E}[\theta_t + \varepsilon_{i,t} | \mathcal{I}_{i,t}] \rightarrow$ Quality $A_{i,t} \approx$ a fn of squared forecast error.

- **state variable: stock of knowledge**

\[
\Omega_{i,t} \equiv \mathbb{E} \left[ \left( \mathbb{E}[\theta_t | \mathcal{I}_{i,t}] - \theta_t \right)^2 \right]^{-1} \text{ (posterior precision)}
\]

**Lemma**

**The optimal sequence of capital / data choices** $\{k_{i,t}, \delta_{i,t}\}$ **solves:**

\[
V(\Omega_{i,t}) = \max_{k_{i,t}, \delta_{i,t}} P_t \mathbb{E}_i \left[ A_{i,t}(\Omega_{i,t}) \right] k_{i,t}^\alpha - \Psi(\Delta \Omega_{i,t+1}) - \pi \delta_{i,t} - rk_{i,t} + \frac{V(\Omega_{i,t+1})}{1 + r}
\]

where $n_{i,t} = z_i k_{i,t}^\alpha$ and (Kalman filter)

\[
\Omega_{i,t+1} = \left[ \rho^2 (\Omega_{i,t} + \tilde{\sigma}_a^{-2})^{-1} + \sigma_\theta^2 \right]^{-1} + \left( n_{it} + \delta_{it} (1 \delta_{it>0} + 1 \delta_{it<0}) \right) \sigma_\varepsilon^{-2}
\]
Understanding Growth. Data Inflows and Outflows

- **inflow**: \( z_i k_{it}^\alpha \sigma_\varepsilon^{-2} \) (# of data points \( \times \) precision)

- **outflow**: data depreciation

```
Growth stops for two reasons: Can’t reduce errors below zero and unlearnable risk.
```
Endogenous Growth

- So far, data added to current productivity. Did not augment a stock of ideas.

- Alternative model: Data $\rightarrow$ idea creation

\[ A_{i,t} = A_{i,t-1} + \max\{0, \Delta A_{i,t}\}, \quad (1) \]
\[ \Delta A_{i,t} = \bar{A} - (a_{i,t} - \theta_t - \varepsilon_{a,i,t})^2. \quad (2) \]

- Data increases step size in a quality ladder $\rightarrow$ growth

- One should distinguish data used for research from data used for process optimization. (Just like capital vs. R&D investment.)

Long run, data looks just like capital
Short Run: Single firm enters a steady state

**Proposition (Convex Data Flow)**

There exist parameters $\alpha$ and $\bar{P}$, and a threshold $\hat{\Omega}$ such that when knowledge is scarce $\Omega_{it} < \hat{\Omega}$, net data flow $d\Omega_{it}$ increases over time.

Could explain firm size divergence, ... and eventually convergence?
Data Barter. Why Produce At a Loss?

- **Barter means**: data is “exchanged” for the good
  - at good price \( P_t = 0 \)

- Reality: lots of data is bartered for services (phone apps)

- Result: Data barter arises early in a firm’s life (depends on pref.s)
  - firms produce goods at a loss to generate data

\[
\frac{\partial V_t}{\partial \Omega_{i,t}} > 0
\]

- GDP is missing lots of digital economic activity because price does not reflect value.

How to correct it?
Welfare, Decentralization and Efficiency

- Household problem: $c_{it}$ is quality units consumed
  \[ \max \sum_{t=0}^{\infty} \beta^t \left( \bar{u} \frac{c_t^{1-\gamma}}{(1-\gamma)} + m_t - \frac{h_t^{1+v}}{(1+v)} \right) \]
  subject to \[ p_t c_t + m_t = \Pi_t + w_t h_t, \quad \forall t \]
- Numeraire good $m$ production linear in labor $h_t$, with wage $w_t$
  \[ \max n_t \bar{m} n_t - w_t h_t \]
- Retail good $c$ produced with data and capital, with $E[A_{i,t}]$ and data law of motion, as before. Firm solves
  \[ \max \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \left( P_t E[A_{i,t} k_{i,t}^\alpha] - \psi(\Omega_{i,t+1}) - \Pi_{i,t} - r k_{i,t} \right) \]
- Consumption and data markets clear: $\sum_i c_{it} = A_{it} k_{jt}^\alpha$
  Numeraire used to pay capital rental at open economy $r$.

→ Efficiency. No externalities.
Lots of data used for advertising. Arguable business stealing.

We can adjust quality so that data processing helps the firm that uses it, but has no social value:

Morris-Shin (2002)

\[ A_{i,t} = \bar{A} - (a_{i,t} - \theta_t - \varepsilon_{a,i,t})^2 + \int_{j=0}^{1} (a_{j,t} - \theta_{j,t} - \varepsilon_{a,j,t})^2 dj \]

- Unchanged: firm choices, firm dynamics, aggregate quality
- Changed: welfare
Key features of the model:
▶ Data is information. Information resolves uncertainty (risk). We model this as uncertain consumer demand for a variety of products.
▶ Firms price risk.
▶ Firms choose an up-front investment and then choose how much to produce (Cournot).

Data’s Competing Effects on Markups

In the paper: Markup Measurement
▶ Data creates composition effects: How one measures markups matters.
▶ Procyclical product and counter-cyclical firm / industry markups
Model (1/2): Firms

- $n_F$ firms, indexed by $i$.

- The product space has $N$ attributes, indexed by $j$. Goods, indexed by $k$, are combinations of attributes with weights $a_{jk}$.

- Firm chooses: investment in lowering marginal cost $c_i$ at price $g(\chi_c, c_i)$, and a quantity to produce $q_i$ (vector) to maximize $E[U_i]$ and $U_i$:

$$U_i = E[\pi_i | I_i] - \frac{\rho_i}{2} \text{Var}[\pi_i | I_i] - g(\chi_c, c_i)$$

$$\pi_i = q'_i (p - Ac_i)$$

- $\rho_i$ is firm $i$’s price of risk. From CAPM? From preferences?
Model (2/2): Demand and Data

- **Demand**: Customers’ willingness to pay decreases in the quantity that all firms produce of attribute $j +$ firm-specific shock $b_i$:

$$
\hat{v}_i = \hat{p} - \frac{1}{\phi} \sum_{i'=1}^{N} \hat{q}_{i'-j} + b_i
$$

attribute demand shock $b_i \sim N(0, I)$ is i.i.d. across firms (relax). $\phi$ is price elasticity of demand.

- **Customers’ value of a good is linear in attributes** $v_k = \sum_{j=1}^{N} a_{jk} \hat{v}_j$.

- **Data is information about demand shocks**: (exogenous)
  - $n_{di}$ data points for firm $i$, independent across firms.
  - Each data point is: $\tilde{s}_{i,z} = b_i + \tilde{\epsilon}_{i,z}$, where $\tilde{\epsilon}_{i,z} \sim N(0, \Sigma)$.
  - Information set: $\mathcal{I}_i := \{ \{ \tilde{s}_{i,z} \}_{z=1}^{n_{di}} \}_{i=1}^{n_F}$ (All info public. Private info in appdx)
Equilibrium

- FOC: Production depends on risk and price impact (denominator) and expected profit (numerator) Kyle ‘89 or Back and Zender ‘93.

$$q_i = \left( \rho_i \text{Var}[p_i|\mathcal{I}_i] + \frac{\partial \mathbb{E}[p_i|\mathcal{I}_i]}{\partial q_i} \right)^{-1} \left( \mathbb{E}[p_i|\mathcal{I}_i] - c_i \right)$$

- Define sensitivity of production to a change in the expected price:

$$H_i := \left( \frac{2}{\phi} I_N + \rho_i \text{Var}[b_i|\mathcal{I}_i] \right)^{-1}$$

Data lowers Var, raises $H_i$.

$H_i$ governs the $\text{cov}(q_i, p_i)$.

Data allows a firm to choose quantities that covary with prices.

- Optimal choice of cost (firm size):

$$\frac{\partial \mathbb{E}[U_i]}{\partial \tilde{c}_{ij}} = \frac{1}{2} \frac{\partial \mathbb{E}[\tilde{p} - \tilde{c}_i]}{\partial \tilde{c}_{ij}} H_i \mathbb{E}[\tilde{p} - \tilde{c}_i] - \frac{\partial g(\chi_c, \tilde{c}_i)}{\partial \tilde{c}_{ij}} = 0 \ \forall j$$

marginal benefit

marginal cost
Product Markups

Product-level markup for $k$ produced by firm $i$:

$$M_{ik}^p := \frac{E[p_i(k)]}{c_i(k)}$$

$$= \frac{1}{a_k'c_i} \left( a_k'p - a_k'A^{-1}(\phi I + \bar{H})^{-1} \left( \sum \hat{H}_{i'j}A(p - \tilde{c}_{i'}) \right) \right)$$

What makes product markups large?

- Having lots of valuable attributes: big $a_k$’s.
- Low price elasticity of demand: Low $\phi$.

New:
- Scarce data or high price of risk. High $\rho Var[p|\eta_j]$ makes $\bar{H}$ low. Markup has a risk premium in there.
**Proposition**

*Data-investment complementarity.* Data lowers choice of $c_i$.

**Proposition**

*Higher investment raises product markups.* Lower $c_i$ increases $M_{ik}^p$.

Single-good duopoly. $g(\chi_c, c_i) = \chi_c (\overline{c} - c_i)^2 / 2$ with $\chi_c = 1$ and $\overline{c} = 3$; $p = 5$, $\phi = 1$, $\sigma_b = 1$.
Markups: Risk Premium Channel

**Proposition**

*For investment cost $\chi_c$ or $\rho$ sufficiently high, data reduces $M_{ik}^p$.***

**Proposition**

*Net effect: Data in(de)creases product markups when risk price or marginal cost of investment is sufficiently low (high).*
Dynamic Imperfect Competition

How to combine the recursive economy with the static imperfect competition model?

Challenges: Every firm’s information matters for price. All information sets are state variables.

Potential payoffs:
Understand new firm dynamics in a digital economy.

How to measure competition when many prices are zero?

How to value firms that have never made a profit?

What if we add firm entry?

Welfare? Should we tax data? How?
Conclusions

- Economists cannot continue to study industrial economies. We live in a knowledge economy. We need modern tools that reflect this reality.

- Knowledge economies are not easy to work with. Production generates data, depreciation not std, semi-rivalry, increasing returns and decreasing returns, information state variables.

- Lots of new directions to explore
  - measurement
  - firms dynamics
  - data pricing and valuation theory